6.036: Introduction to Machine Learning

Lecture start: Tuesdays 9:35am

Lecture team: Prof. Tamara Broderick & TA Elizabeth Zou

Questions? Ask on Piazza

Materials: slides, video will all be available on Canvas

Live Zoom feed: https://mit.zoom.us/j/94238622313

Today's Plan

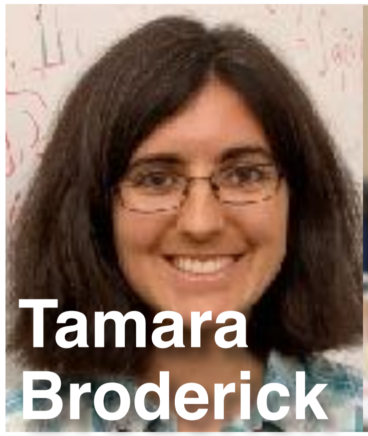
I. Meet the team

II. Machine learning setup

III. Linear regression

IV. Regularization

6.036: Introduction to Machine Learning



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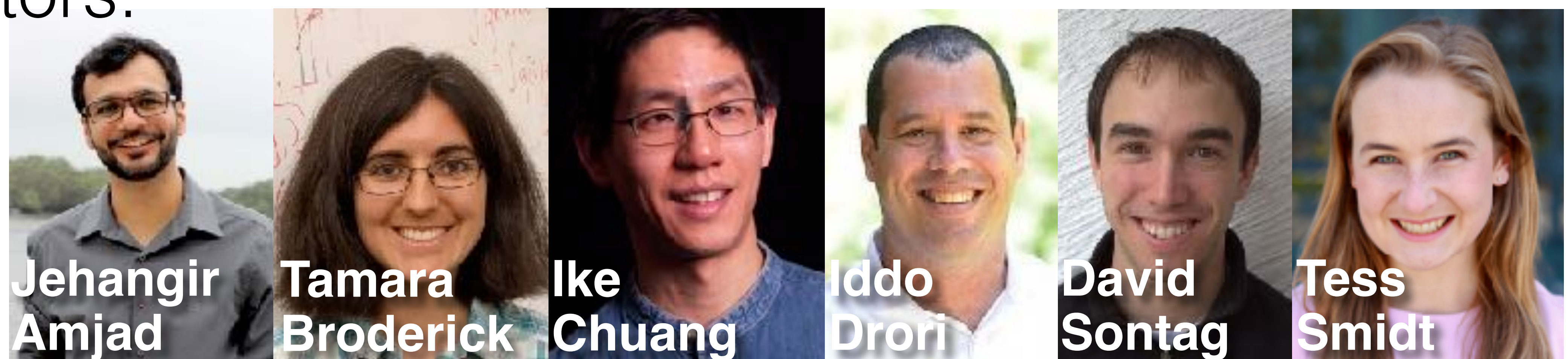

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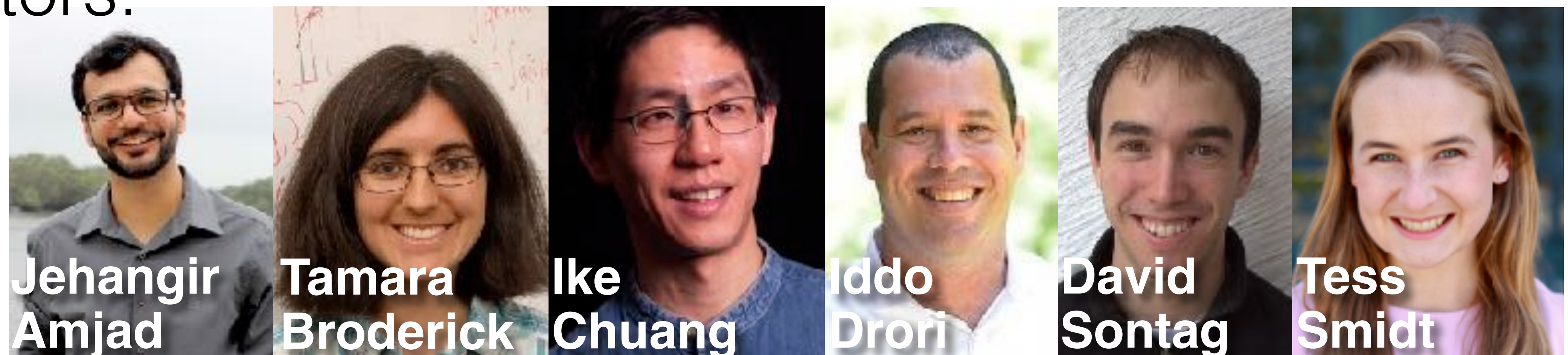
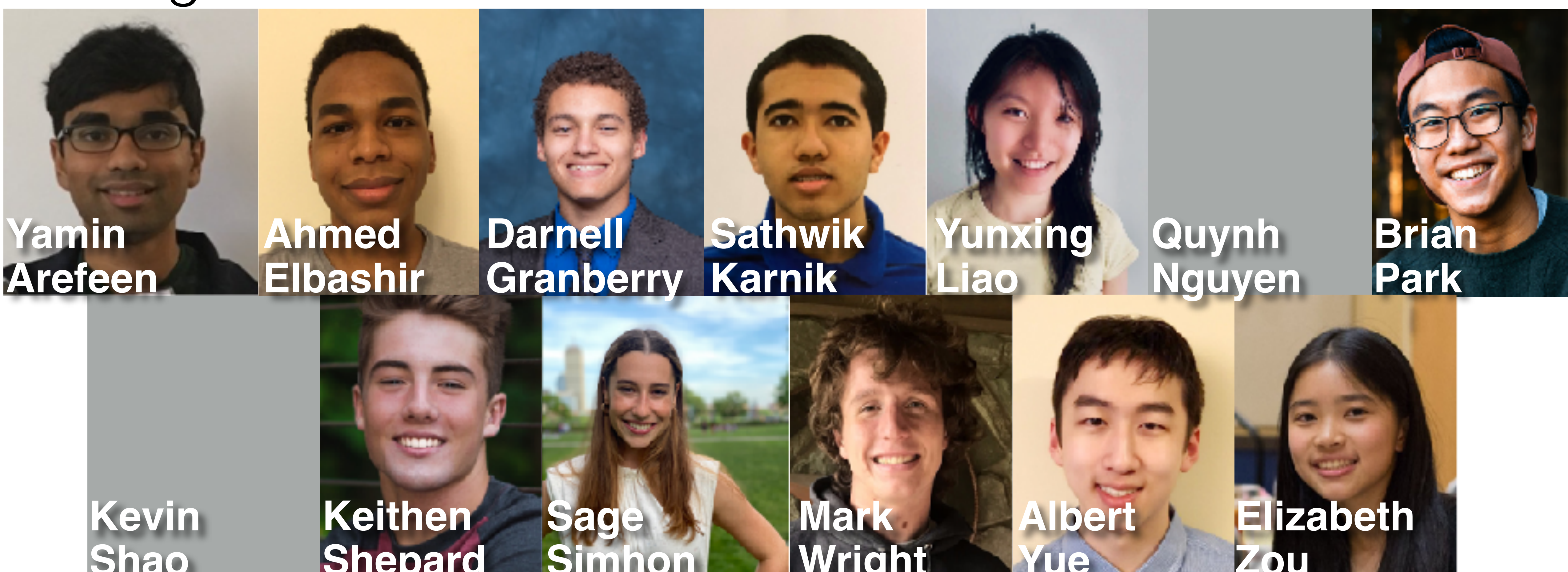


Teaching Assistants:



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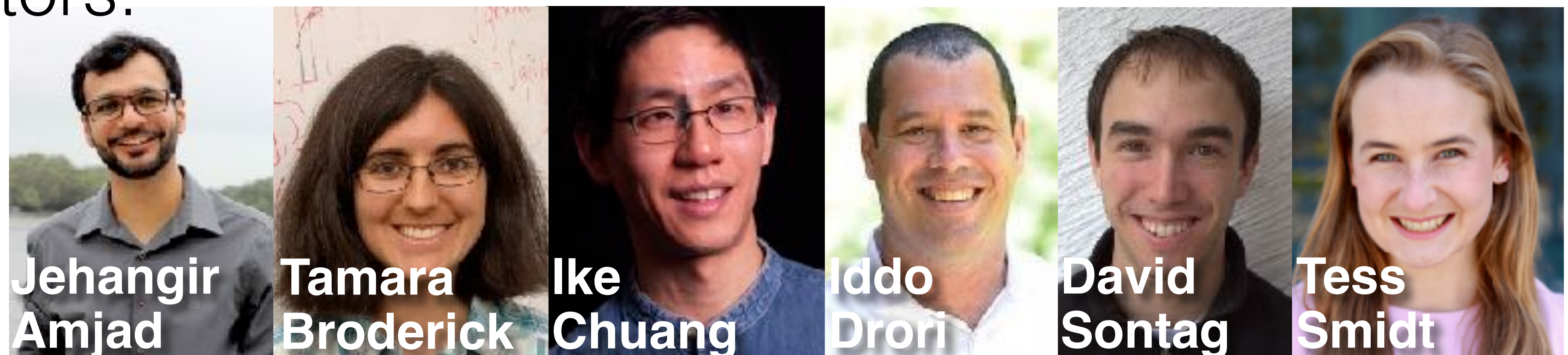

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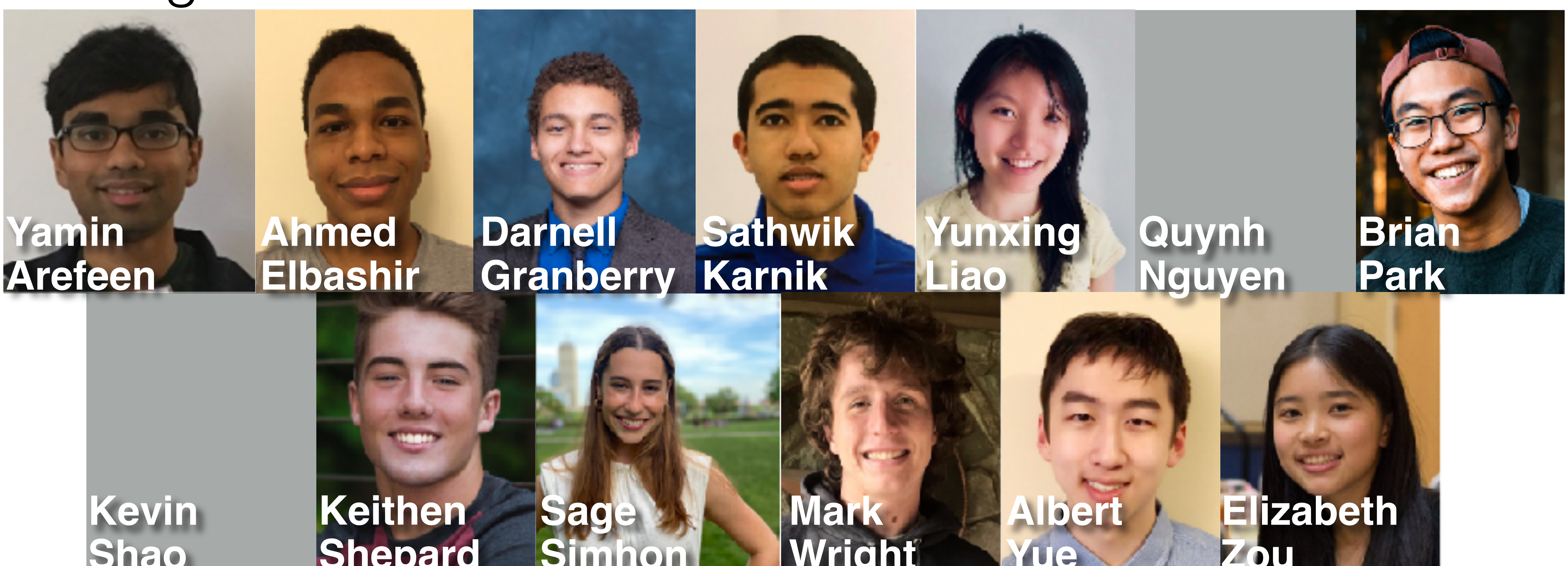

And Lab Assistants!

Machine learning (ML): why & what

Machine learning (ML): why & what

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NEWS | 22 July 2021

DeepMind's AI predicts structures for a vast trove of proteins

AlphaFold neural network produced a 'totally transformative' database of more than 350,000 structures from *Homo sapiens* and 20 model organisms.

Ewen Callaway

Underpinning the latest version of AlphaFold is a novel machine learning approach that incorporates physical and biological knowledge about protein structure, leveraging multi-sequence alignments, into the design of the deep learning algorithm.

[https://www.nature.com/articles/d41586-021-02025-4; https://www.nature.com/articles/s41586-021-03819-2]

Machine learning (ML): why & what

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TechRepublic.

Machine learning algorithm confirms 50 new exoplanets in historic first

f in

by **R. Dallon Adams** in **Innovation** on August 26, 2020, 9:07 AM PST

A new machine learning technique can be used to sift through massive datasets to discern exoplanets from false positives.

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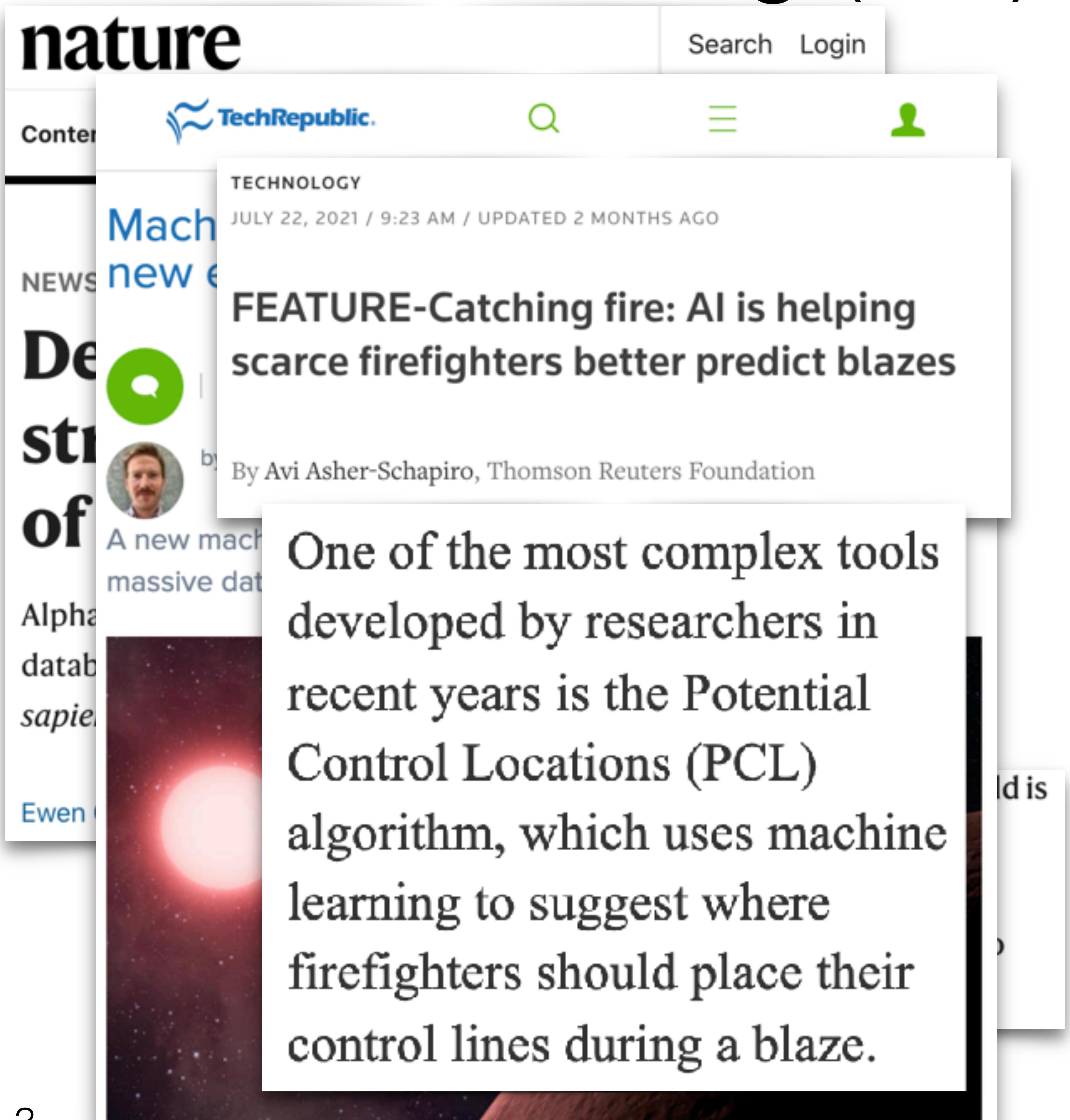
[https://www.techrepublic.com/article/machine-learning-algorithm-confirms-50-new-exoplanets-in-historic-first/]

Machine learning (ML): why & what



TECHNOLOGY

JULY 22, 2021 / 9:23 AM / UPDATED 2 MONTHS AGO

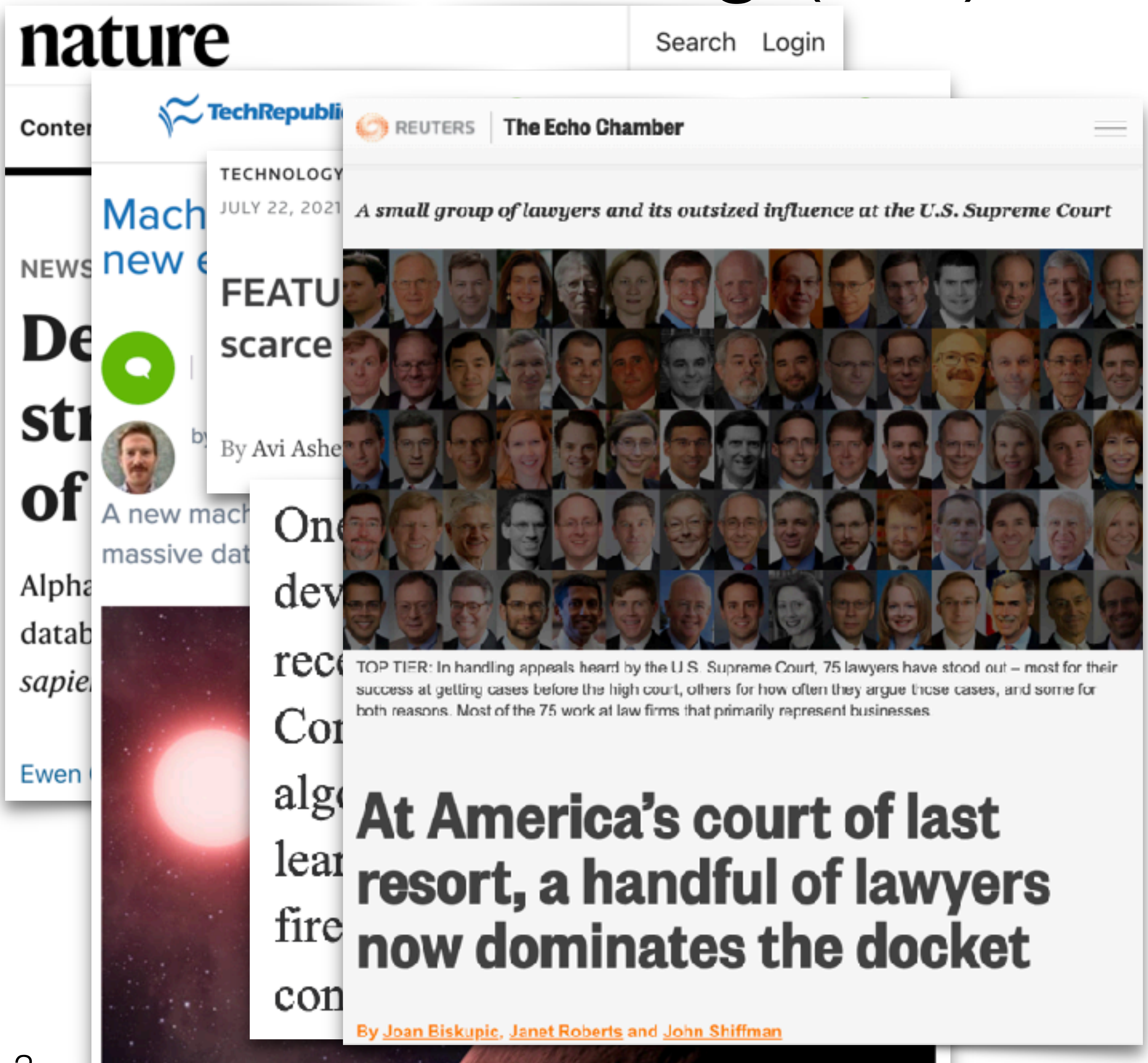
FEATURE-Catching fire: AI is helping scarce firefighters better predict blazes

By Avi Asher-Schapiro, Thomson Reuters Foundation

One of the most complex tools developed by researchers in recent years is the Potential Control Locations (PCL) algorithm, which uses machine learning to suggest where firefighters should place their control lines during a blaze.

[https://www.reuters.com/article/tech-environment-wildfire-climate-change-idUSL8N2OW008]

Machine learning (ML): why & what



[https://www.reuters.com/investigates/special-report/scotus/]

Machine learning (ML): why & what

[https://www.npr.org/2021/05/07/982709480/massachusetts-pioneers-rules-for-police-use-of-facial-recognition-tech]

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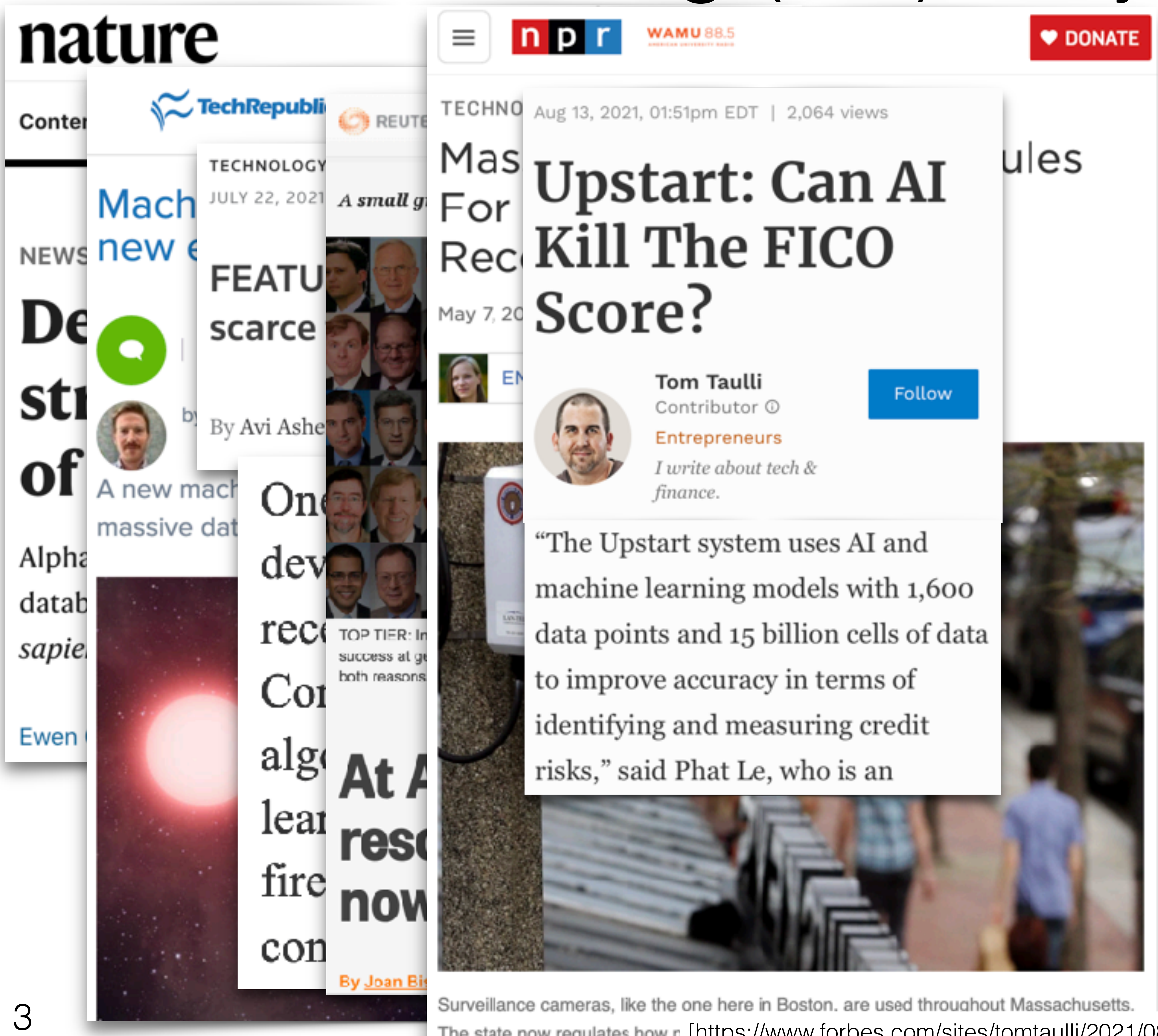

[https://www.forbes.com/sites/tomtaulli/2021/08/13/upstart-can-ai-kill-the-fico-score/]

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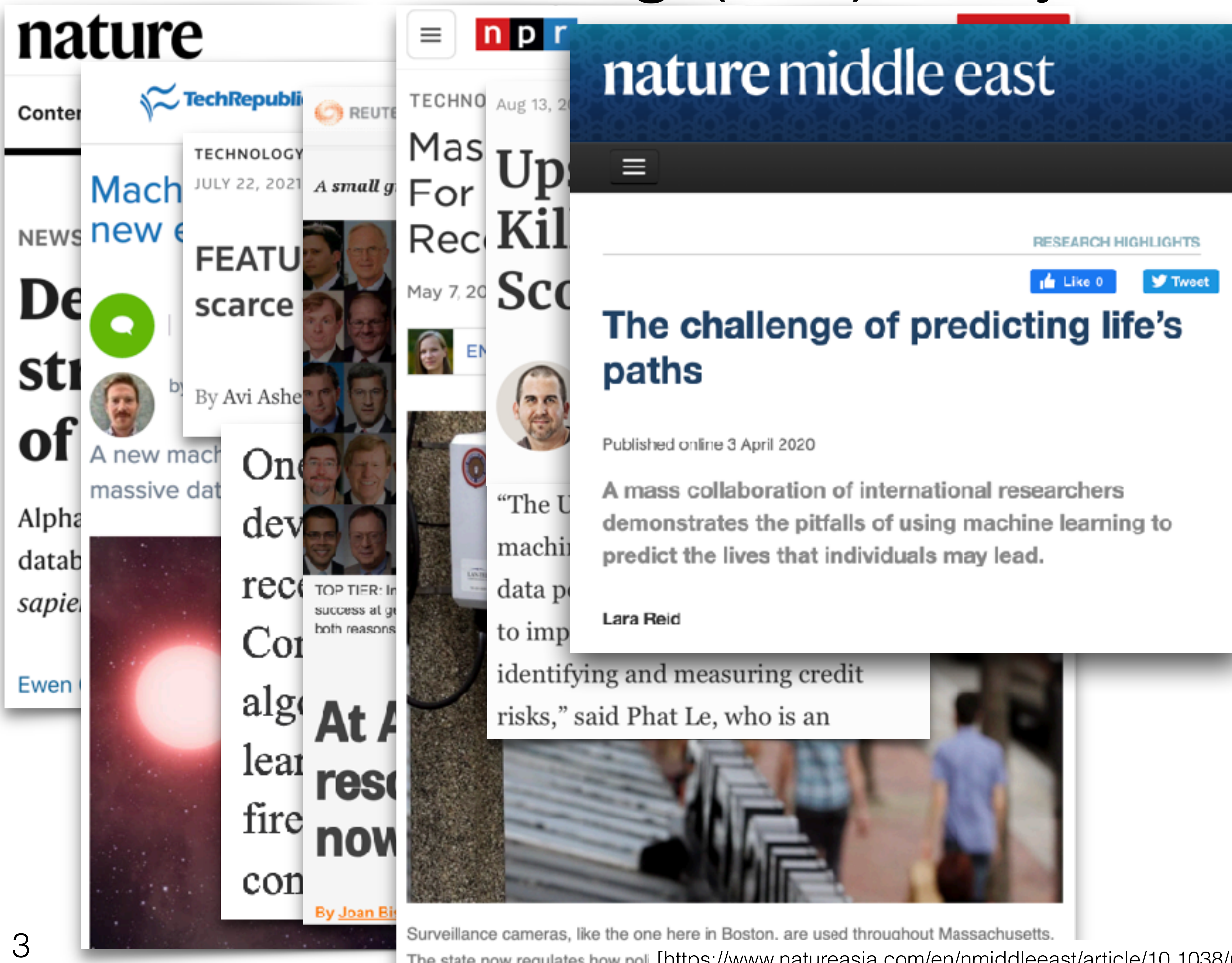

[https://www.natureasia.com/en/nmiddleeast/article/10.1038/nmiddleeast.2020.46]

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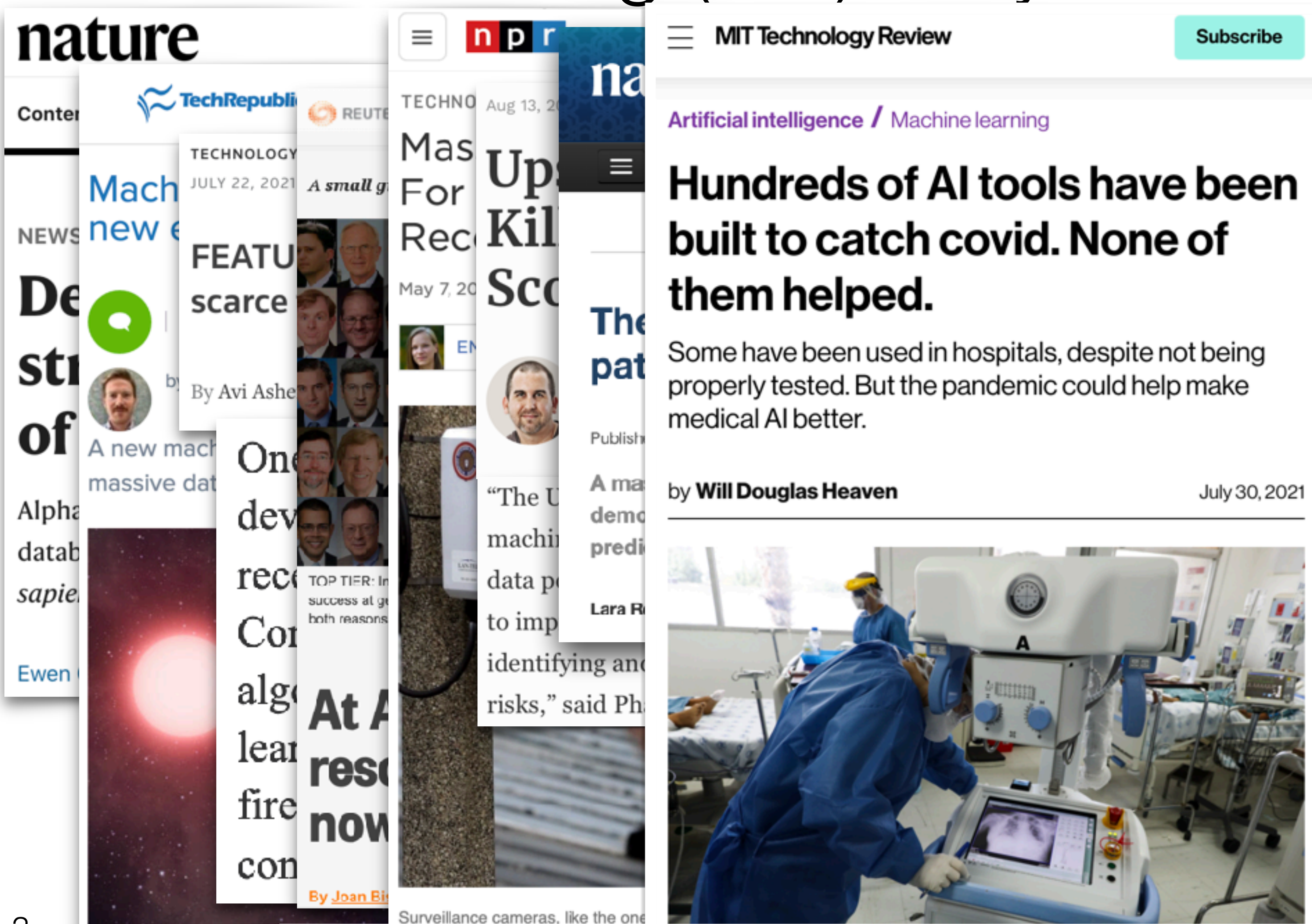

[https://www.technologyreview.com/2021/07/30/1030329/machine-learning-ai-failed-covid-hospital-diagnosis-pandemic/]

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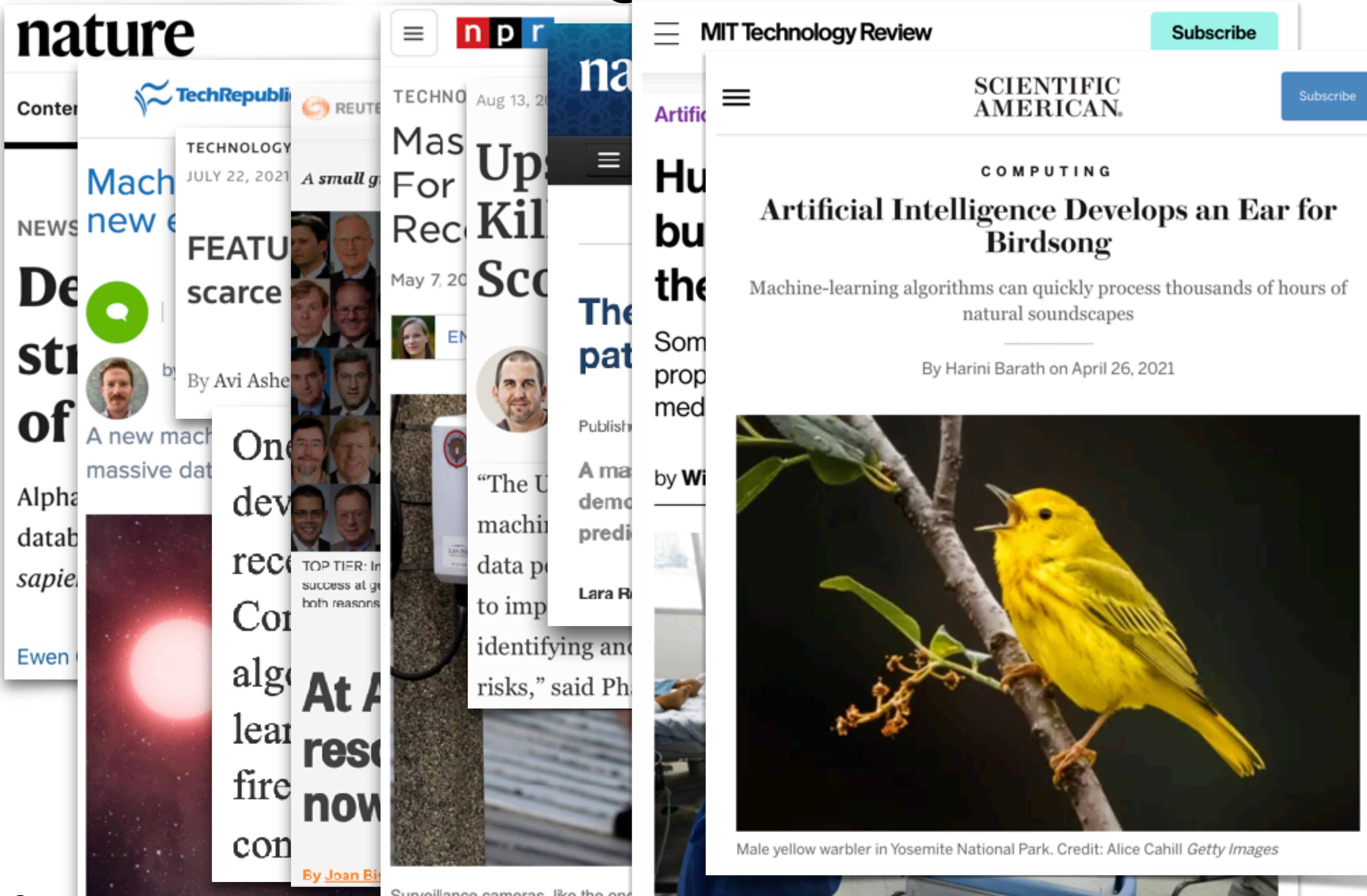

[https://www.scientificamerican.com/article/artificial-intelligence-develops-an-ear-for-birdsong1/#]

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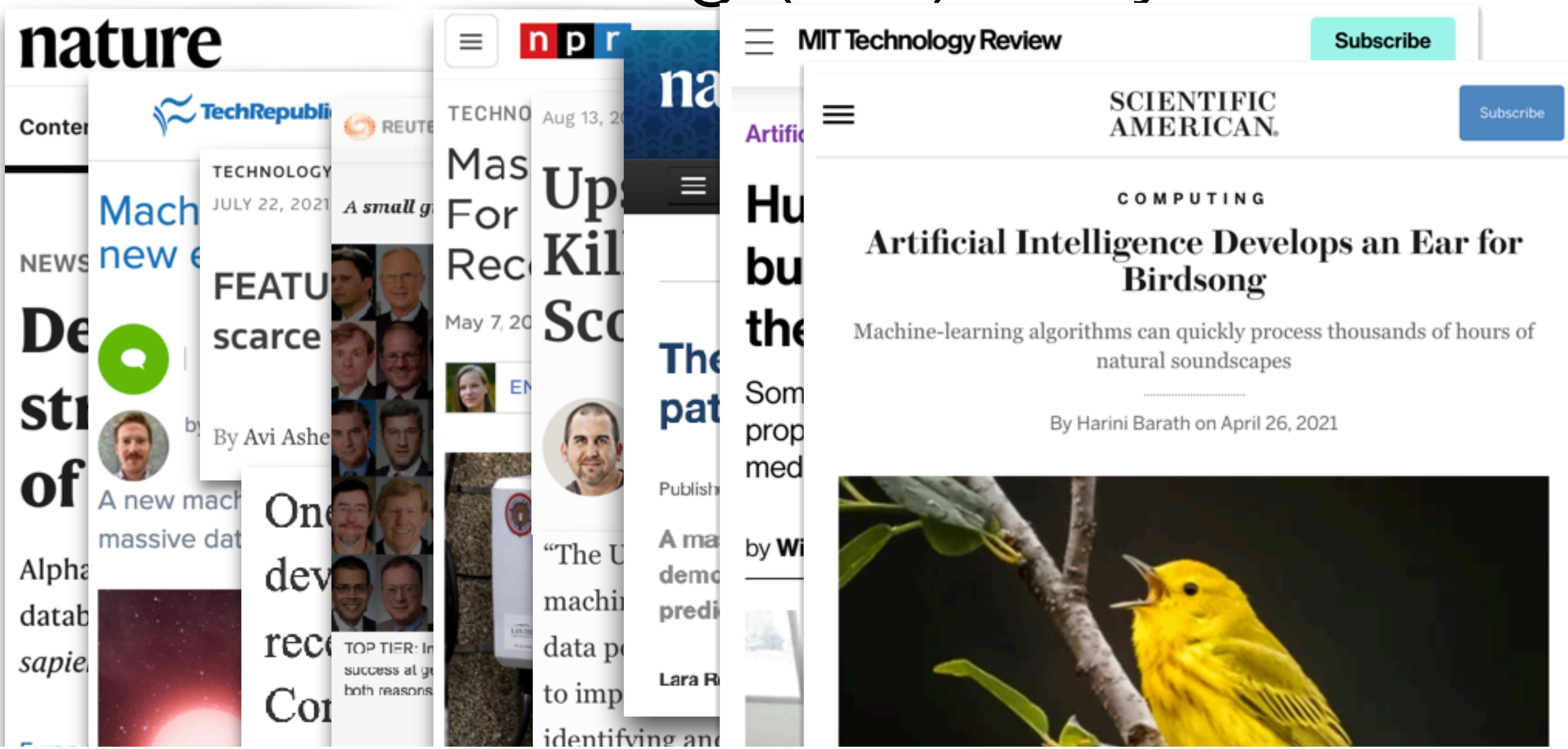
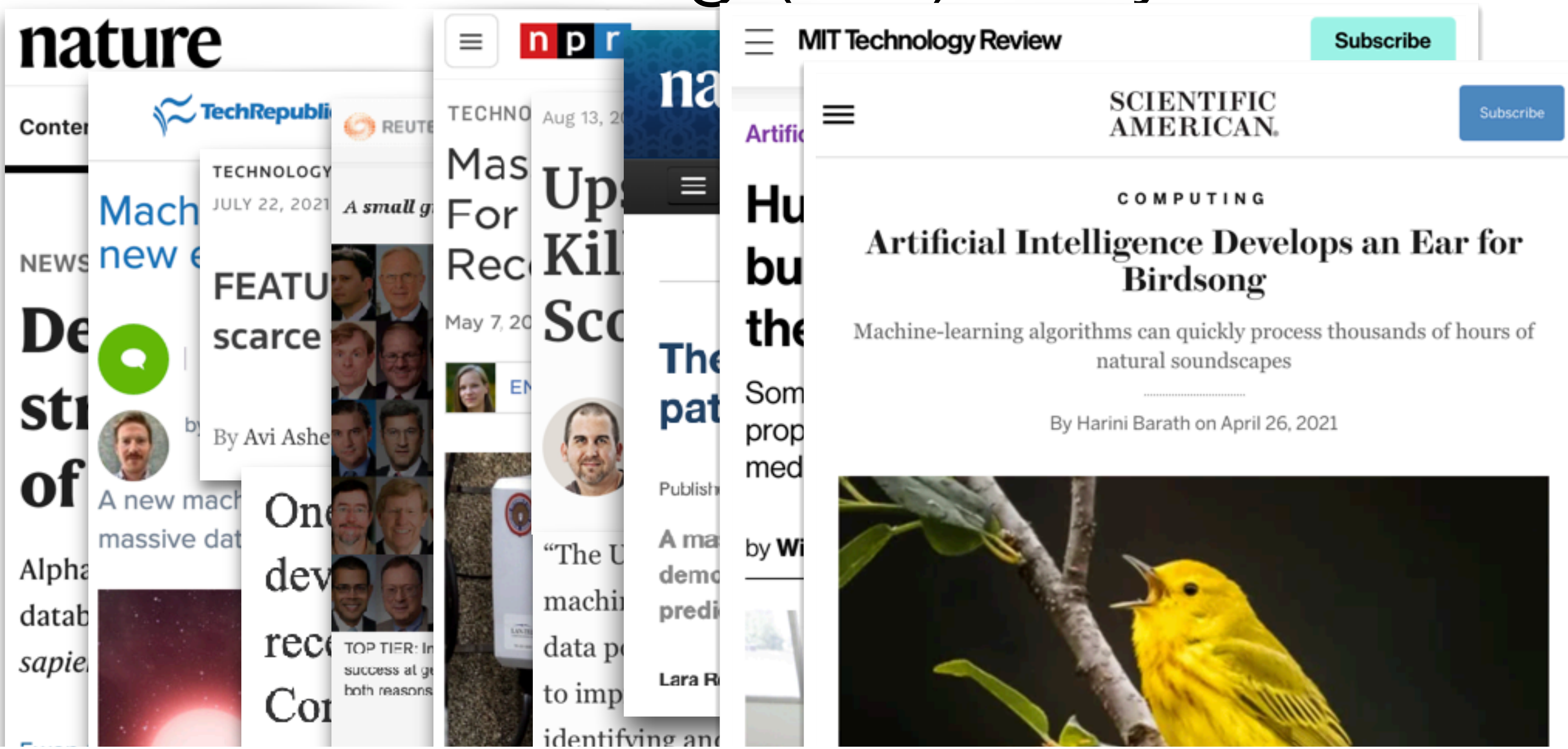


- **What is ML?**

Machine learning (ML): why & what



- **What is ML?** A set of methods for making decisions from data. (See the rest of the course!)

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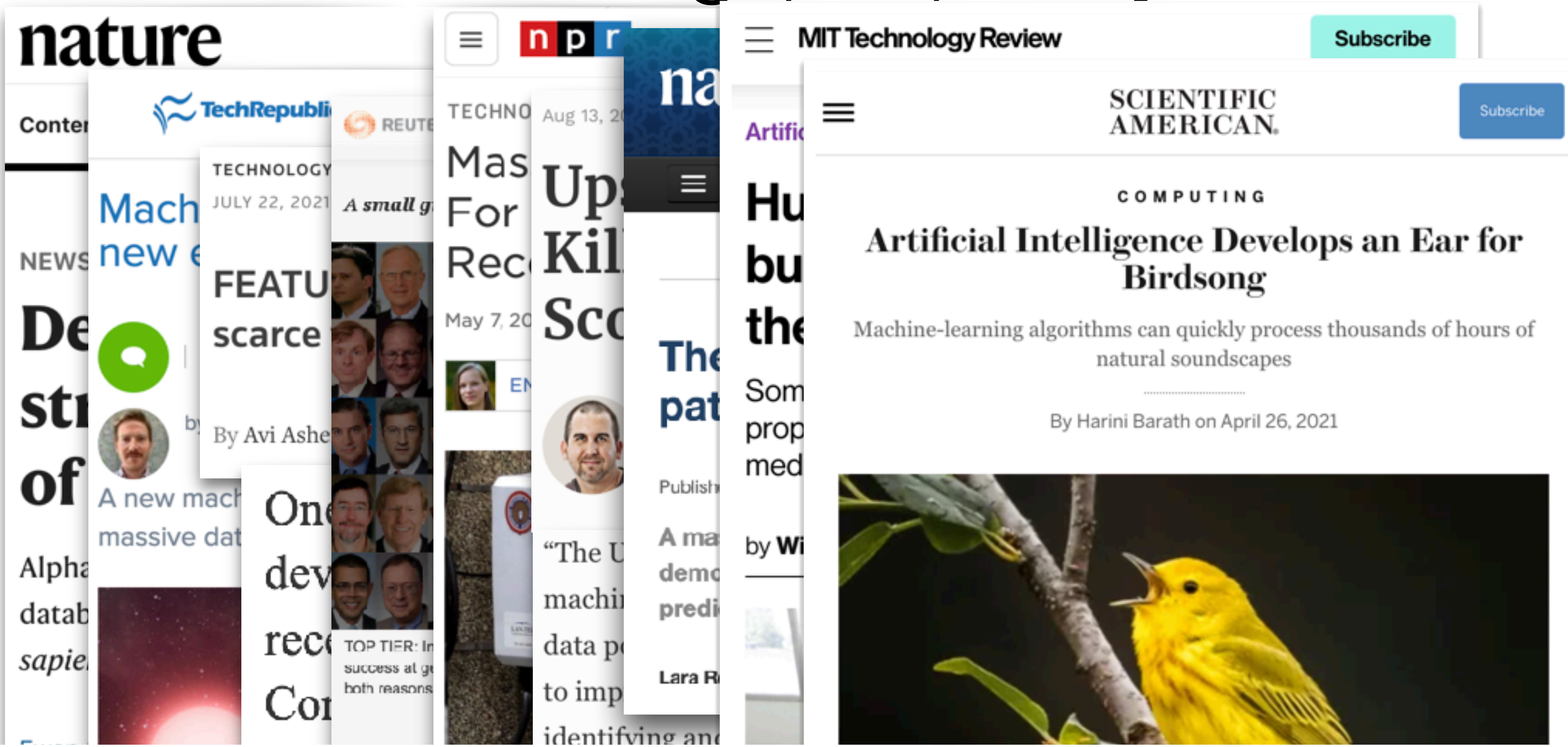

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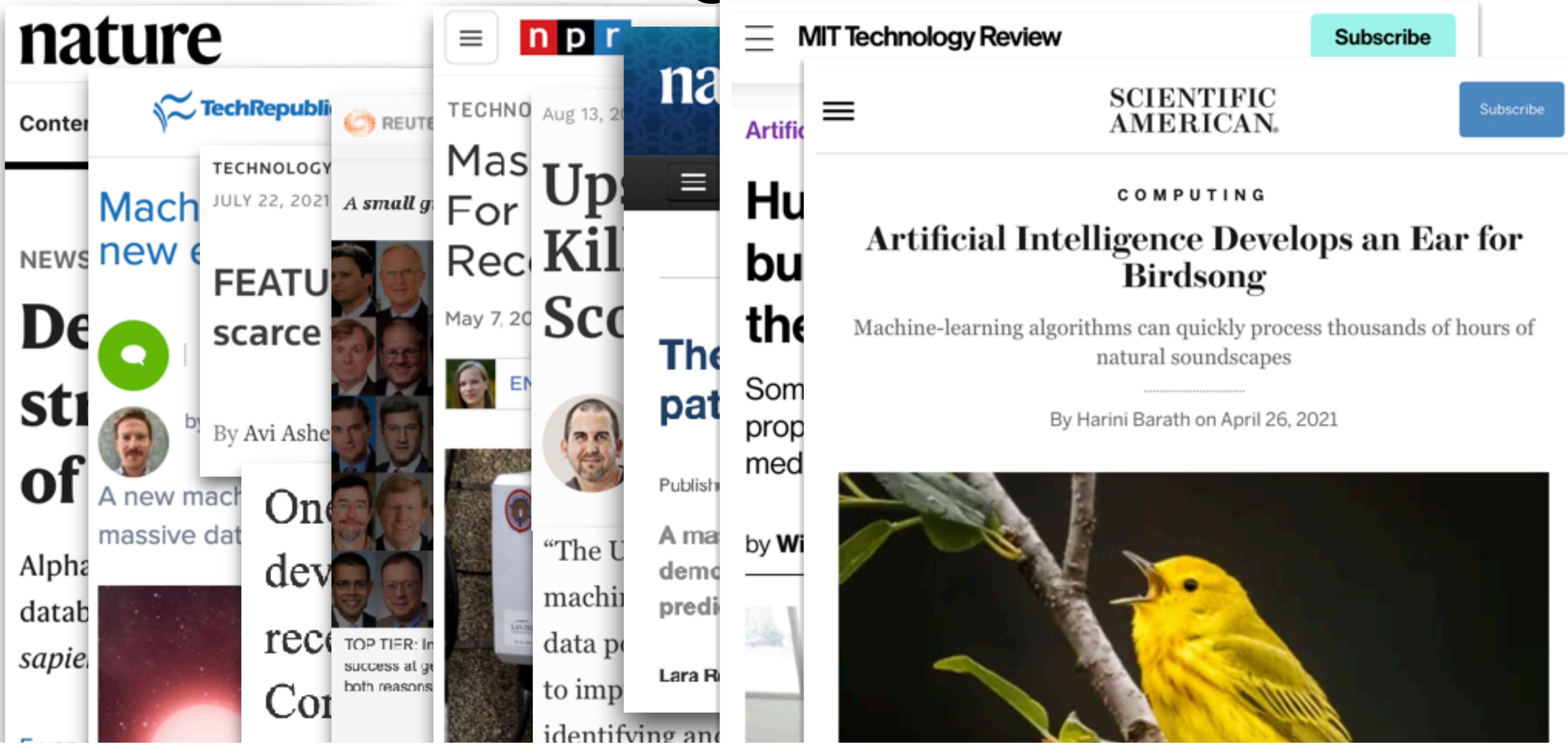

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- **Notes:** ML is a tool with pros & cons. ML is built on math

Getting started: regression

Getting started: regression

Example: predict pollution level

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What do we have?

