6.036: Introduction to Machine Learning

Lecture start: Tuesdays 9:35am

Who's talking? Prof. Tamara Broderick

Questions? Ask on Piazza: "lecture (week) 3" folder

Materials: slides, video will all be available on Canvas

Live Zoom feed: https://mit.zoom.us/j/94238622313

Last Time

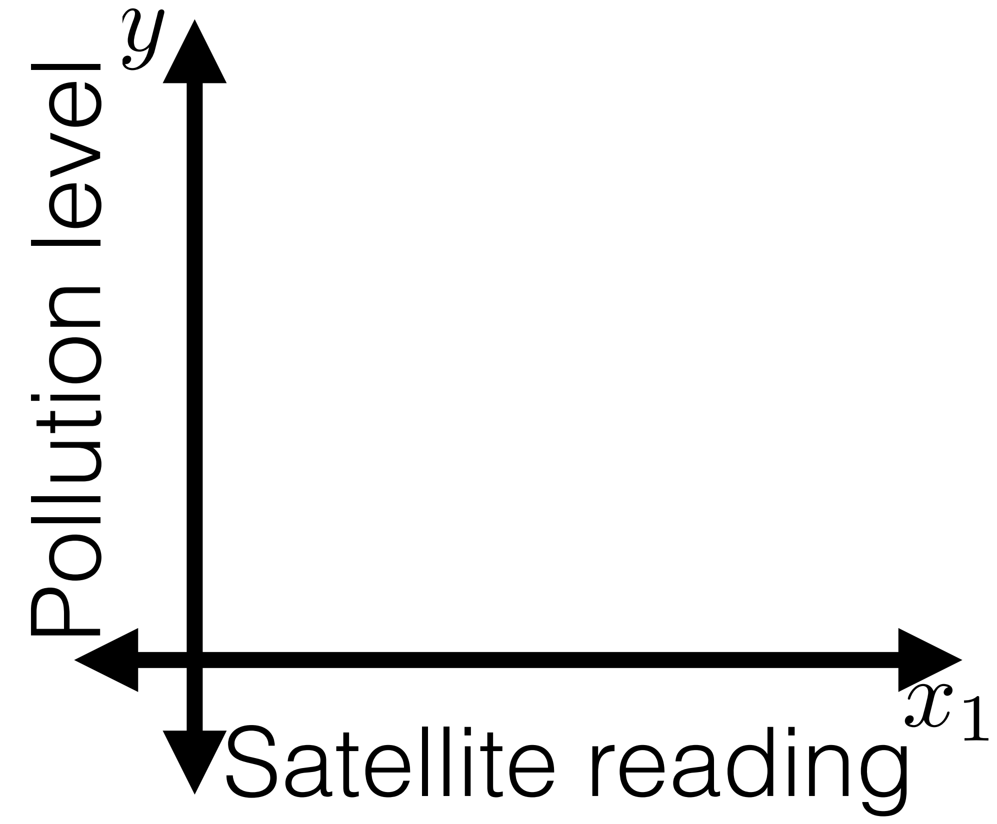
I. Machine learning setup
II. Linear regression
III. Regularization