Getting started: regression

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What do we have? (Training) data

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- *n* training data points

Getting started: regression

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Satellite reading

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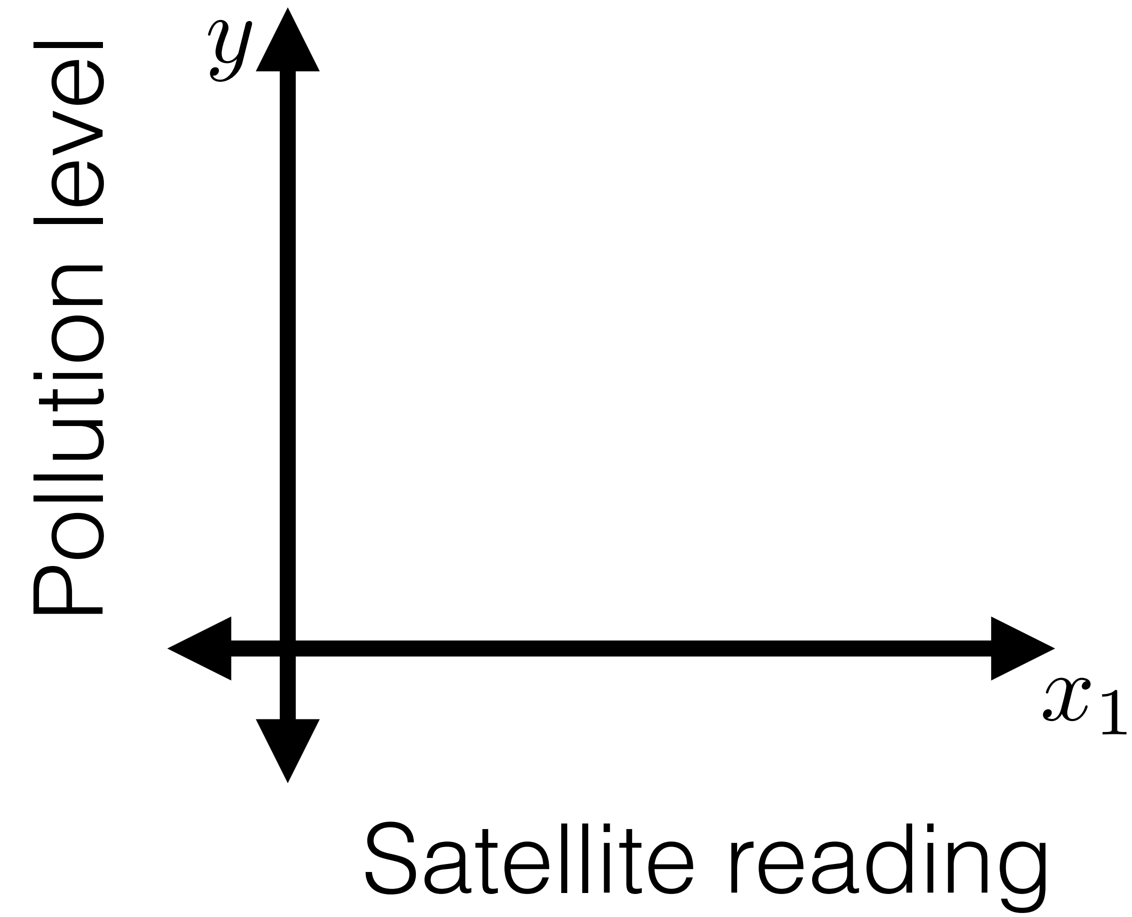

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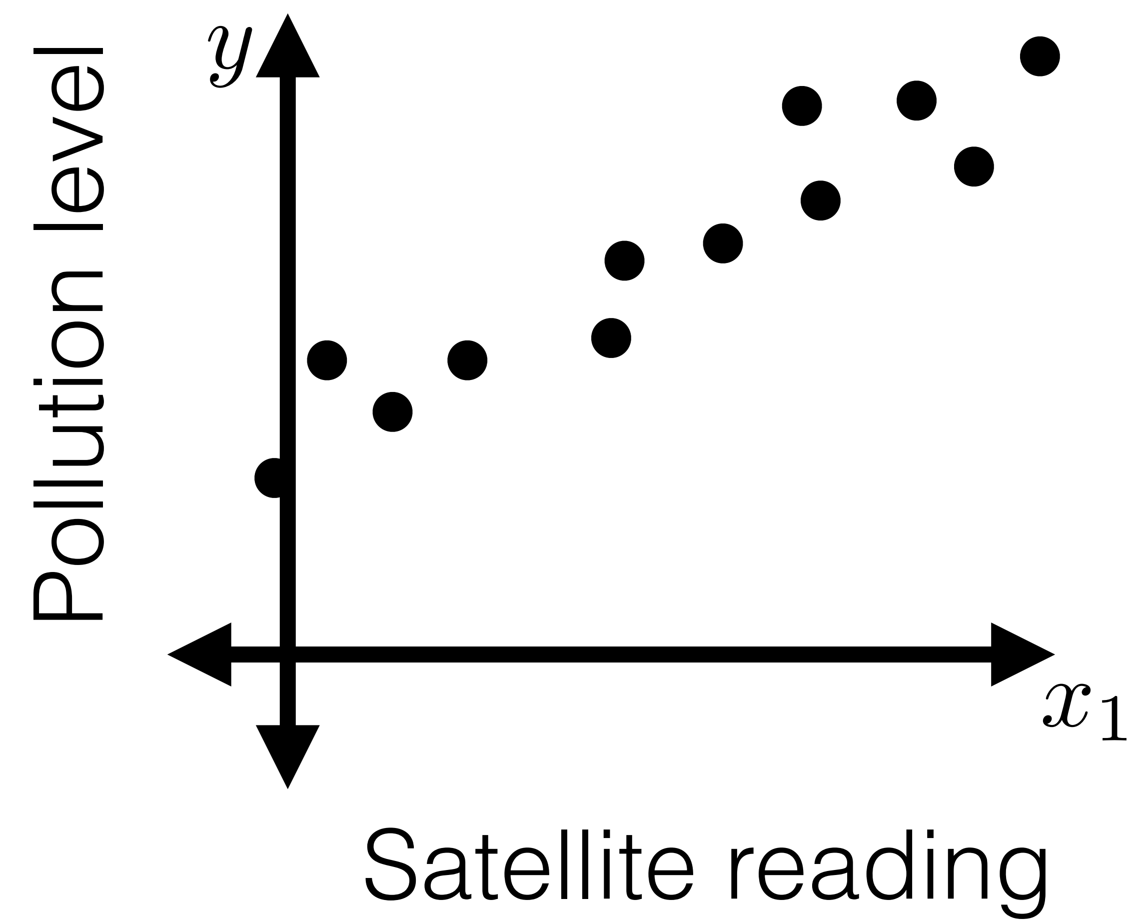

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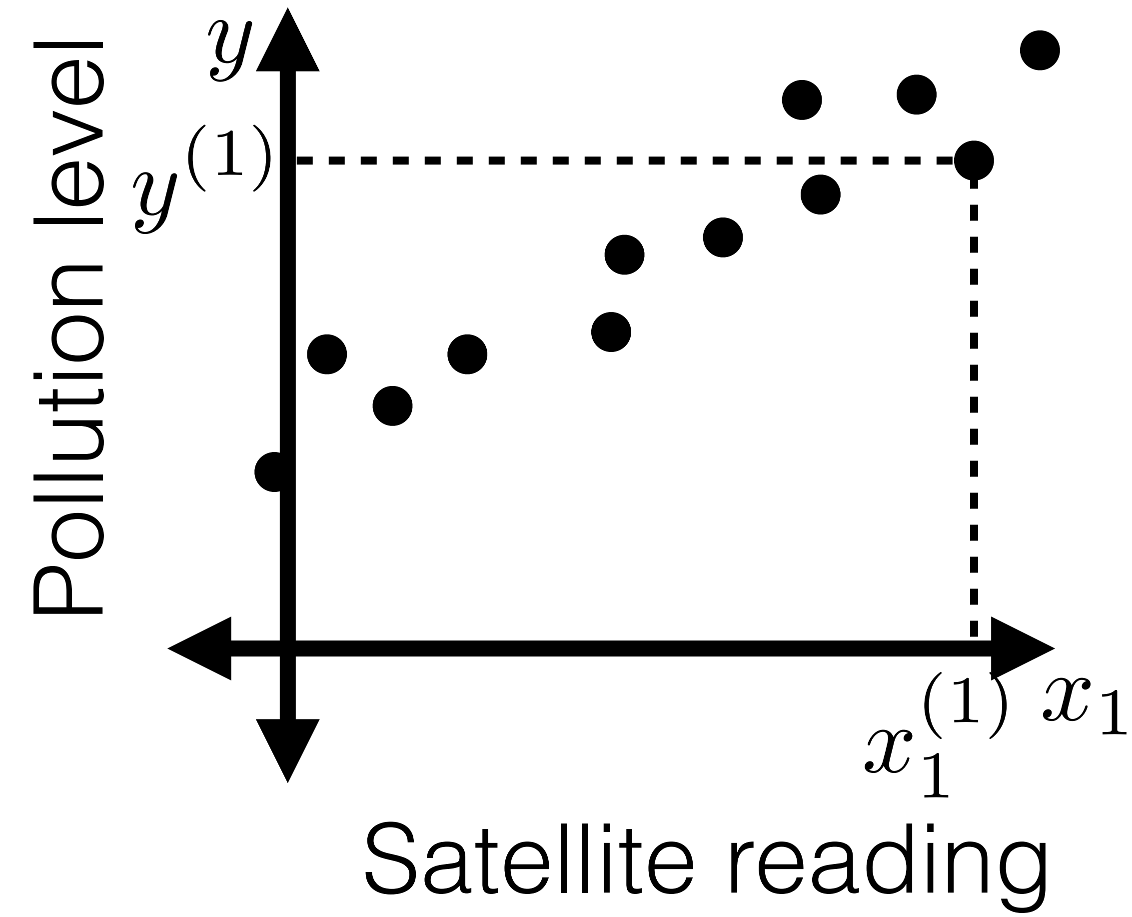

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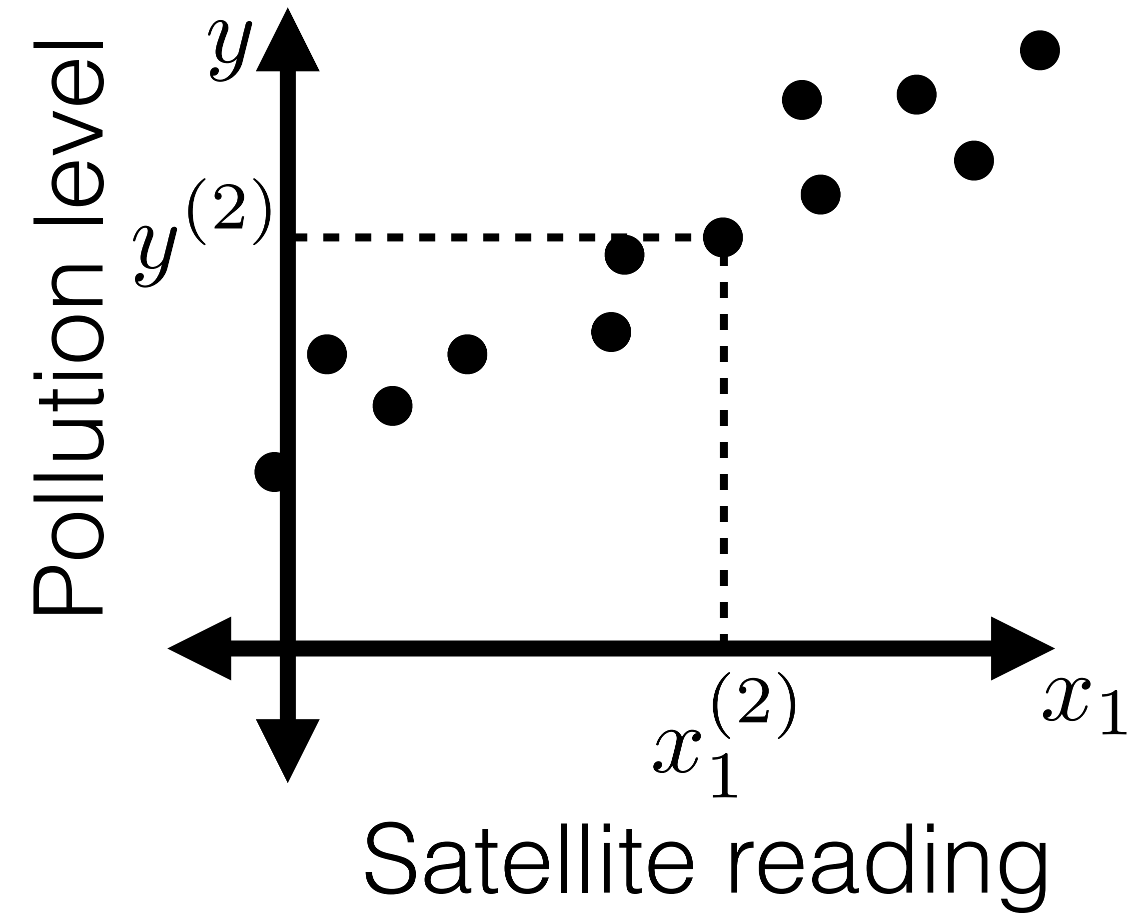

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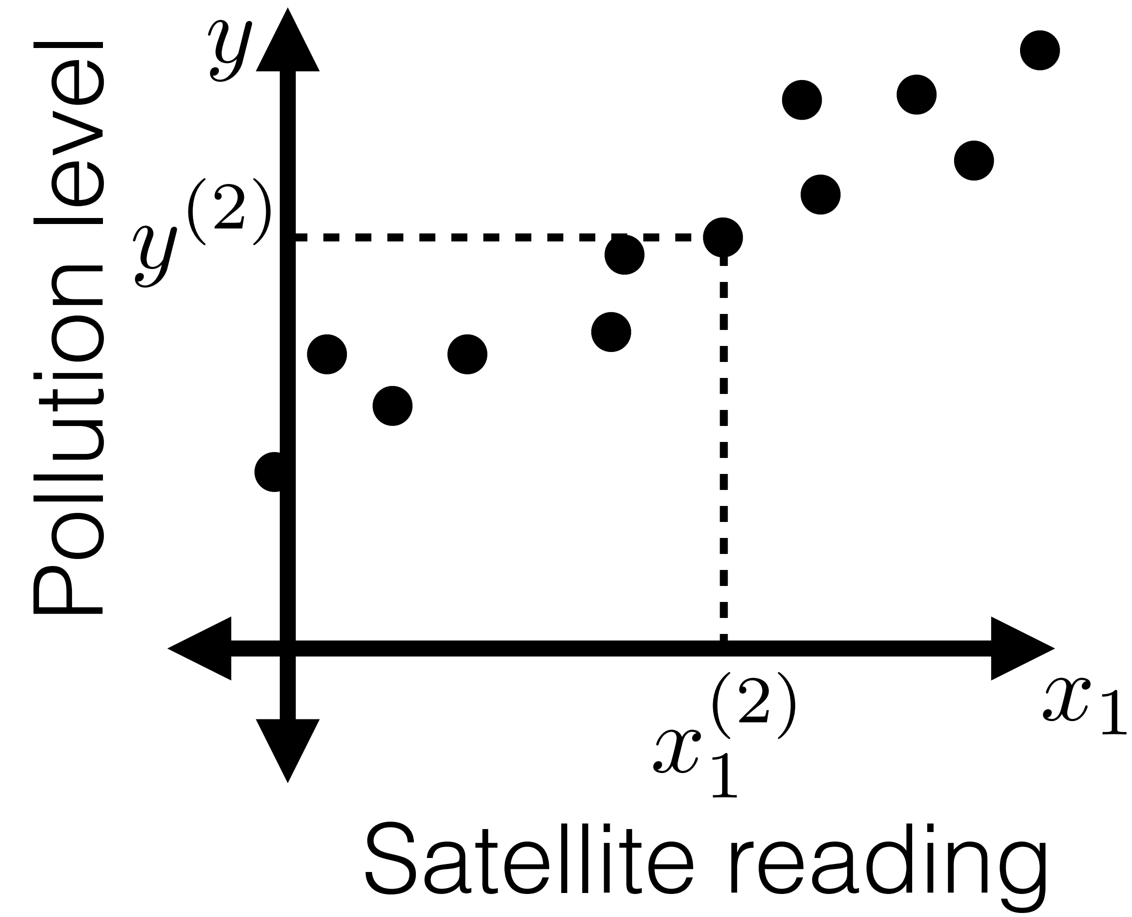

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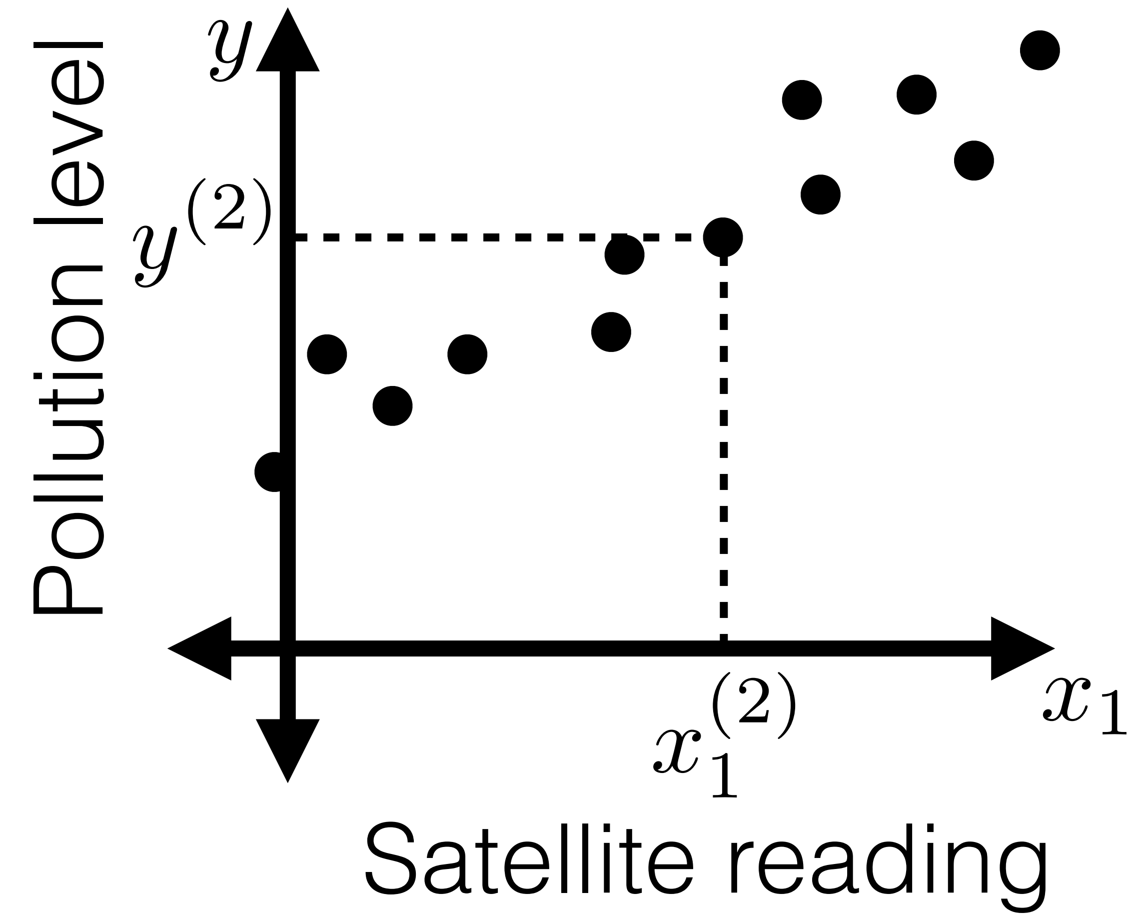

What do we want?

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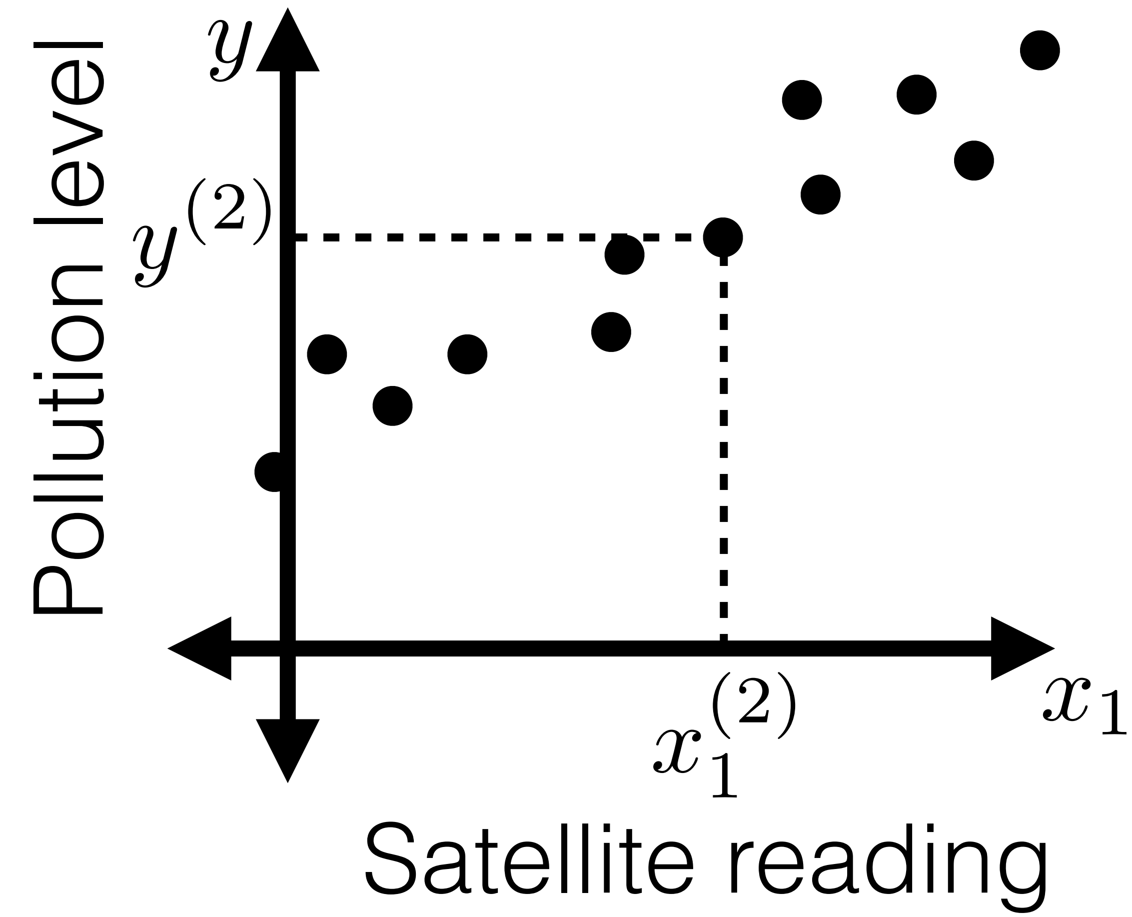

What do we want? A good way to label new points

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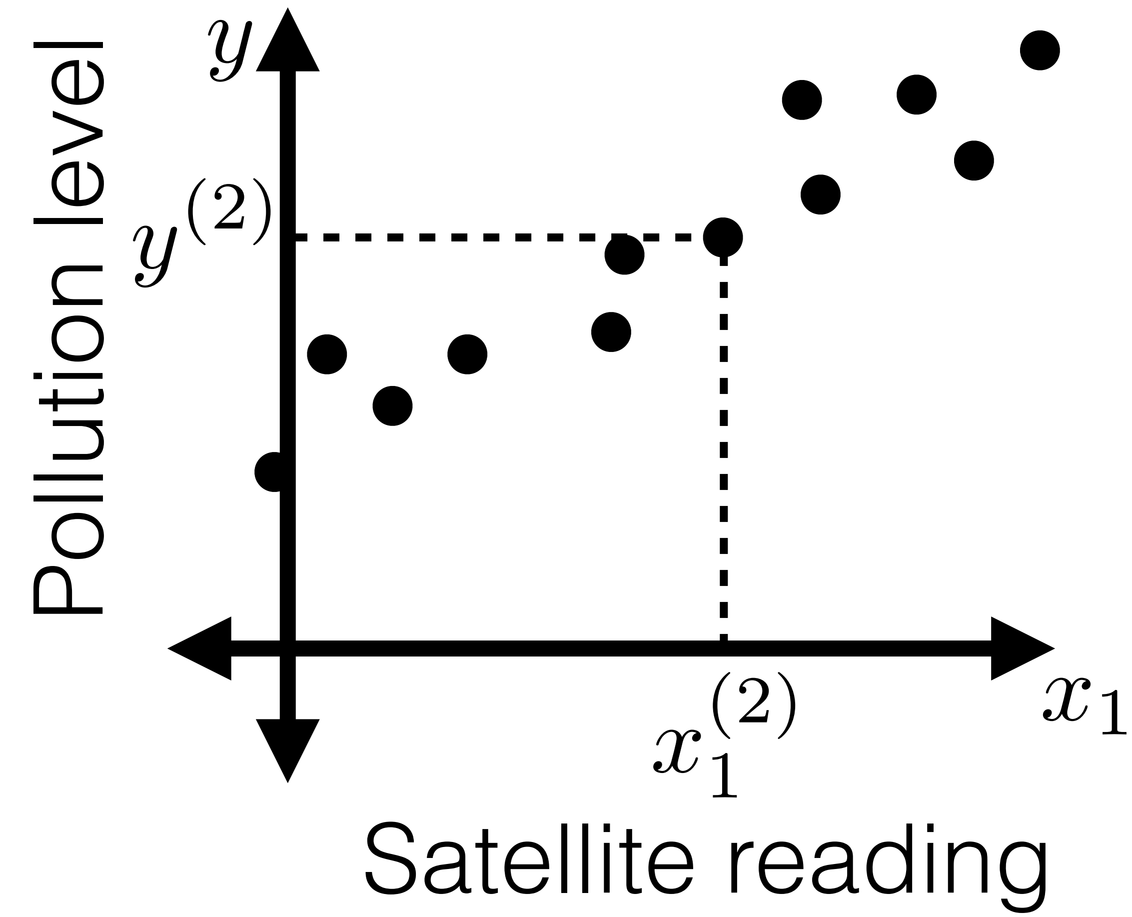
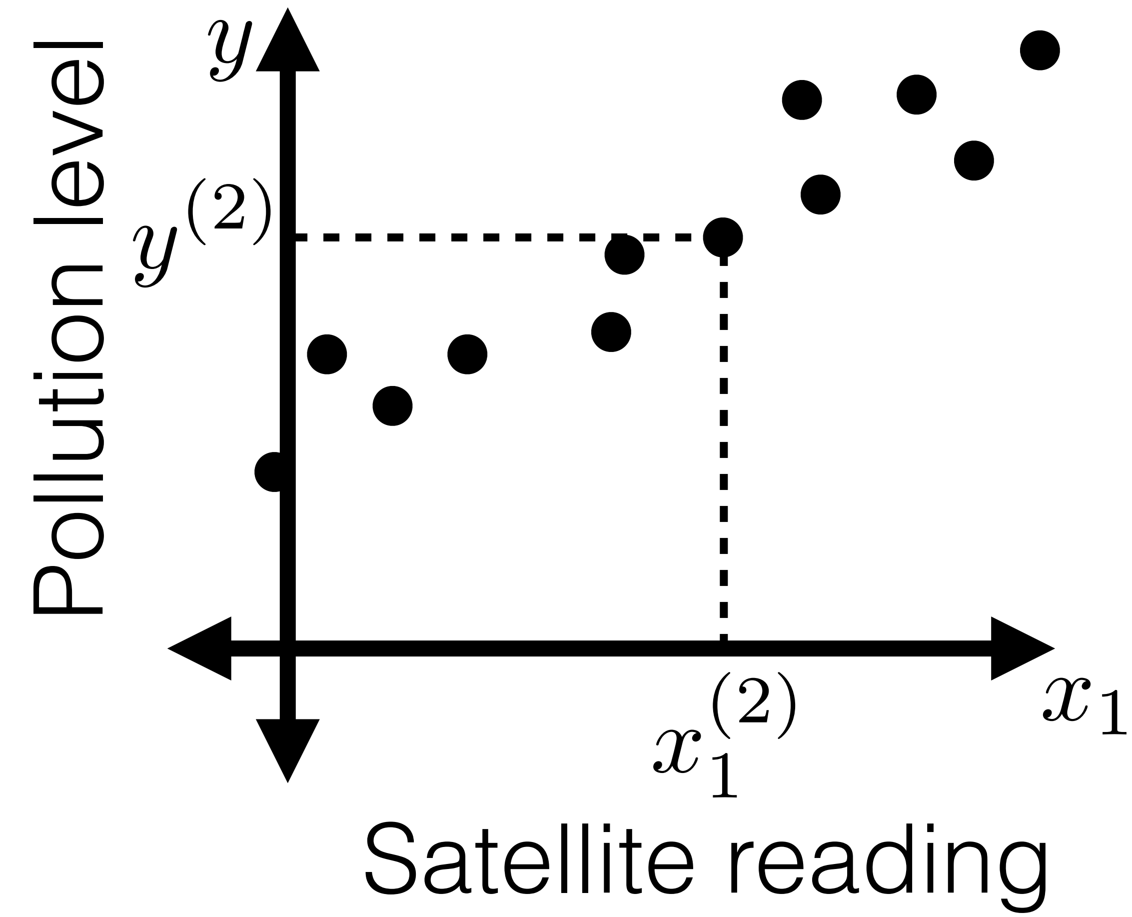


What do we want? A good way to label new points

- How to label?

Getting started: regression



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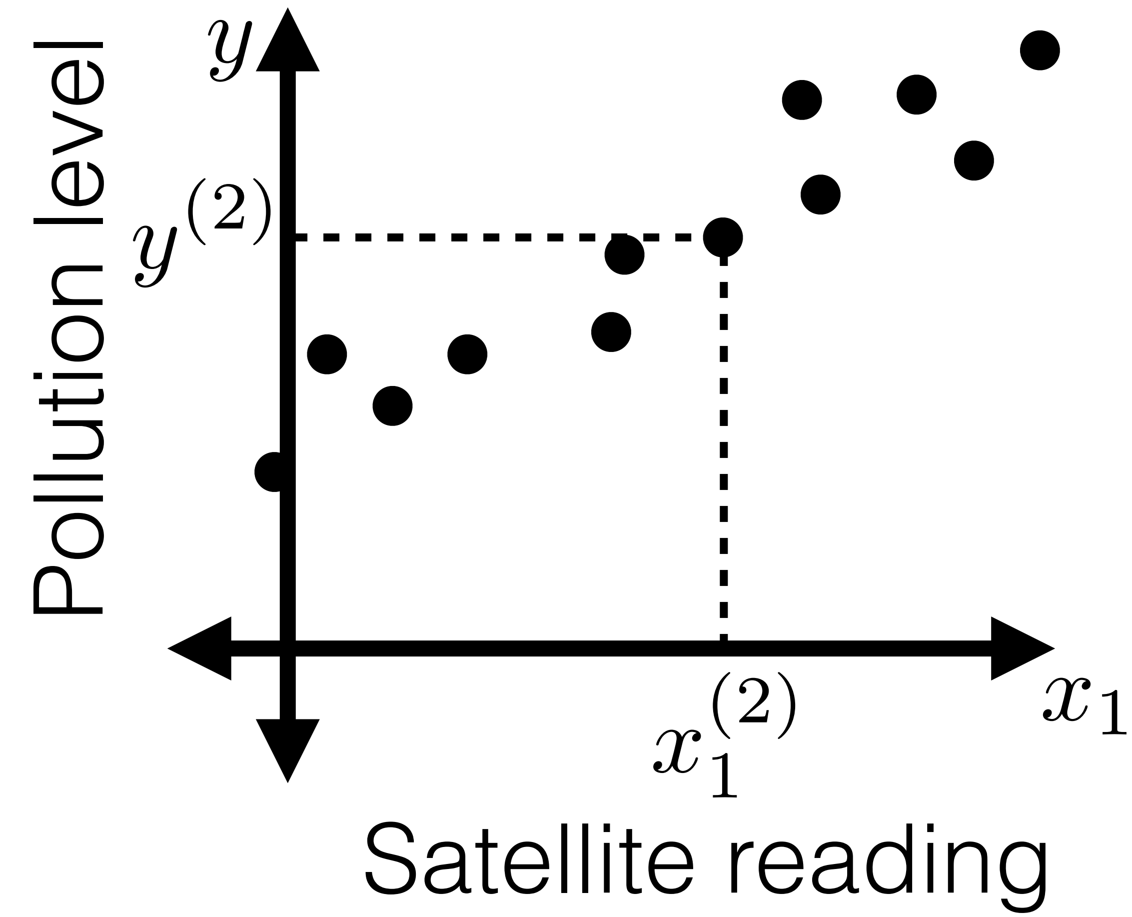

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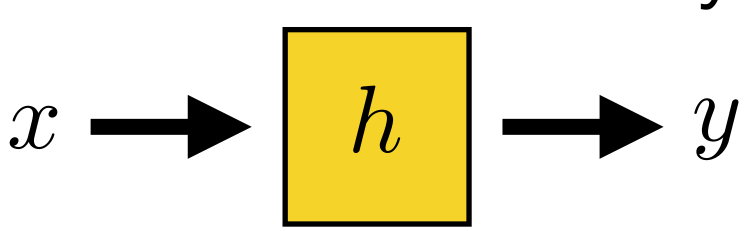

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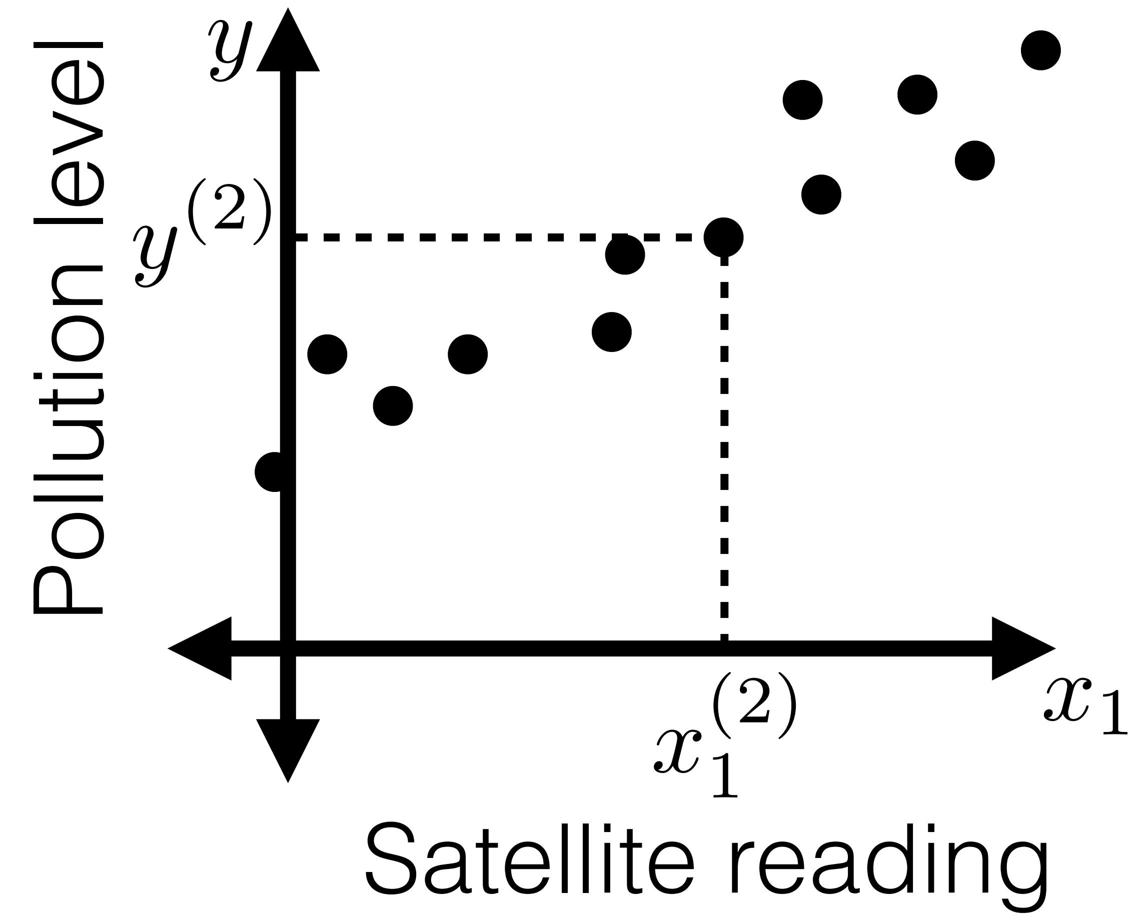

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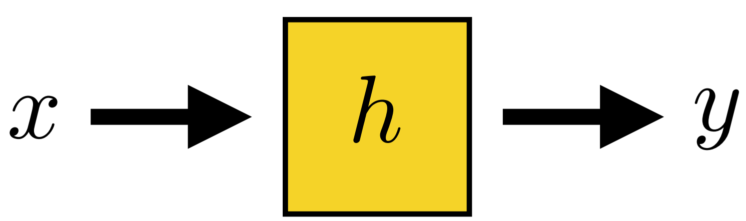

  - Example *h*: For any *x*, *h*(*x*) = 1,000,000

Getting started: regression

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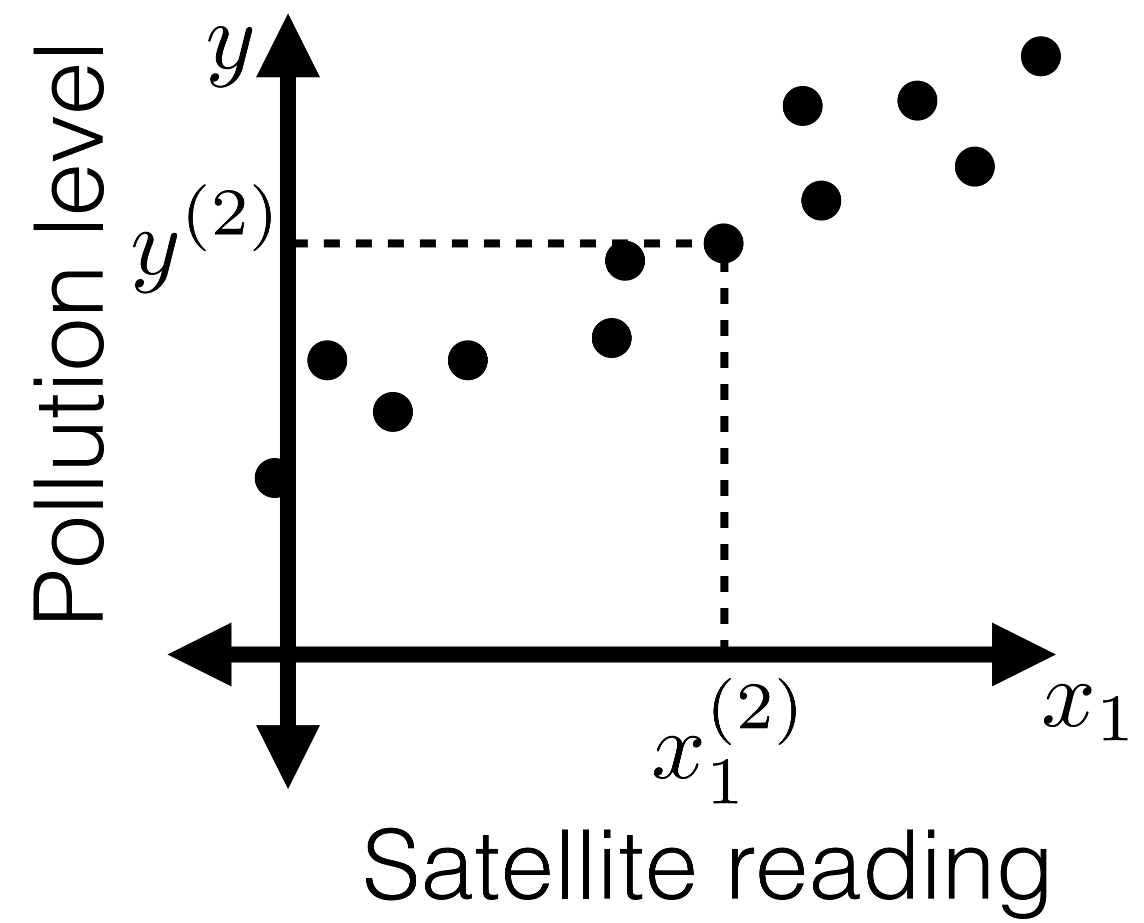

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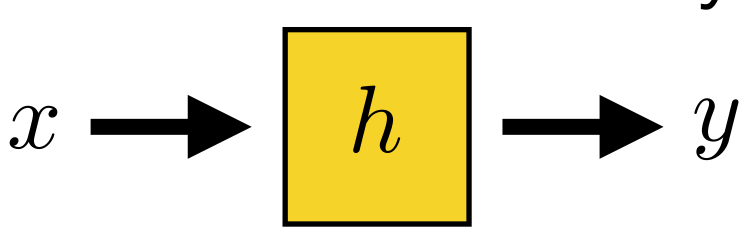

Is this a hypothesis?

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Getting started: regression

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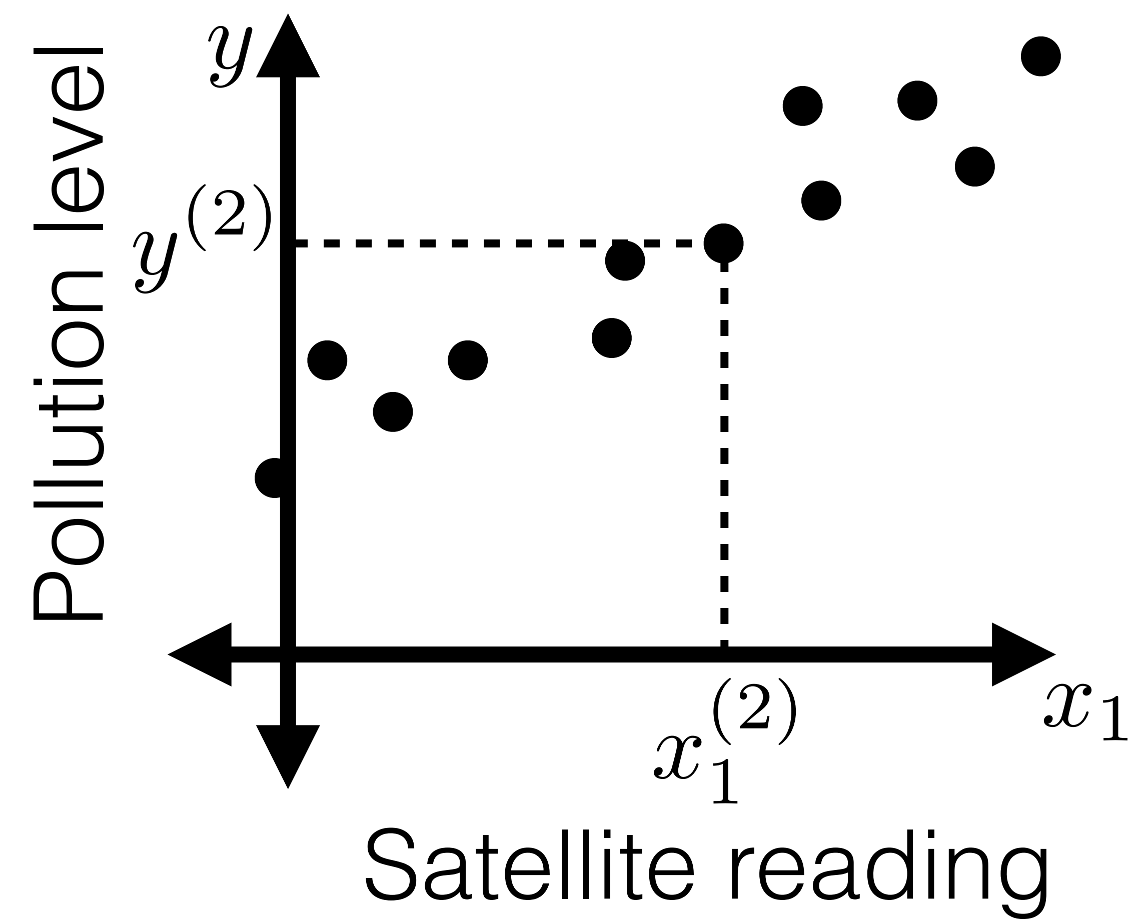

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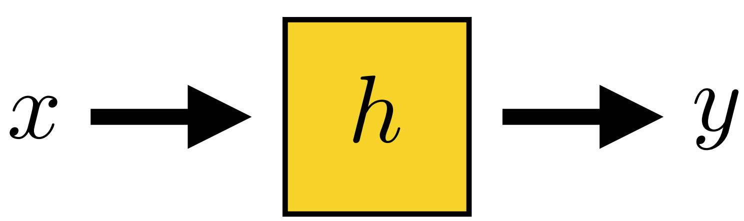

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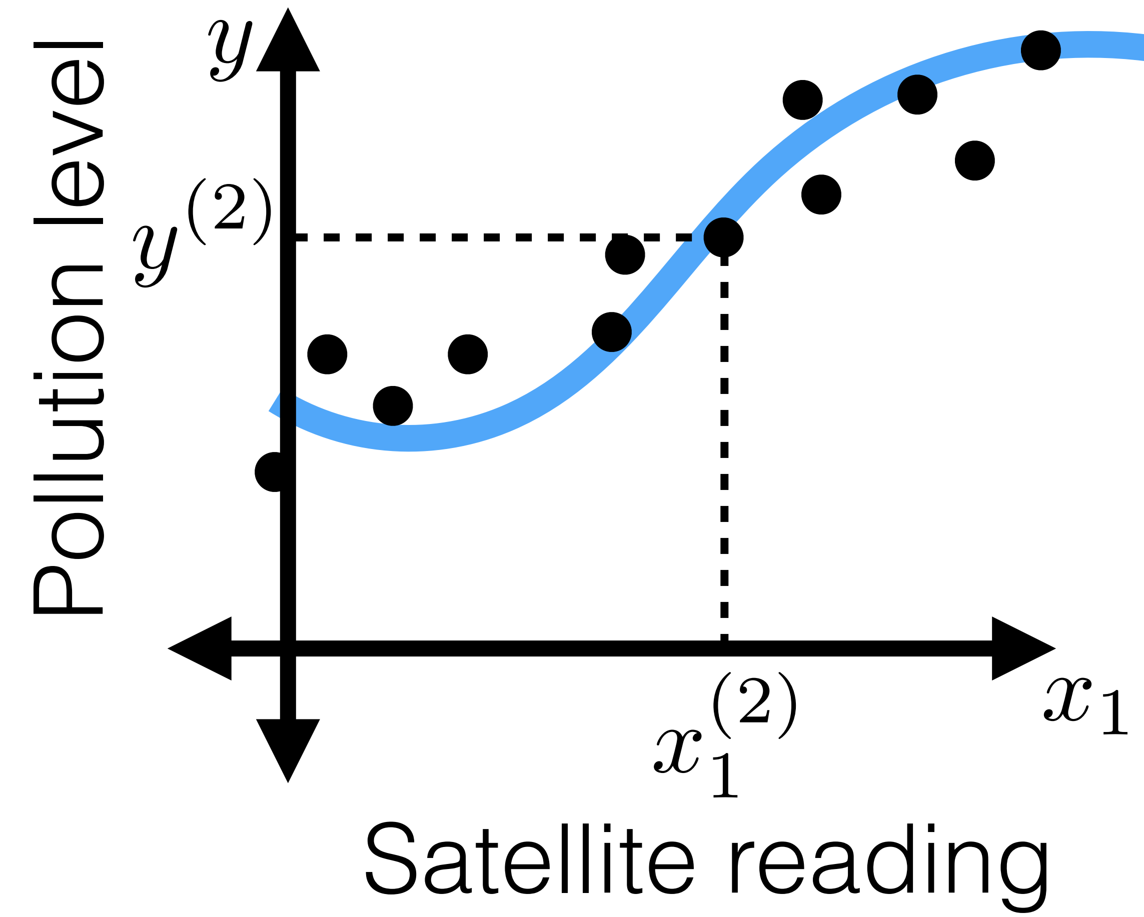
Getting started: regression



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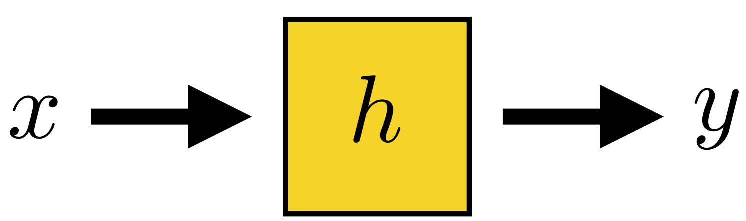

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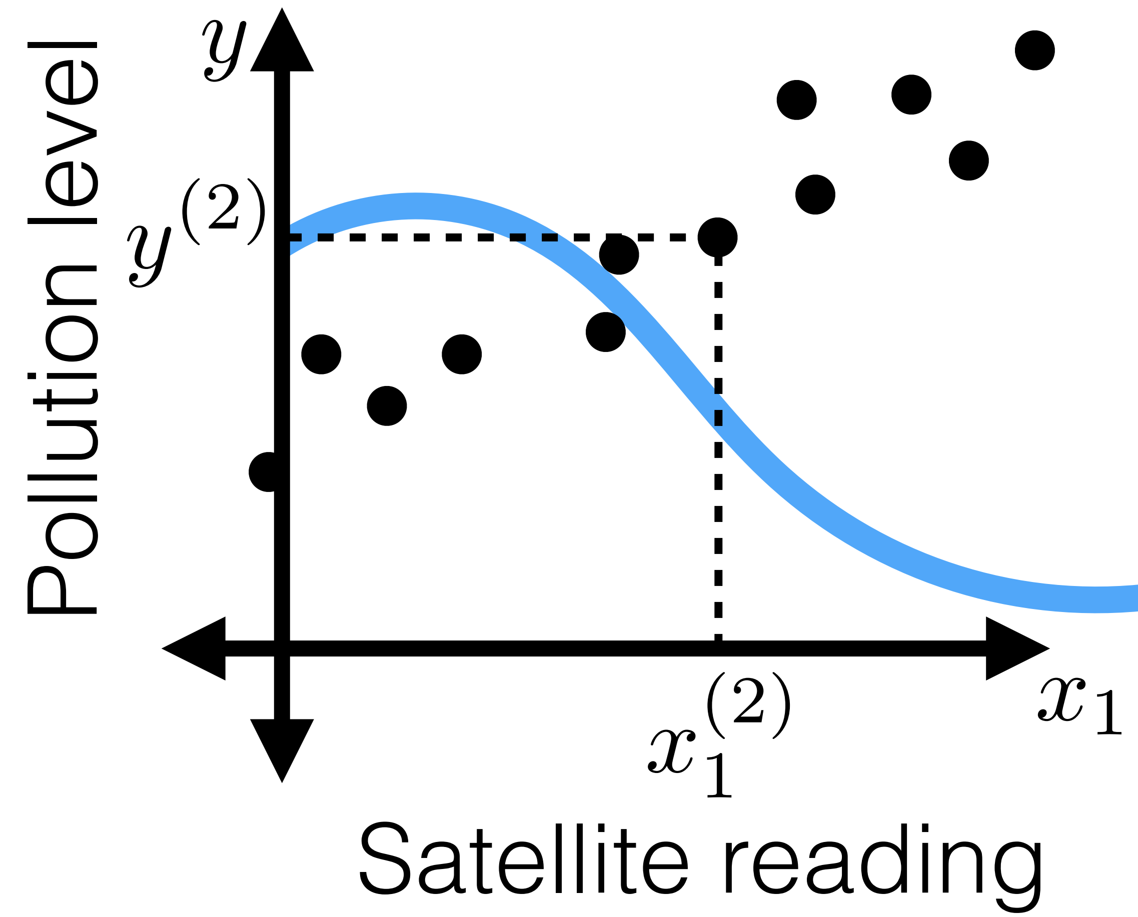

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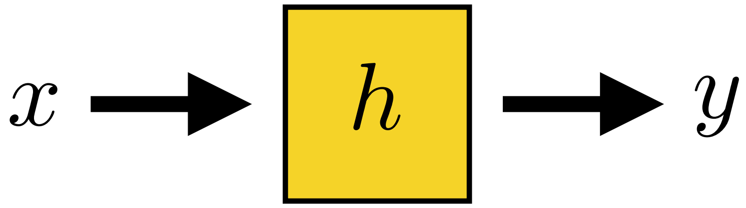
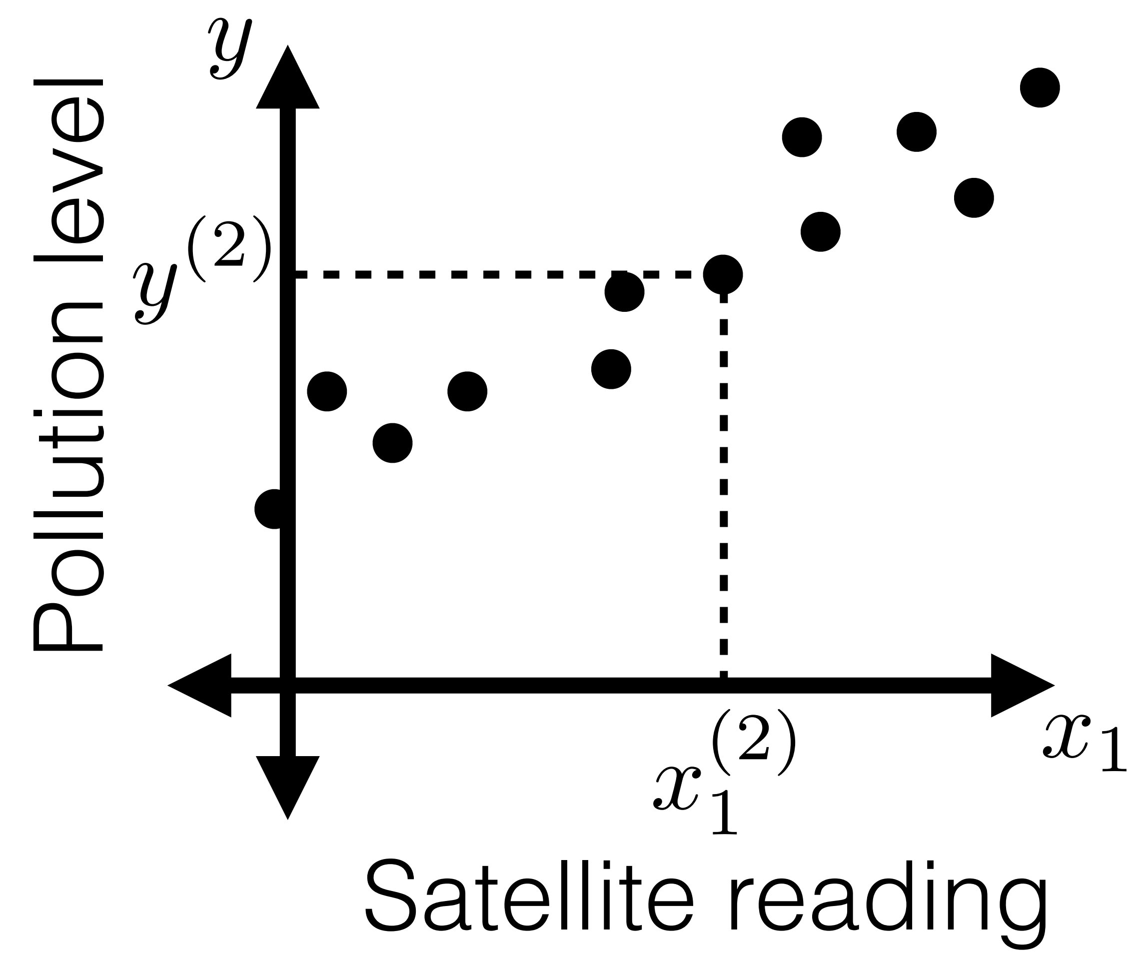


Is this a ***good*** hypothesis?

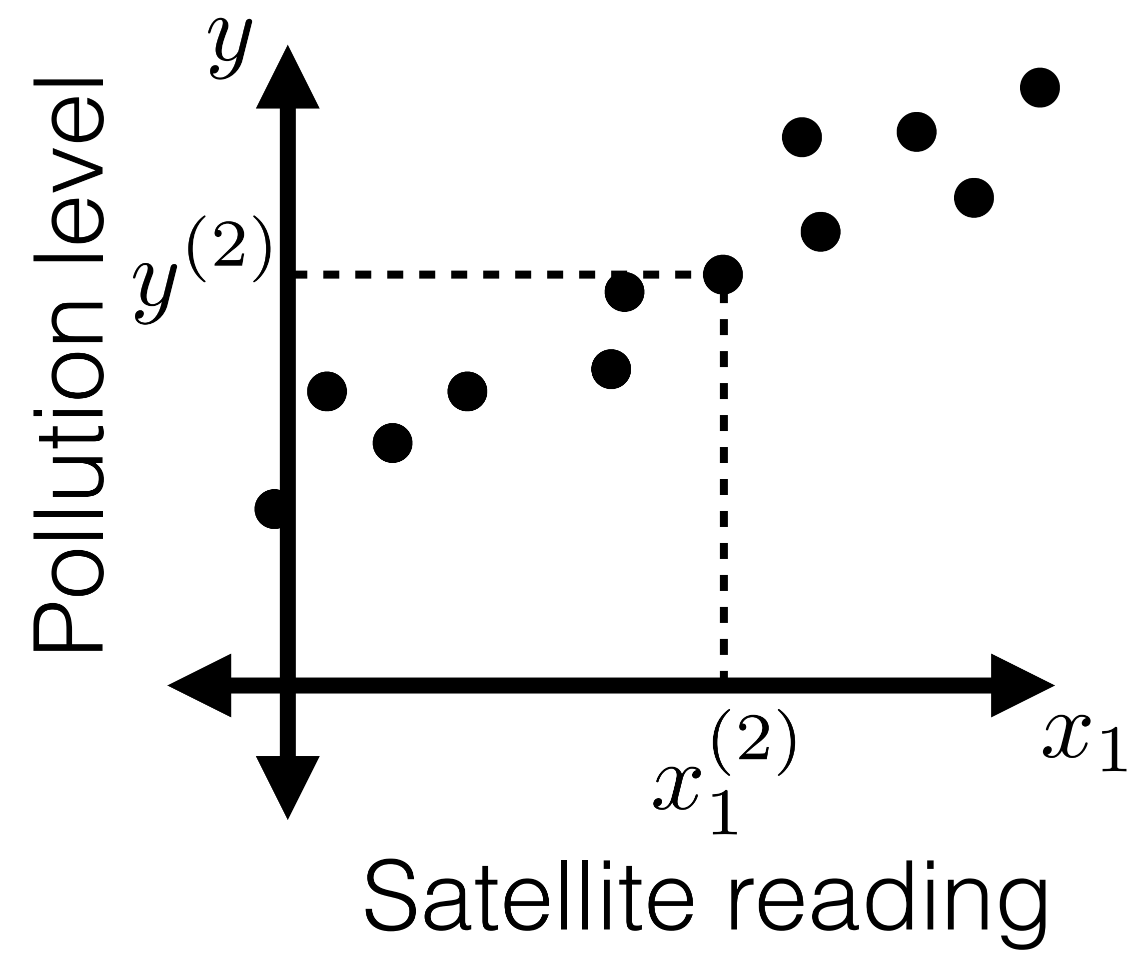
- Example h: For any x, $h(x)$ = 1,000,000

Linear regressors



Linear regressors

- Hypothesis class $\mathcal{H}$: set of h



Linear regressors

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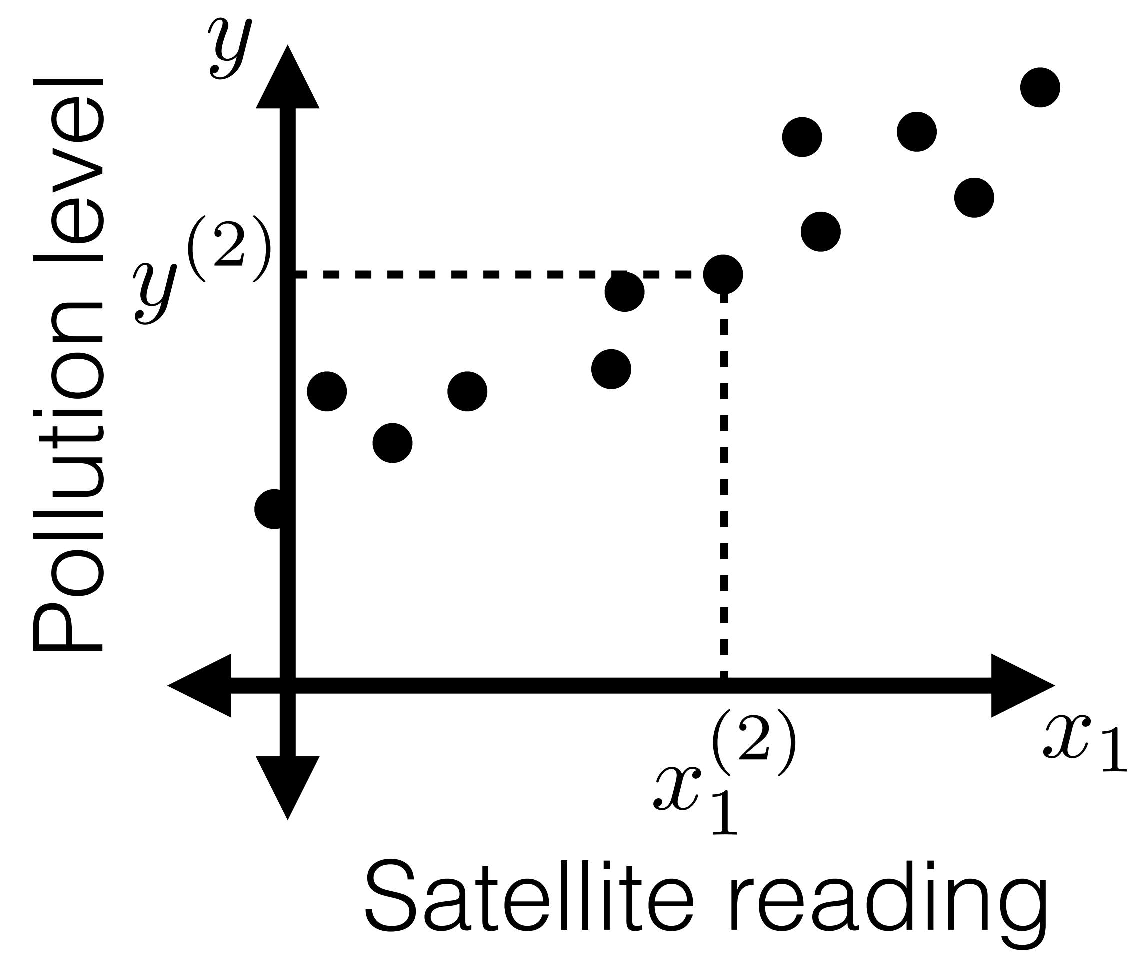

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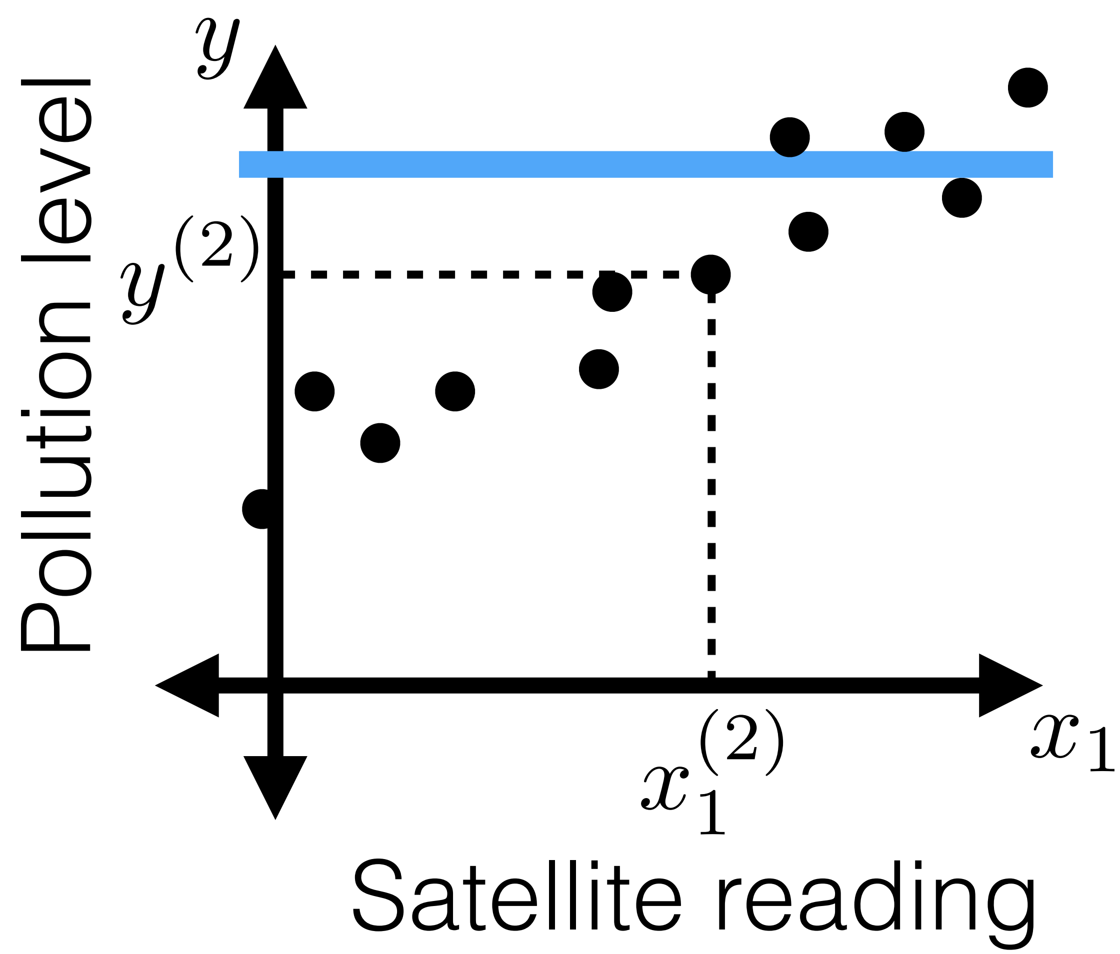

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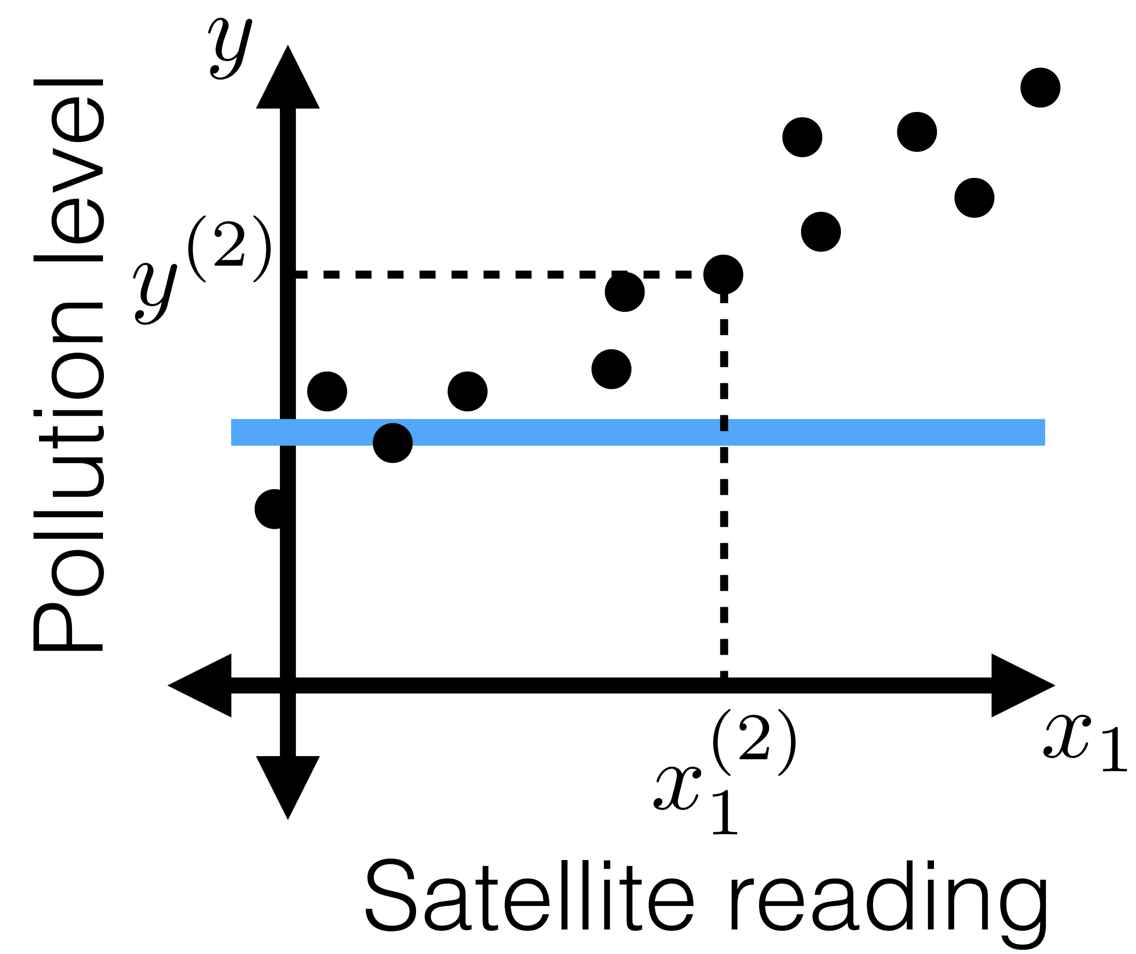

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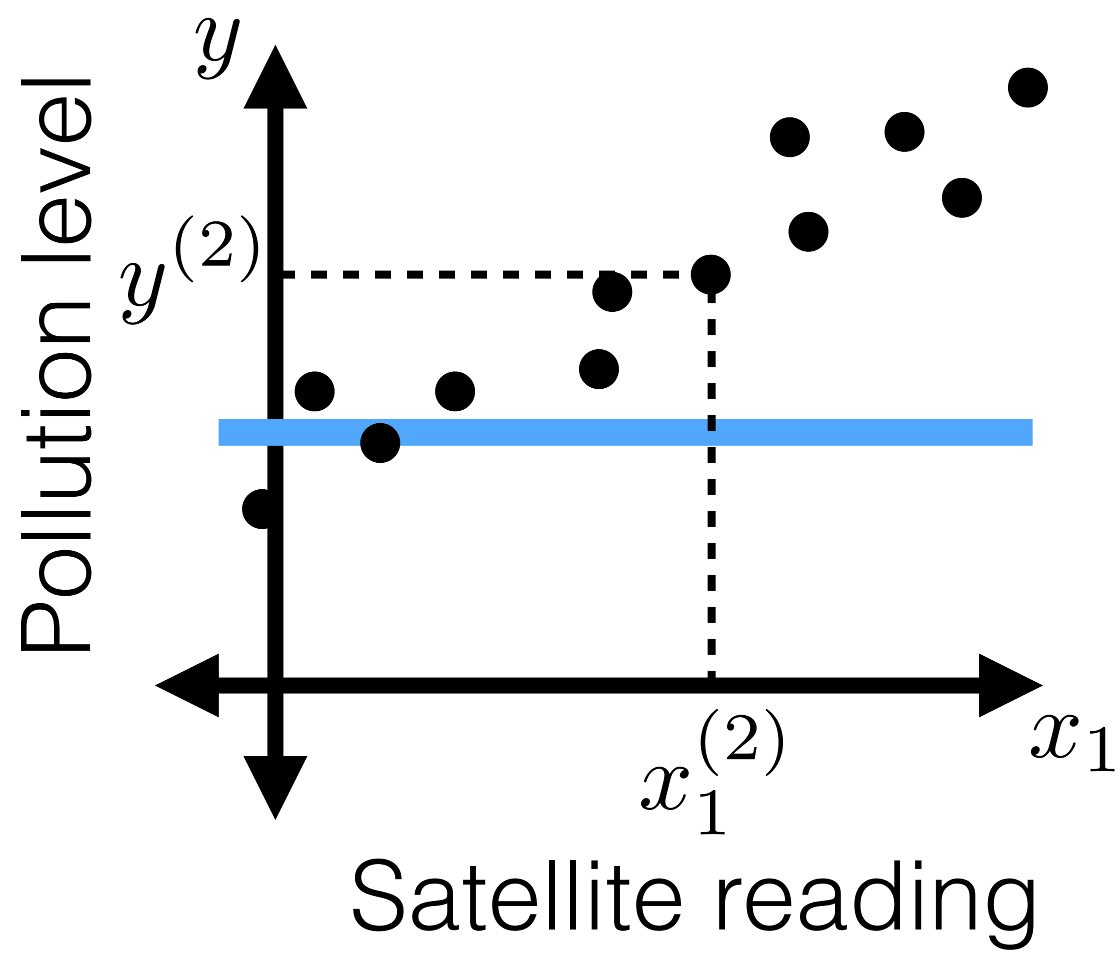

Linear regressors

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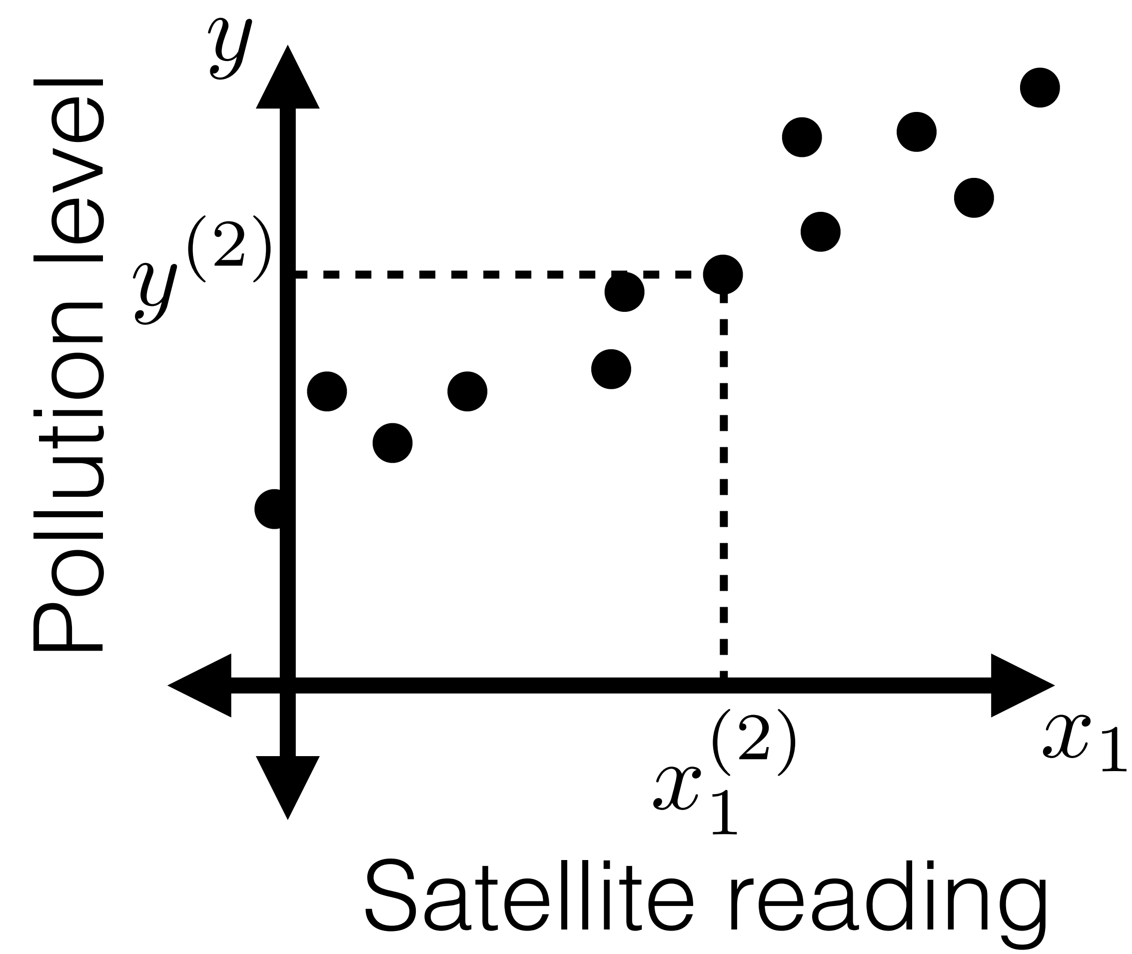
$$h(x) = \theta x + \theta_0$$



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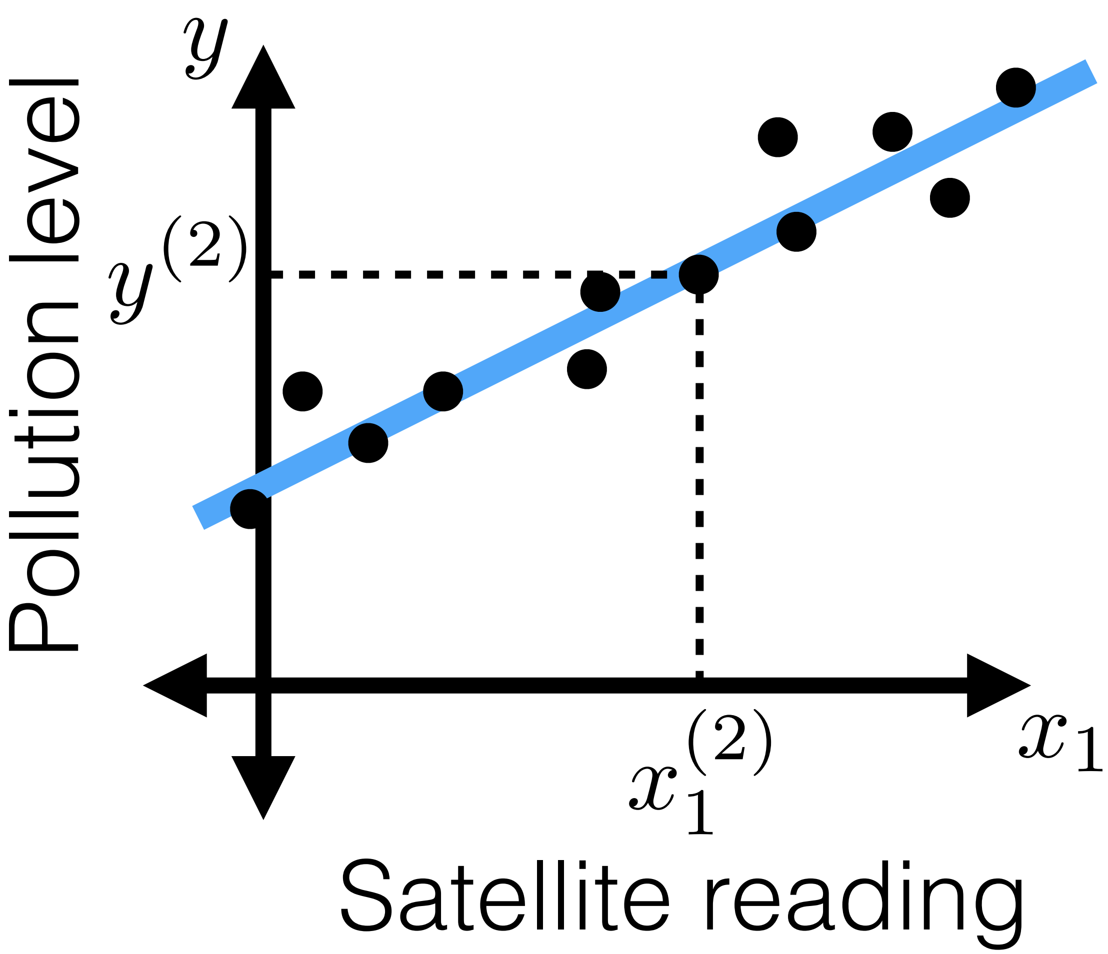

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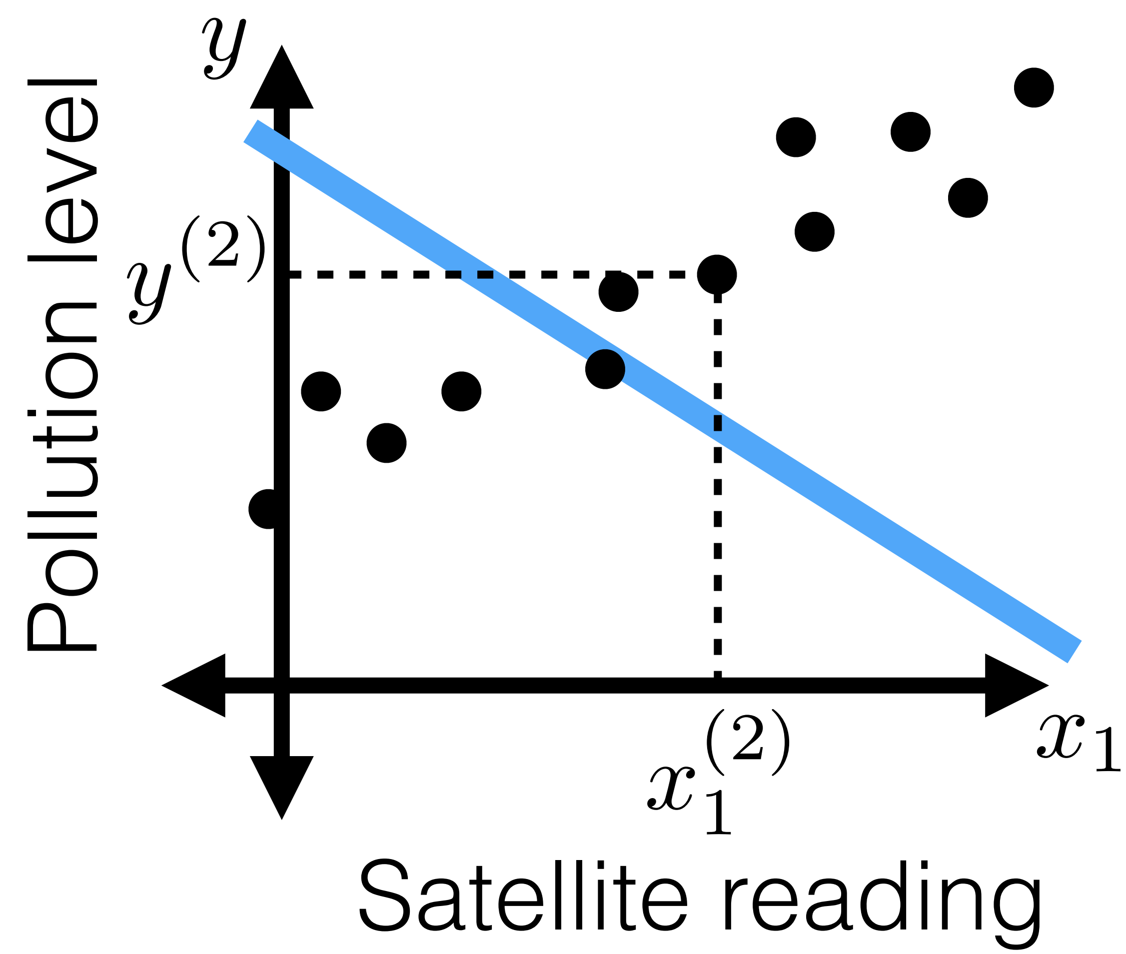

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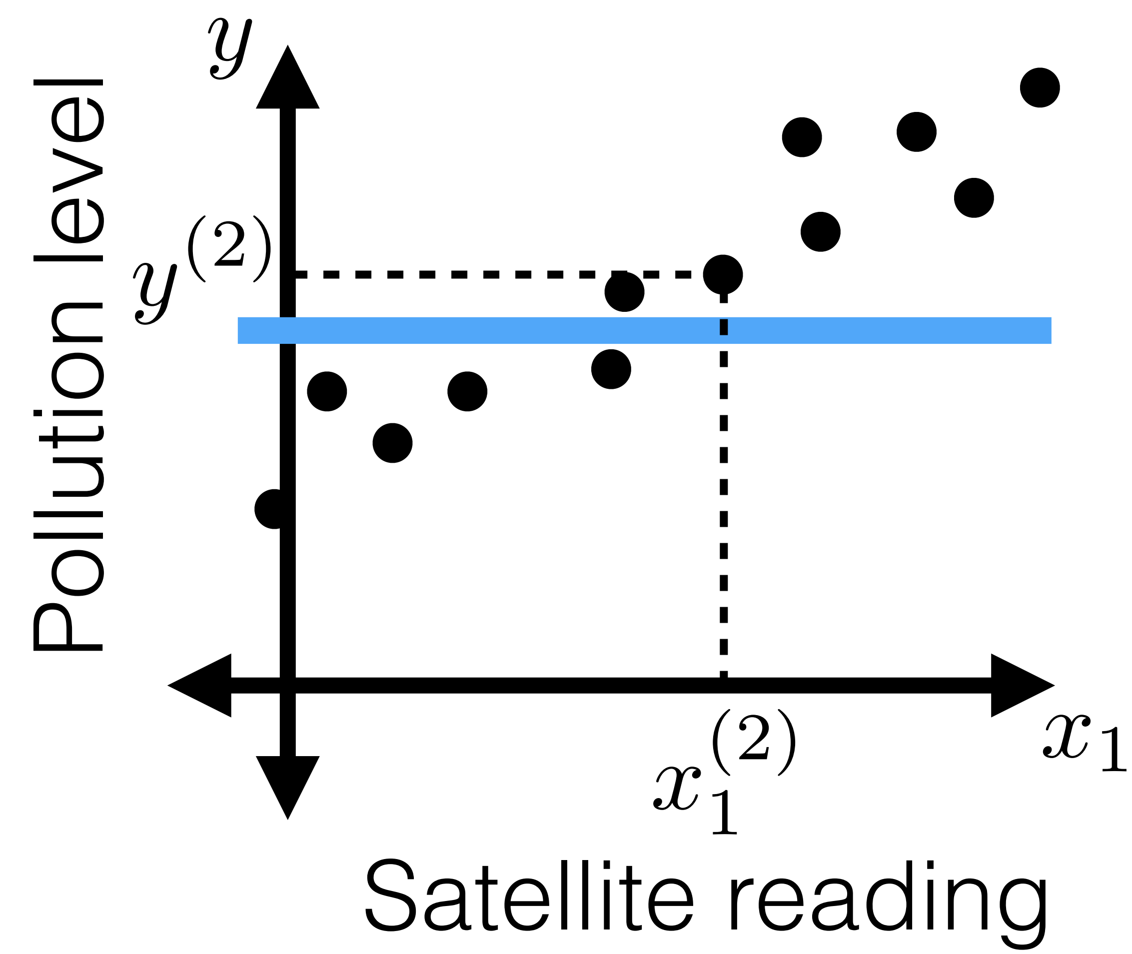

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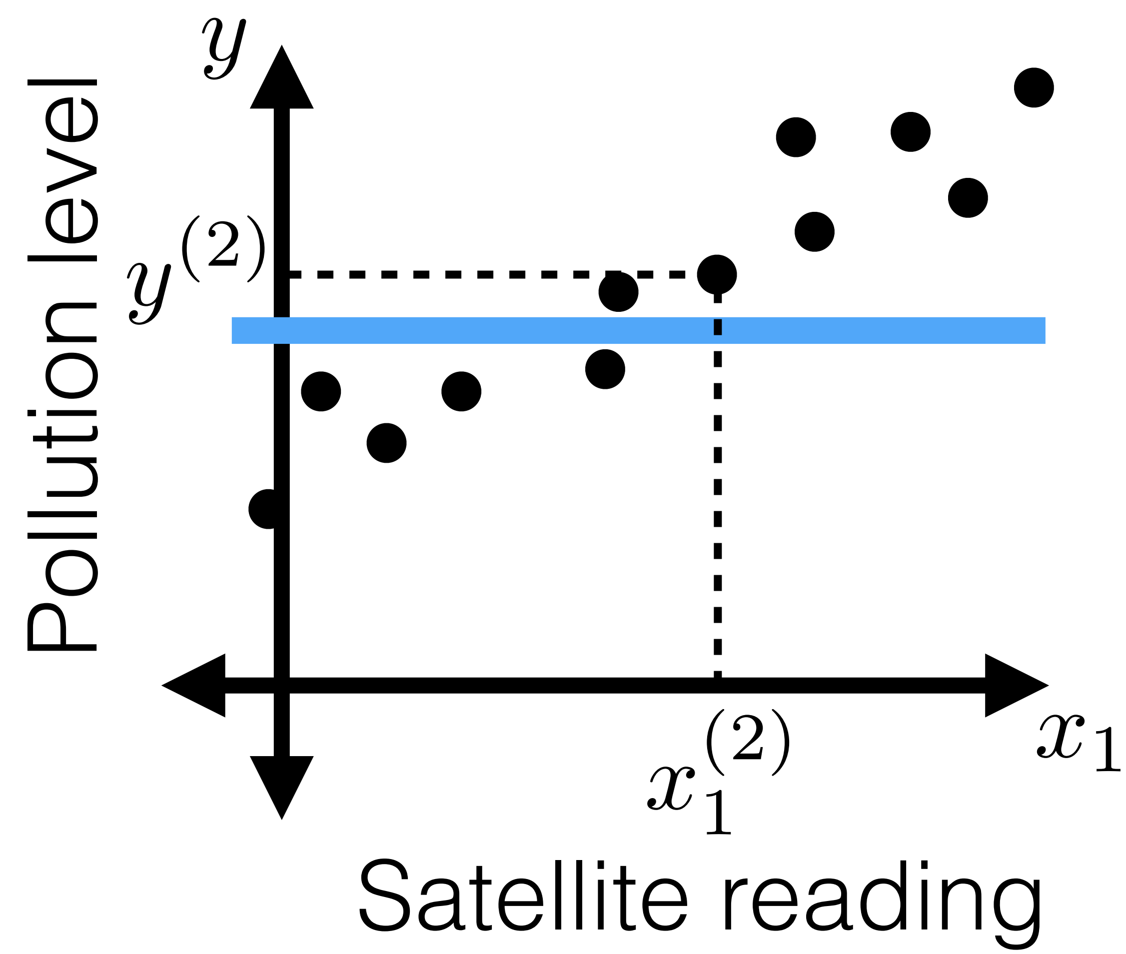
$$h(x; \theta, \theta_0) = \theta x + \theta_0$$



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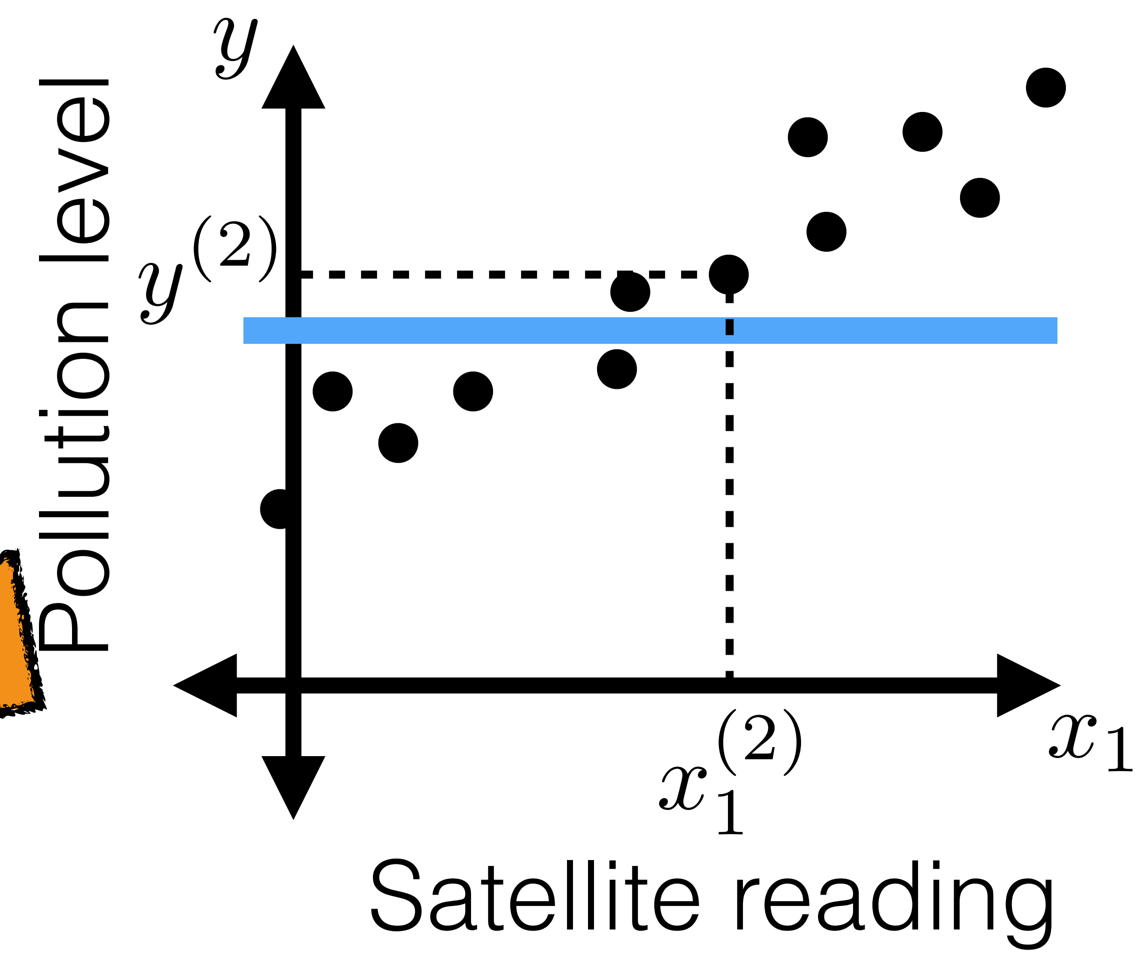

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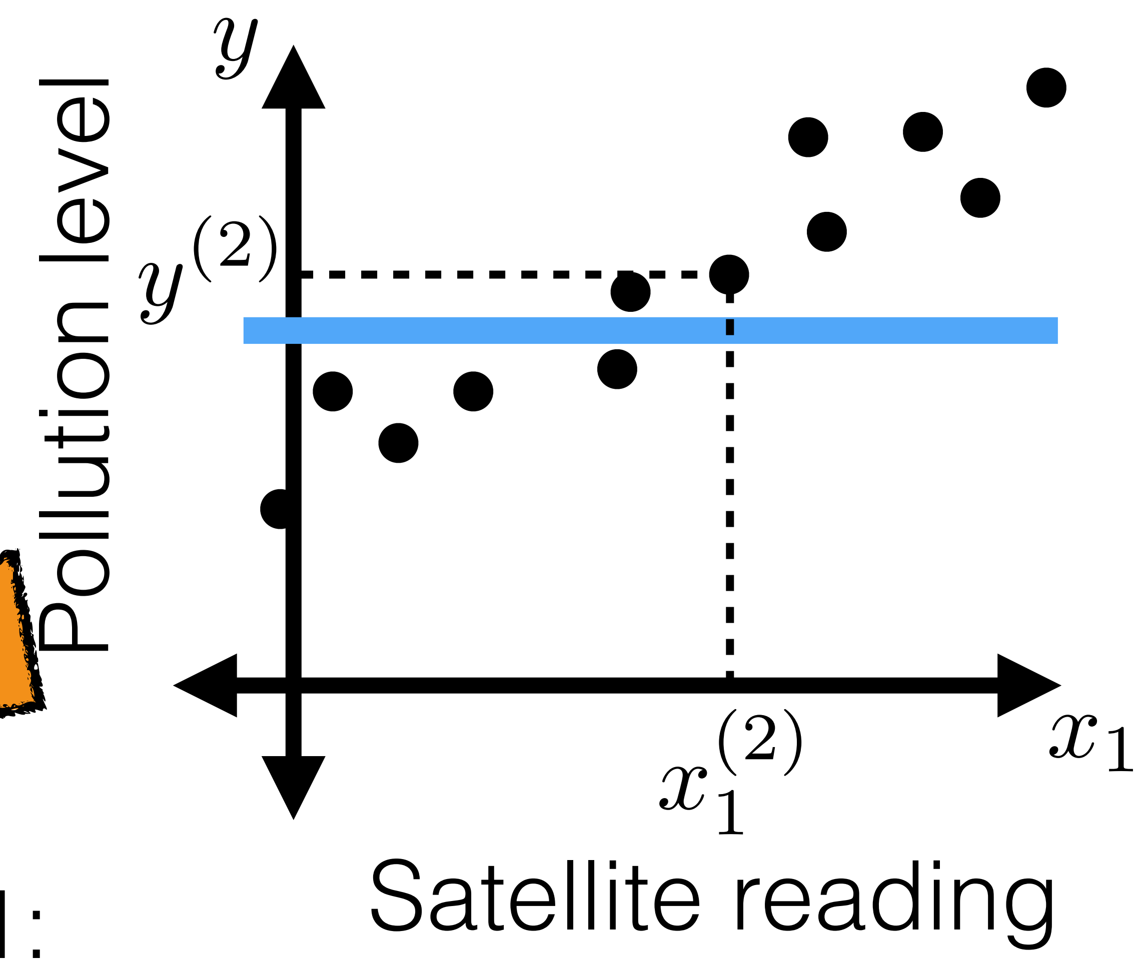

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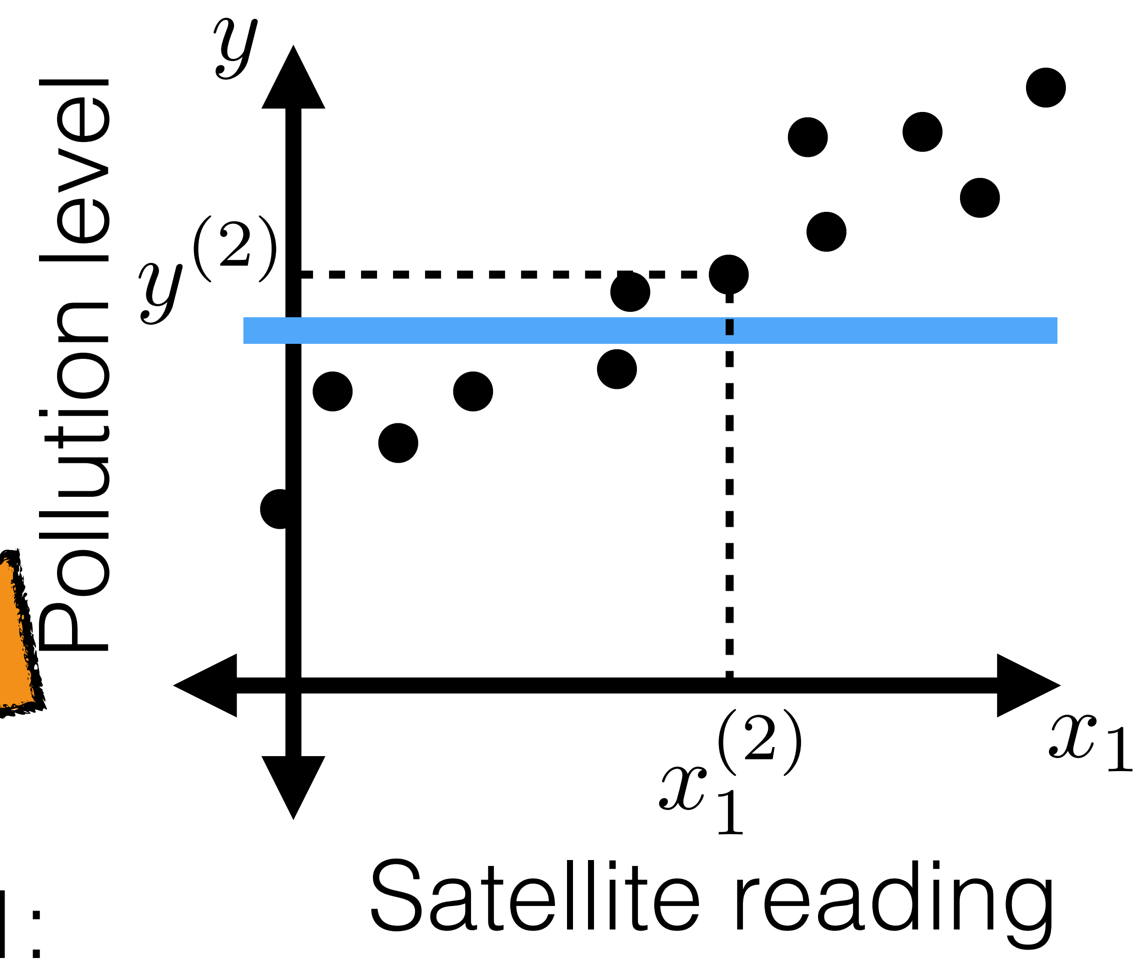

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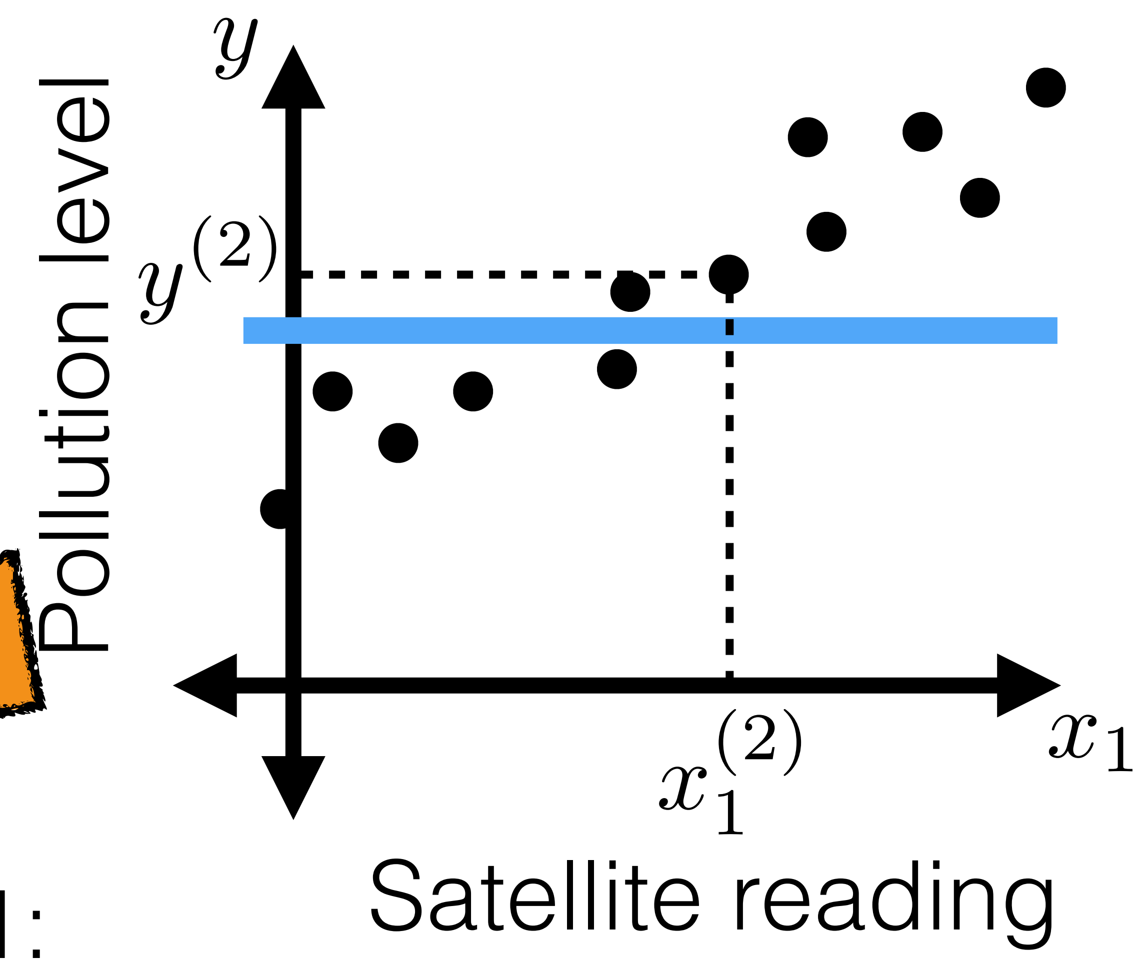