Today's Plan

I. Gradient descent
II. Stochastic gradient descent (SGD)

Recall

Recall



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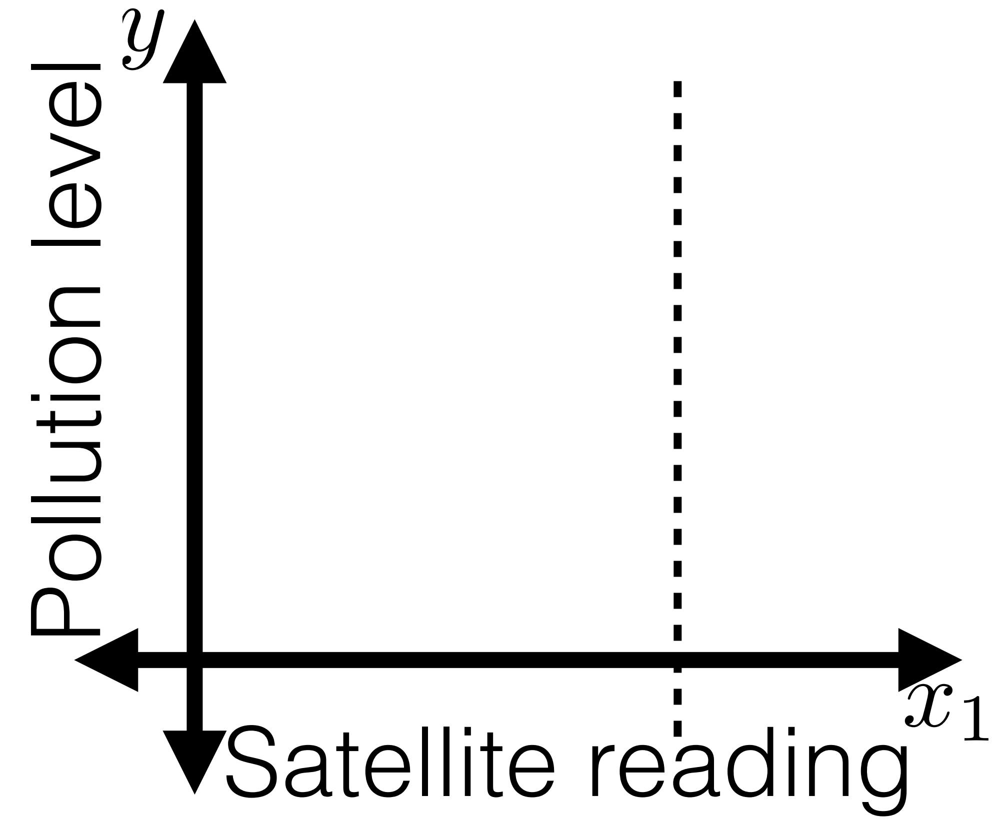

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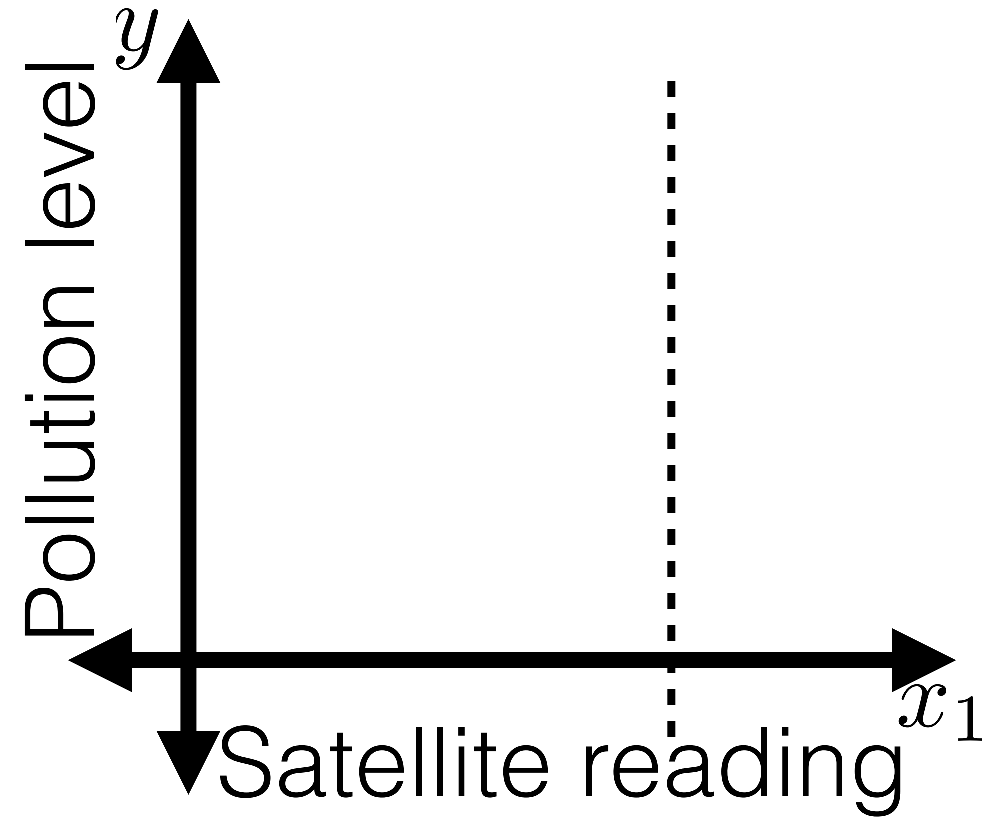