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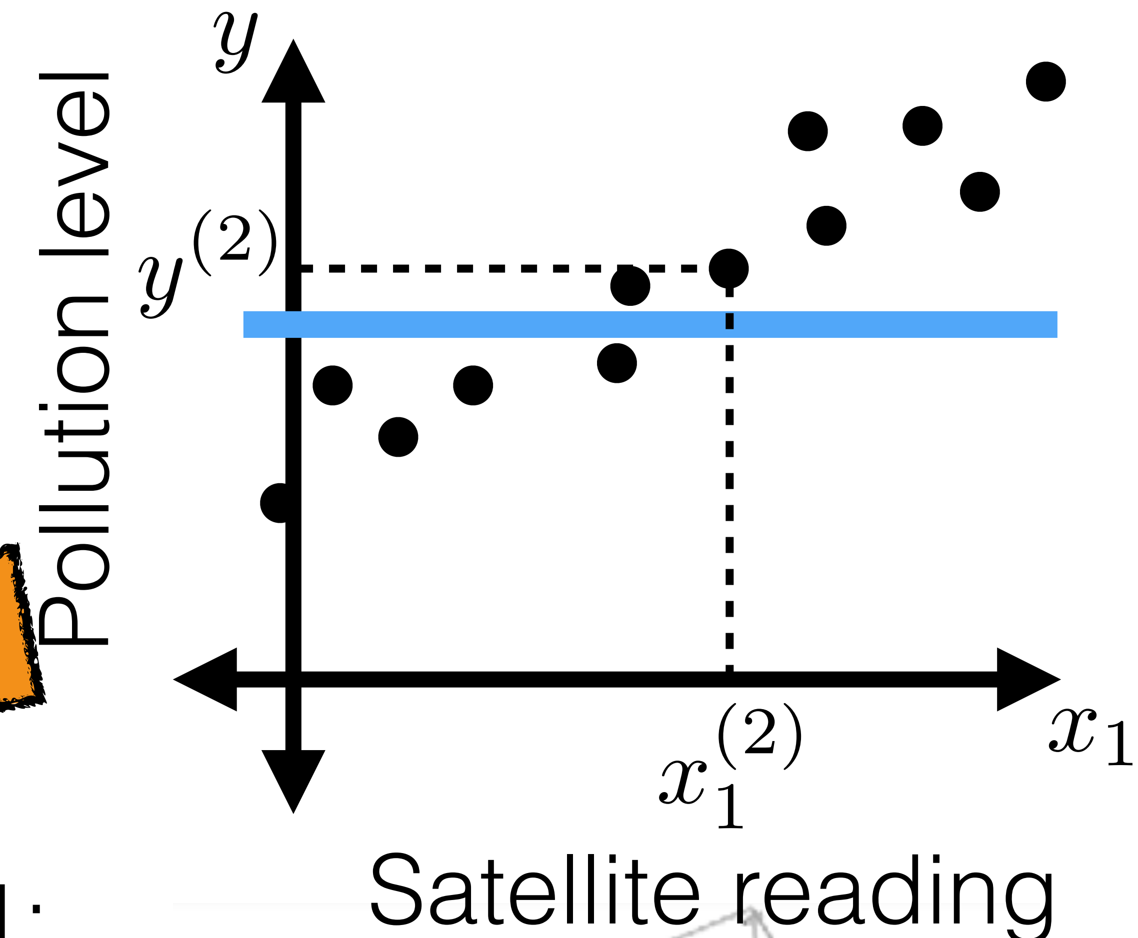

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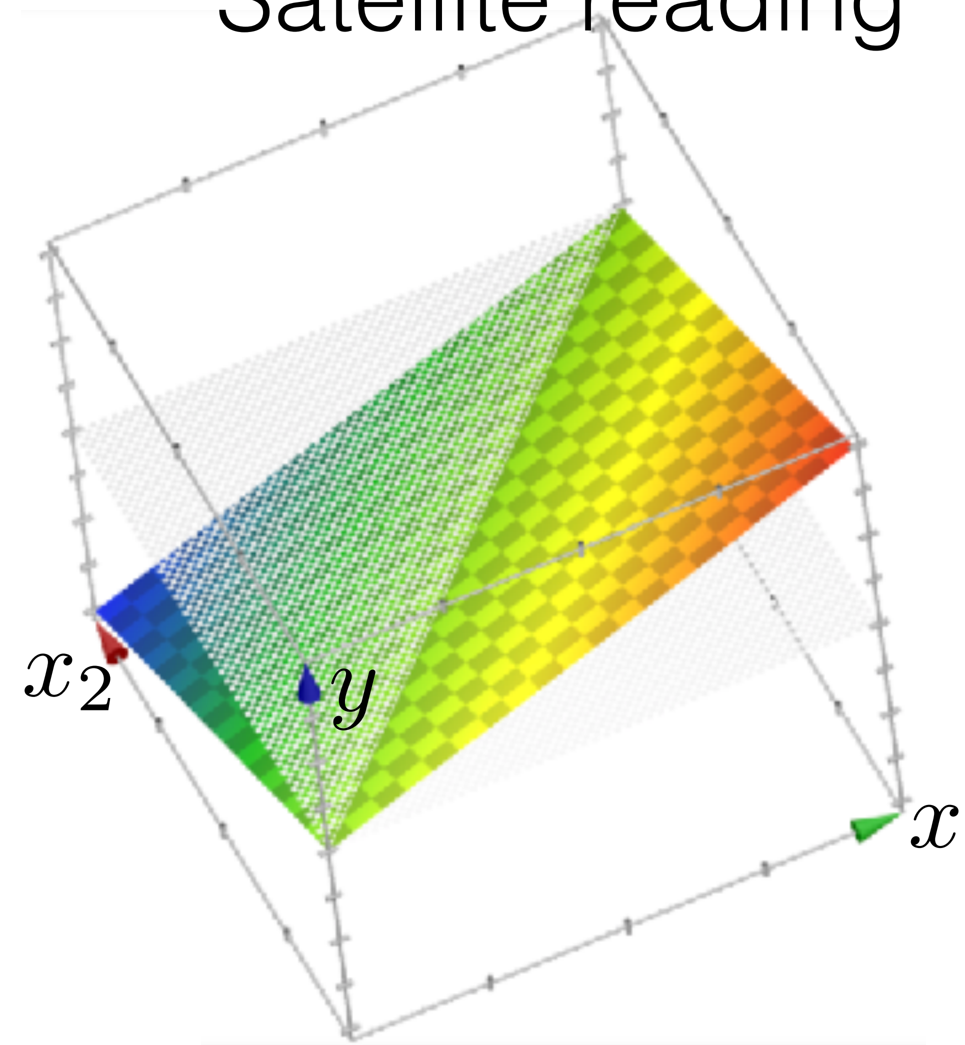

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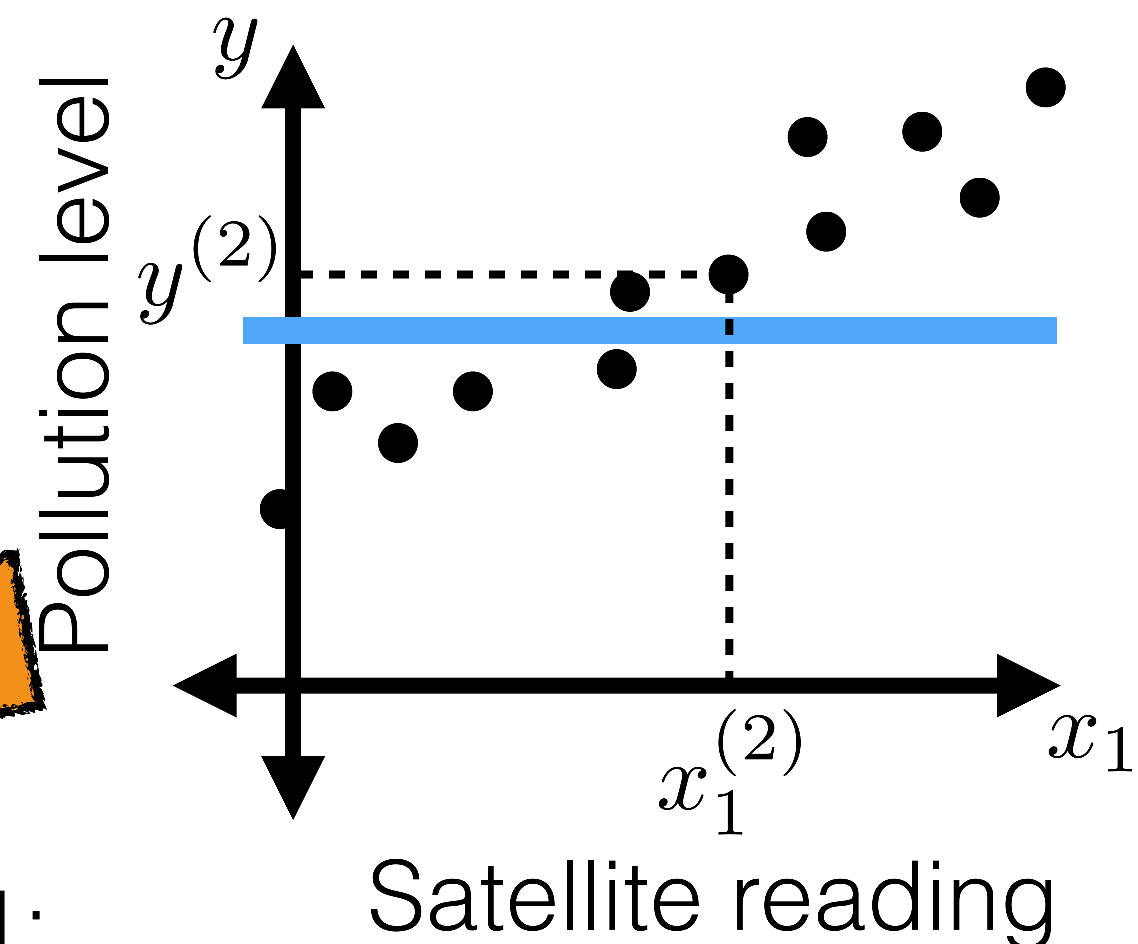

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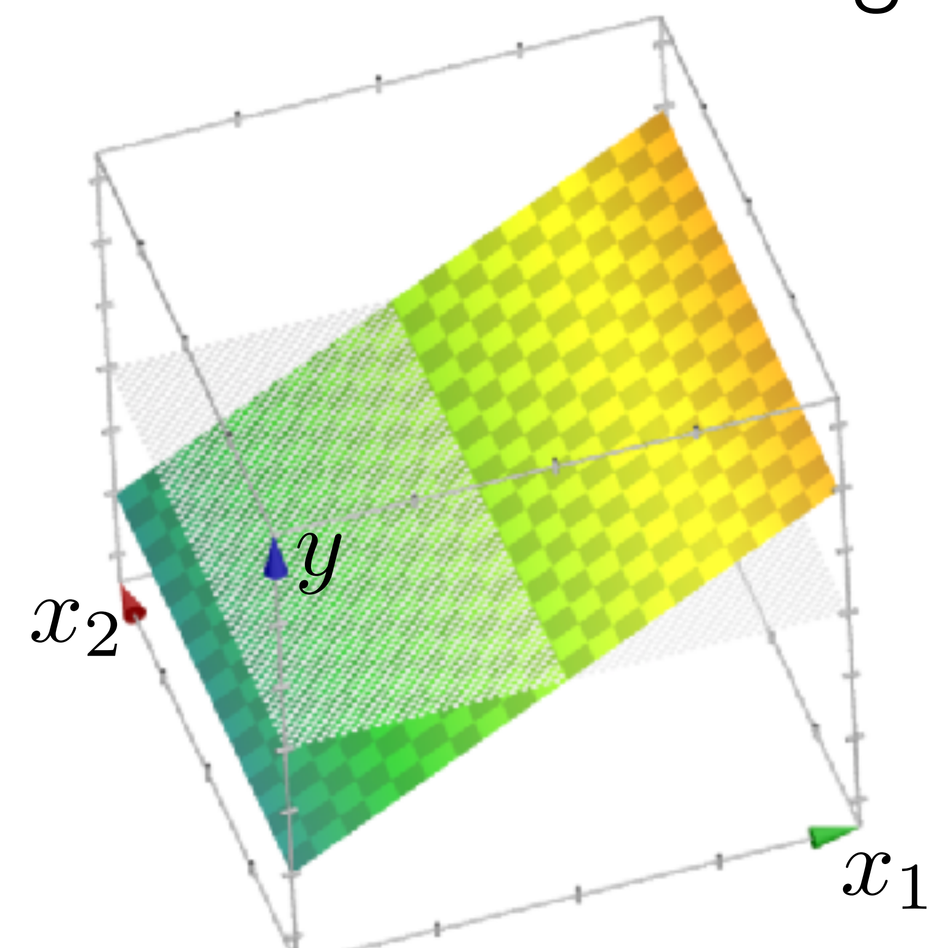

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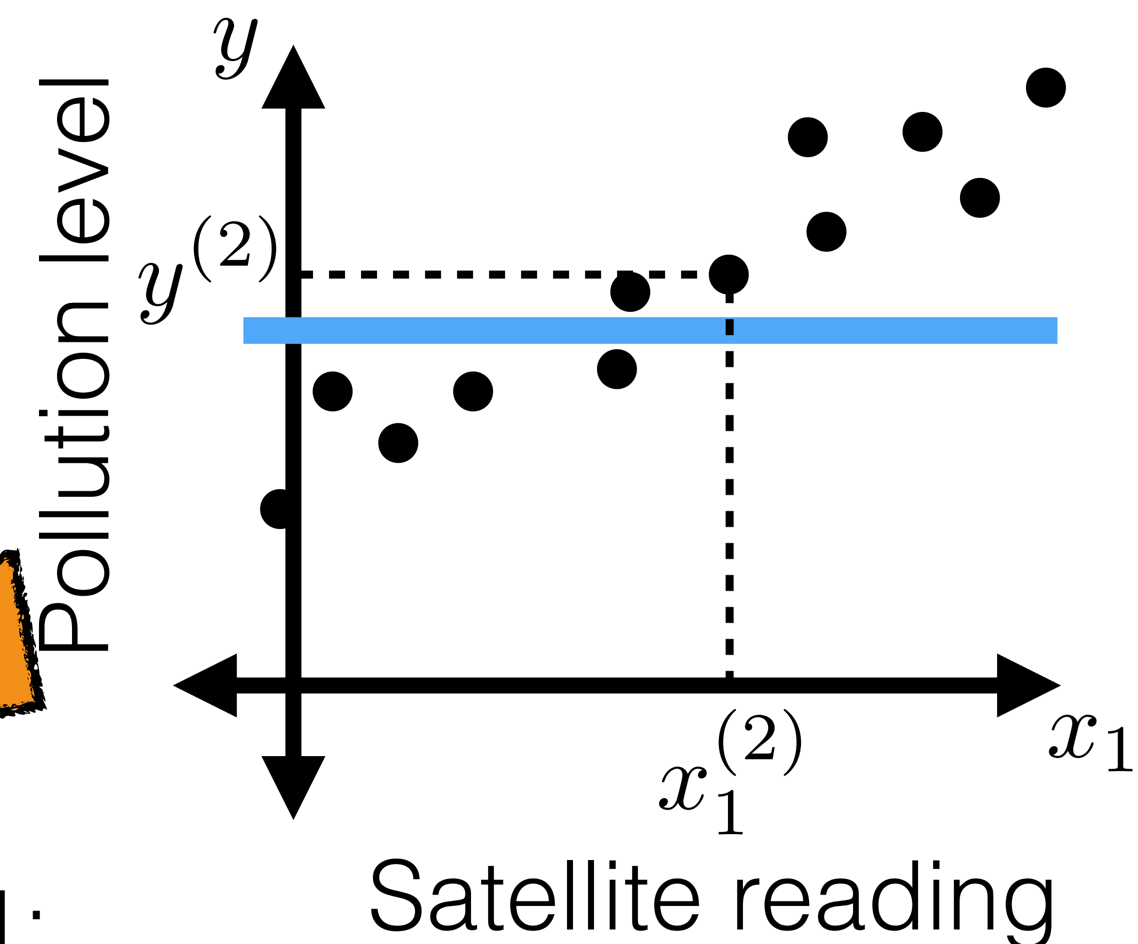

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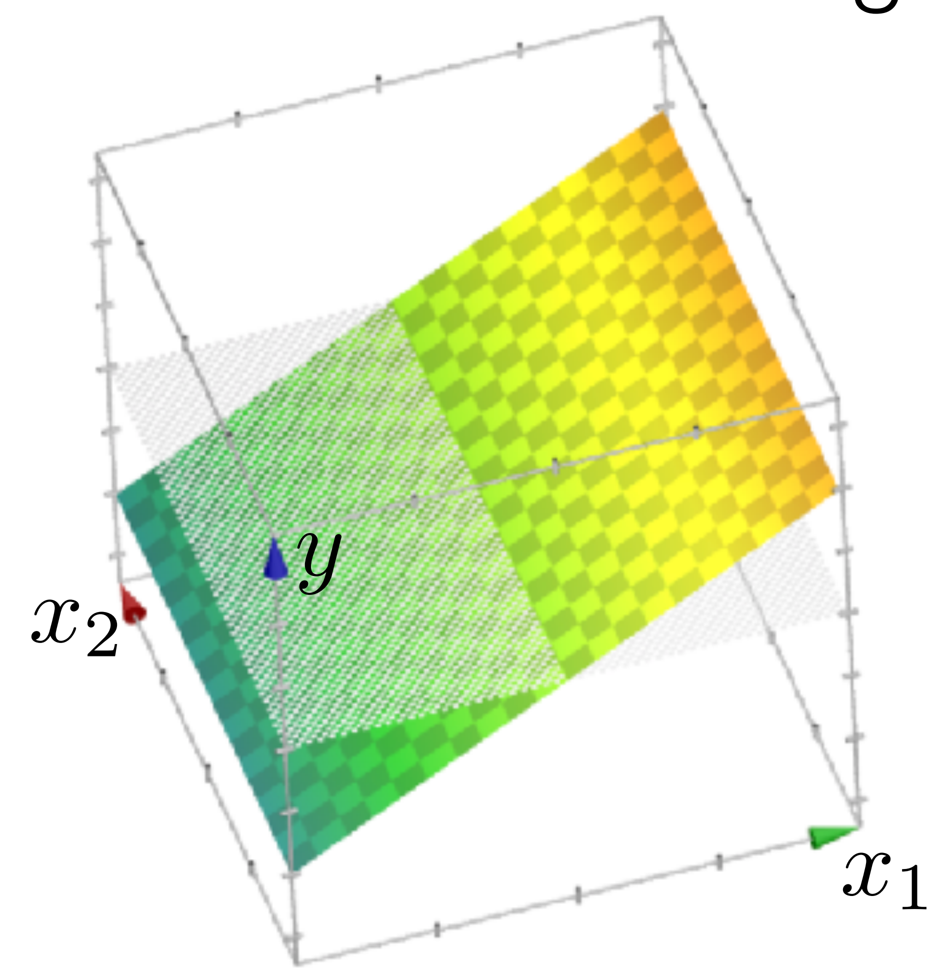

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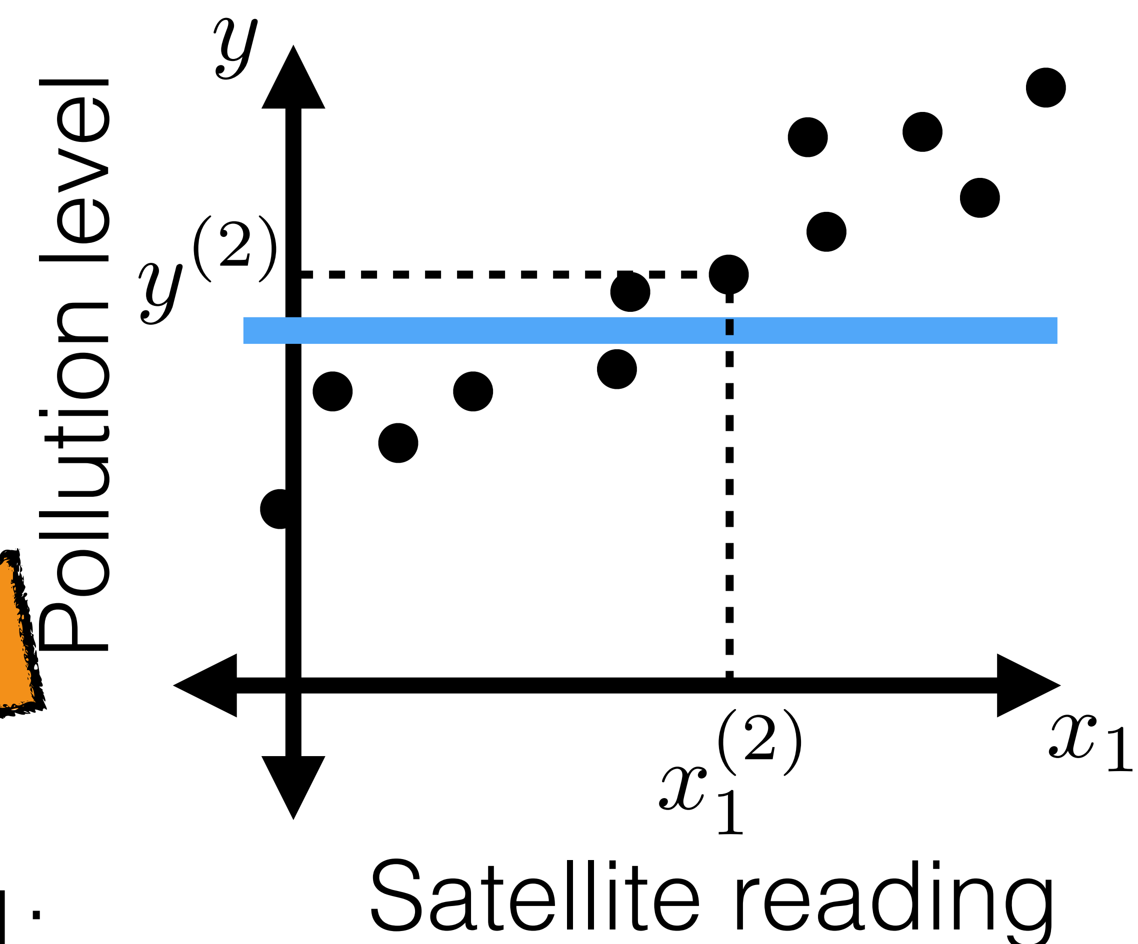

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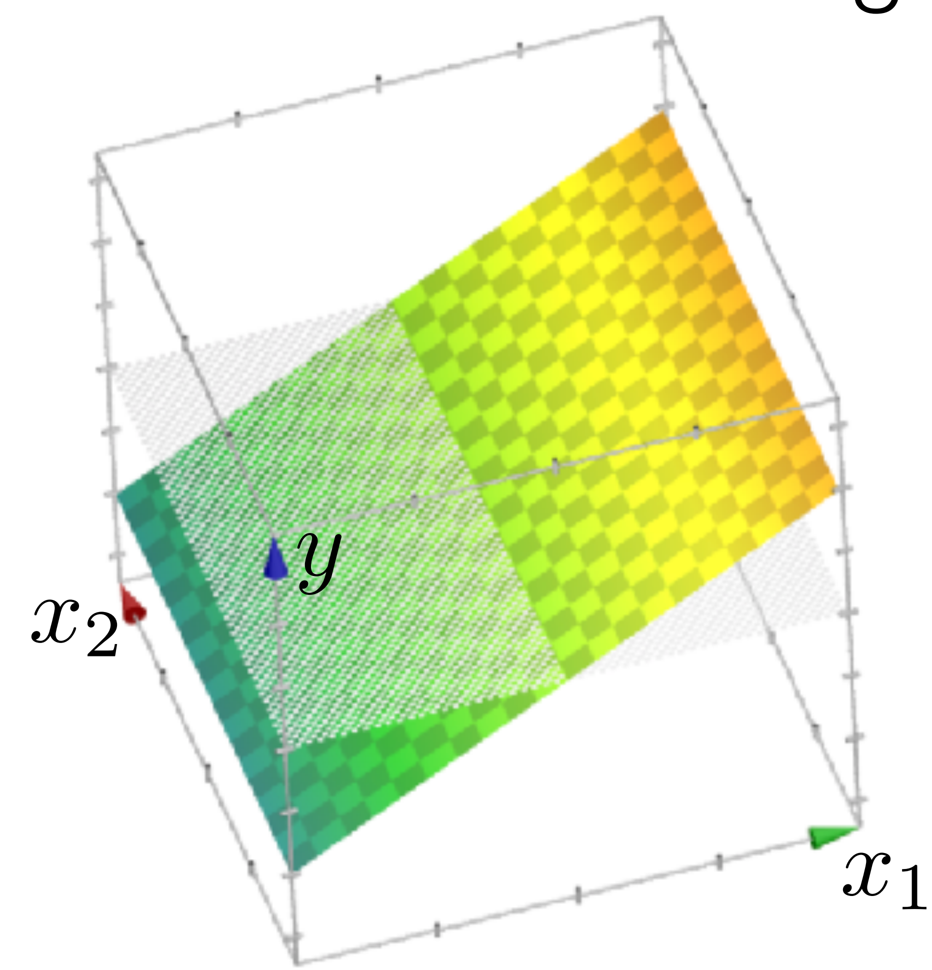

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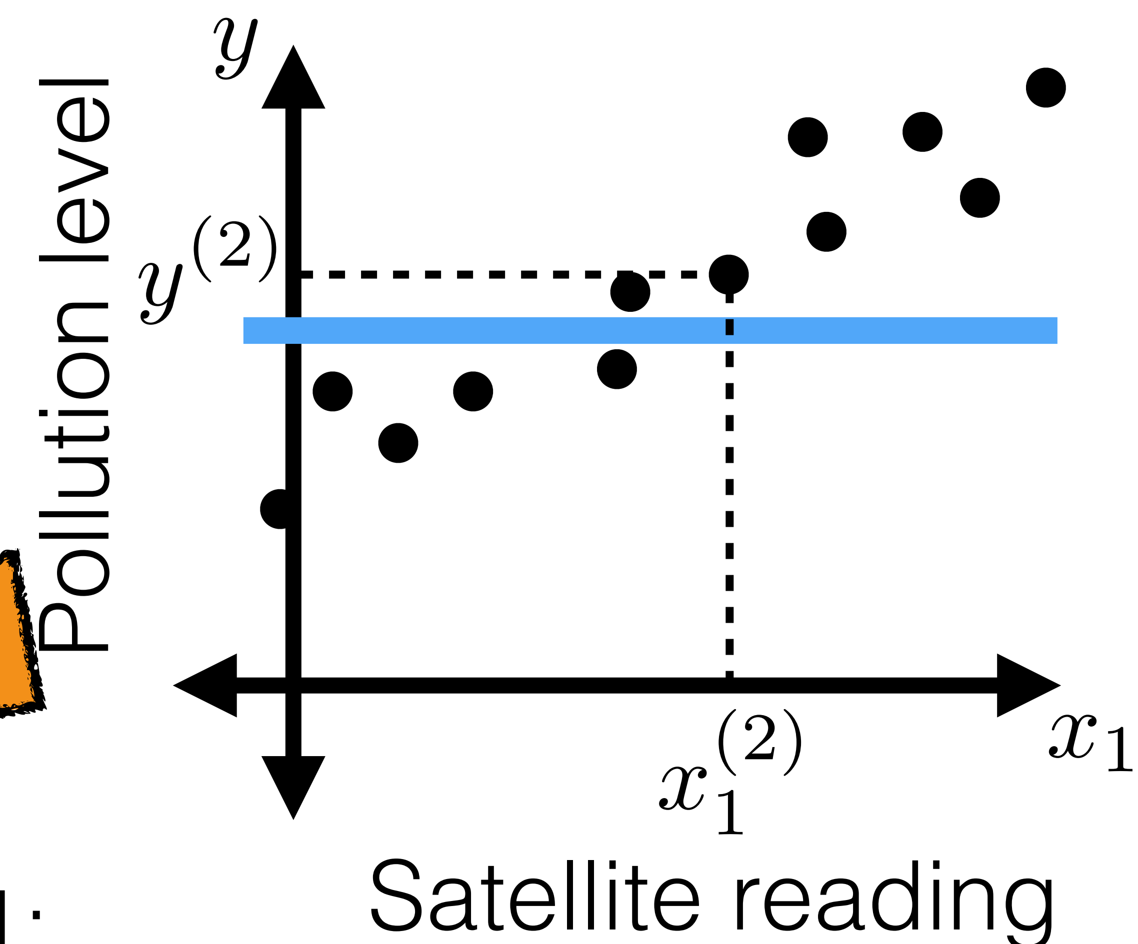

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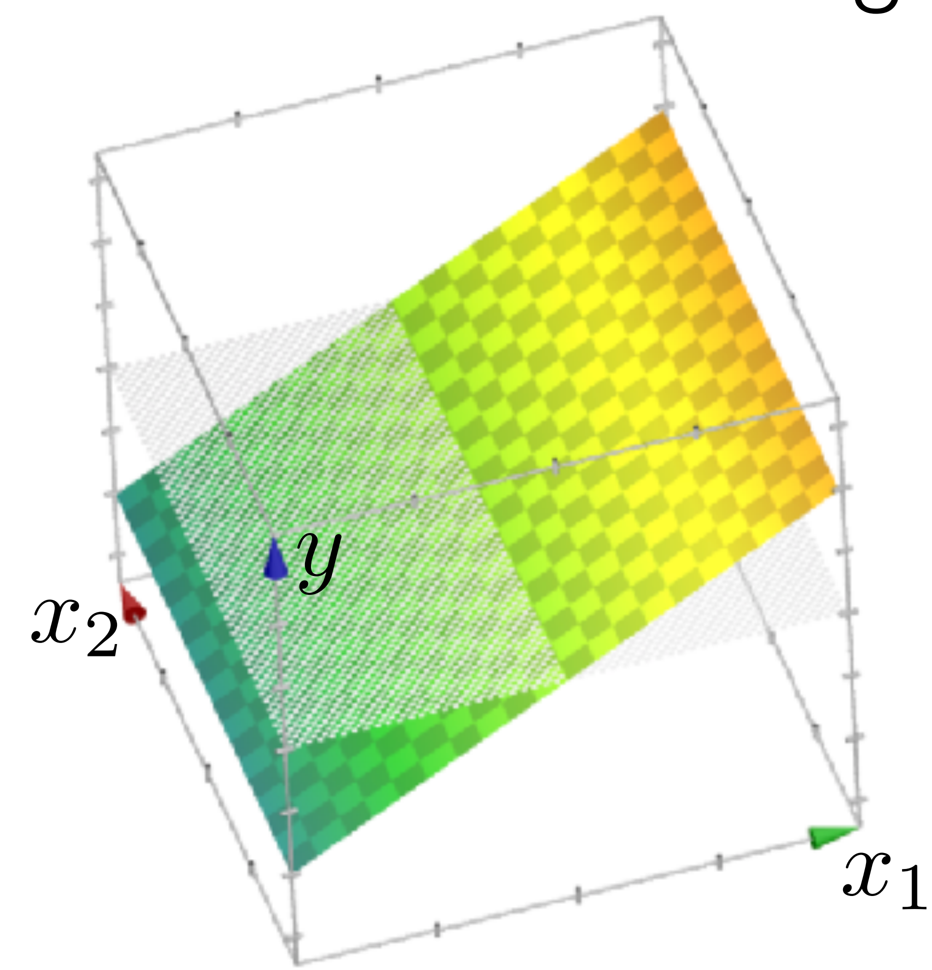

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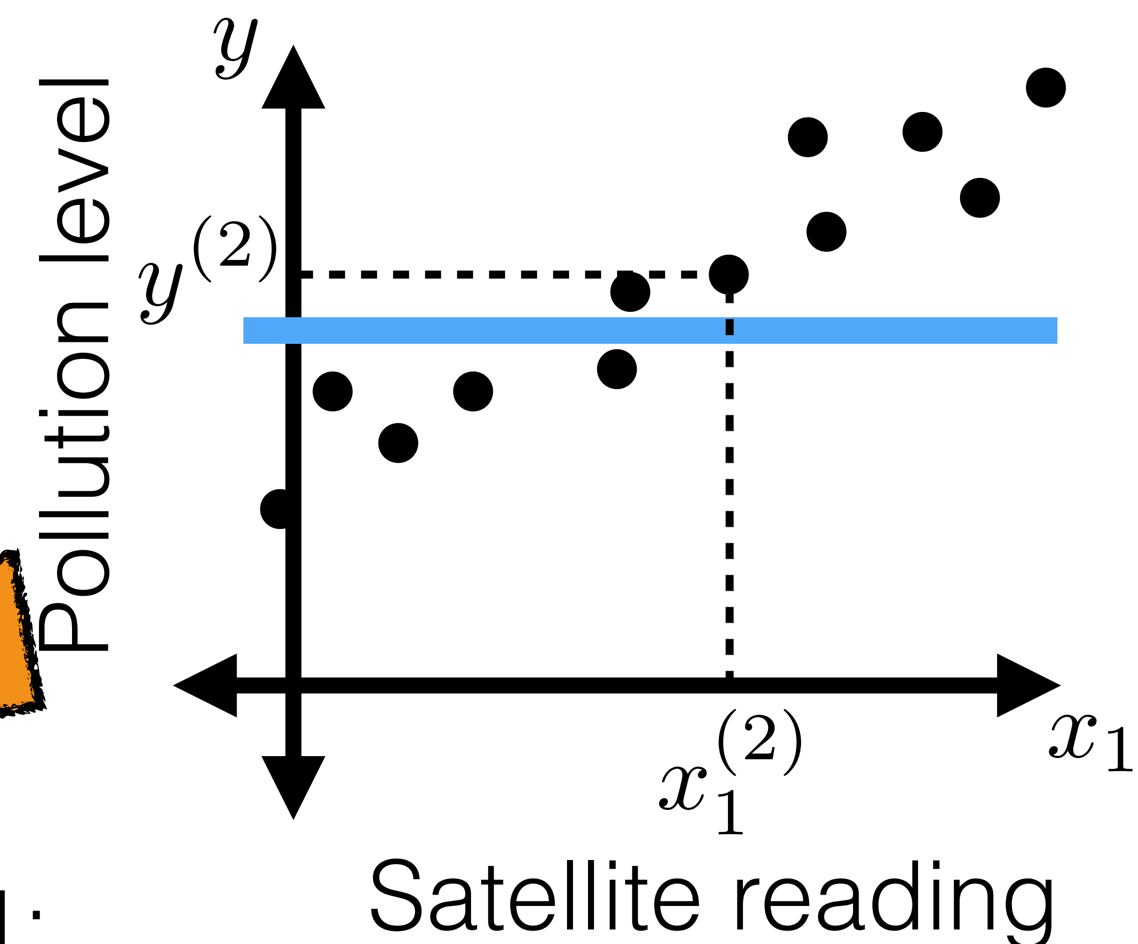

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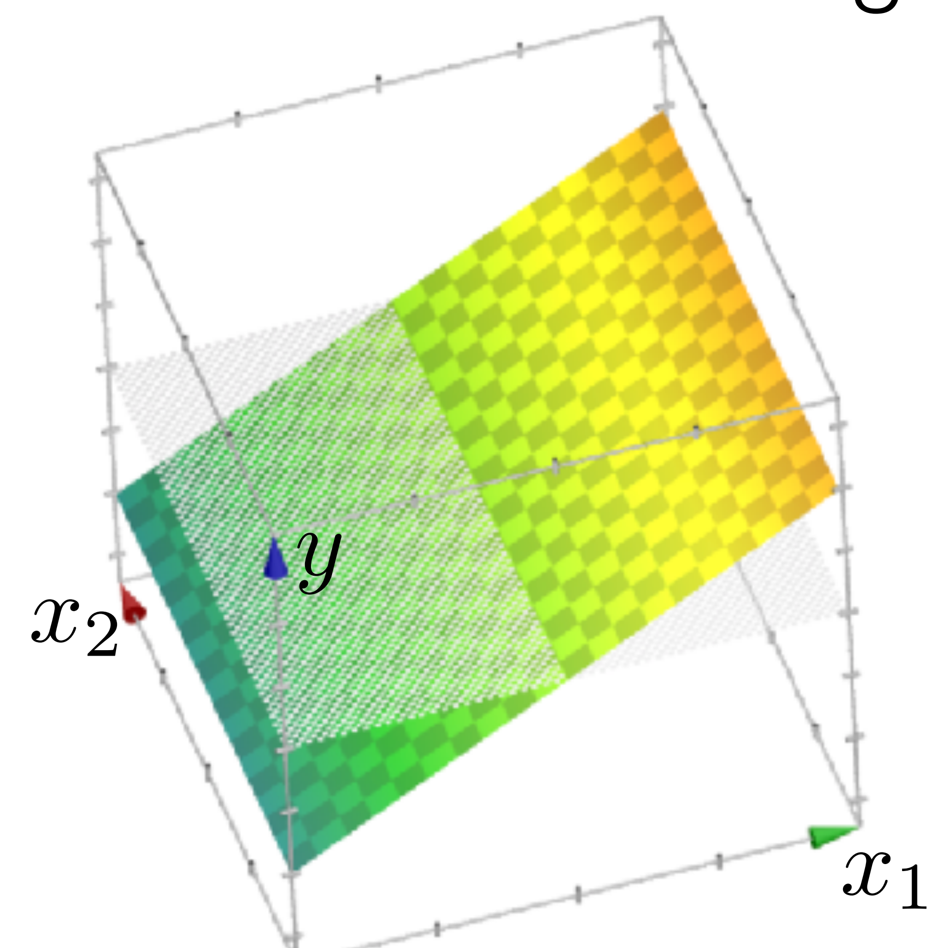

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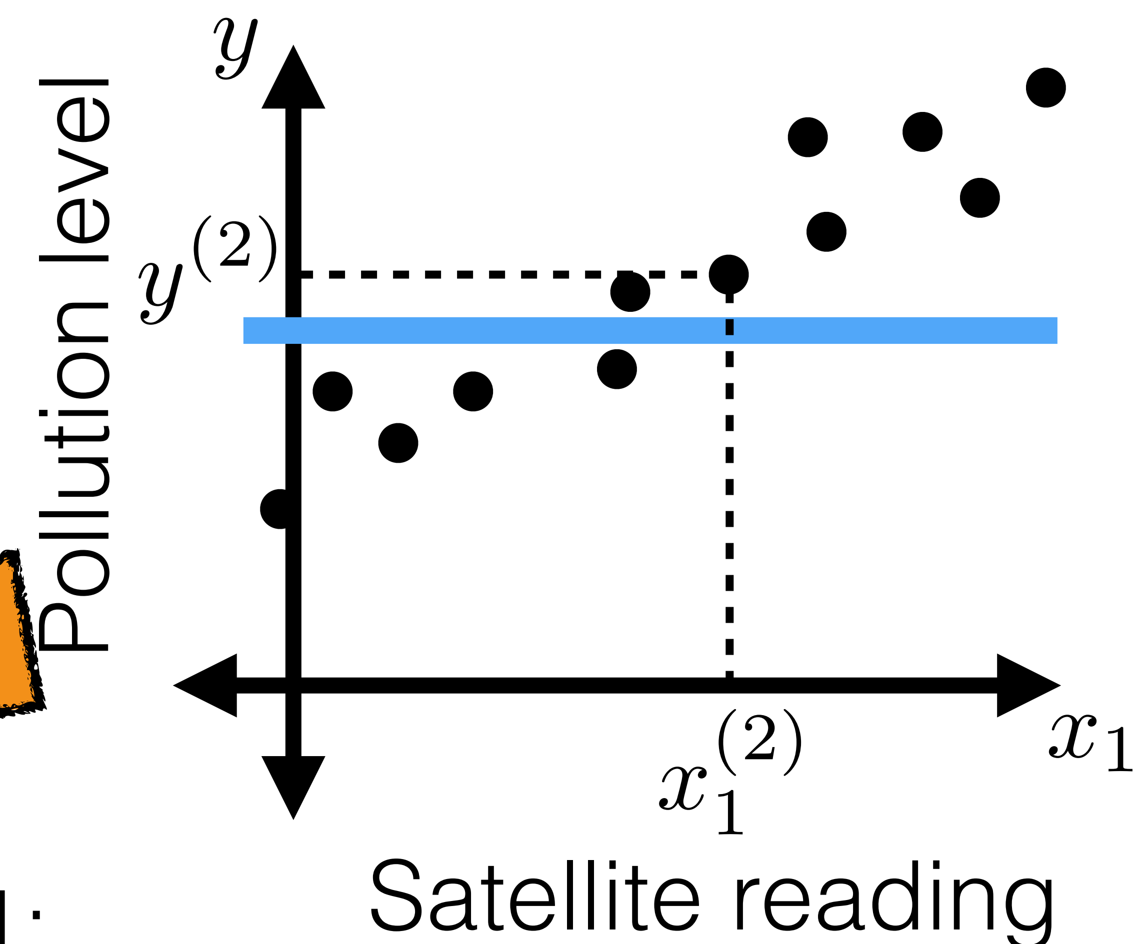

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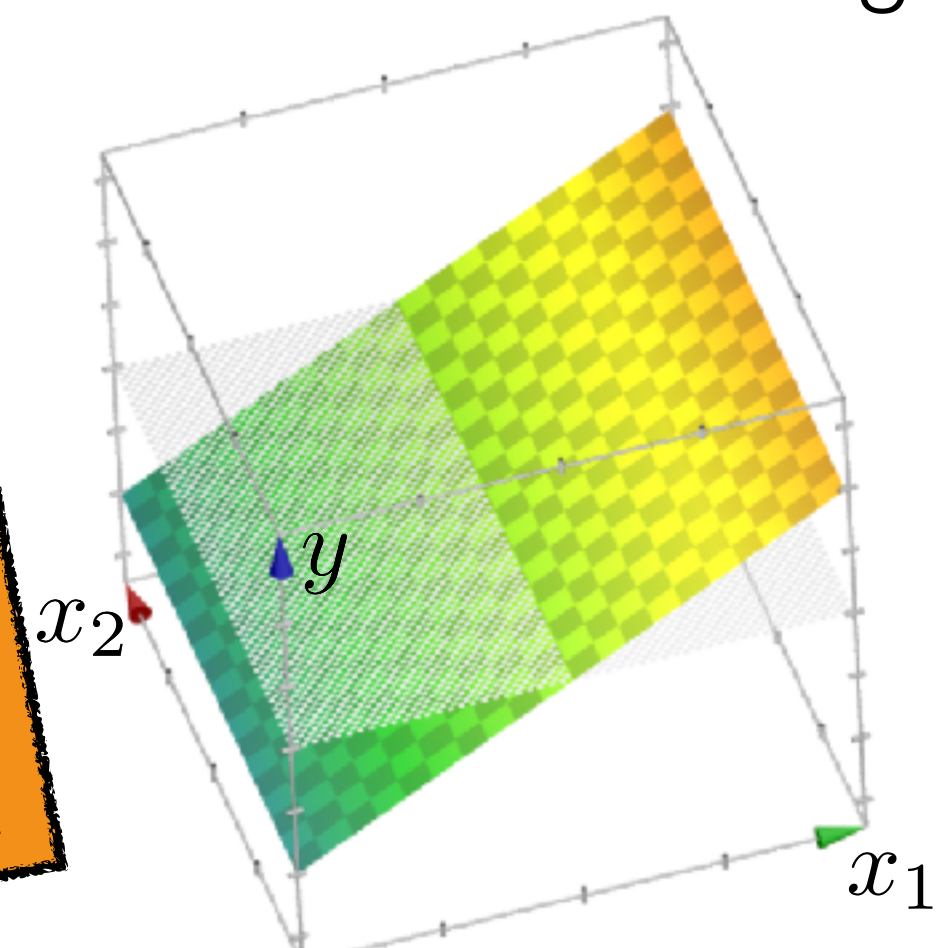

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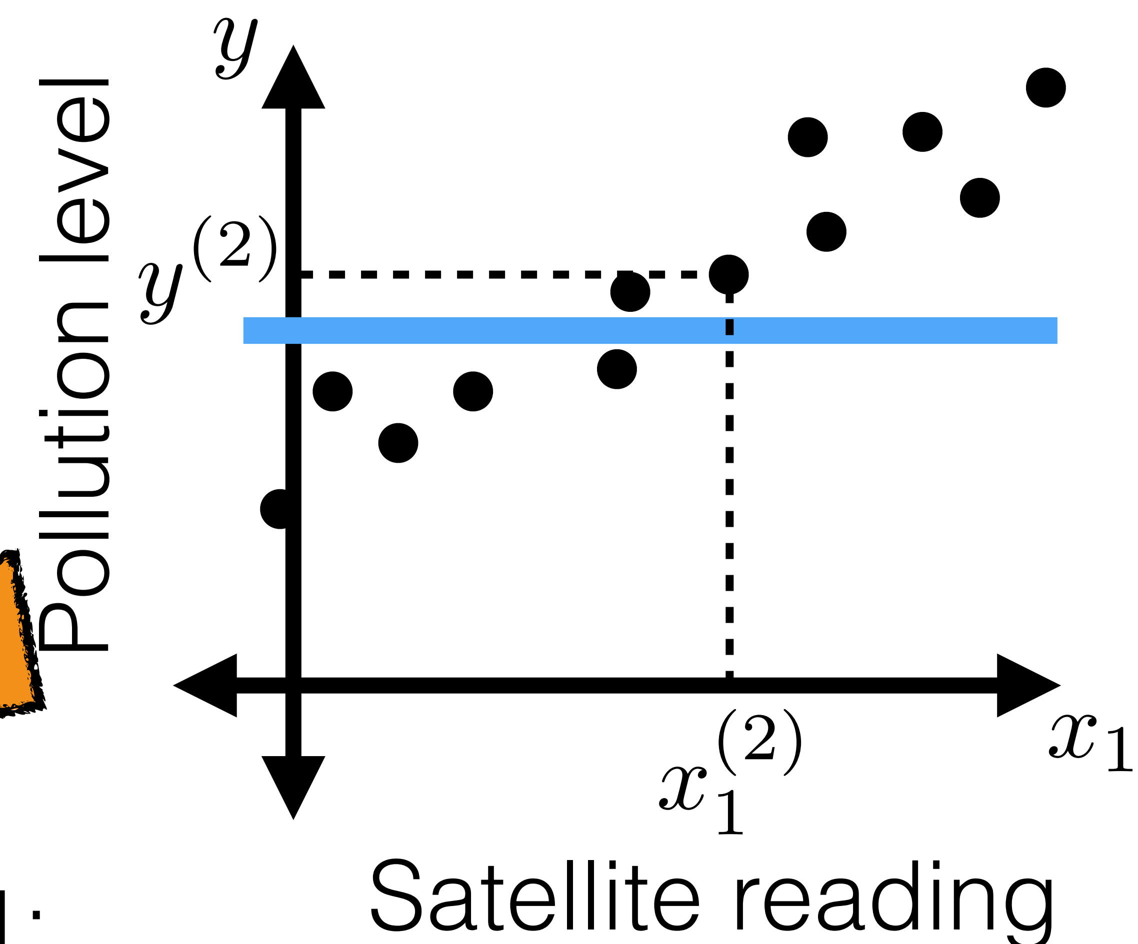

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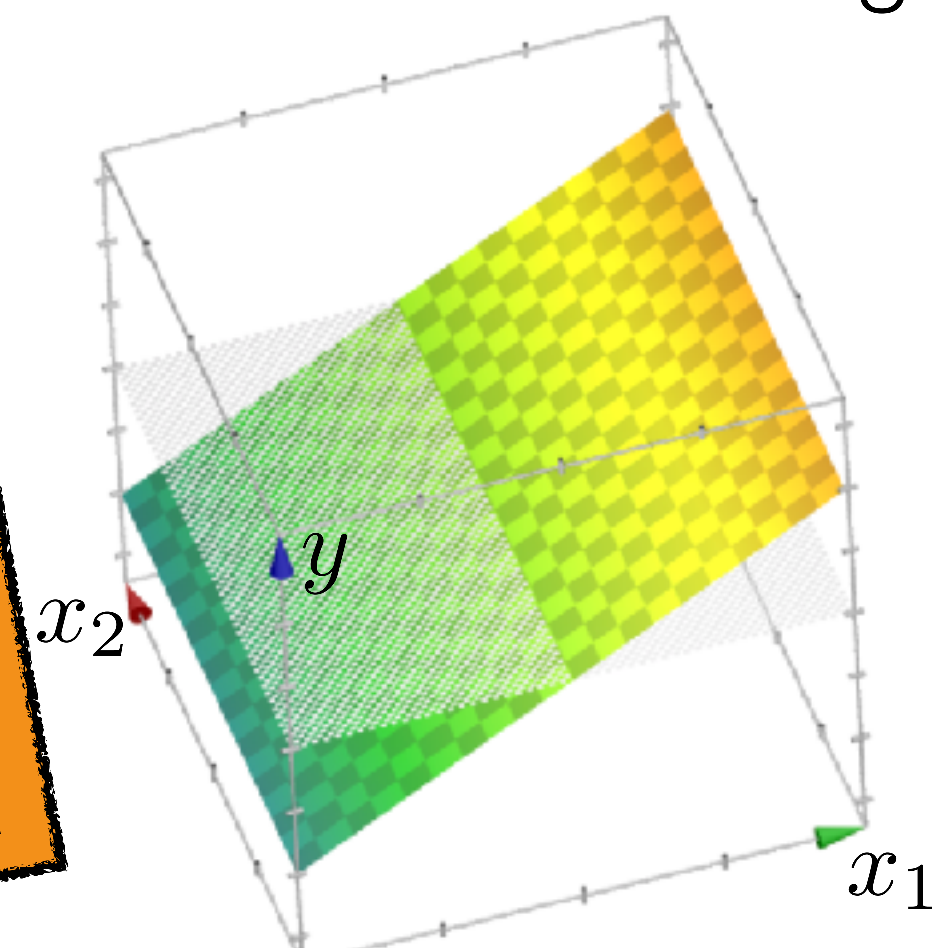

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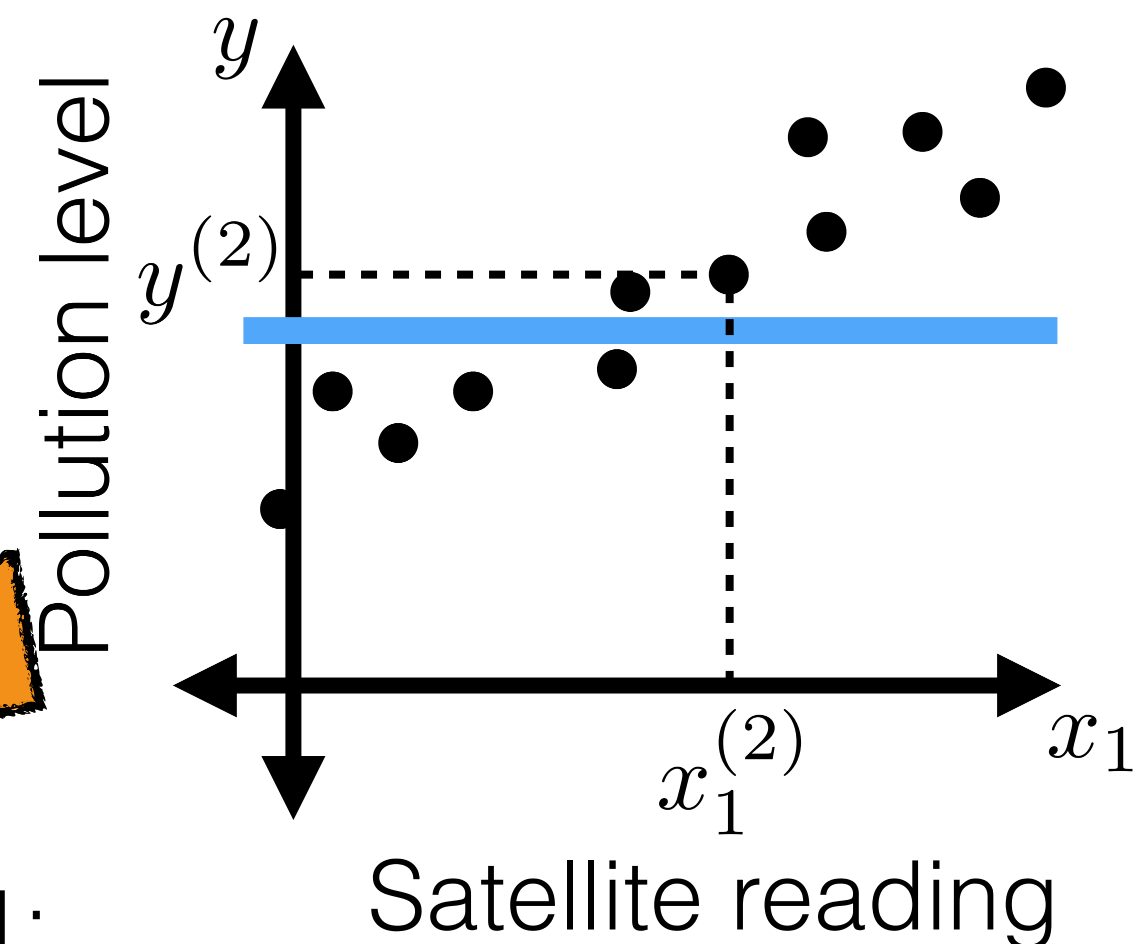

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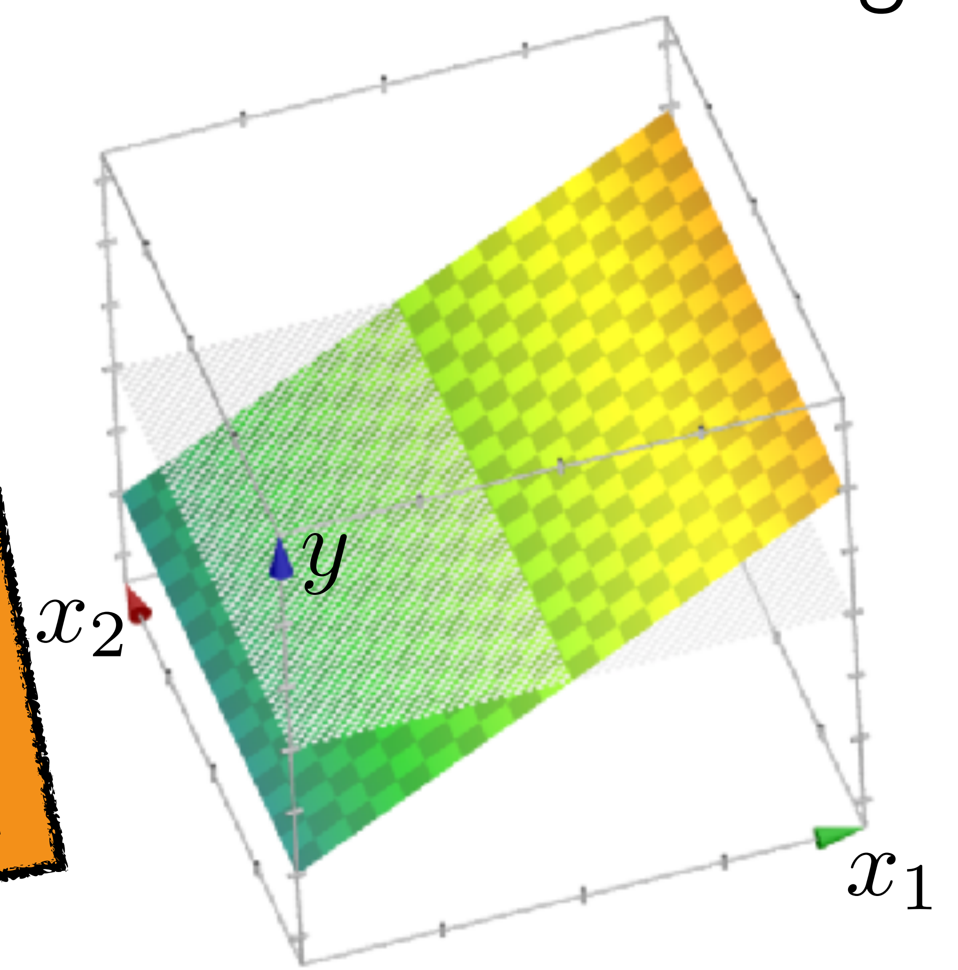

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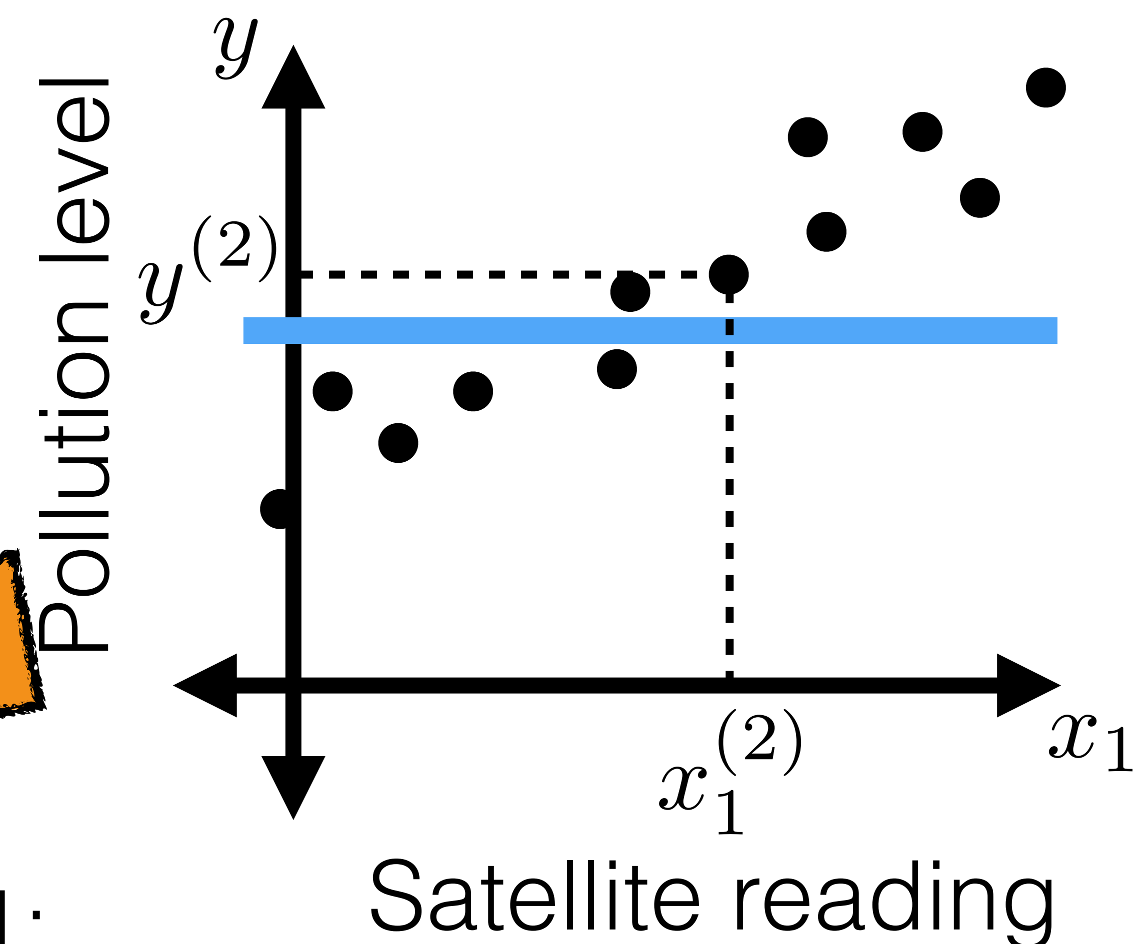

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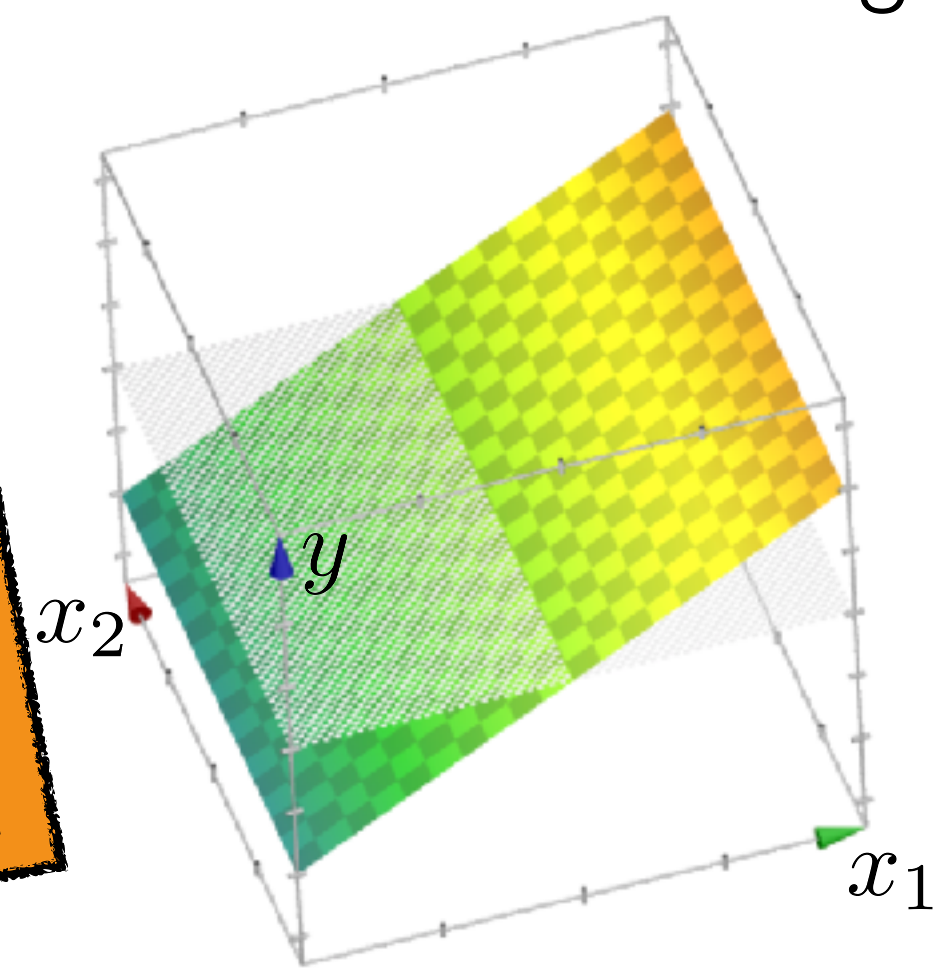

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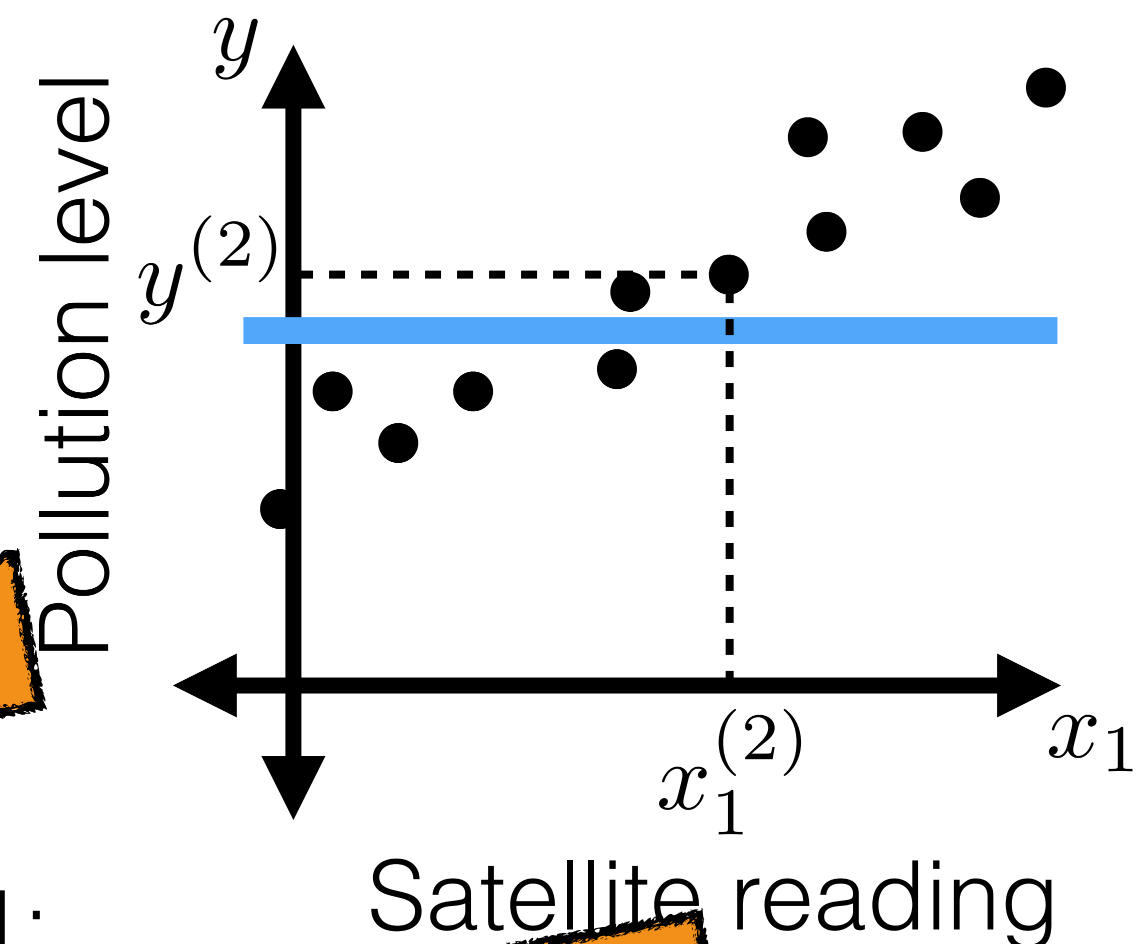

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Hypothesis is a "hyperplane"

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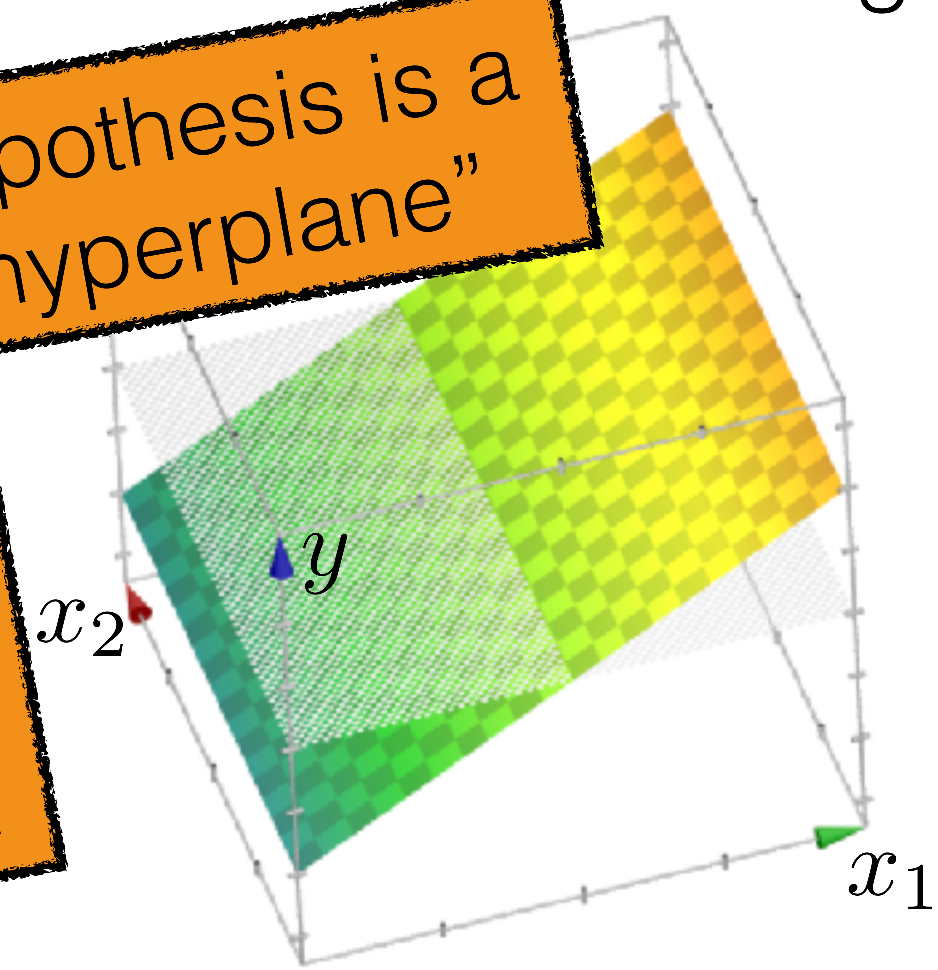

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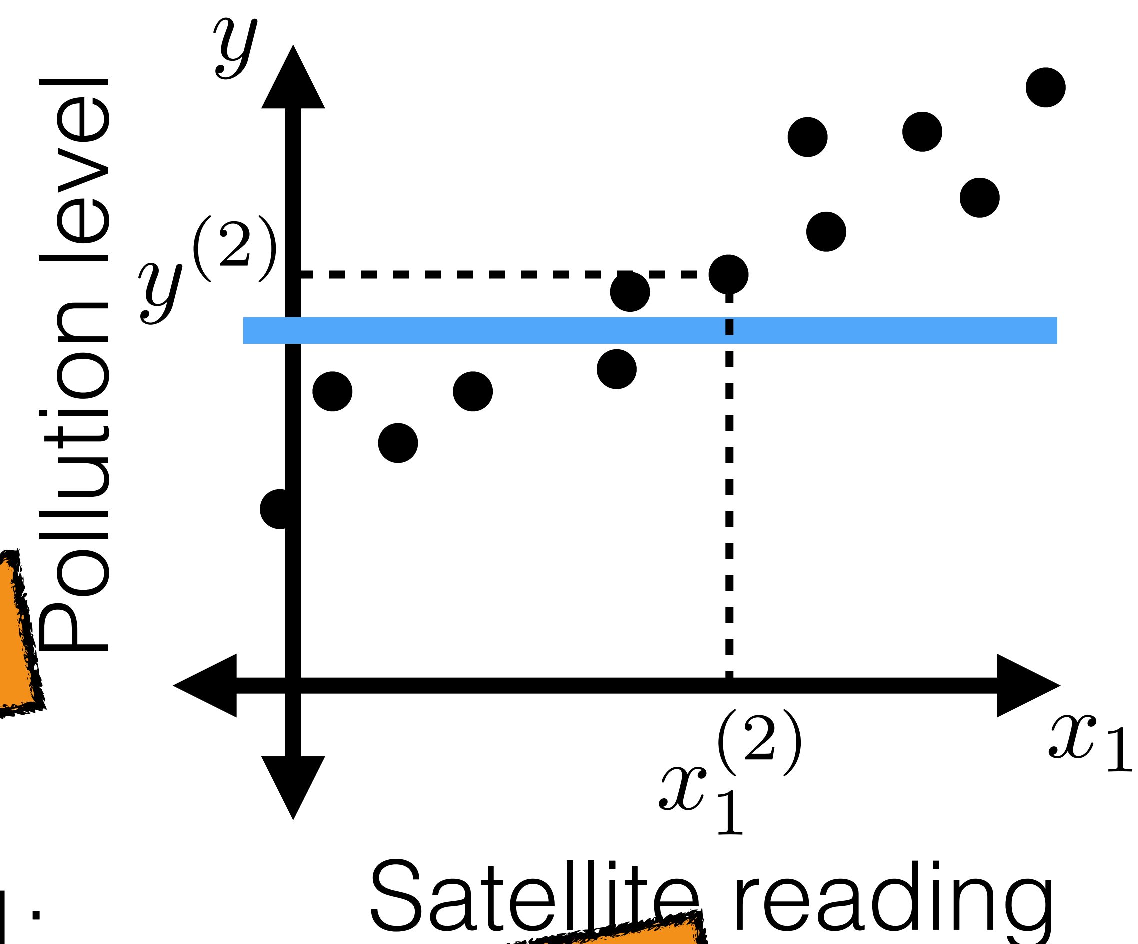

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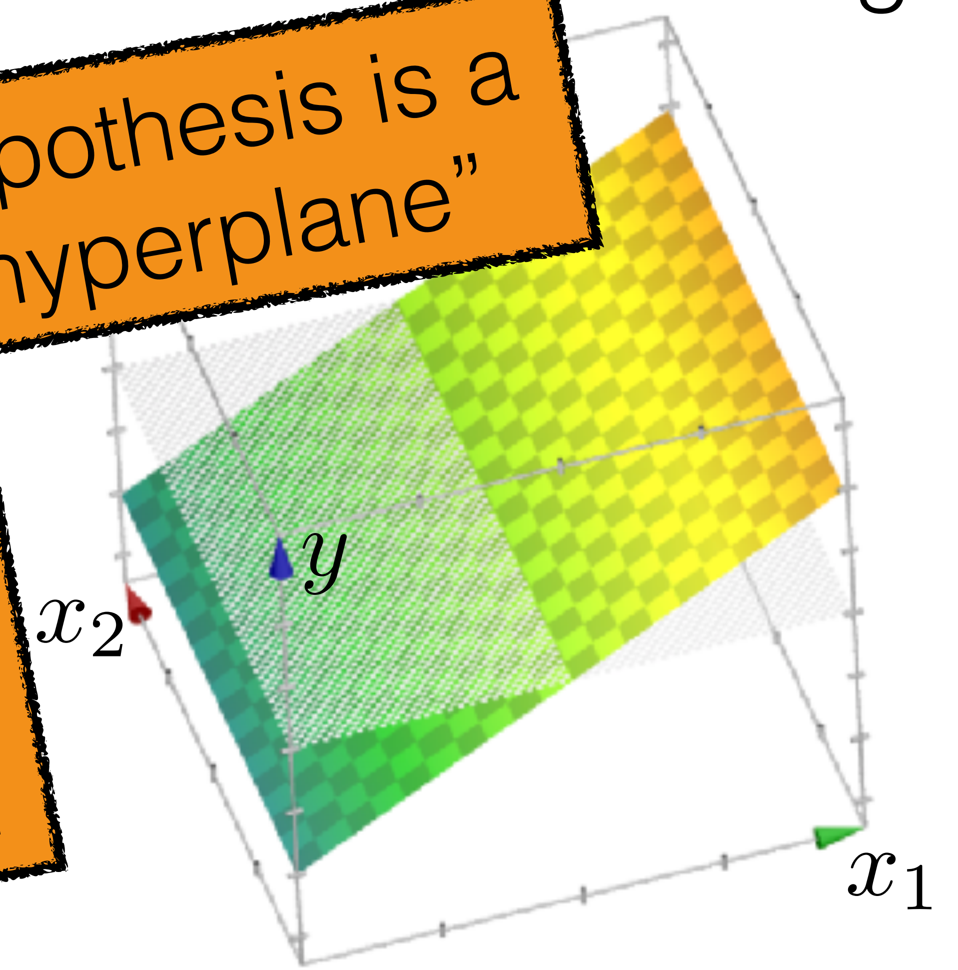

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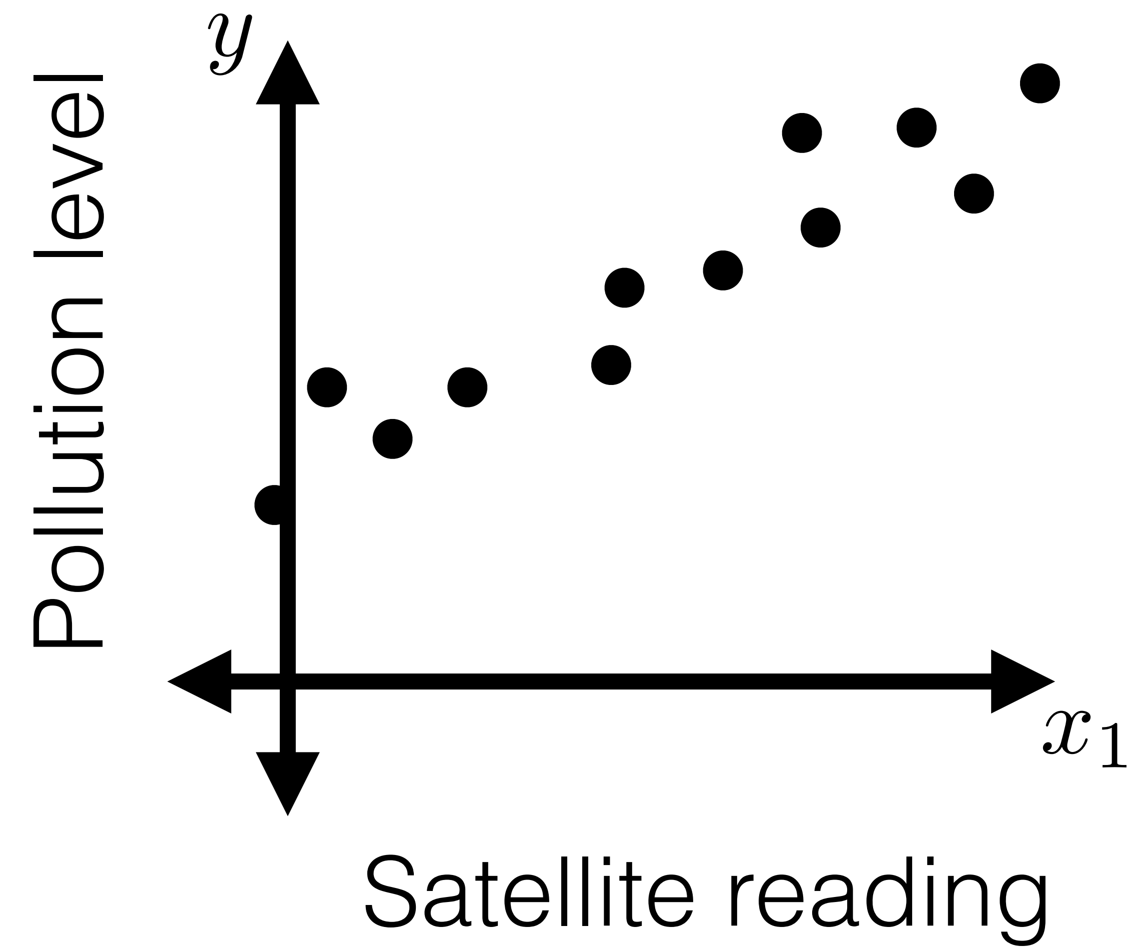
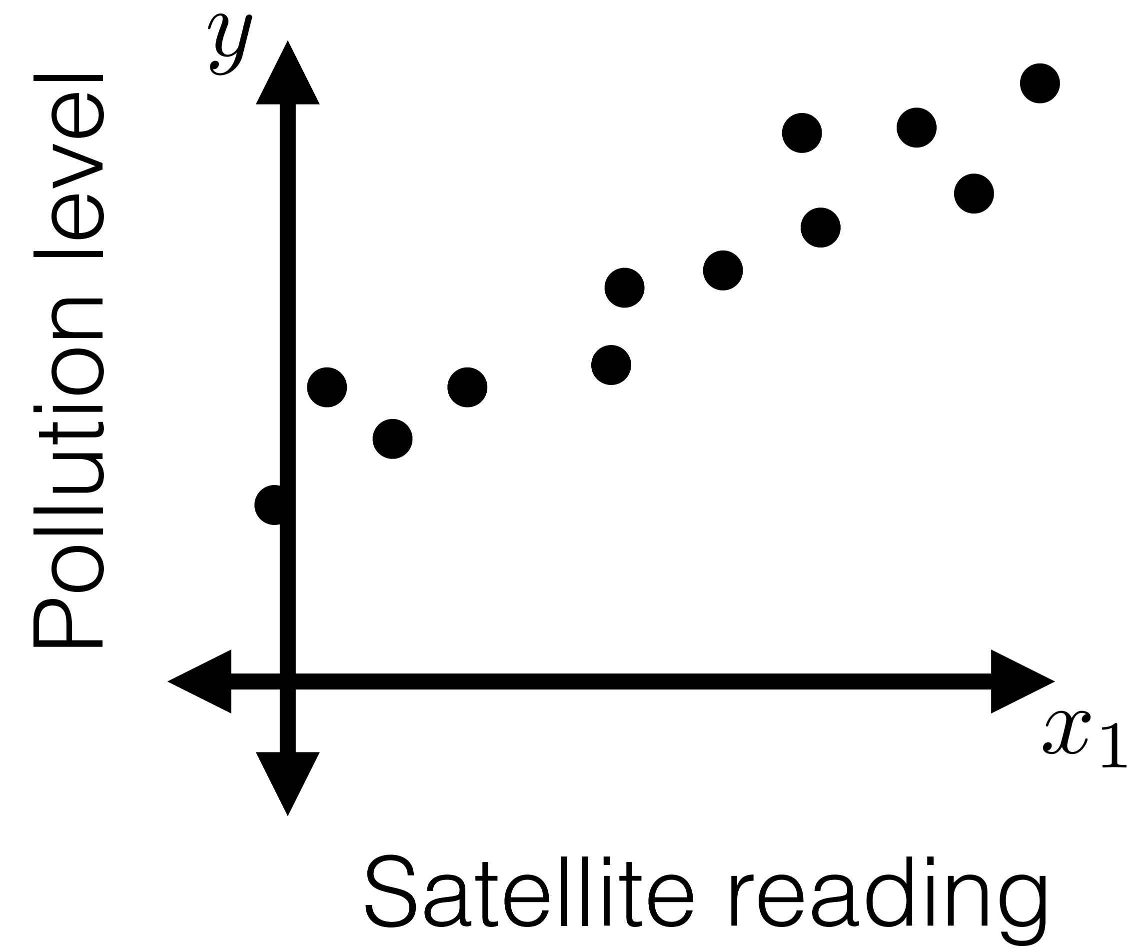
- Our hypothesis class in linear regression will be the set of all such h

How good is a regression hypothesis?



How good is a regression hypothesis?

- Should predict well on future data



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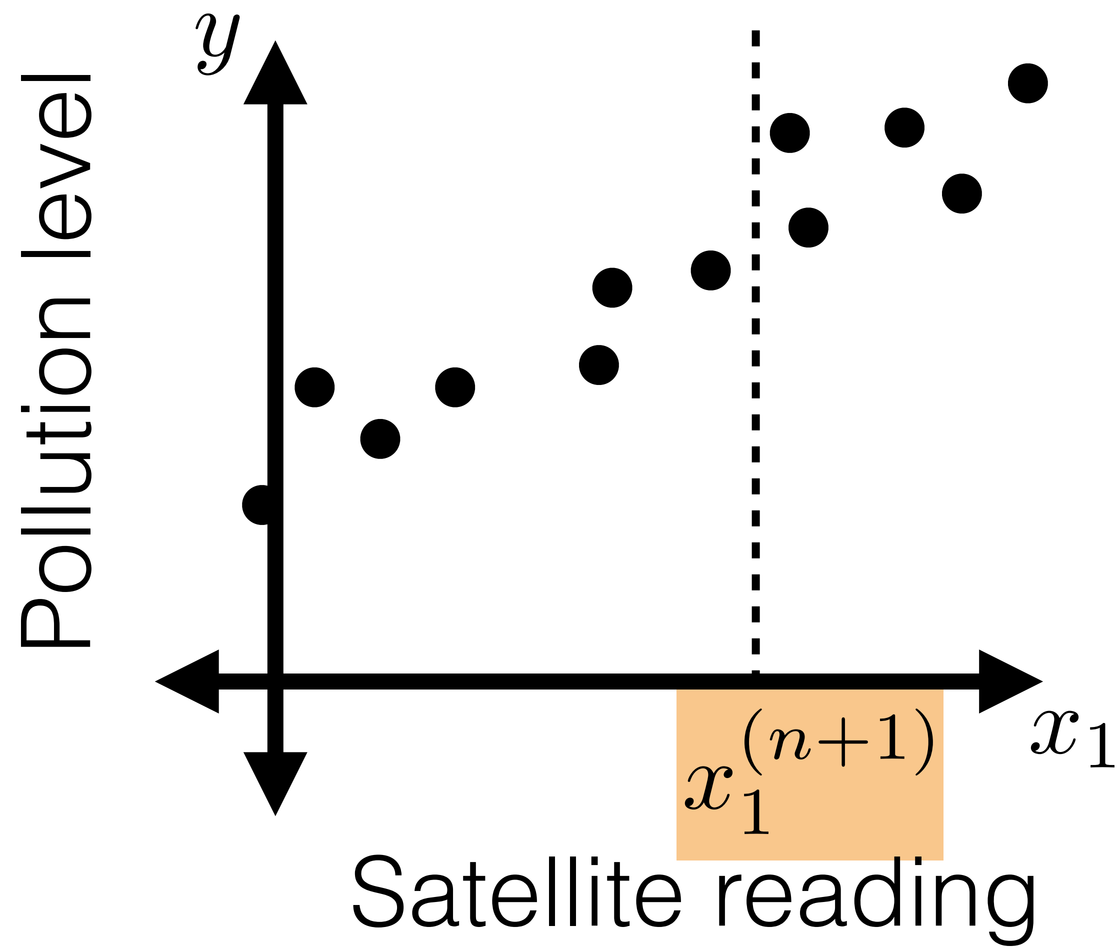

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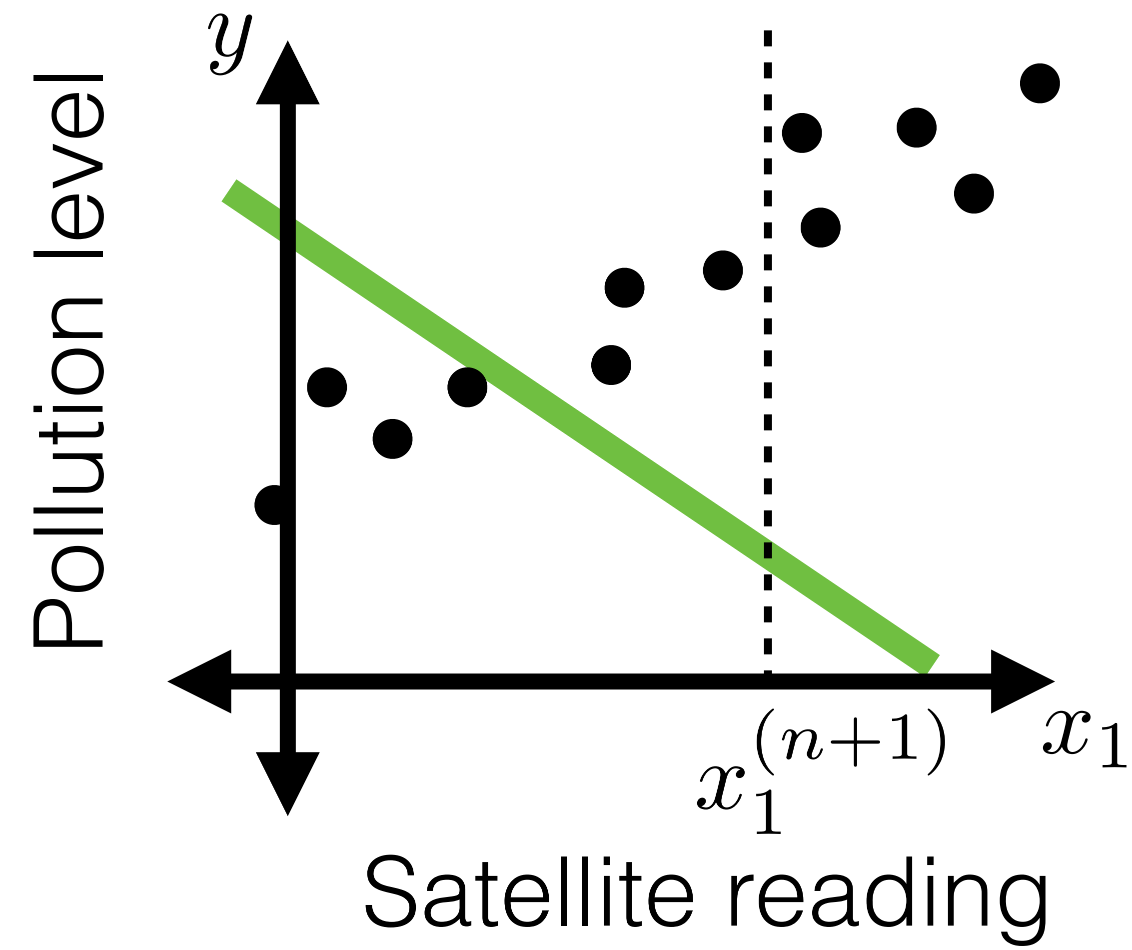

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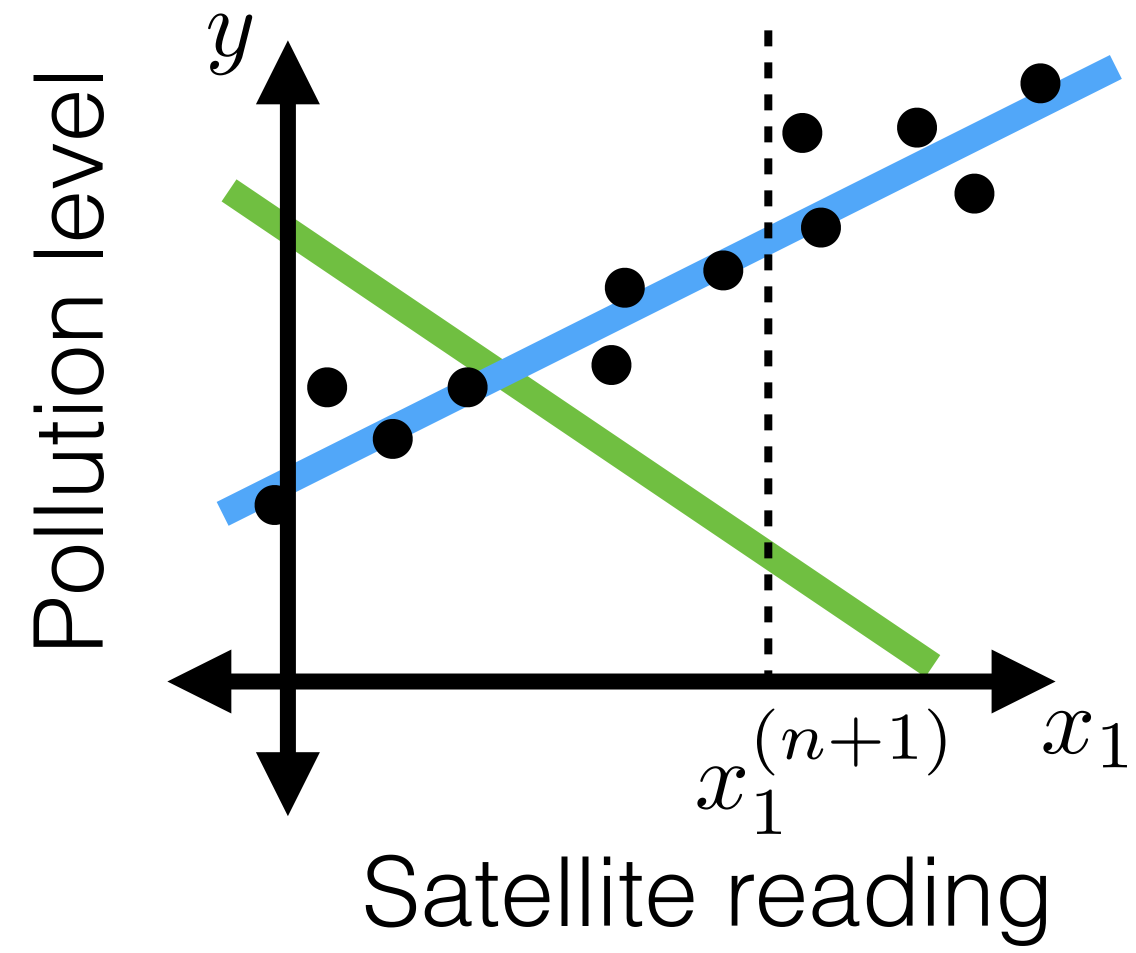

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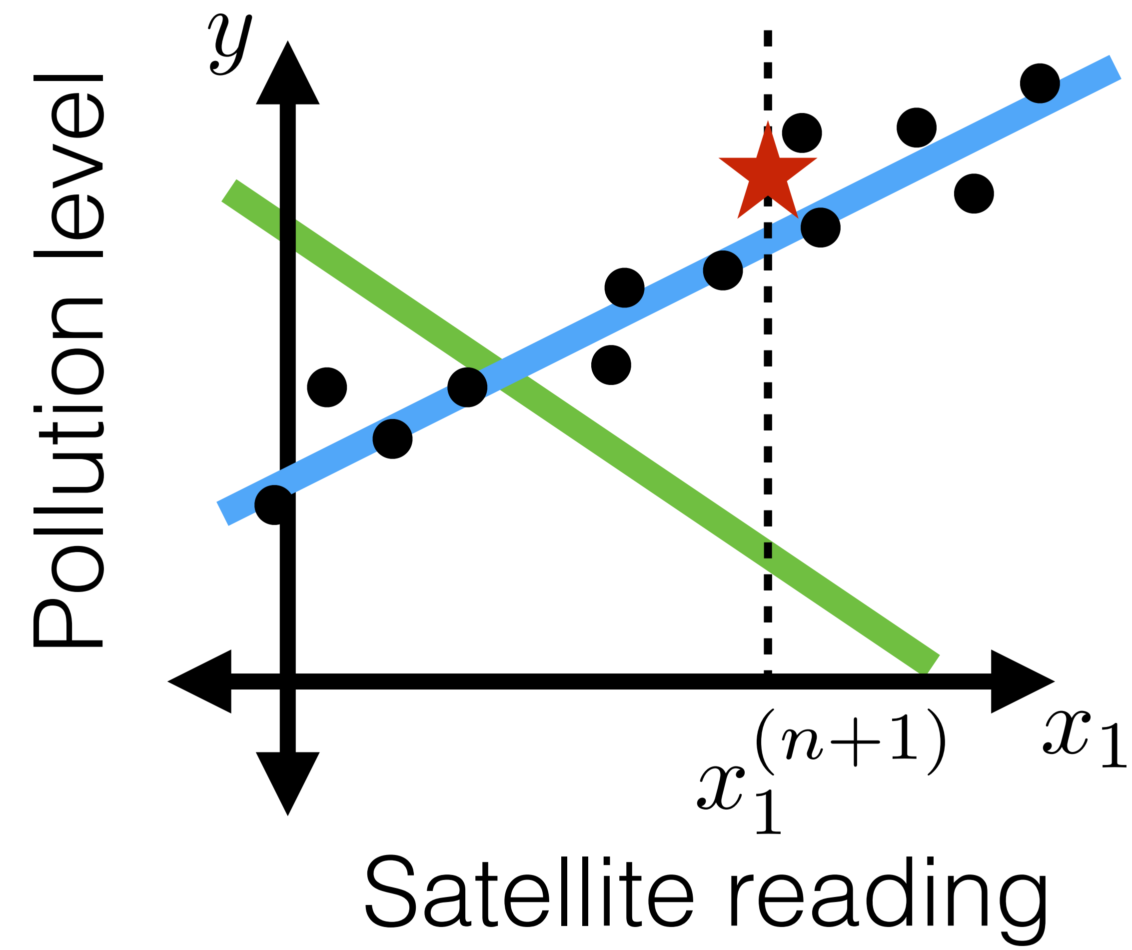

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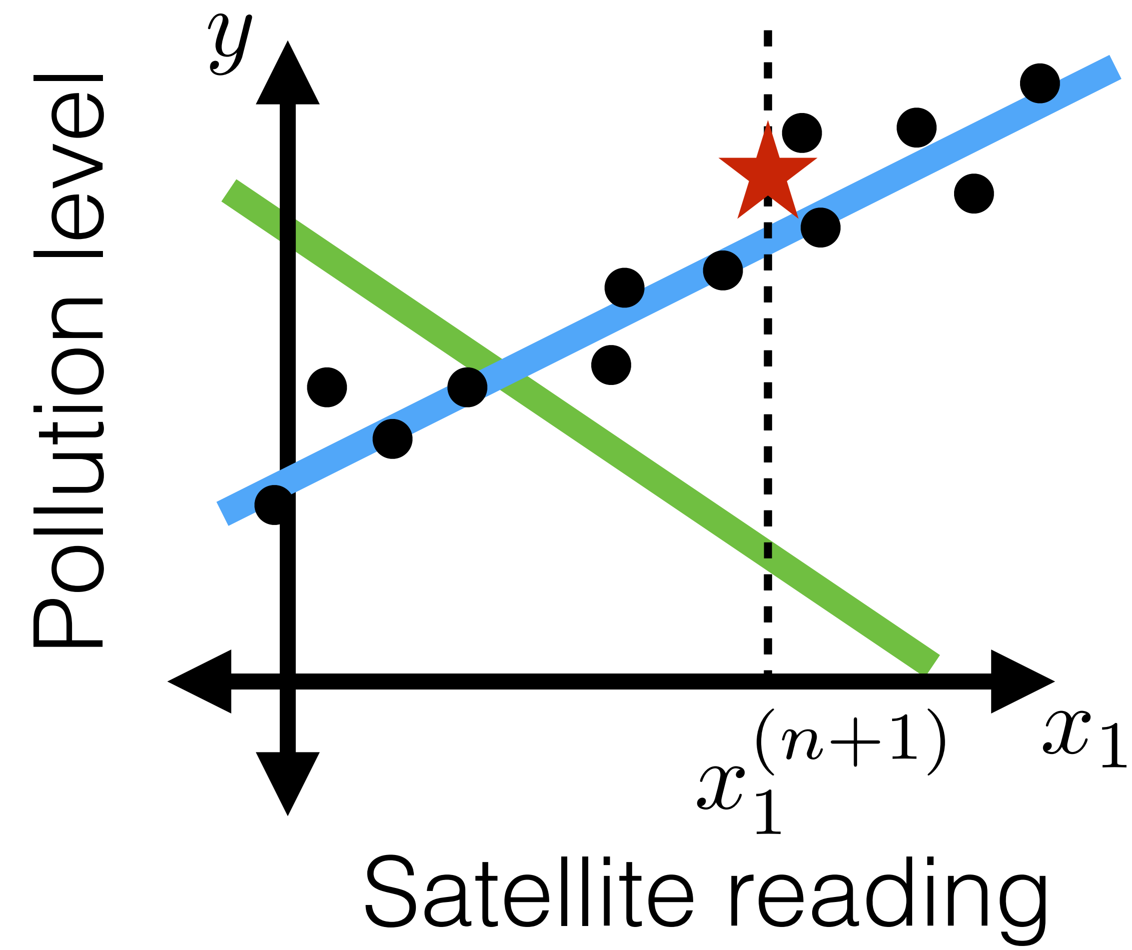

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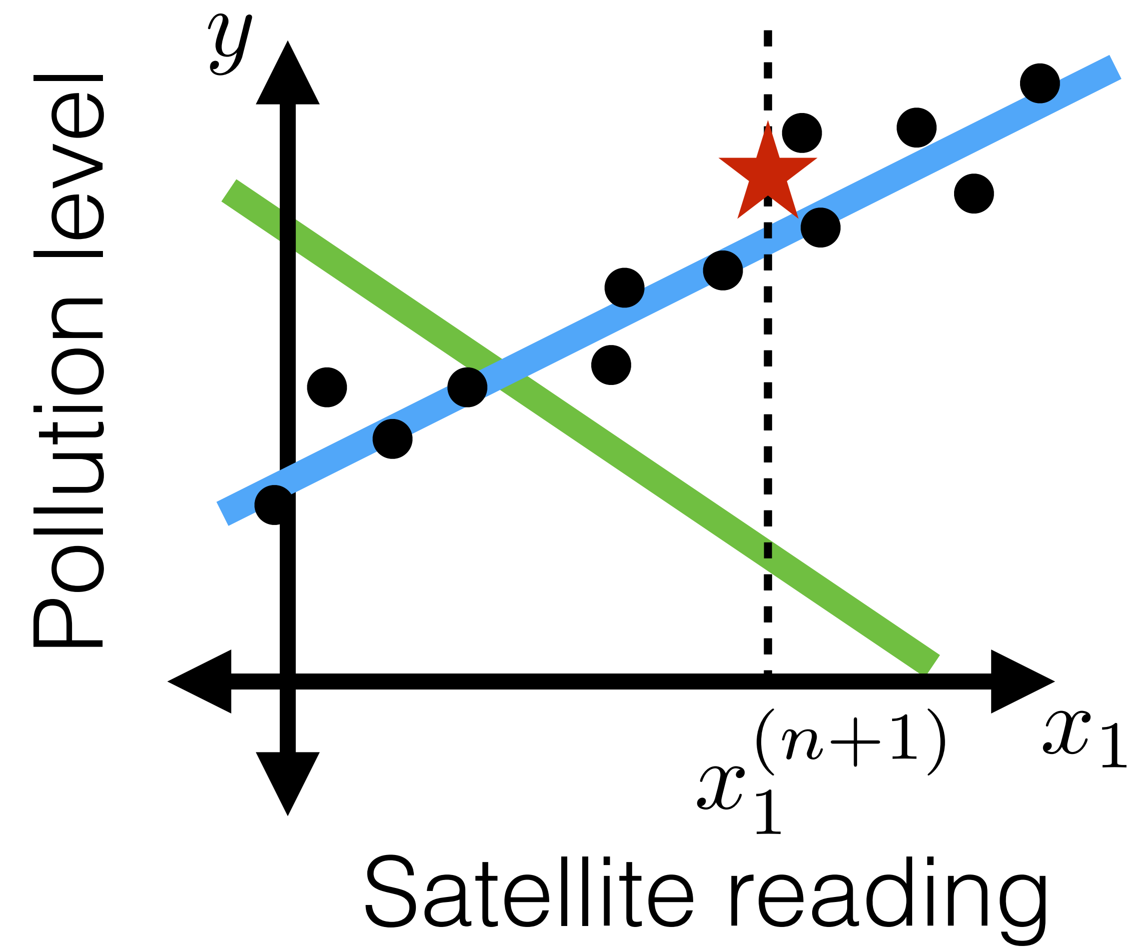