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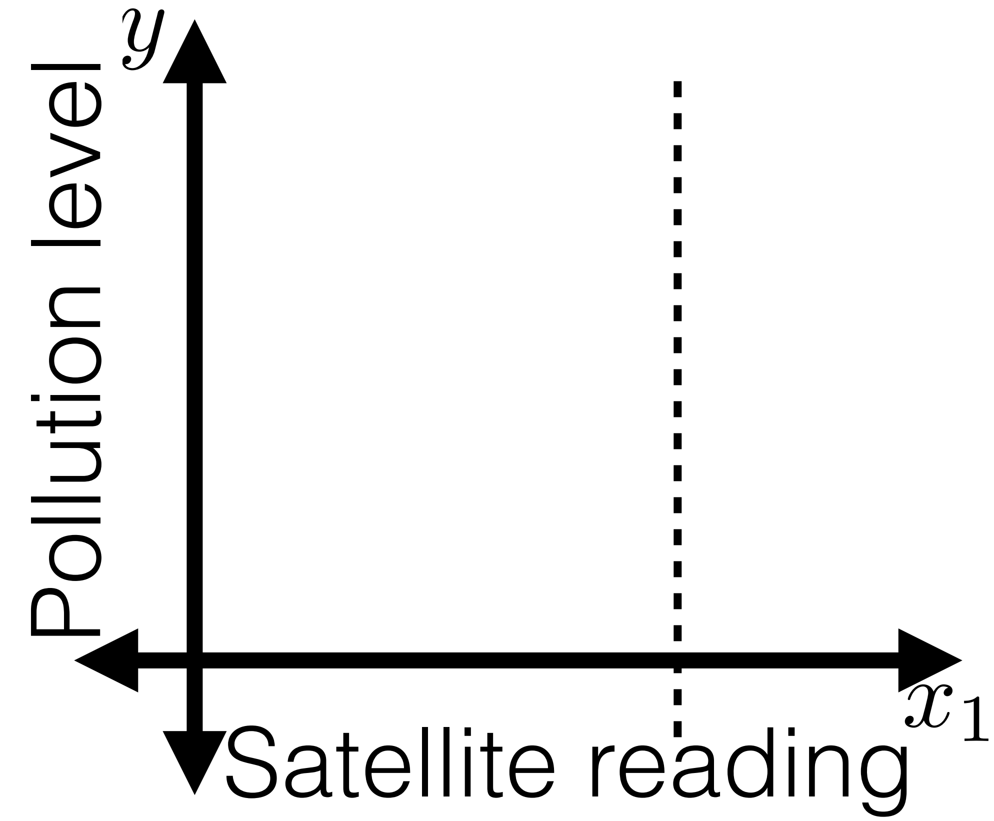