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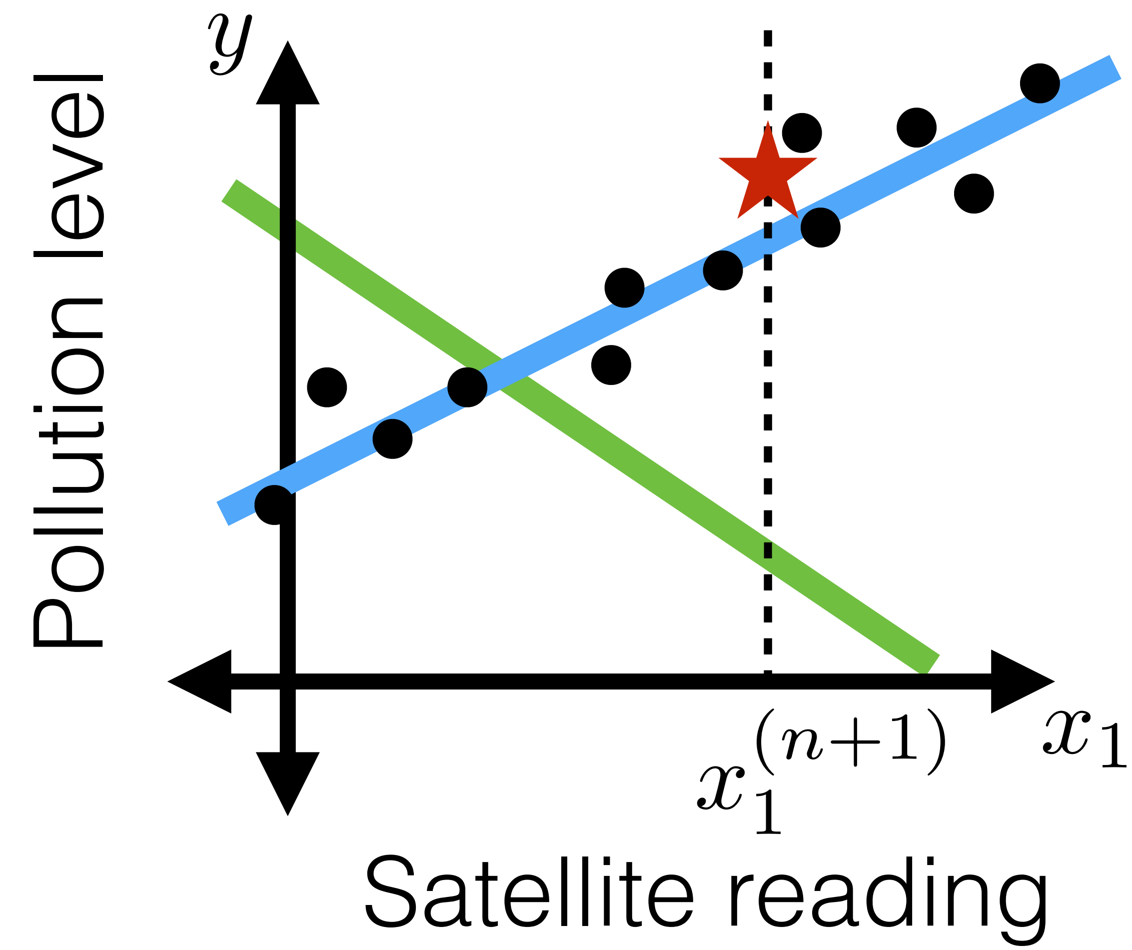

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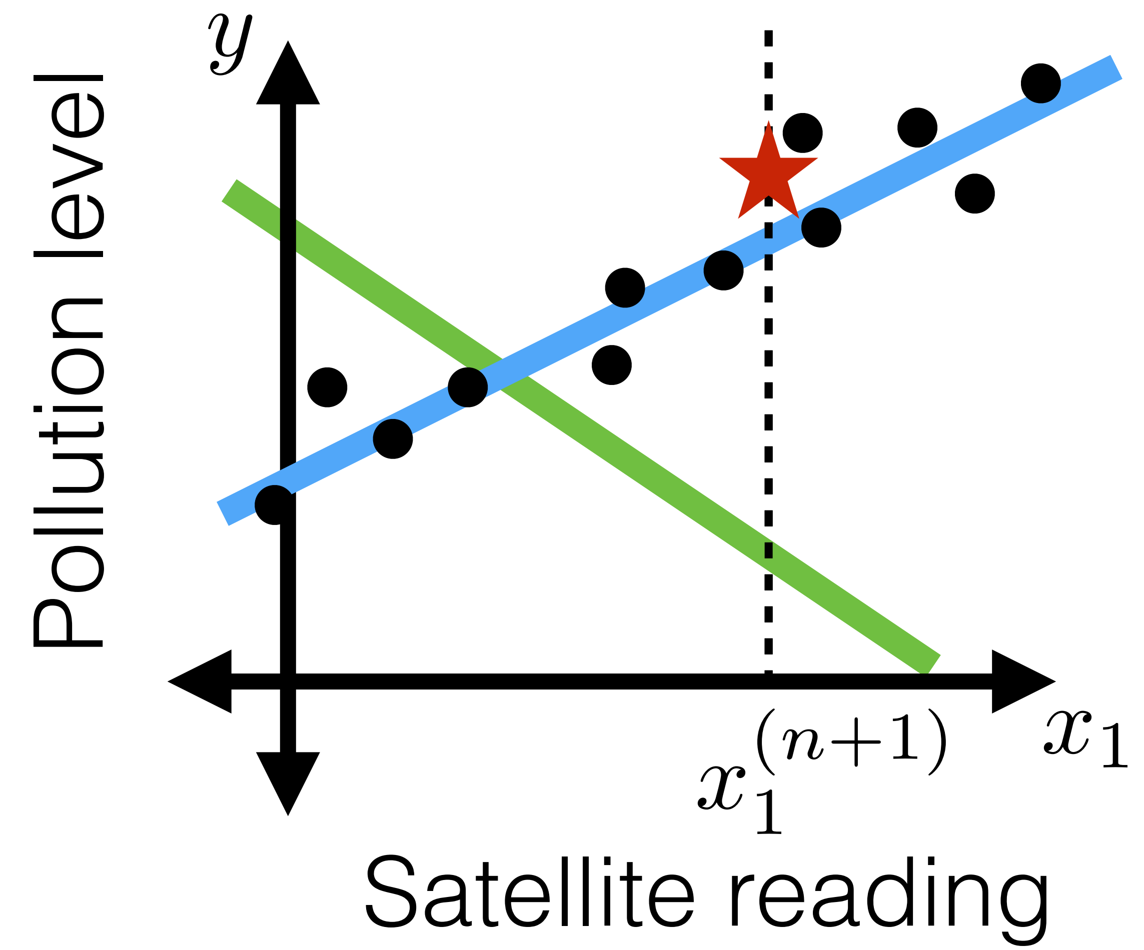

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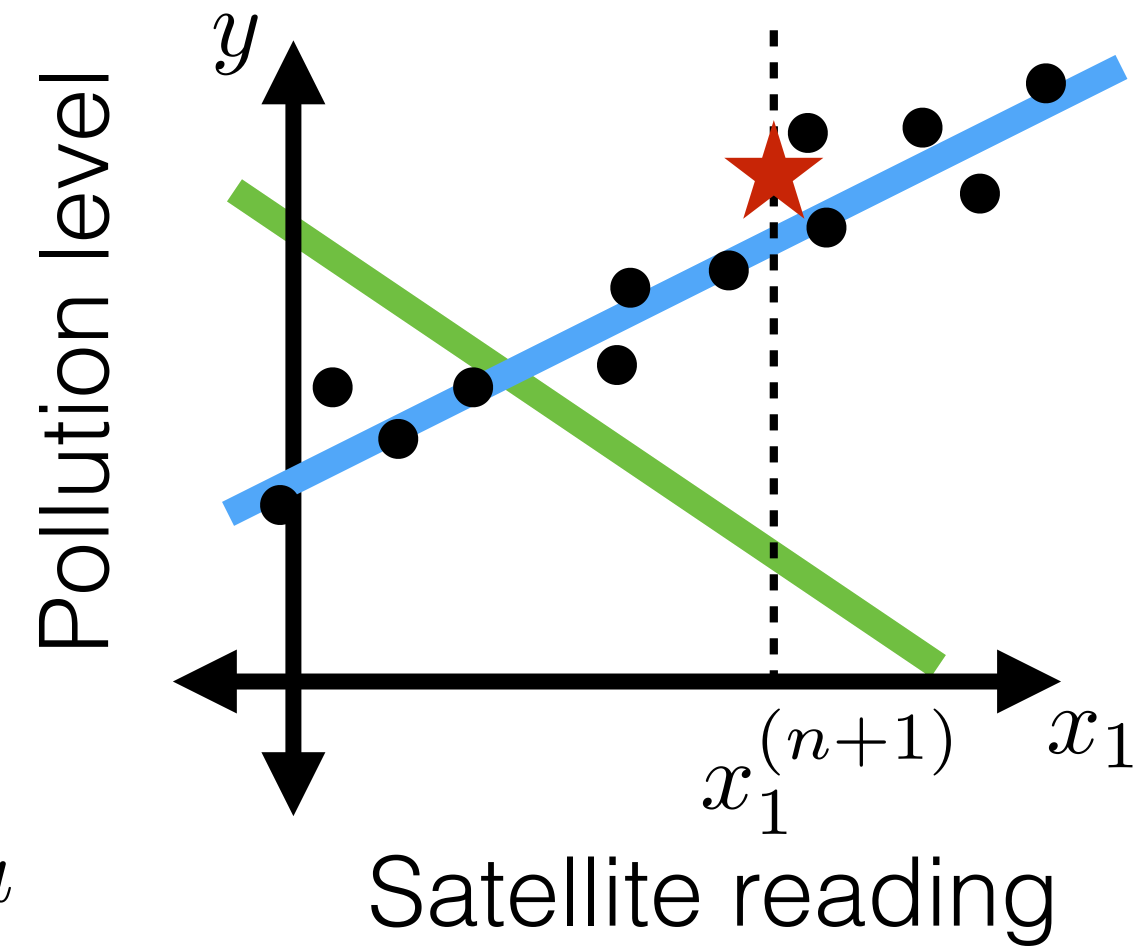

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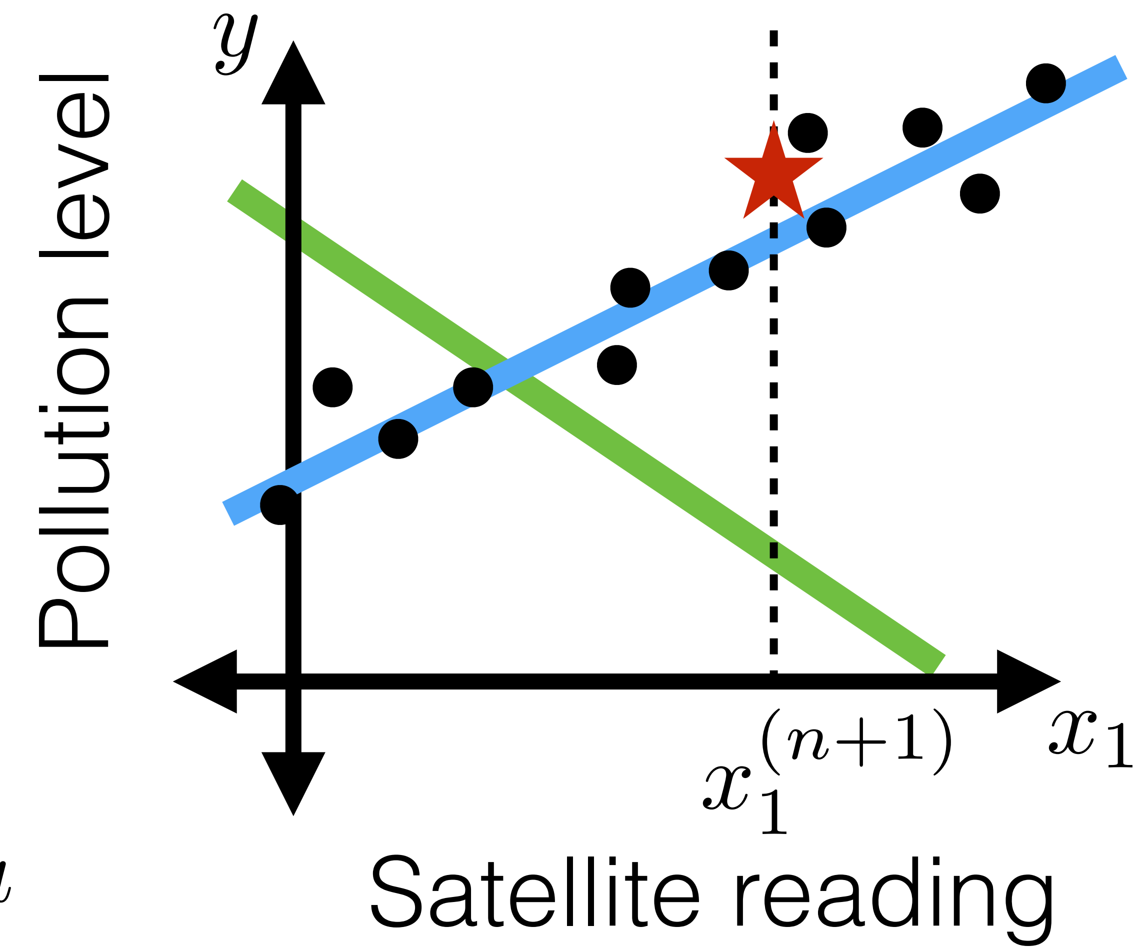

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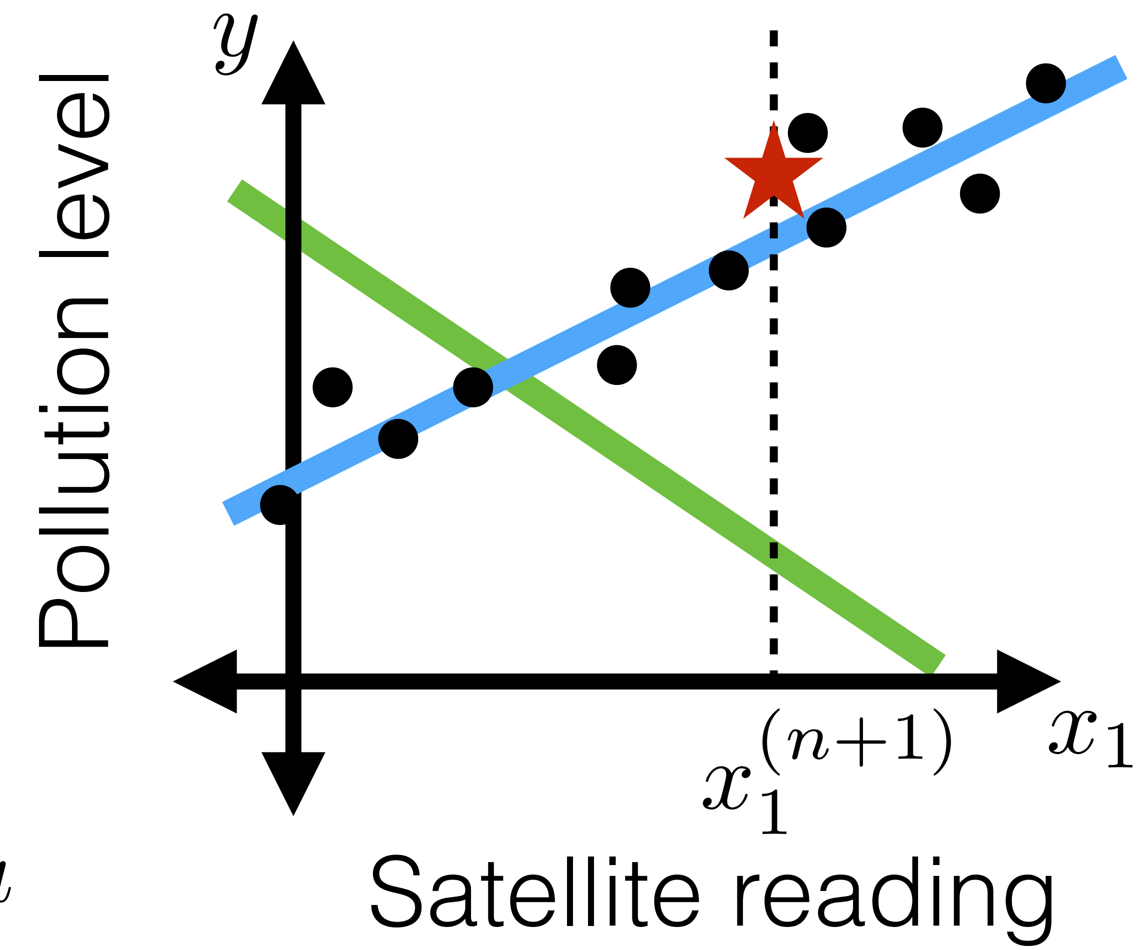

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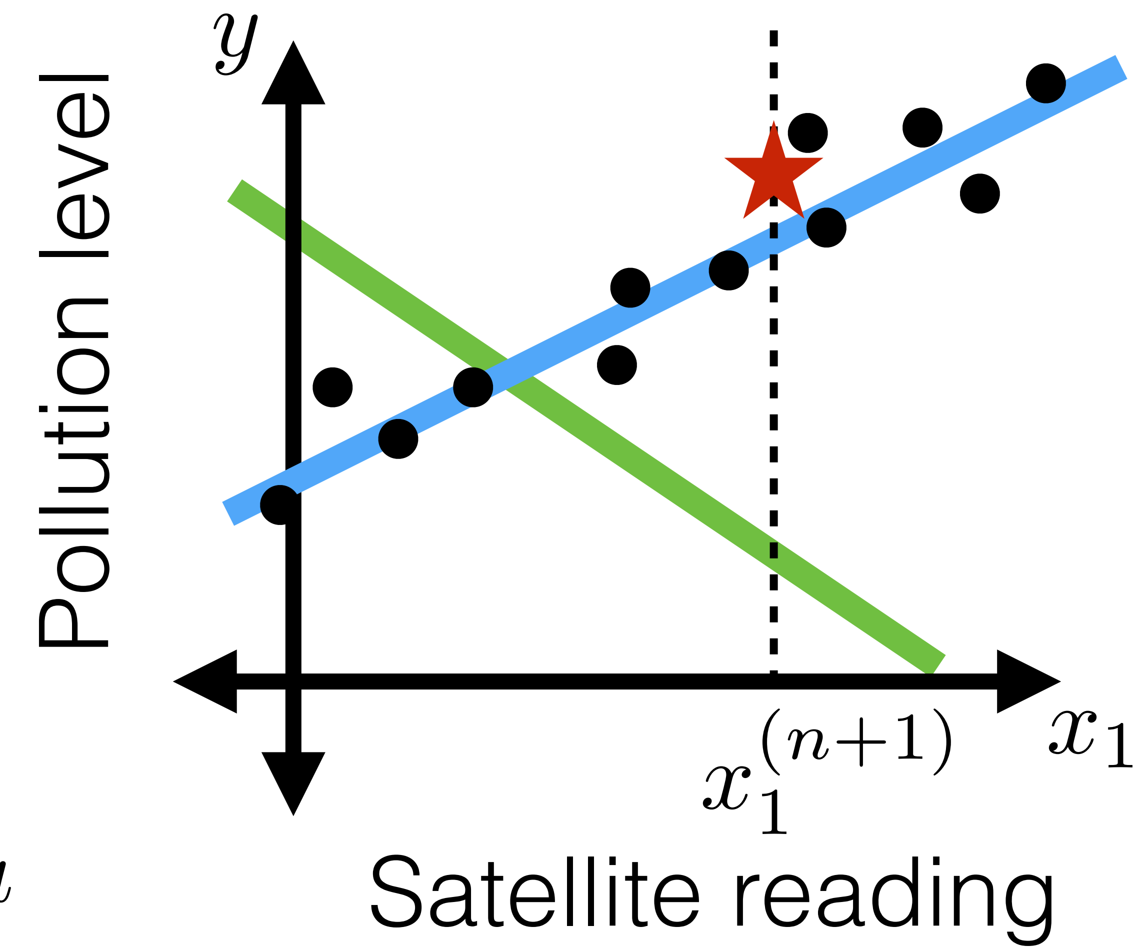

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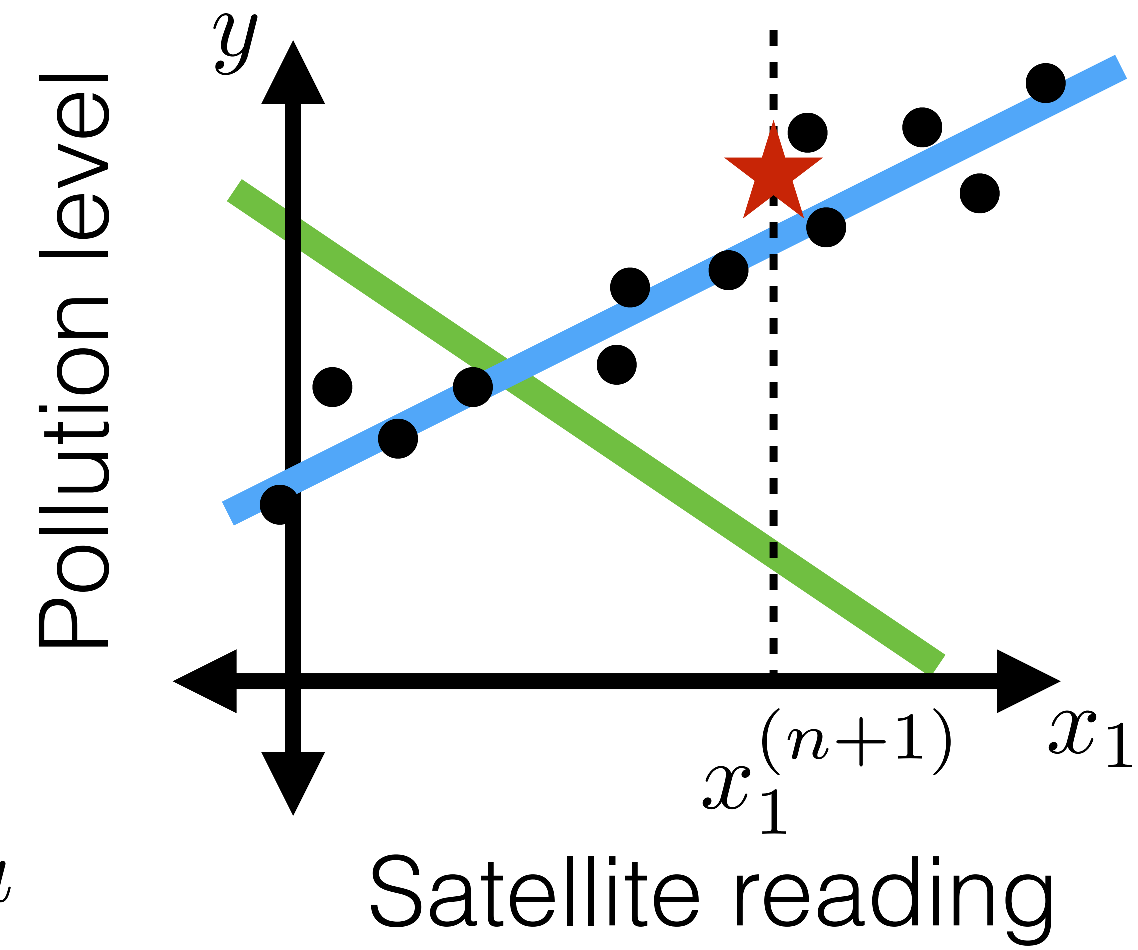

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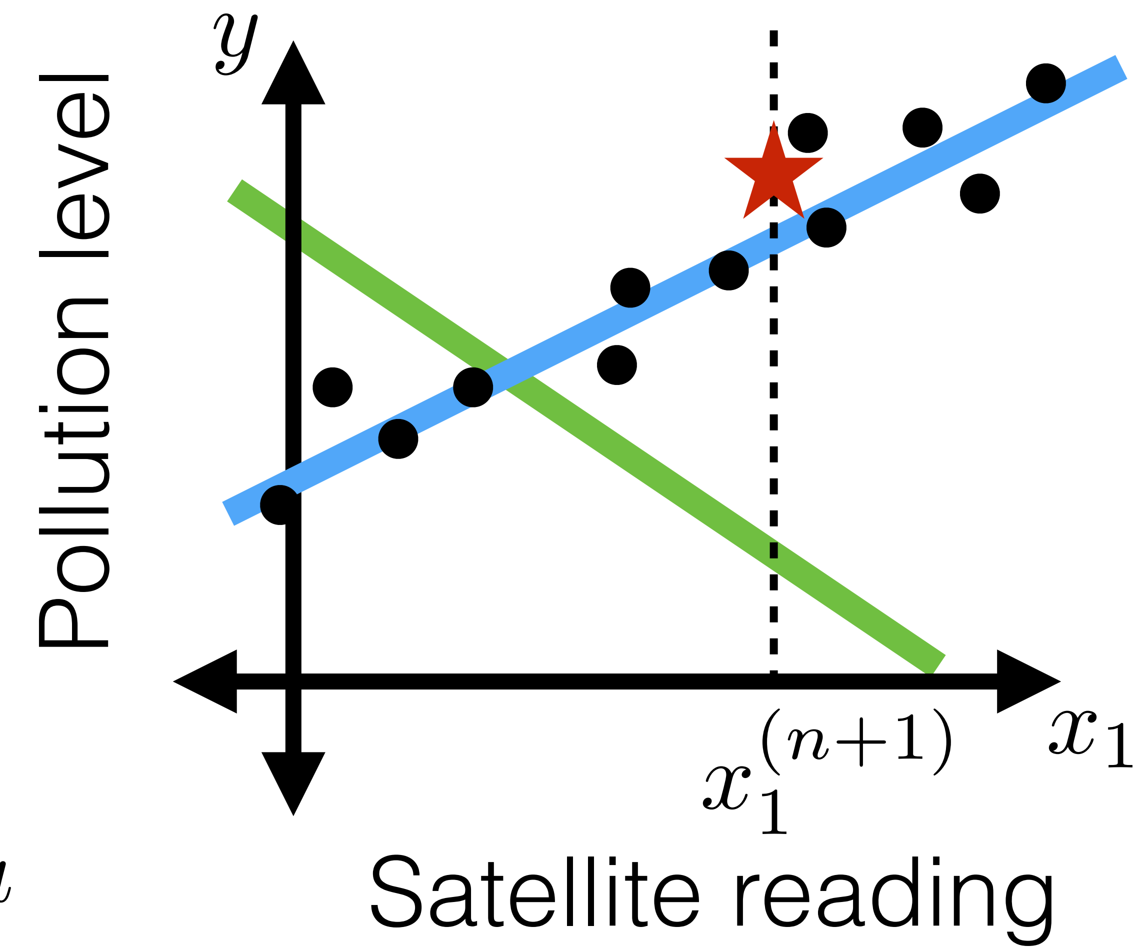

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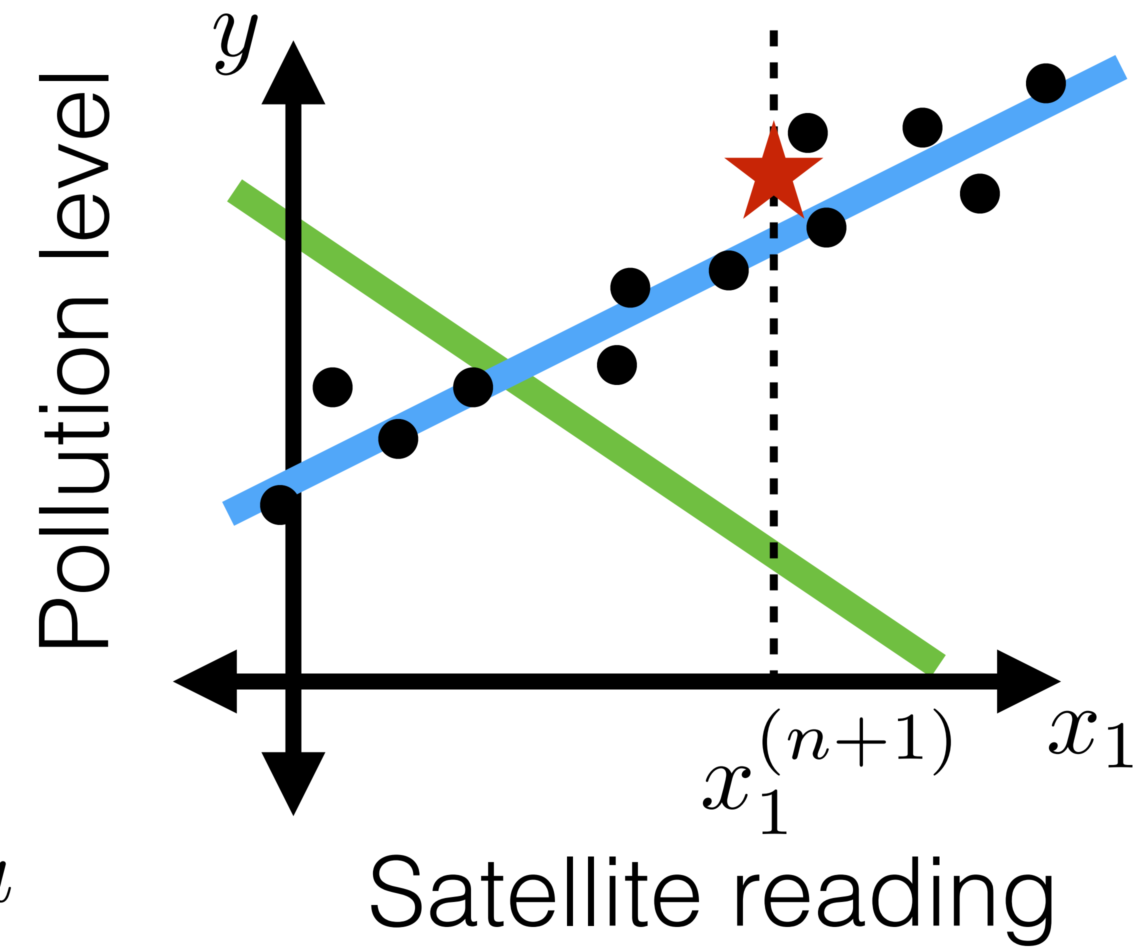

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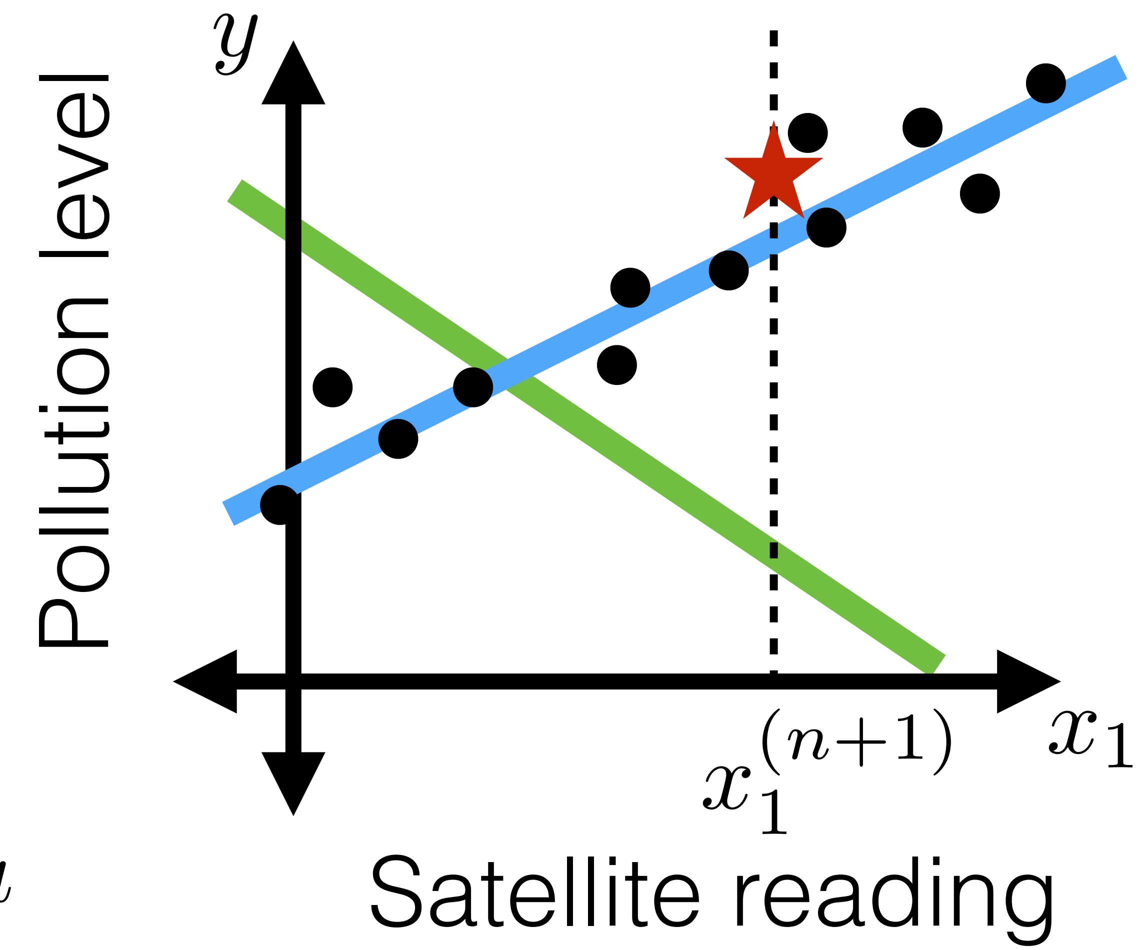

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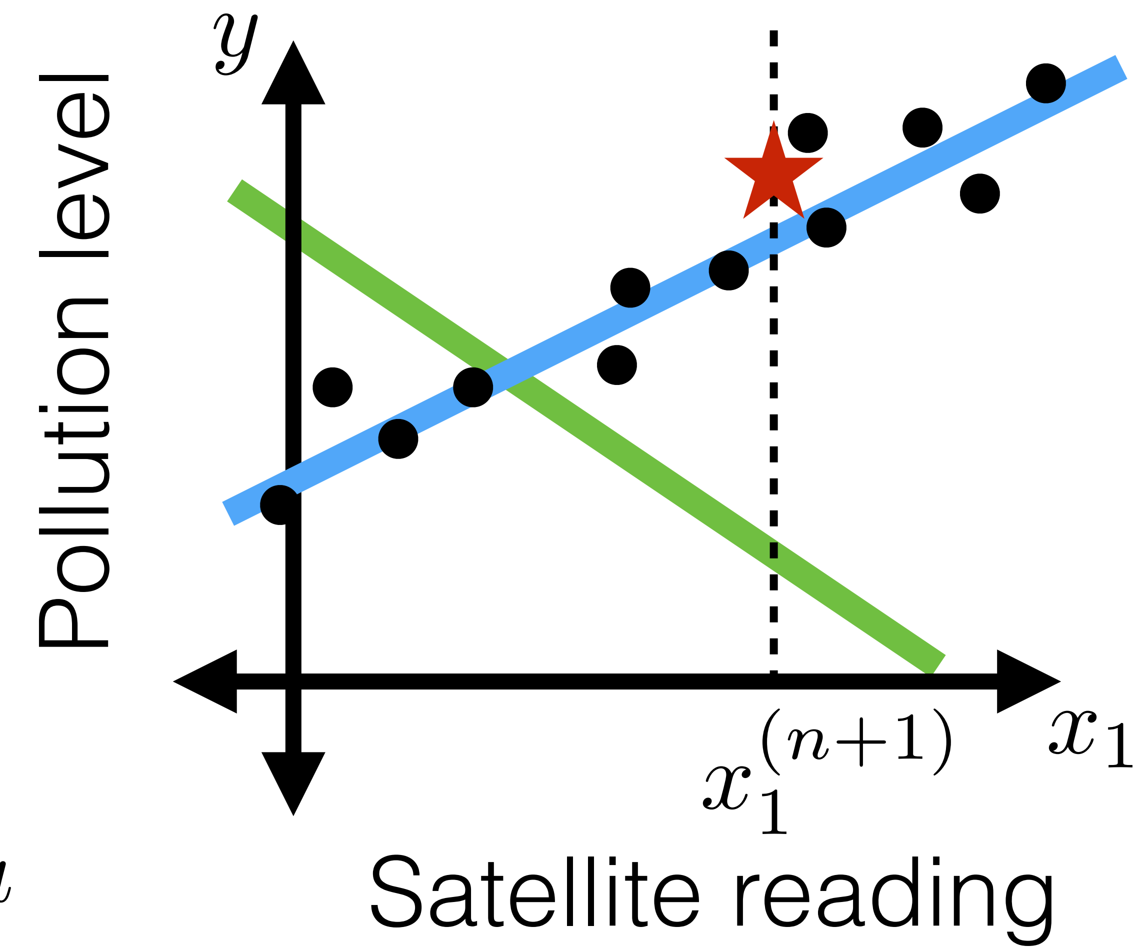

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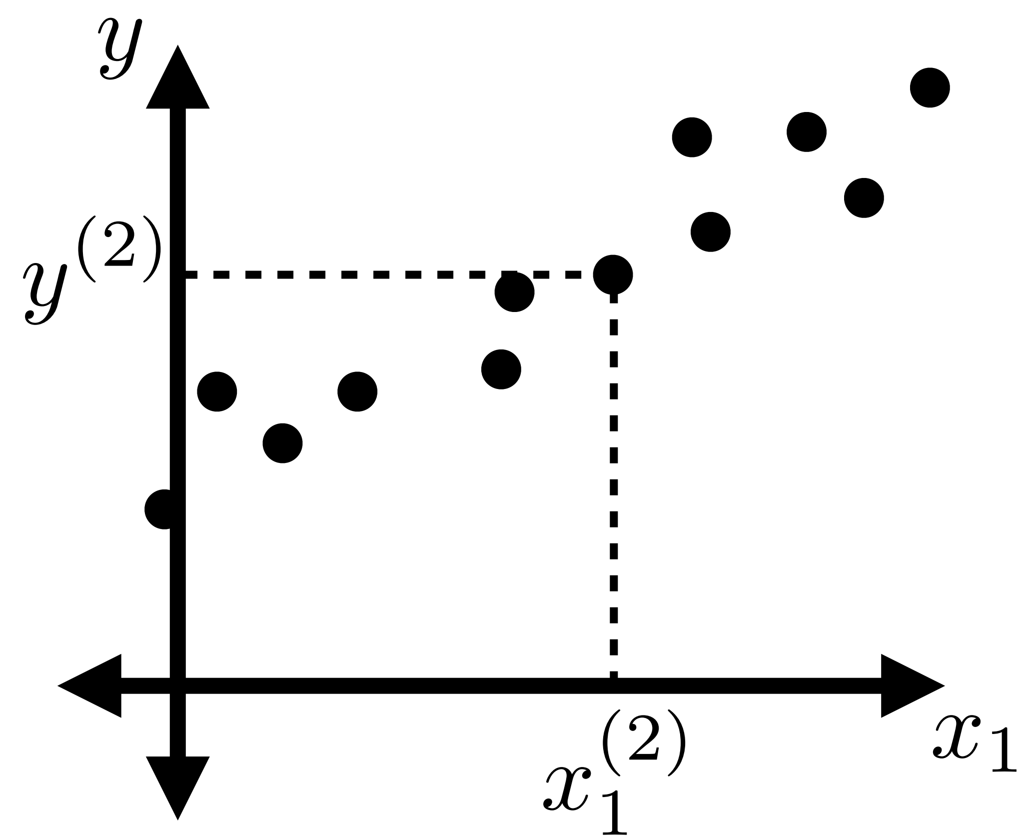
Learning a regressor



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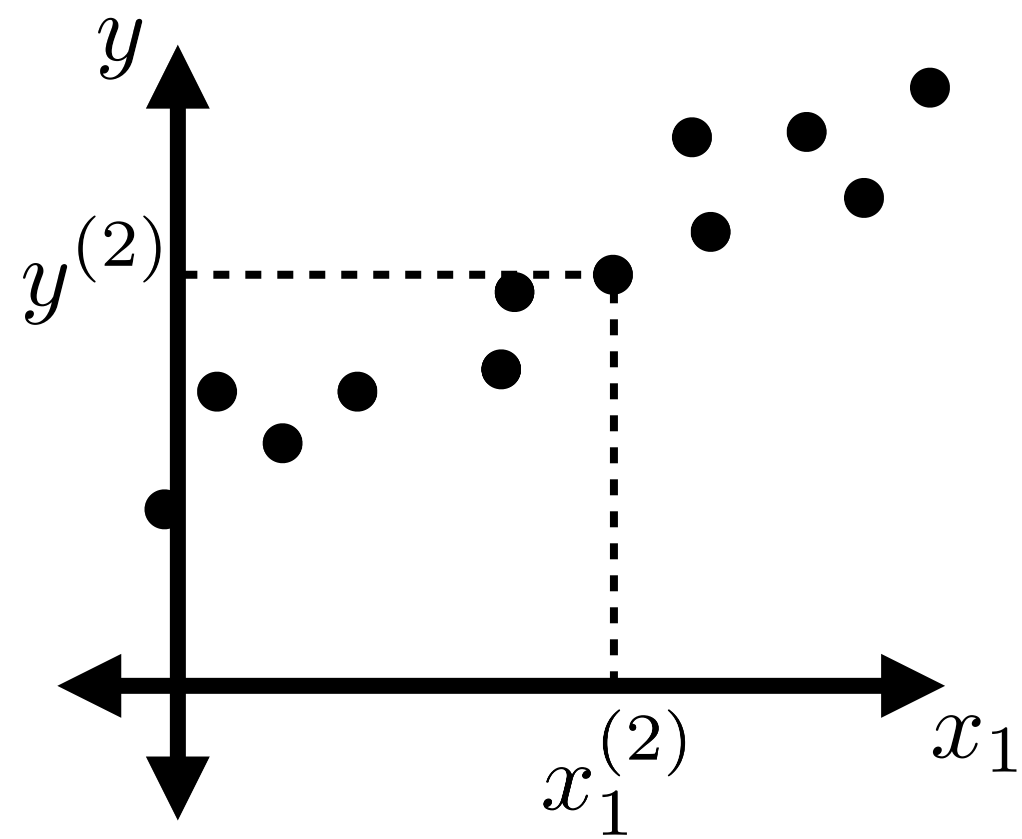

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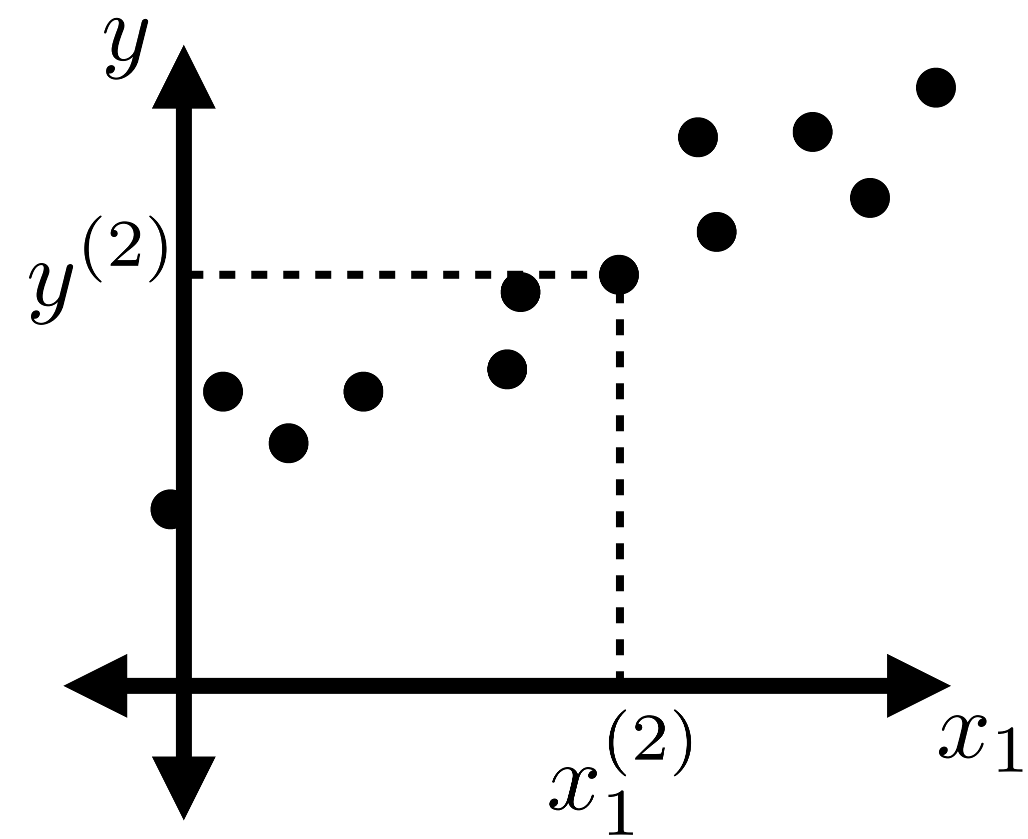

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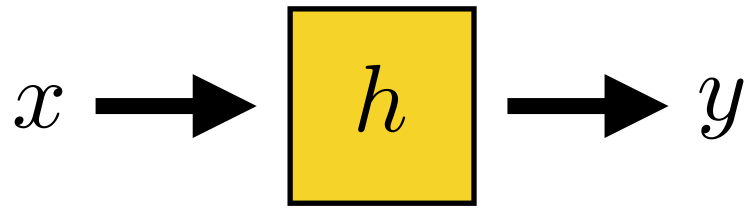
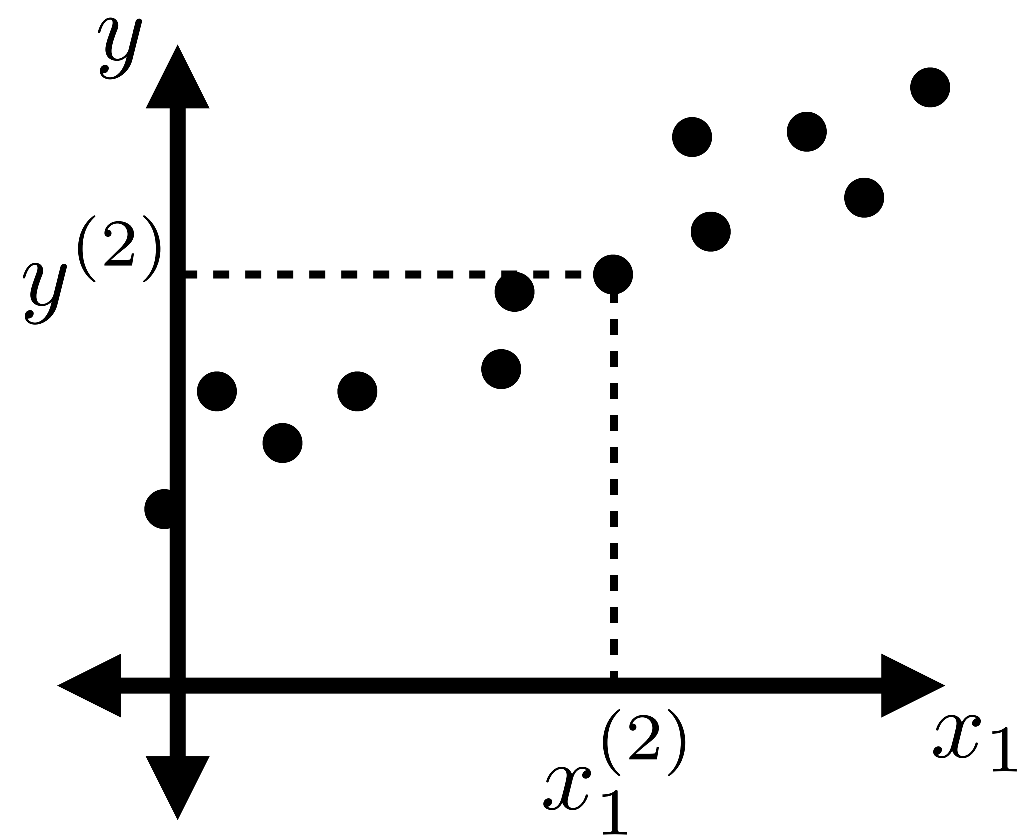
  - Recall: 



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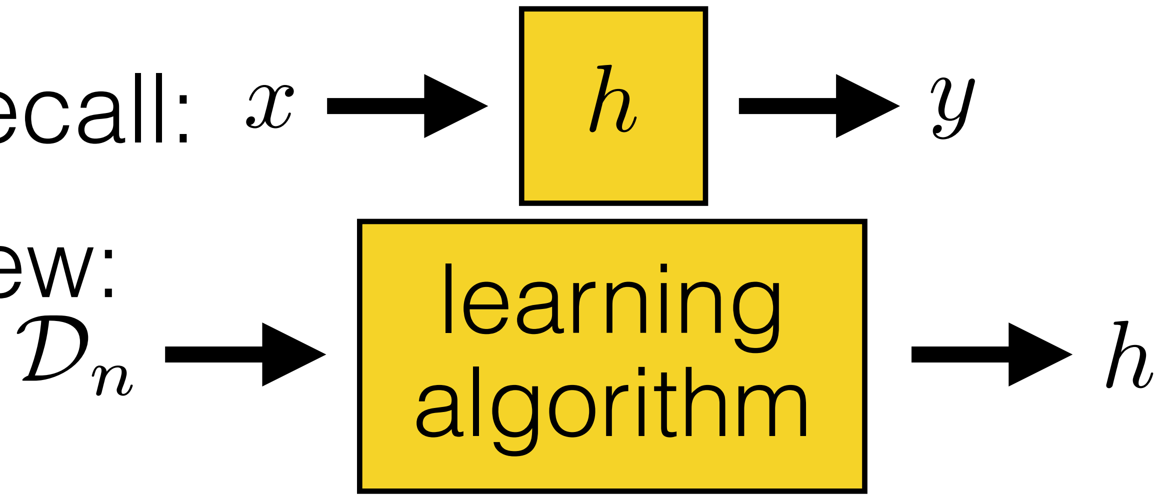

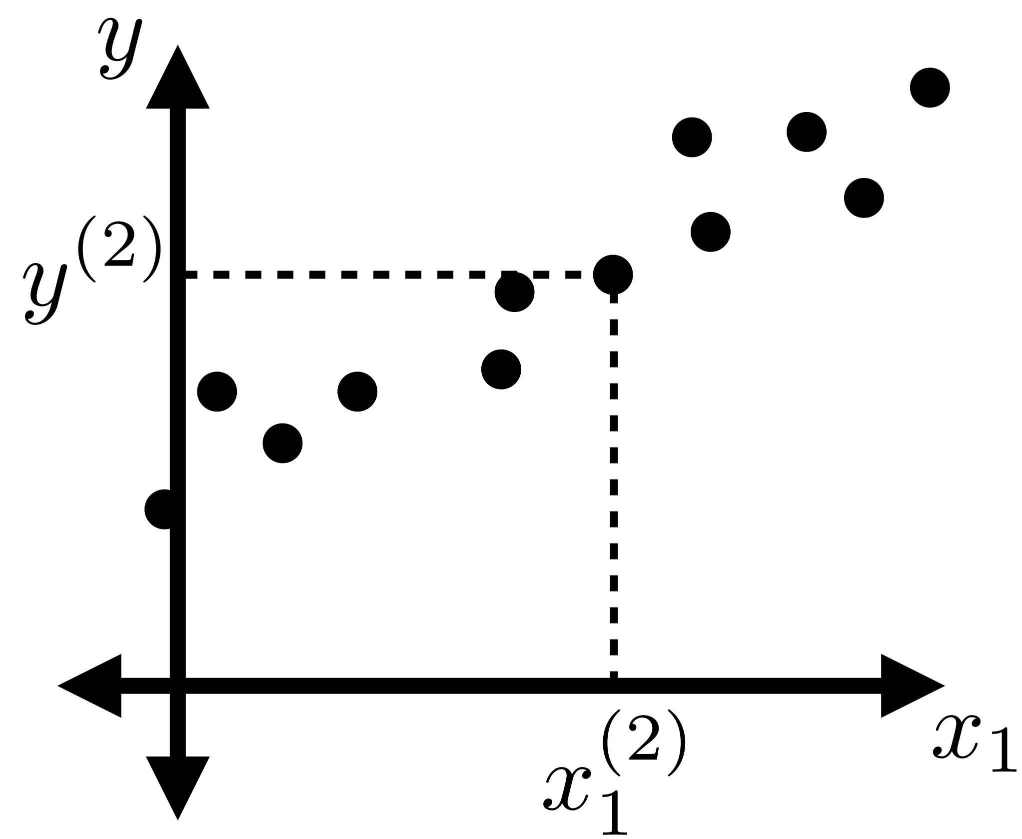

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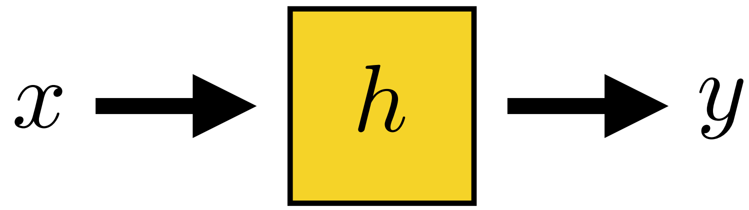
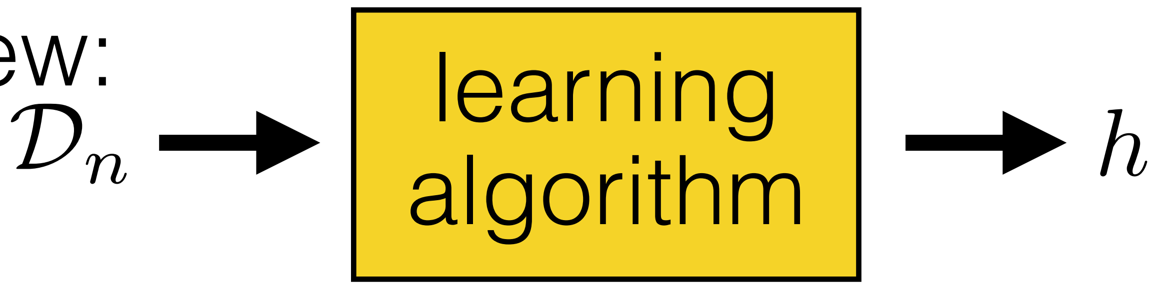
    - Recall: 

    - New: 

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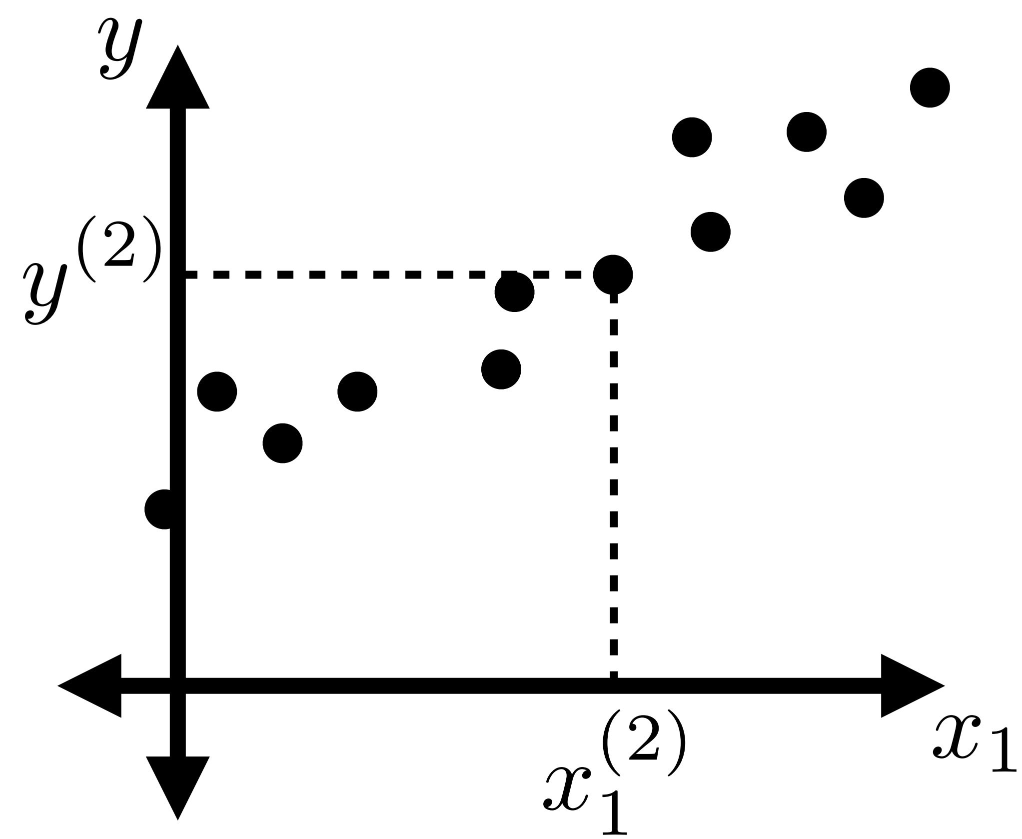

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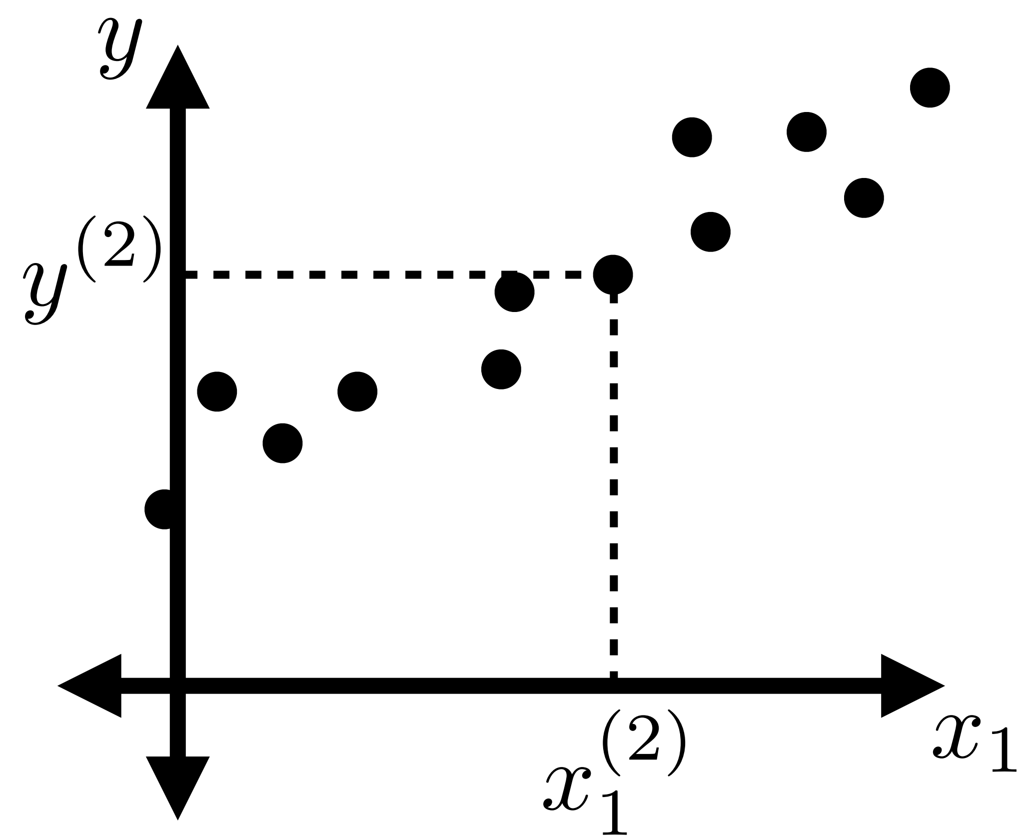

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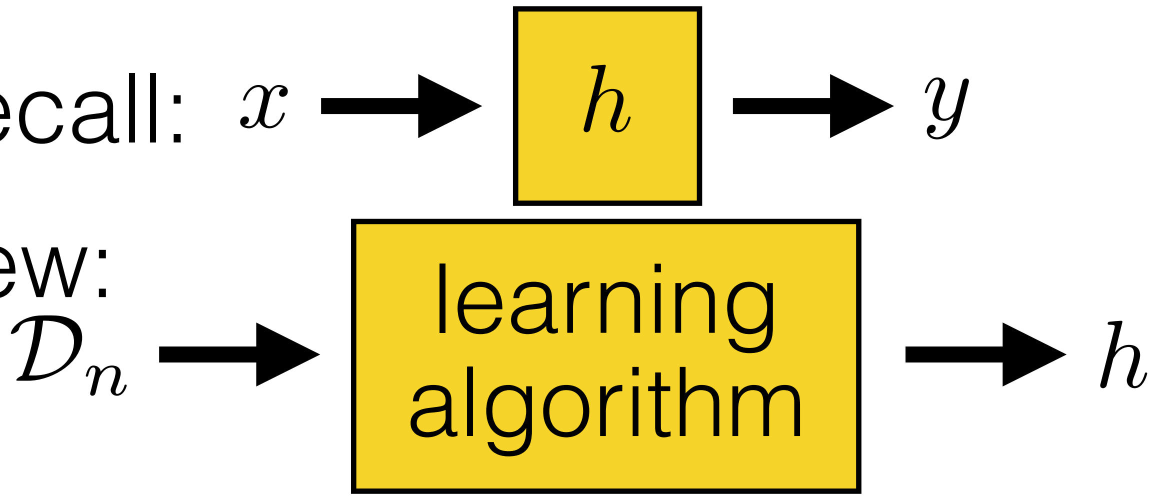

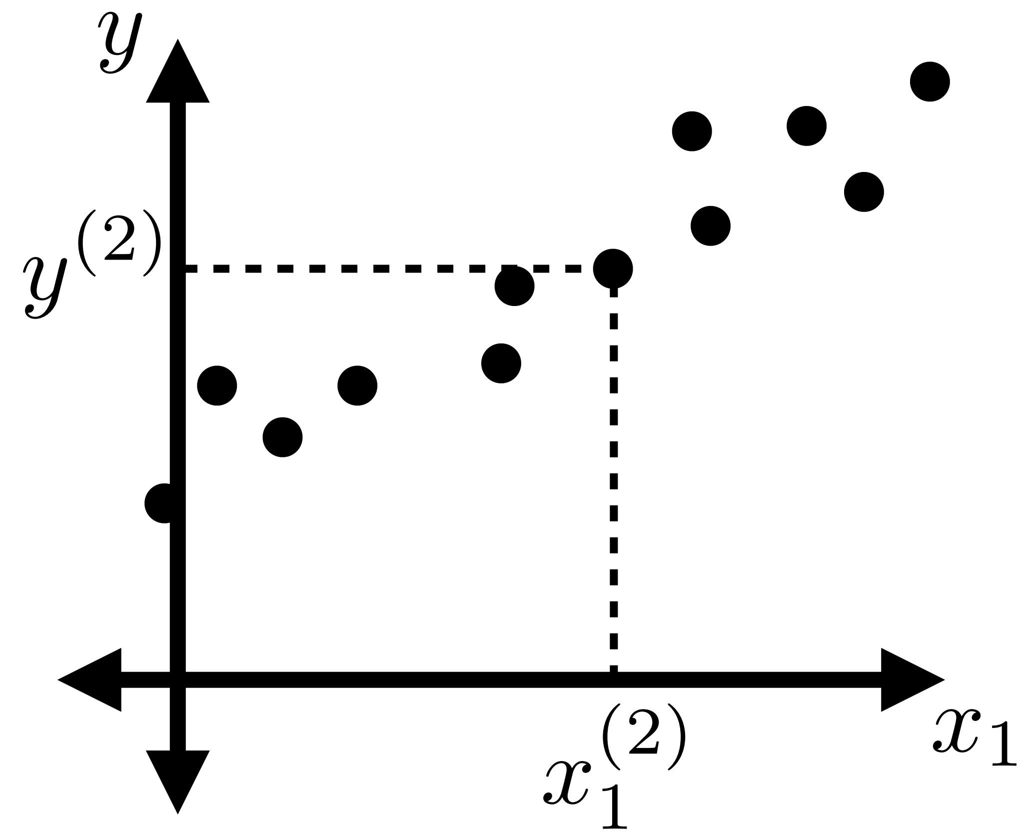

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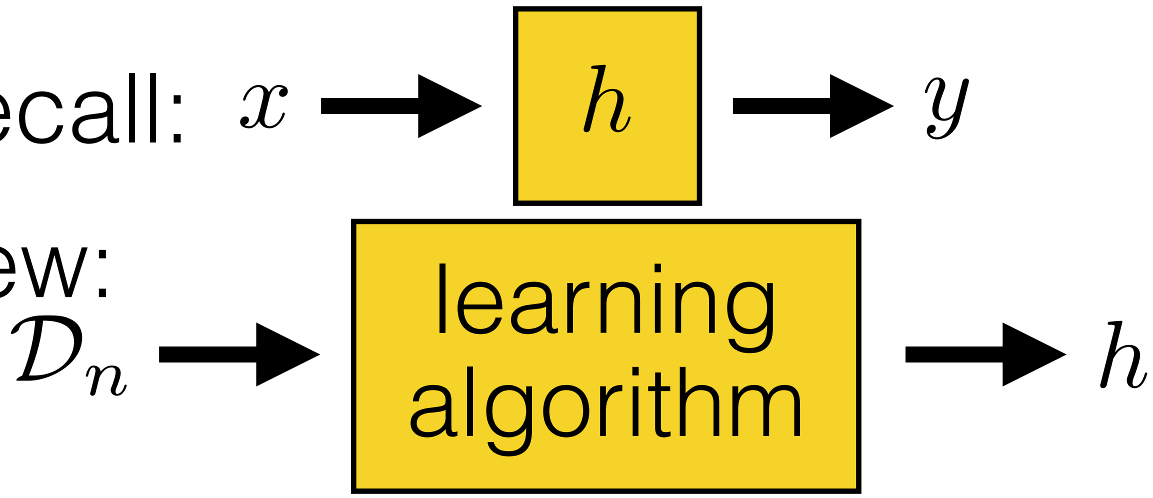

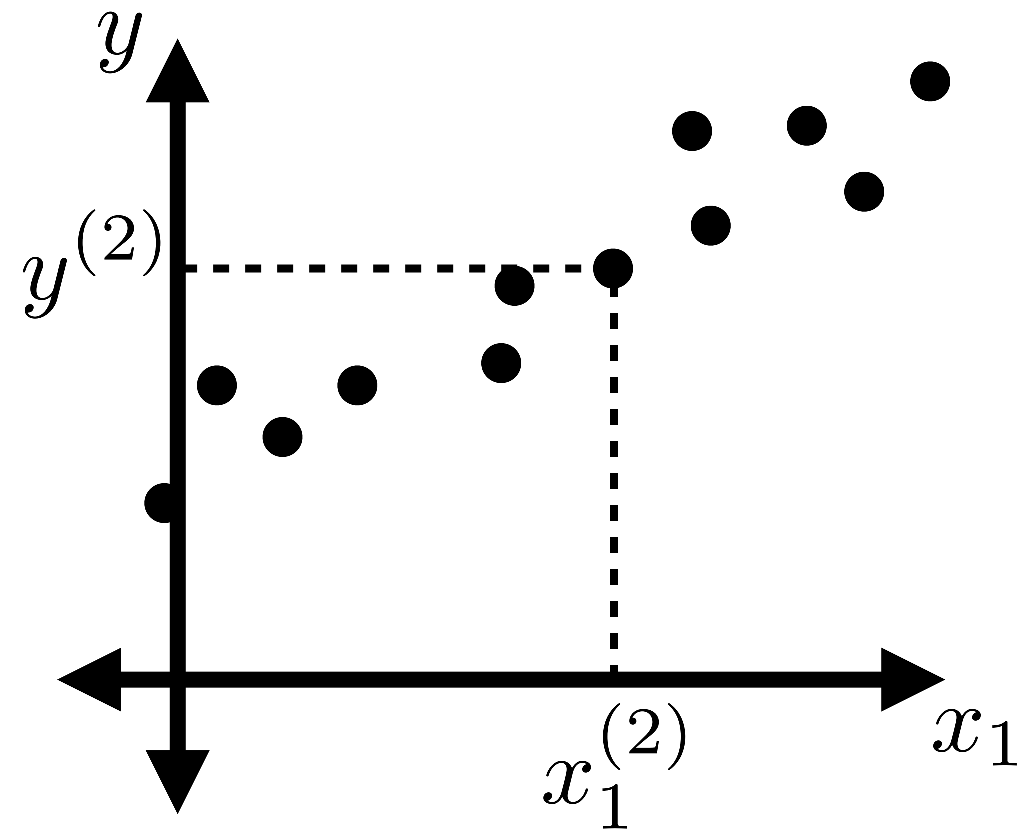

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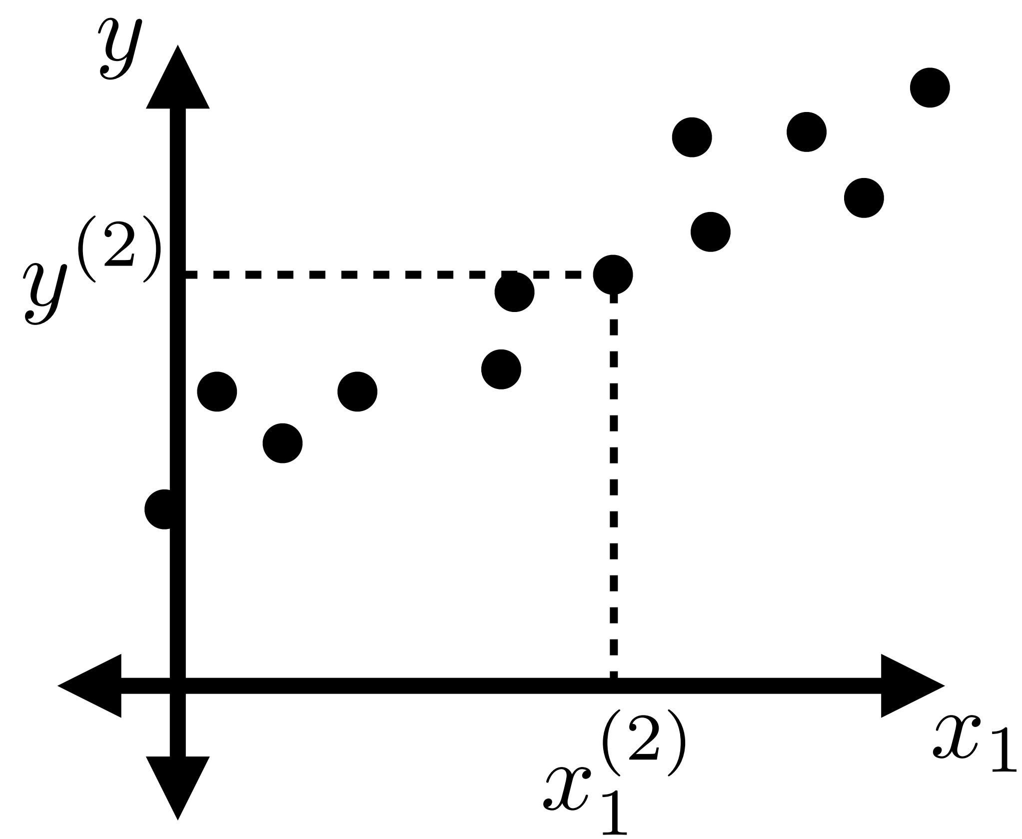

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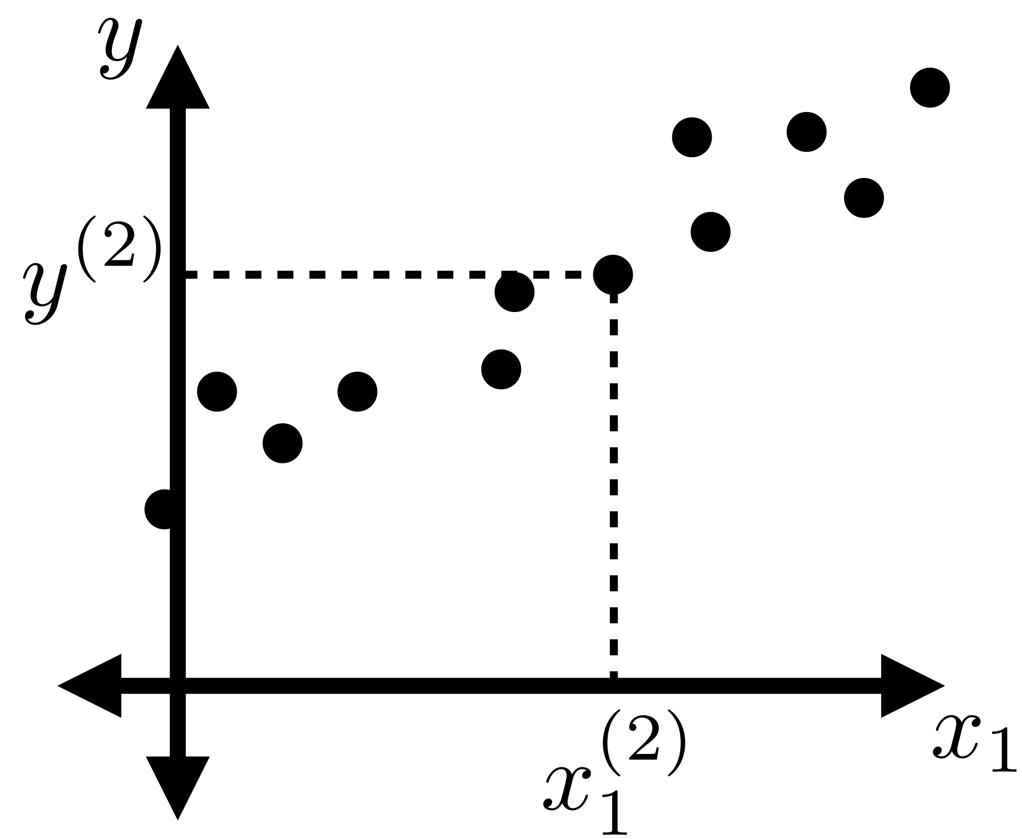

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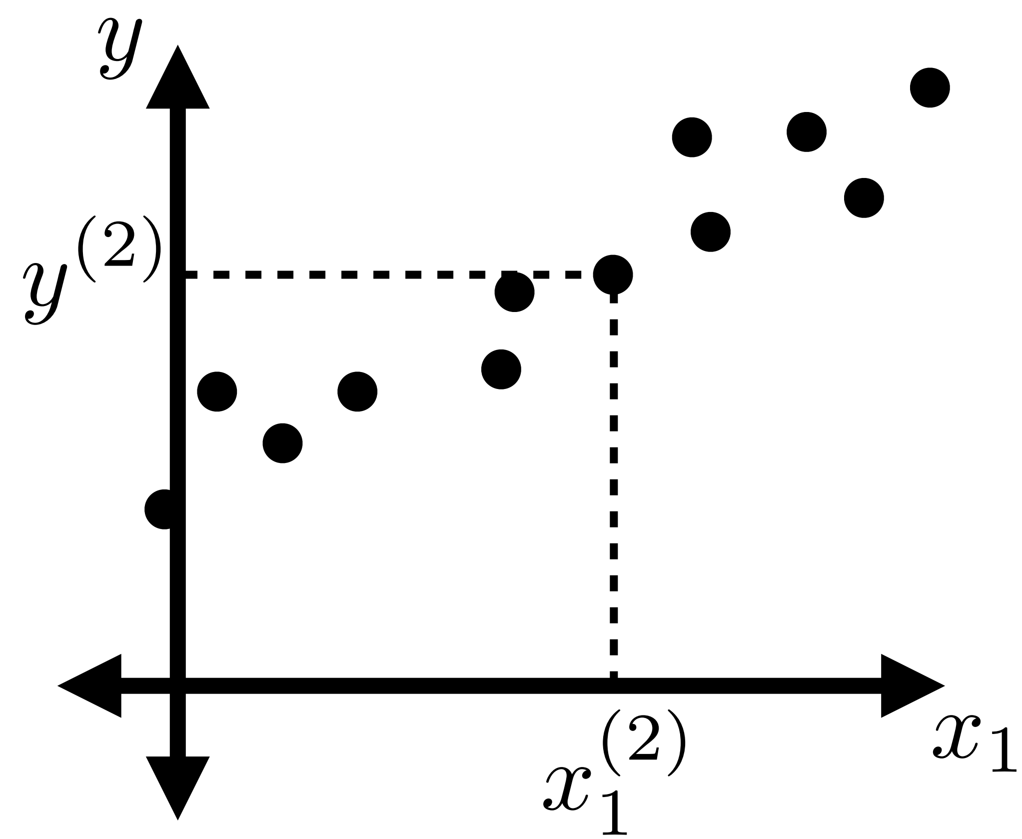

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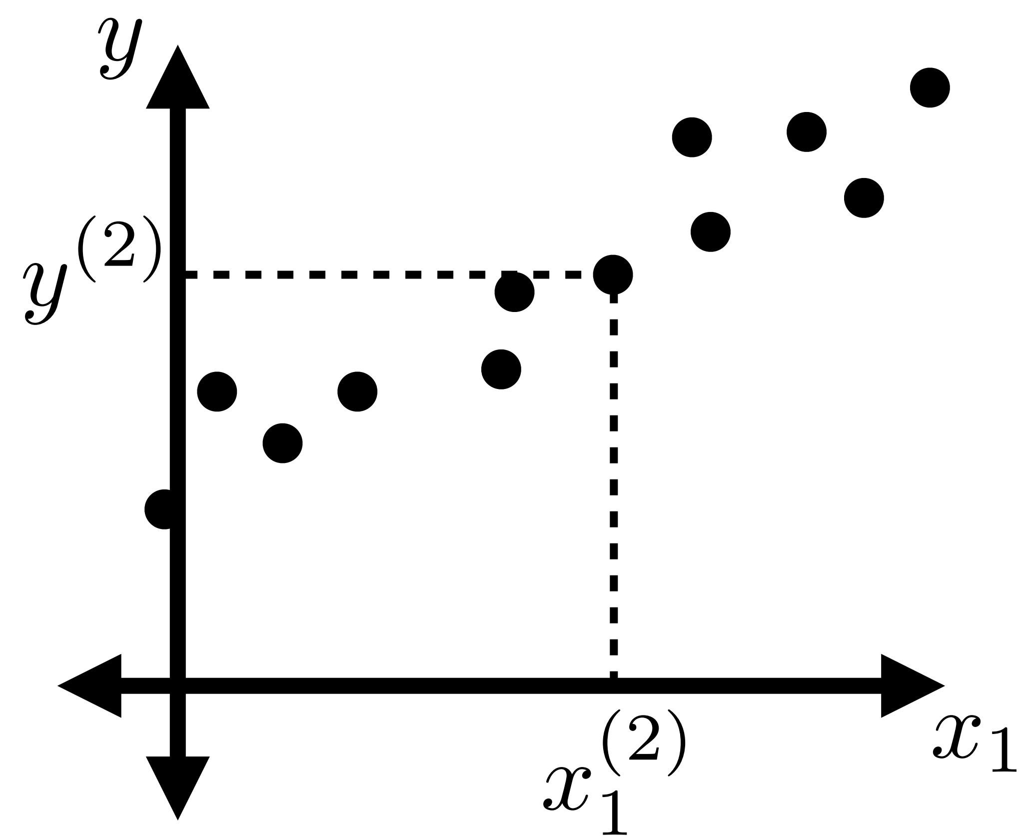

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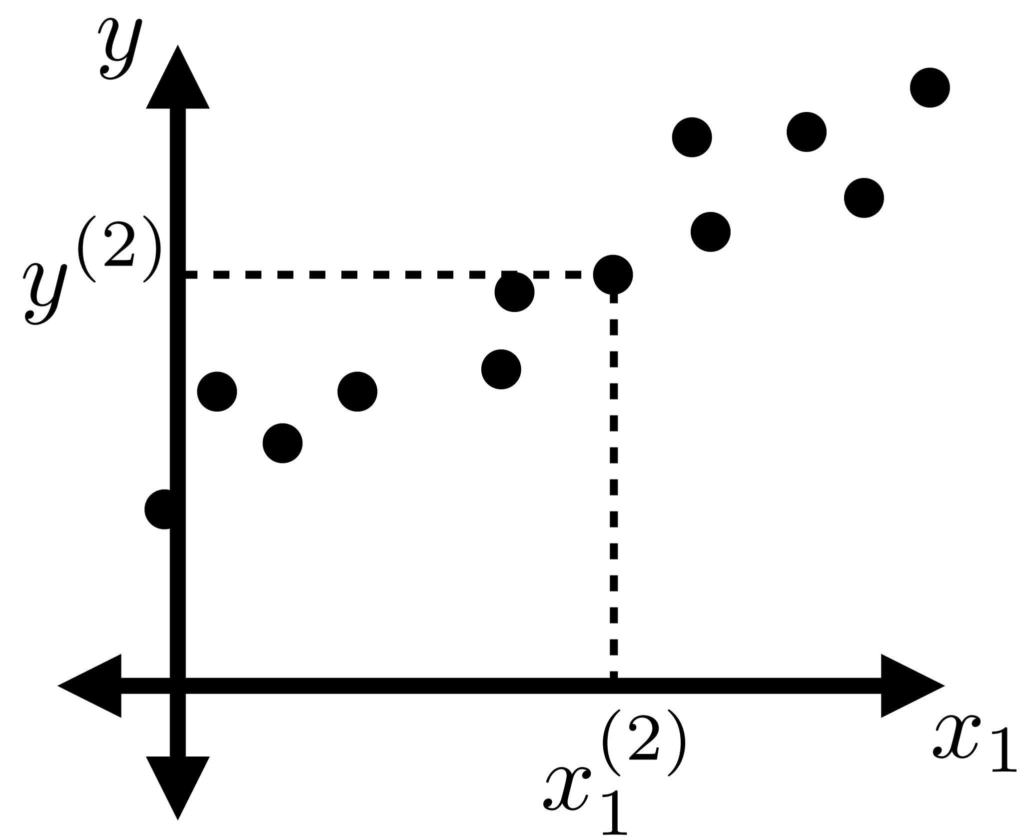

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- How does training error of `Ex_learning_alg`($\mathcal{D}_n$;1) compare to the training error of `Ex_learning_alg`($\mathcal{D}_n$;2)?

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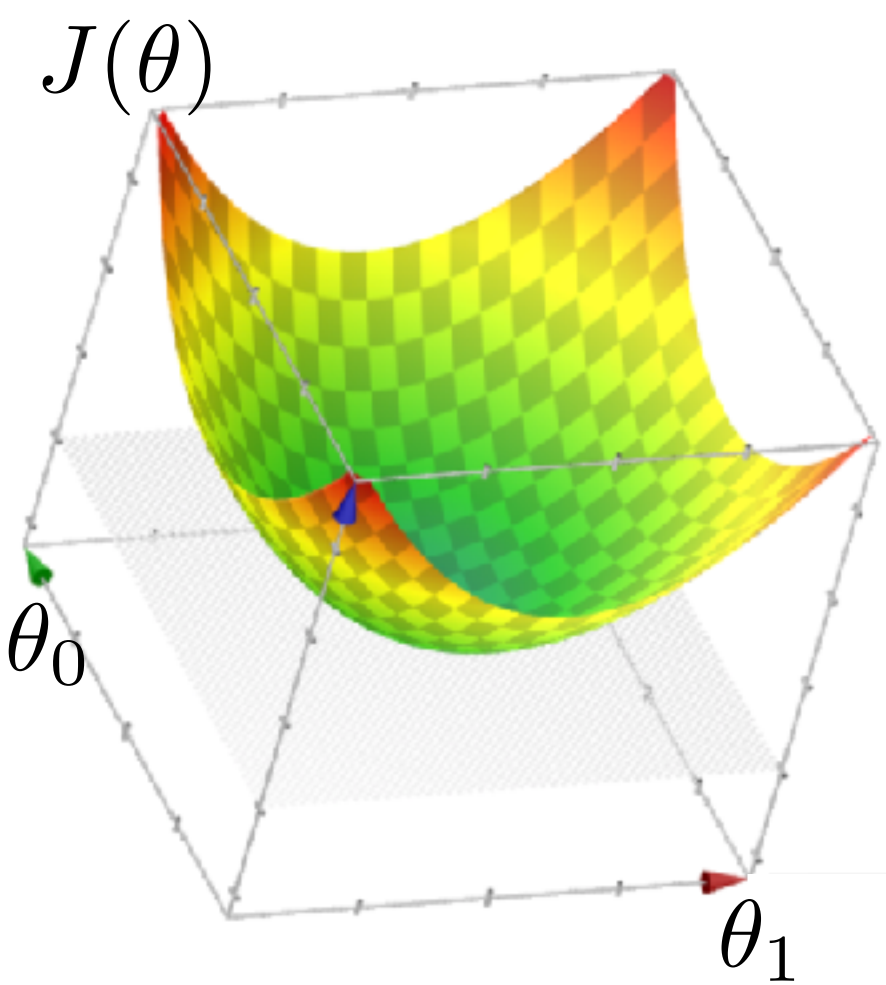

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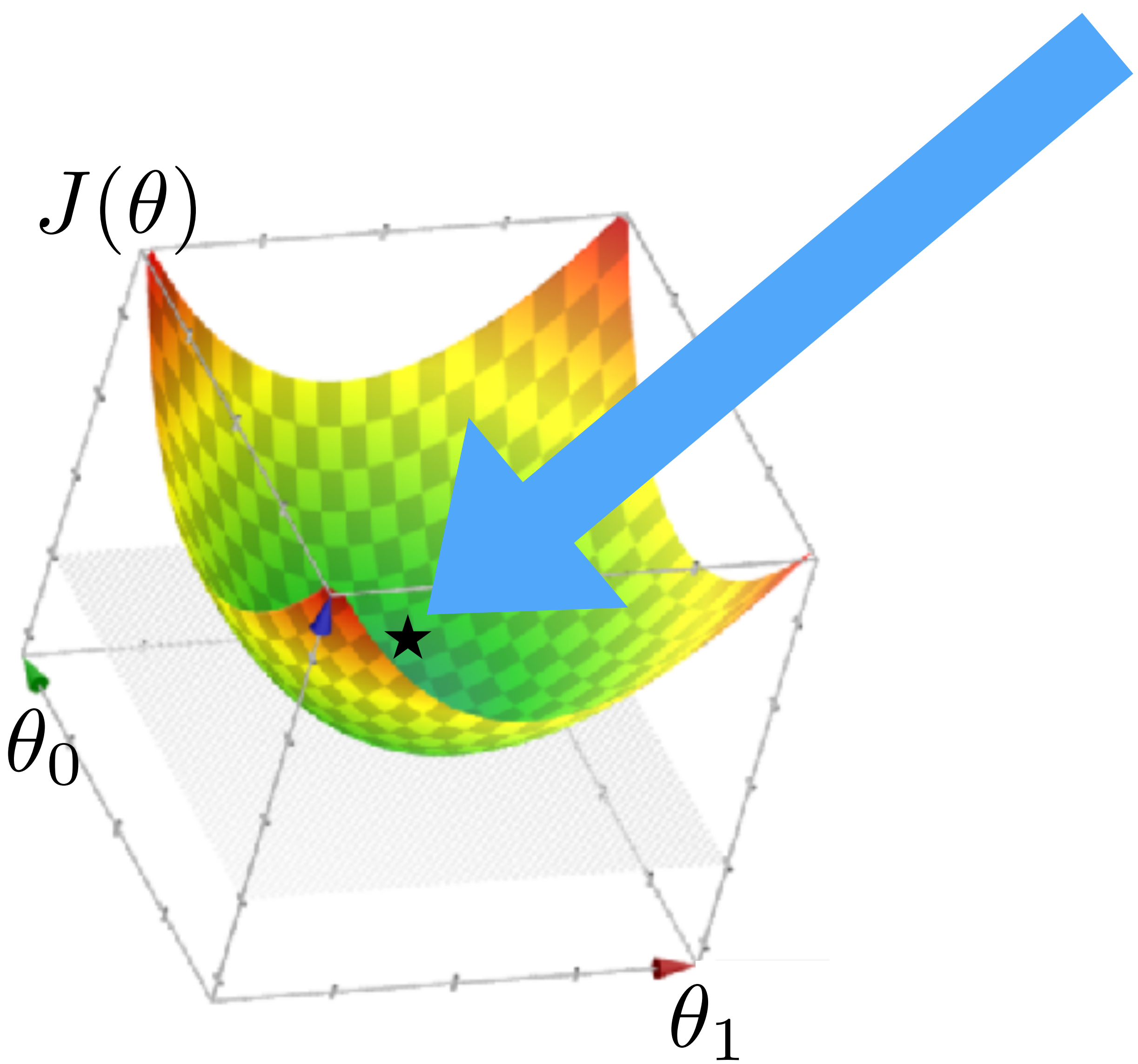

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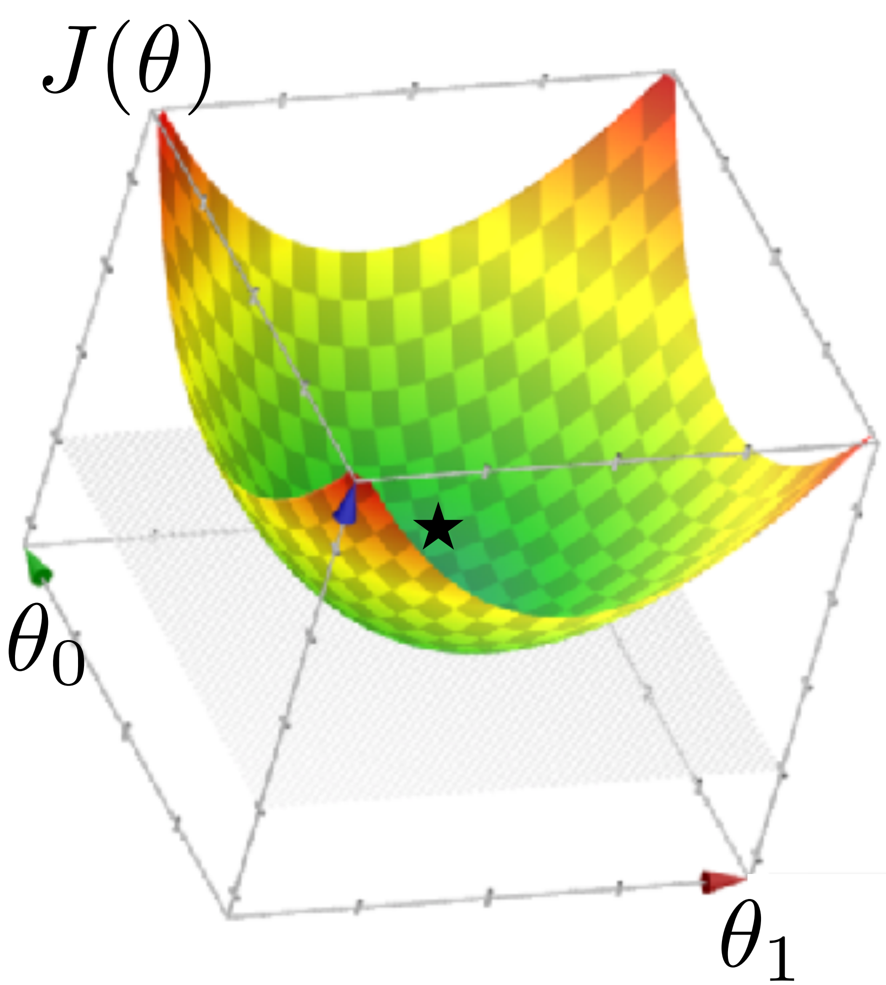

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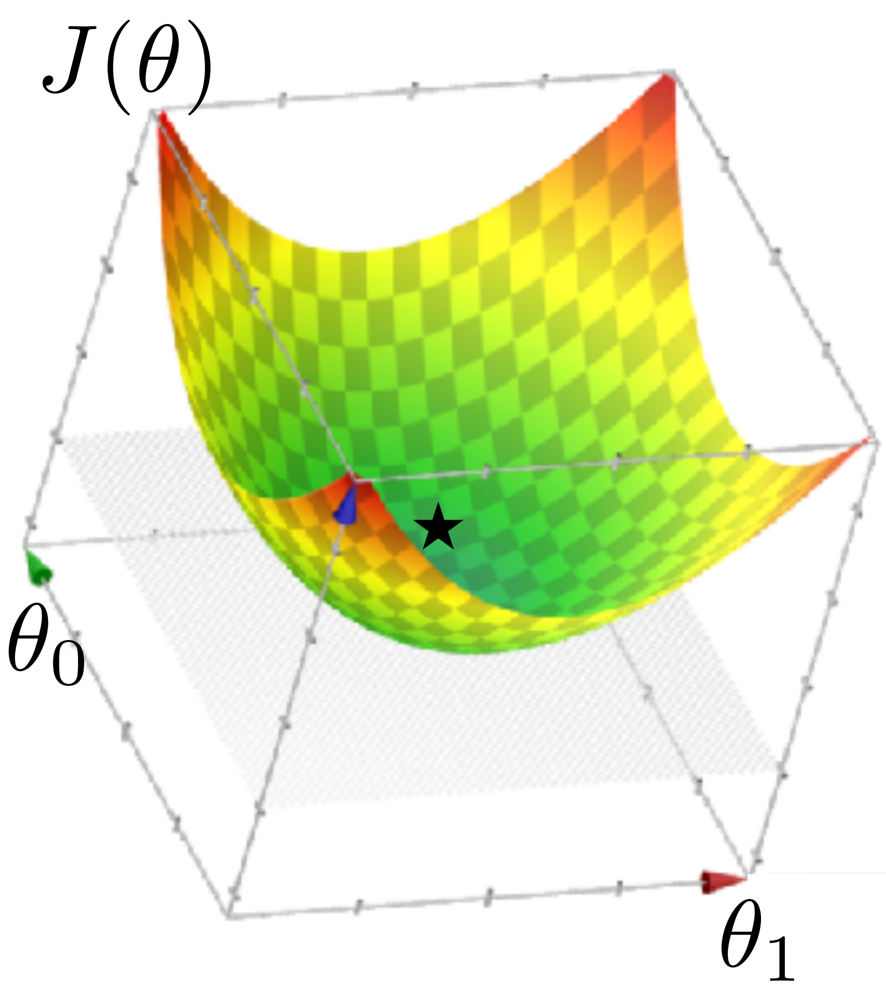

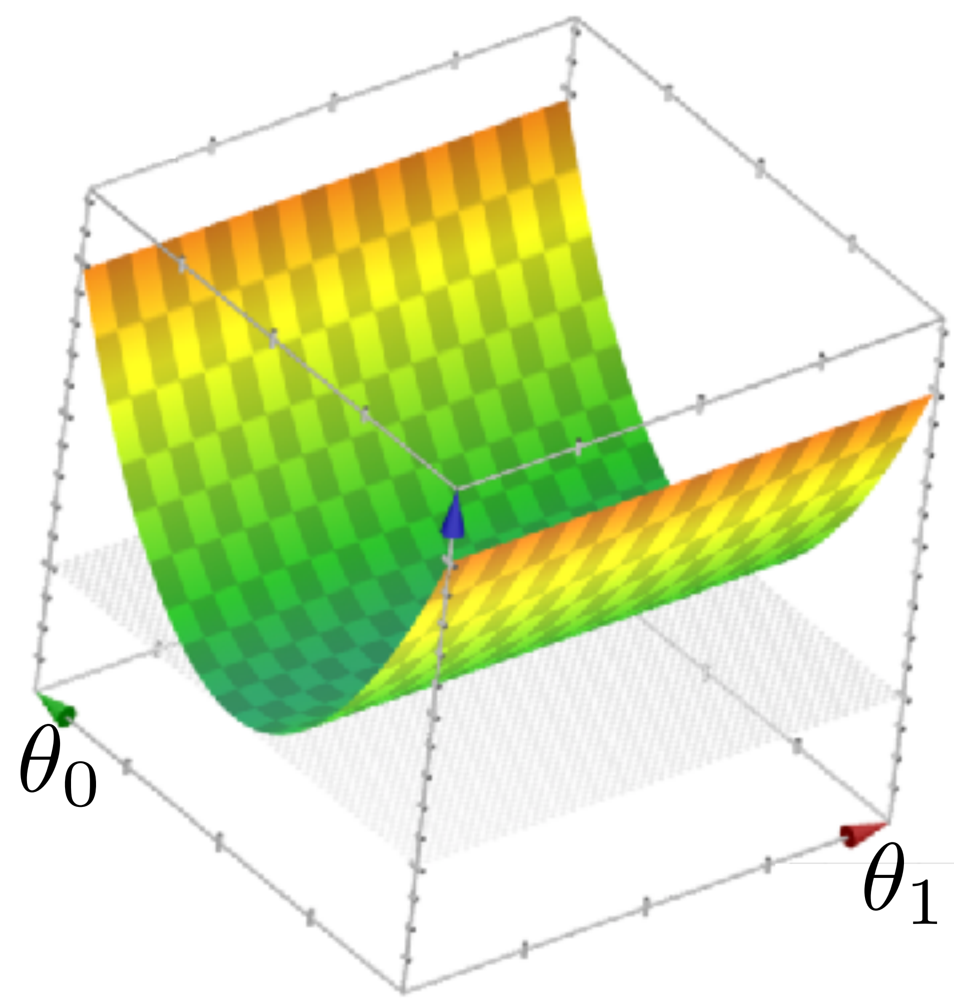

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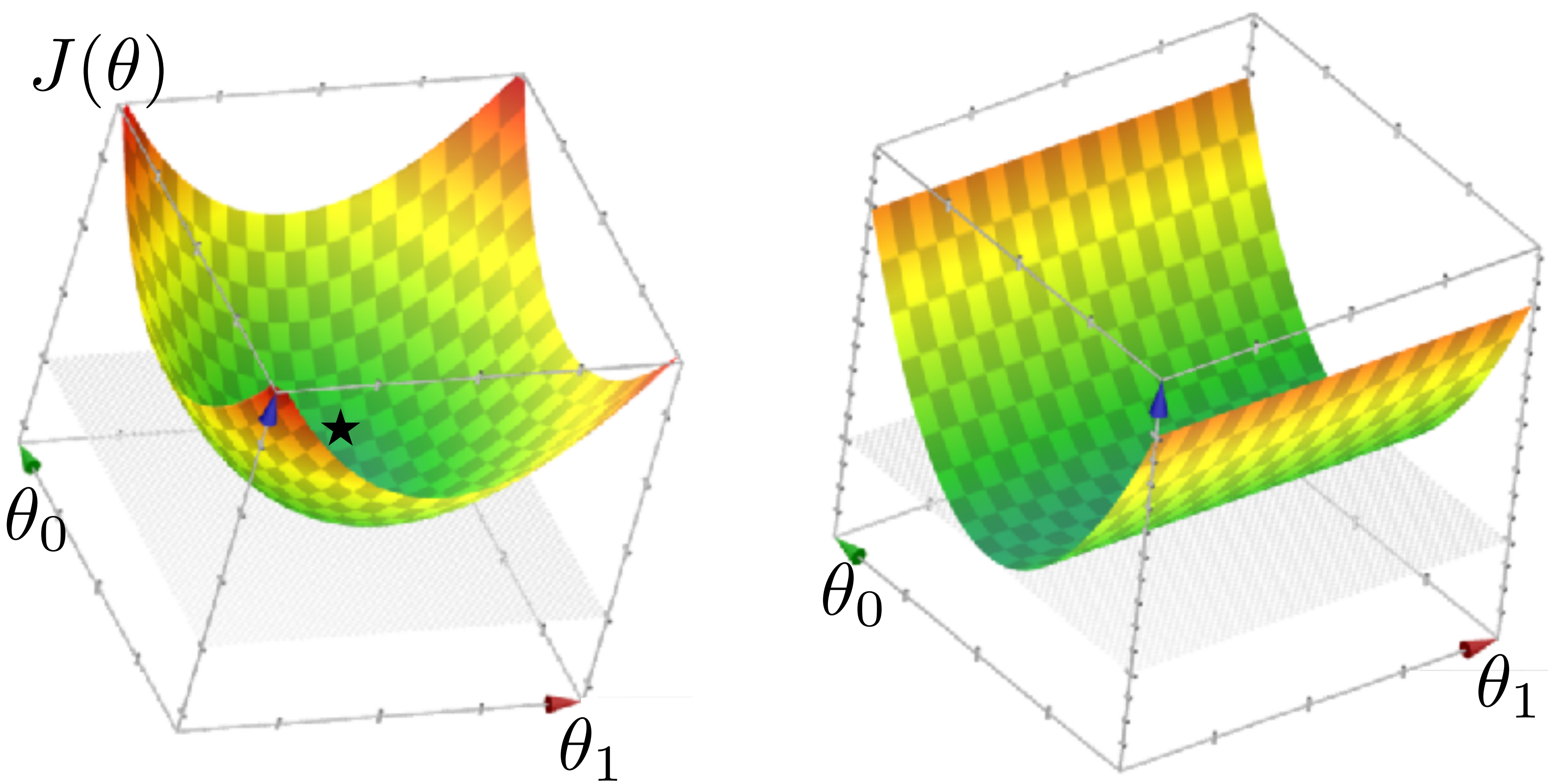

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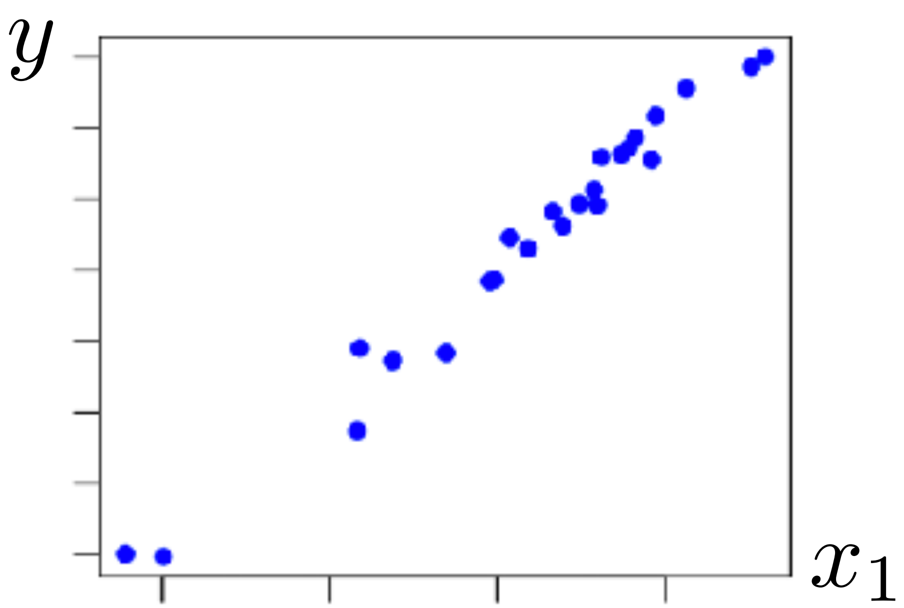

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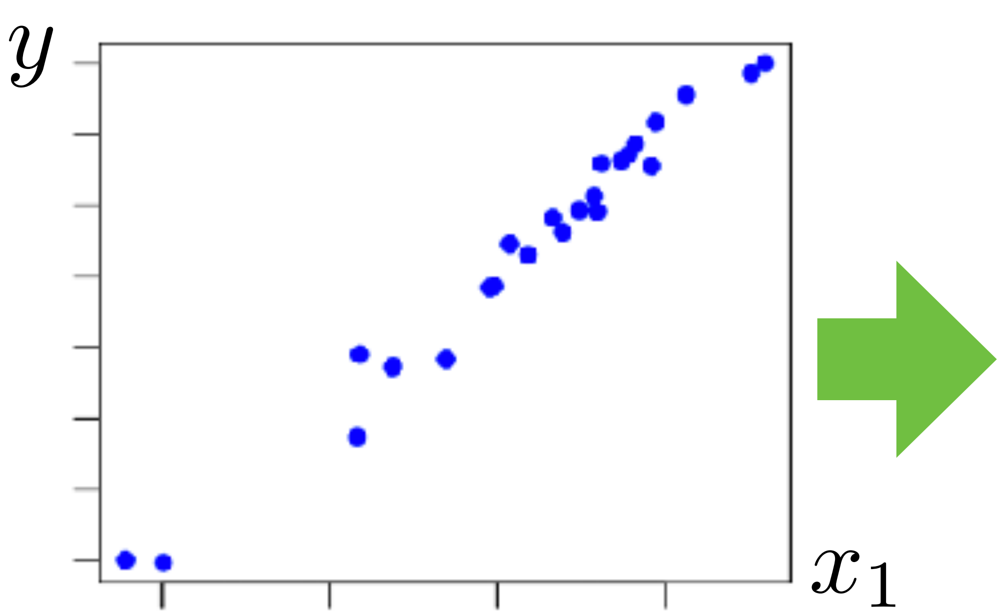

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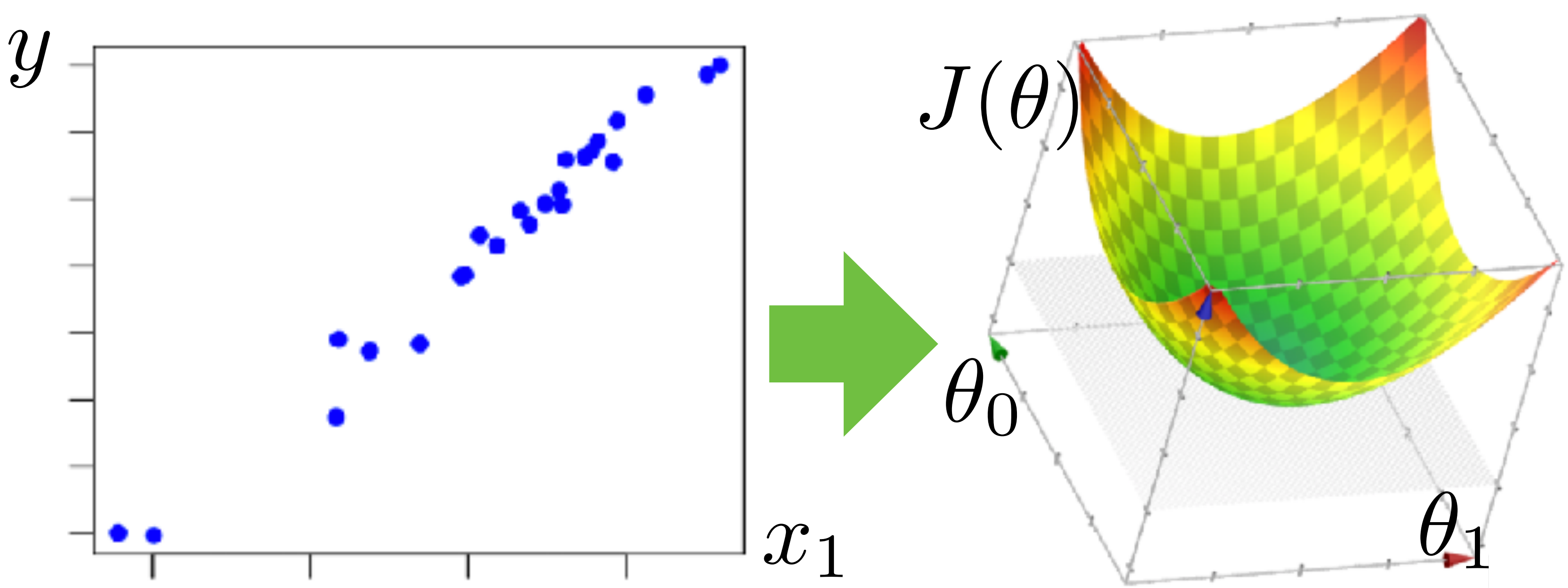

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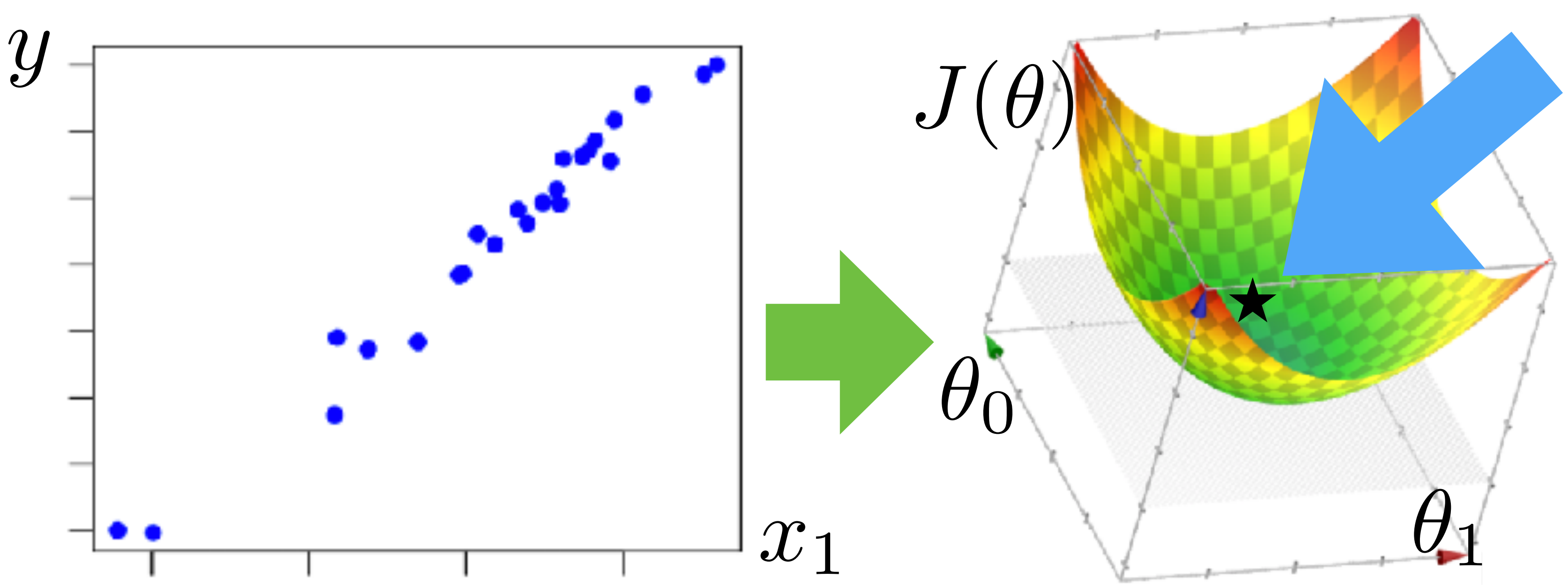

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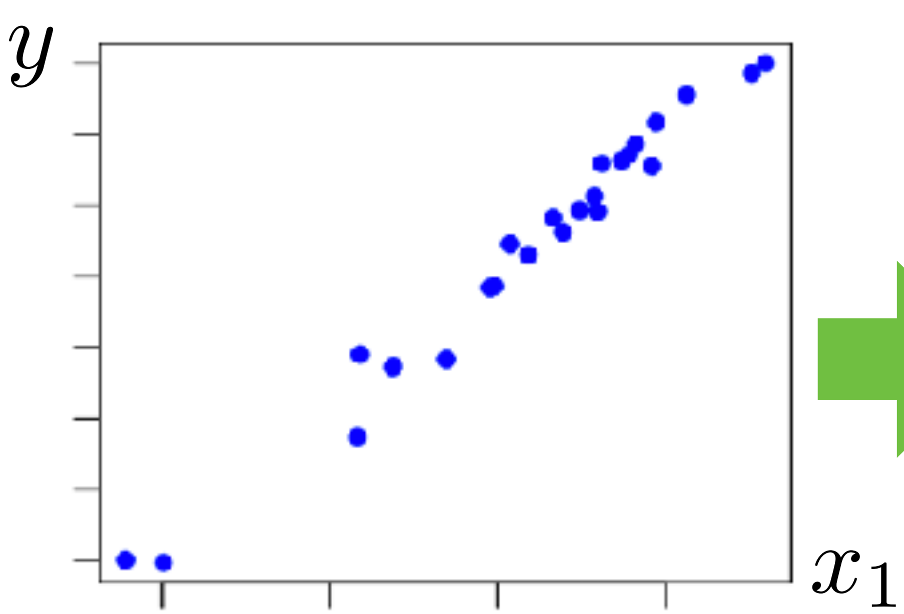

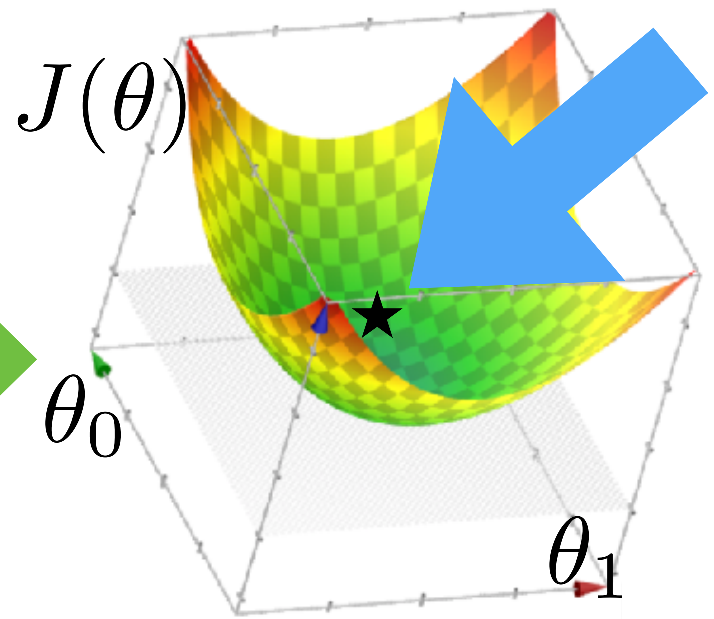

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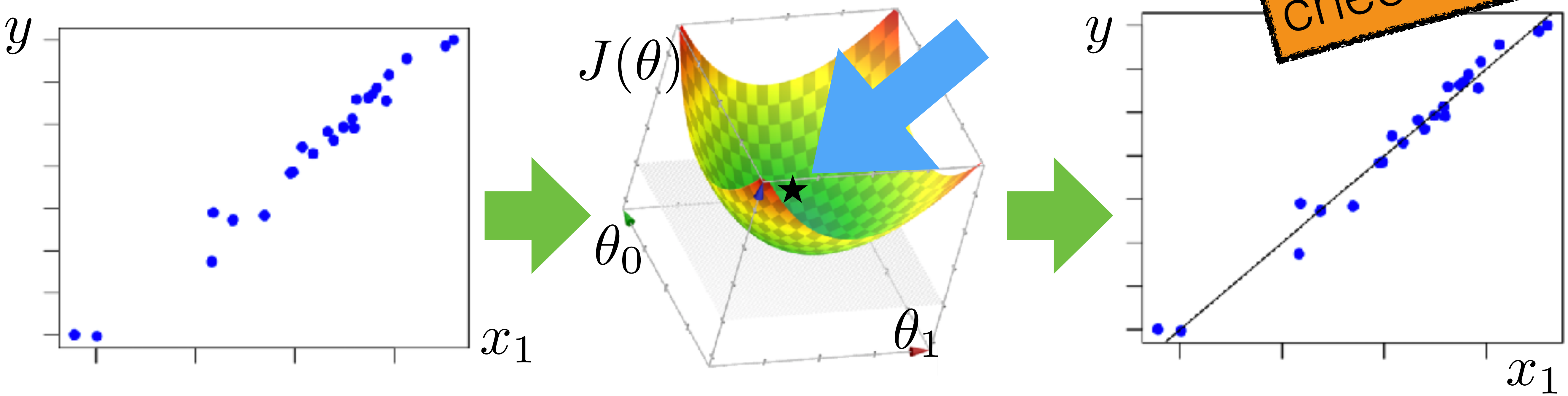

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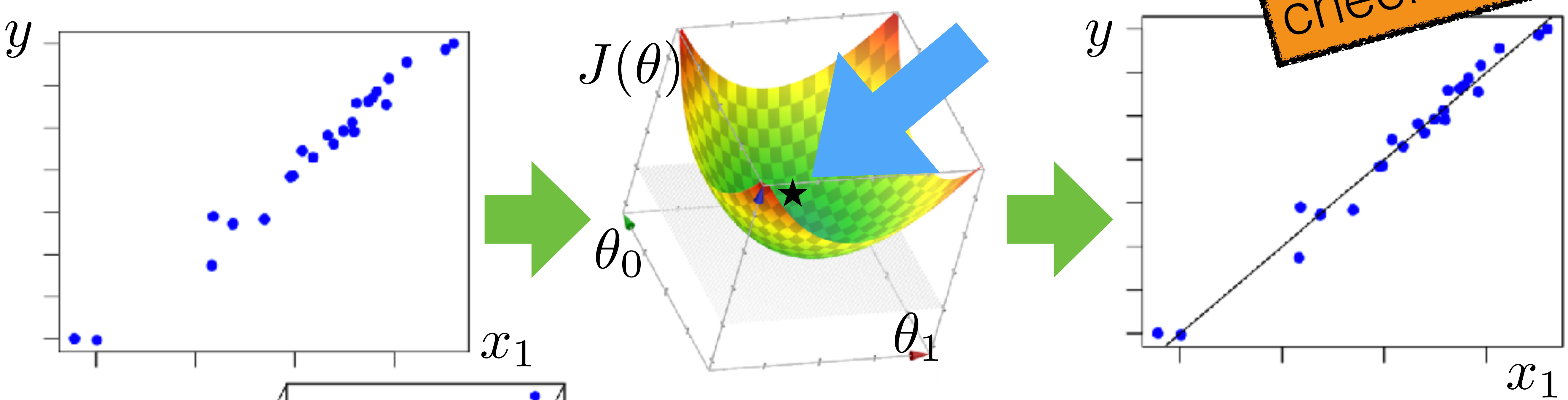

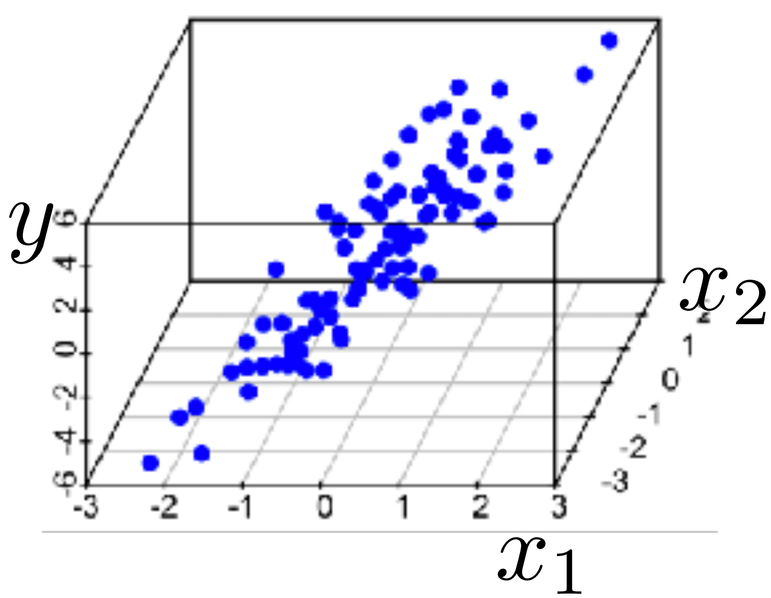

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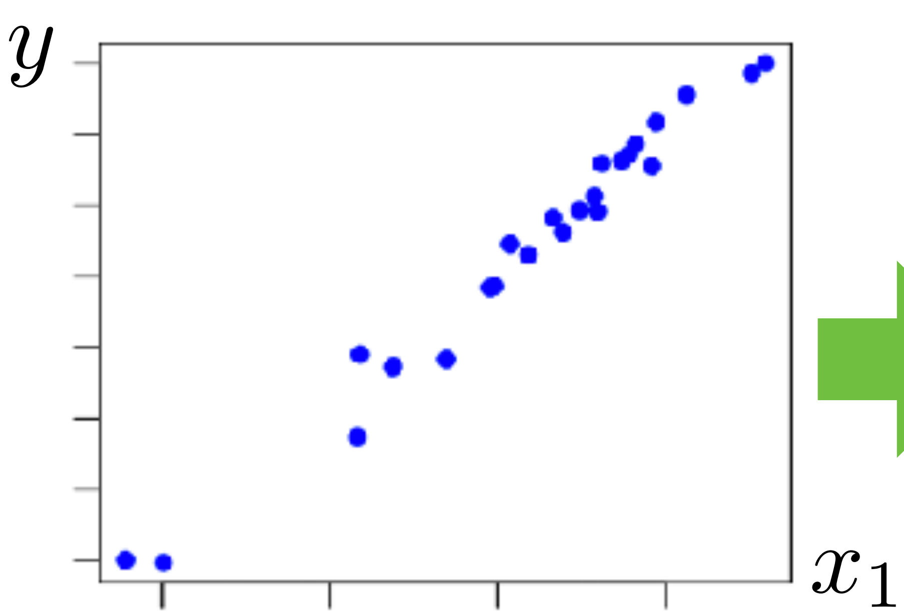

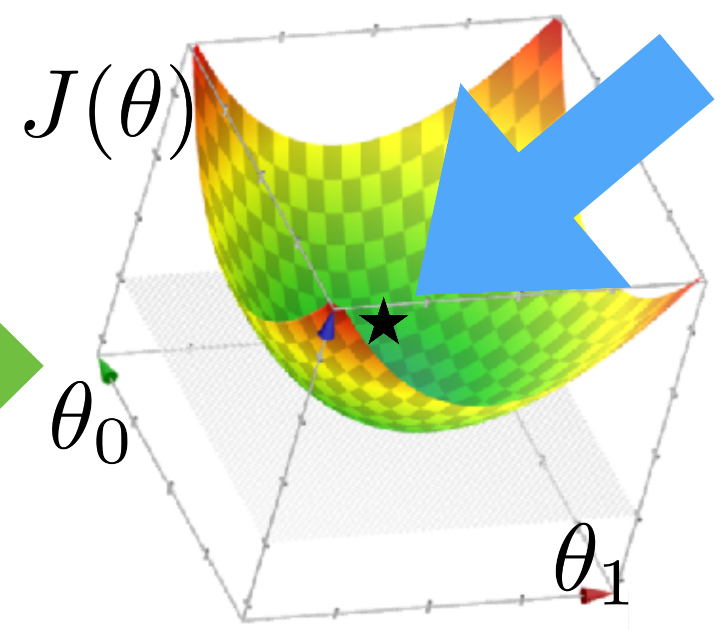

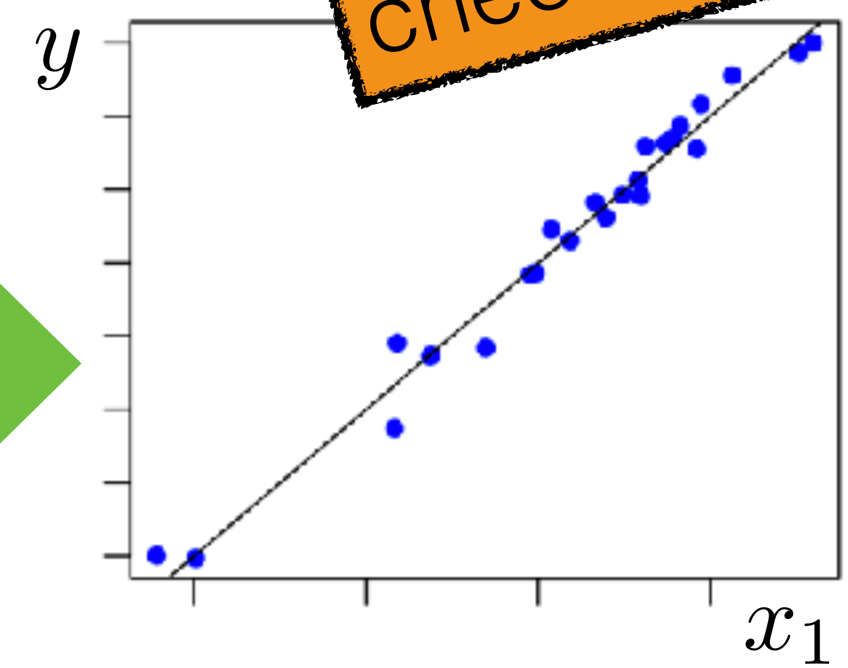

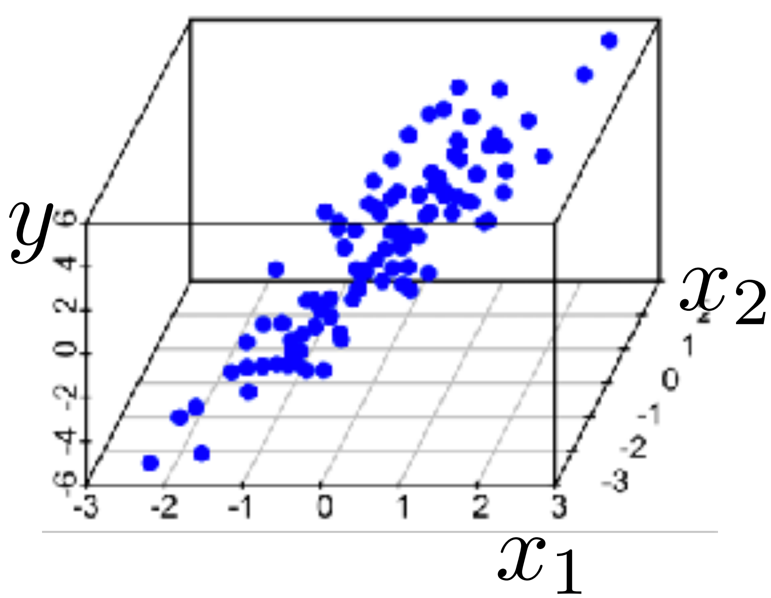

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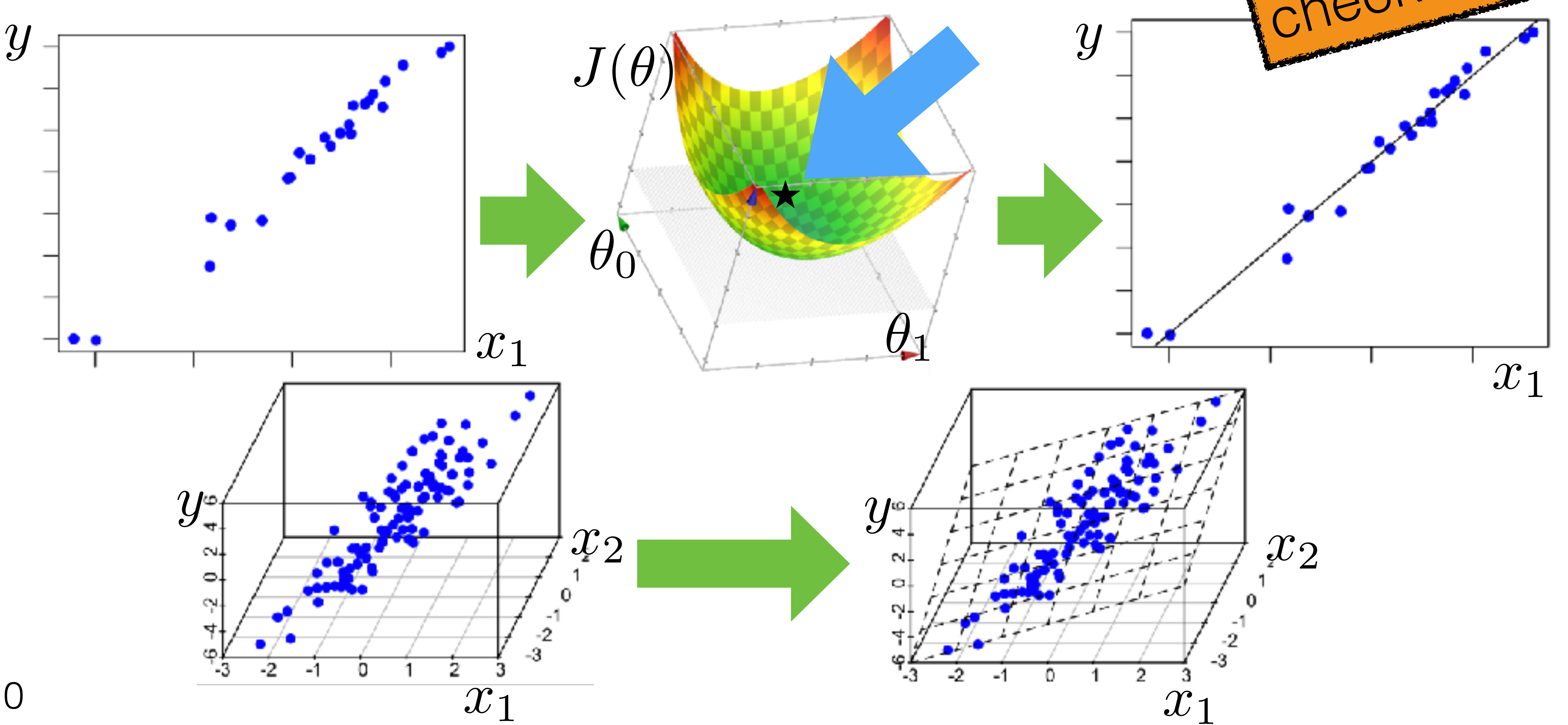

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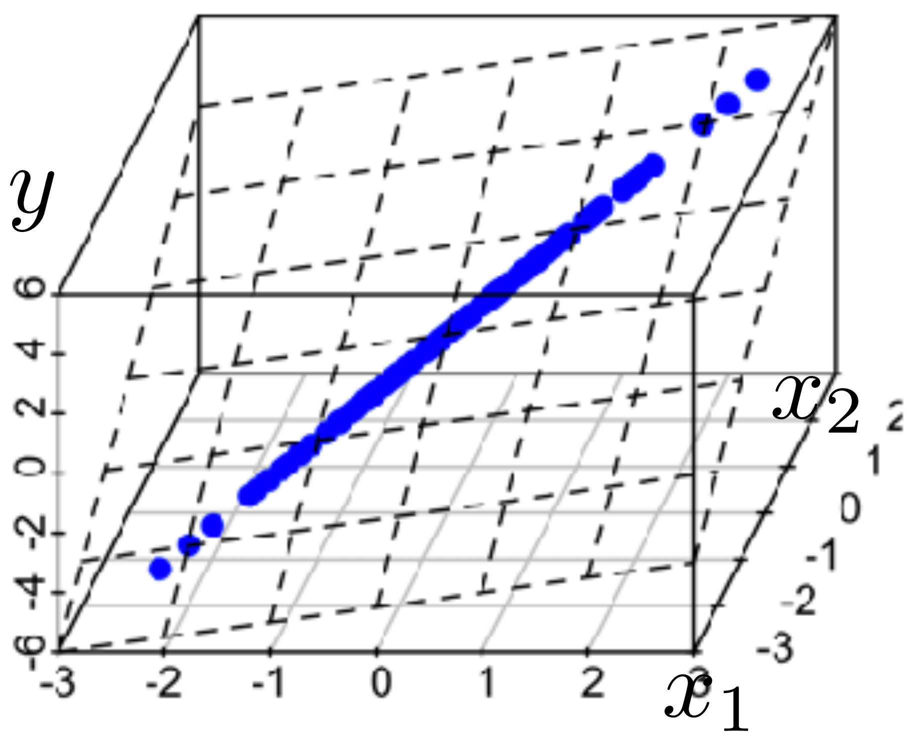

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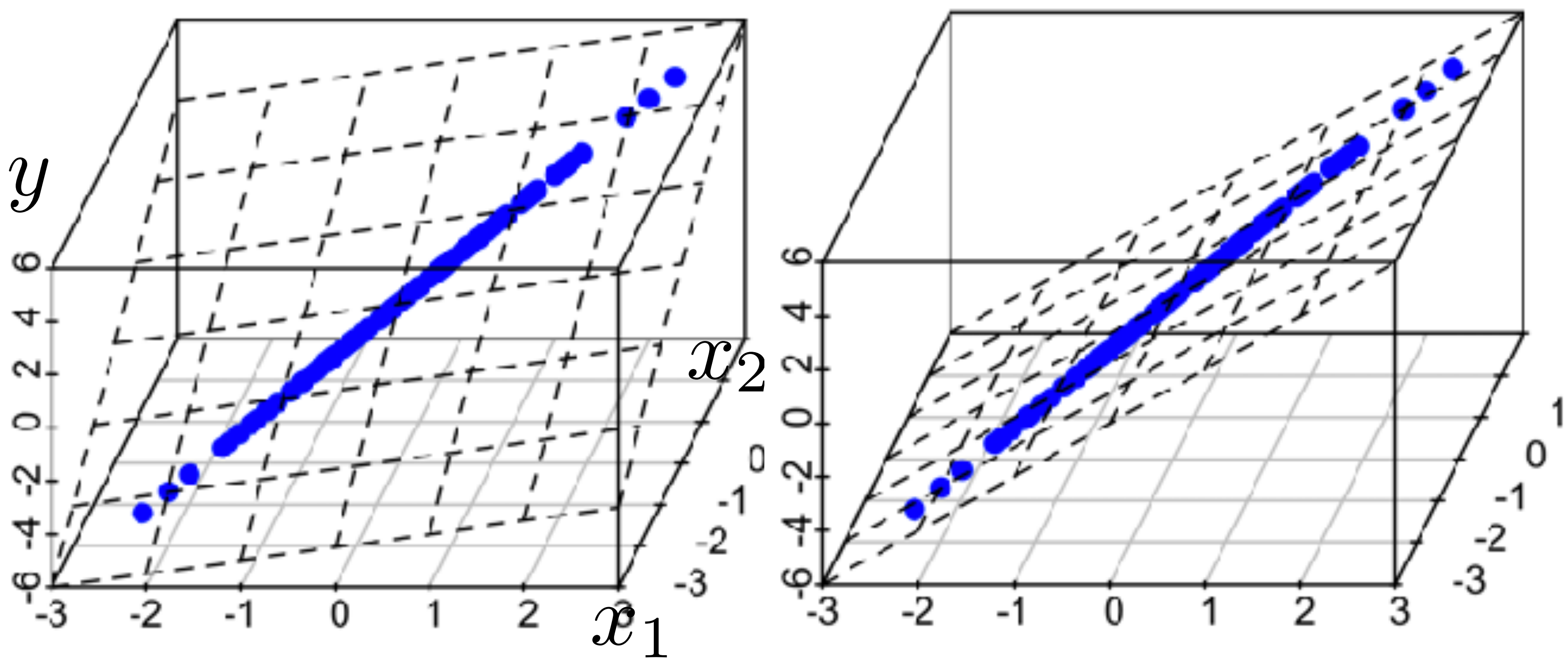

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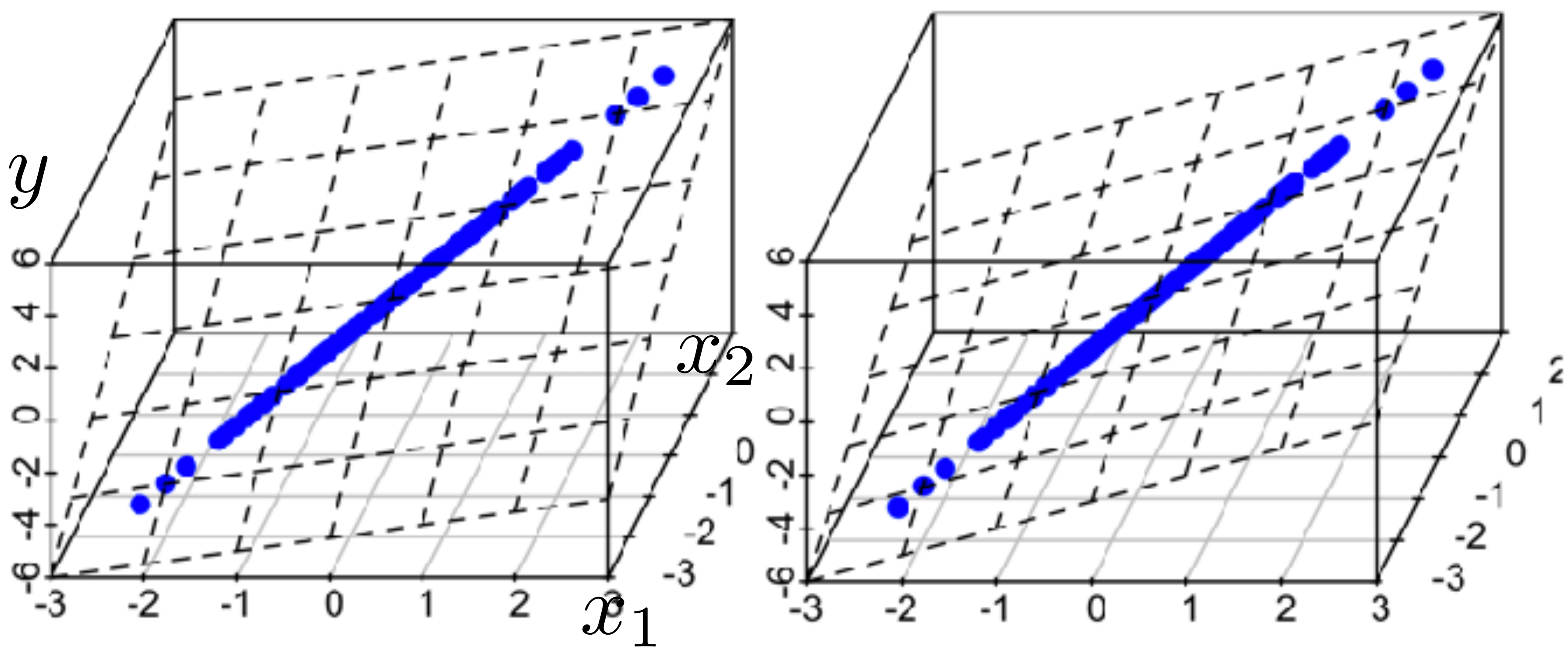

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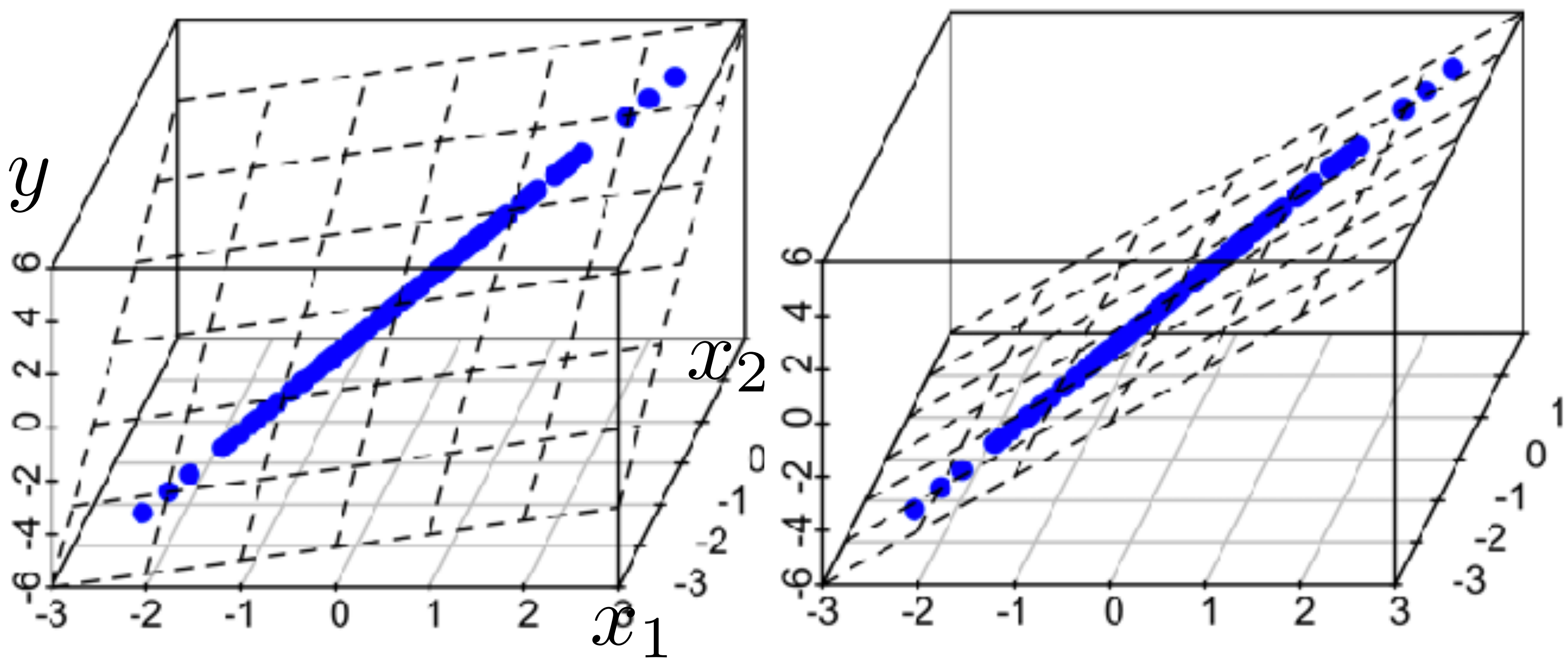

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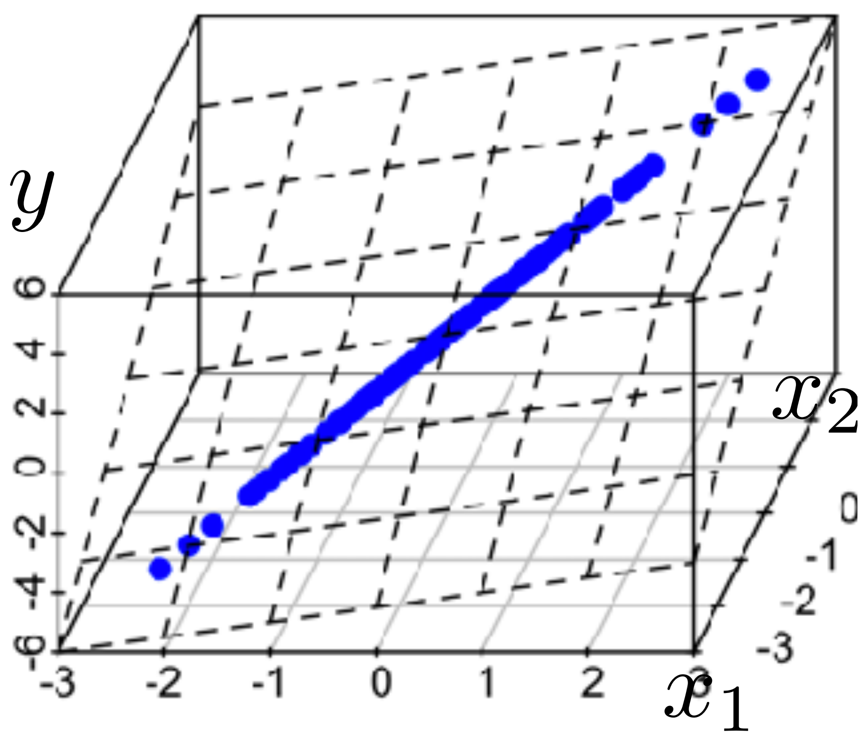

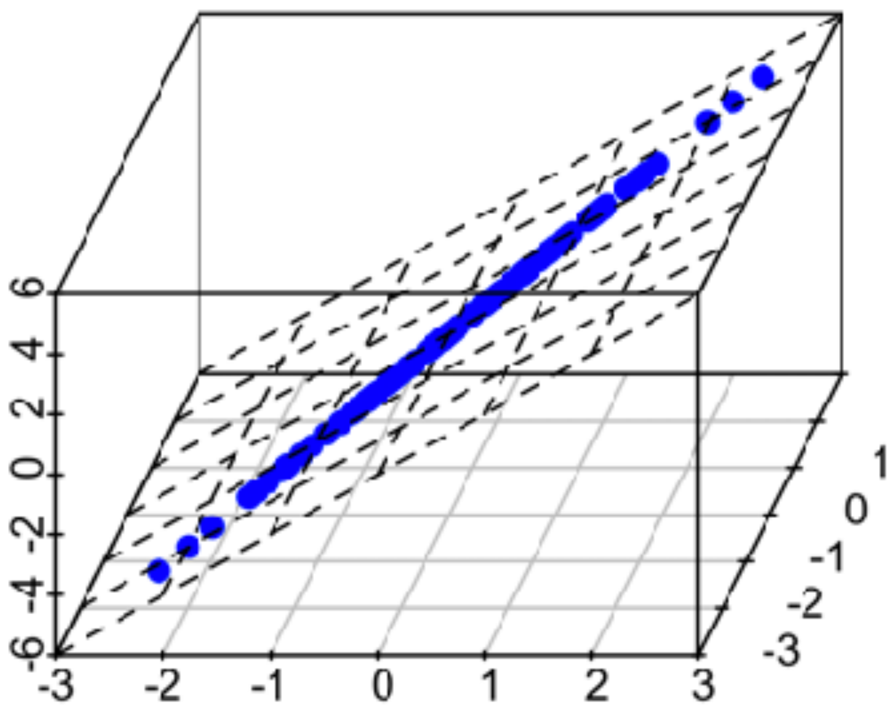

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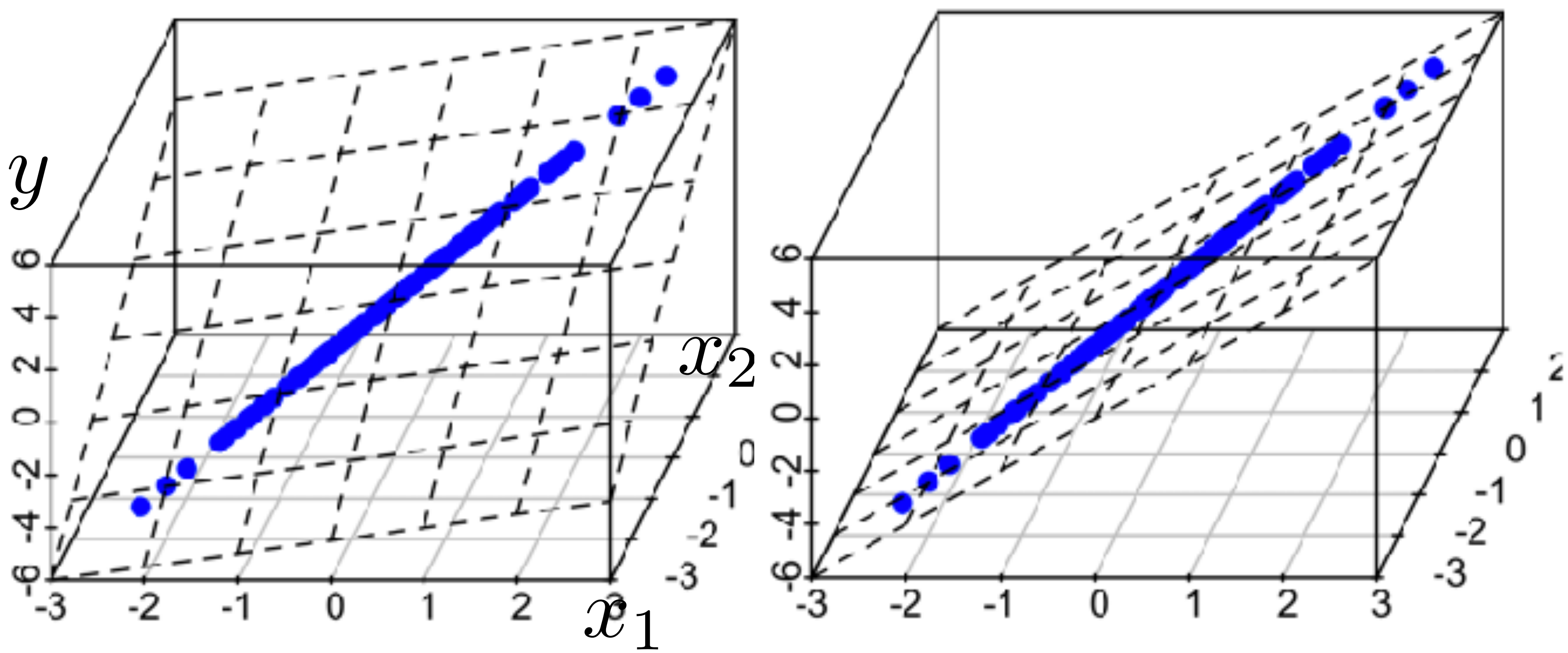

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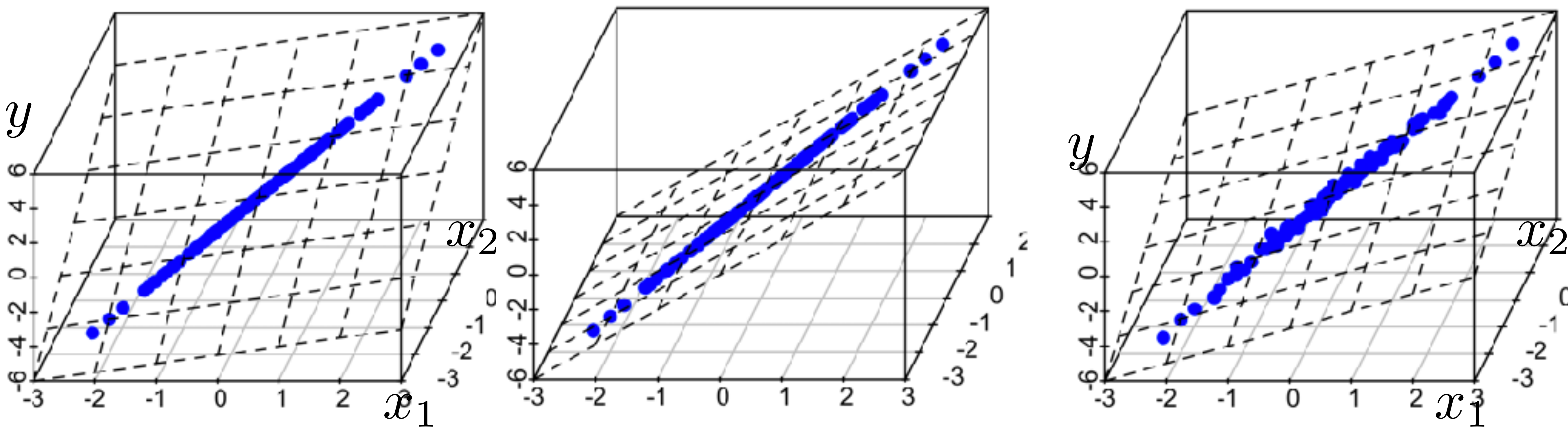

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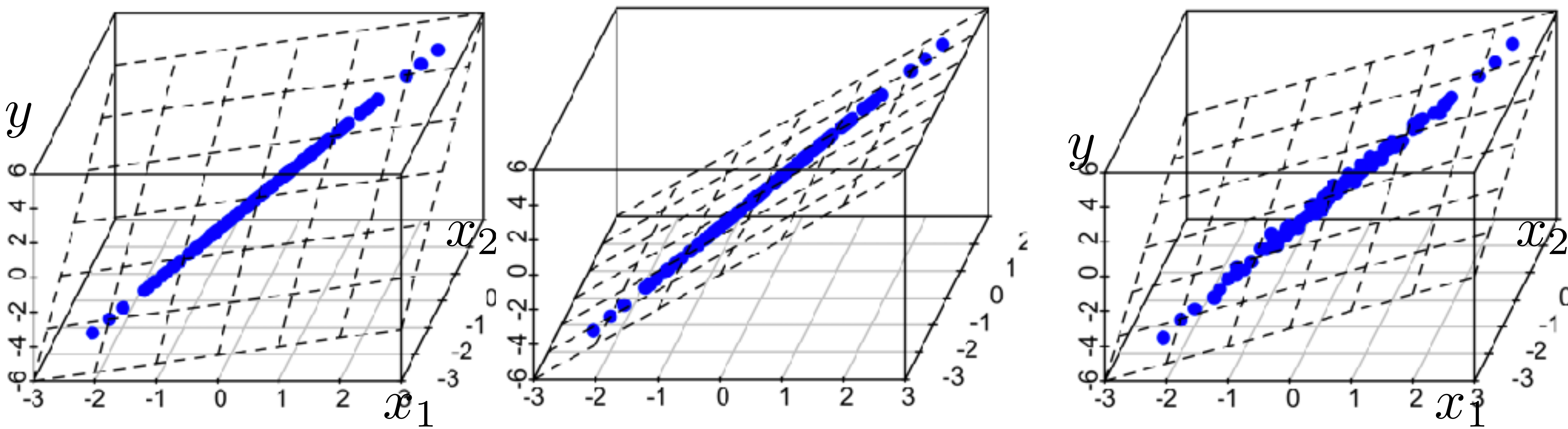

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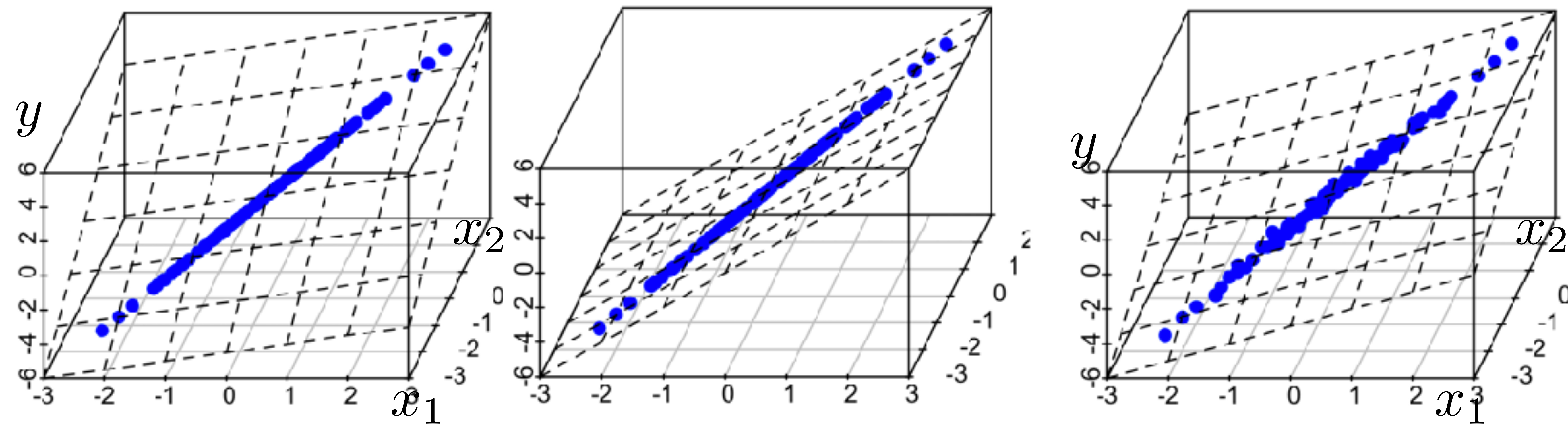

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- How to choose among hyperplanes? Preference for θ components being near zero

Regularizing linear regression

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- Linear regression with square penalty: ridge regression

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- Can think of λ as hyperparameter of a learning algorithm
- How to choose λ? One option: cross validation (see HW!)