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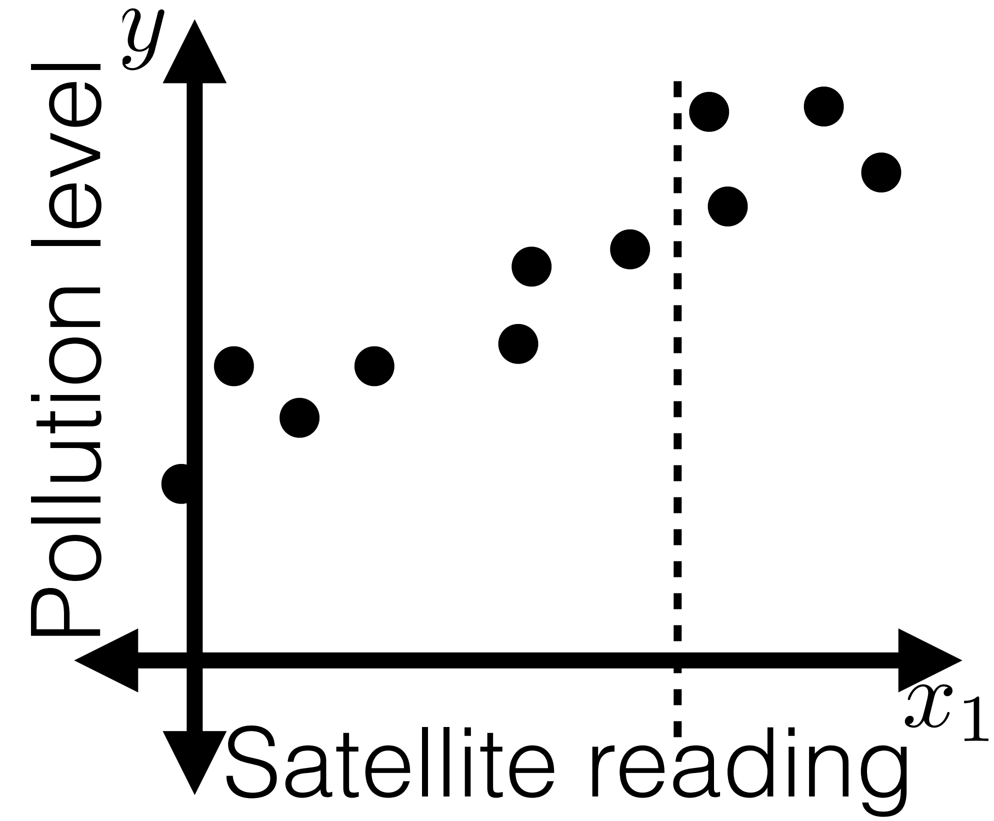

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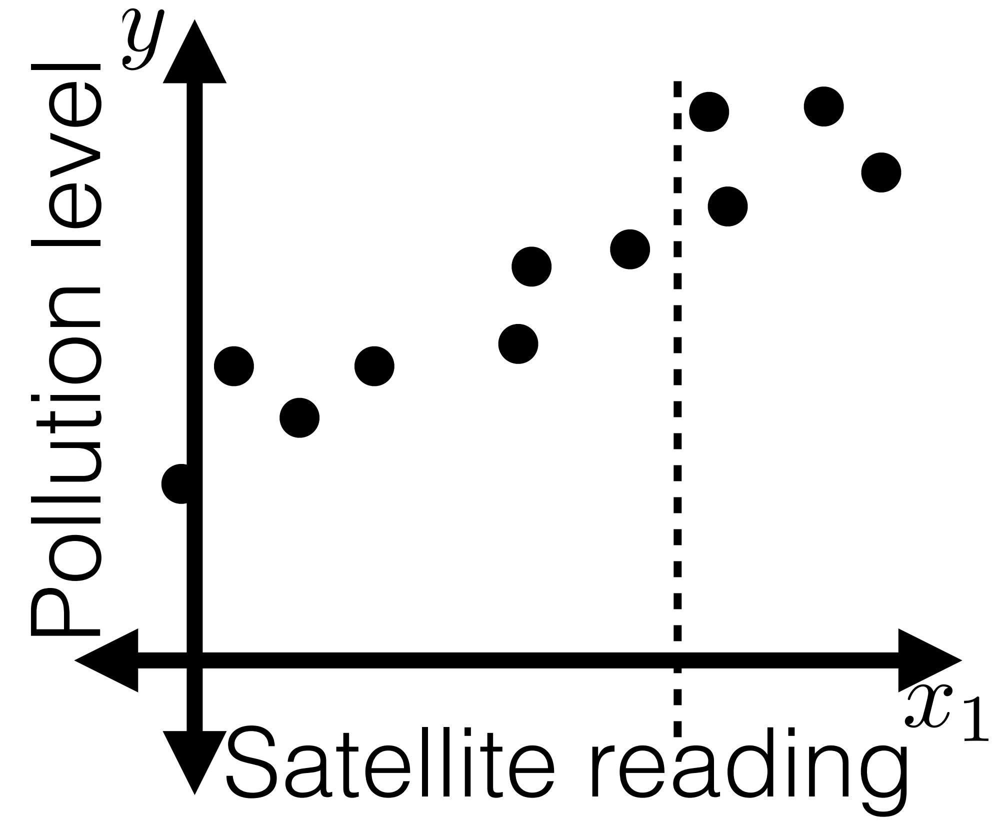

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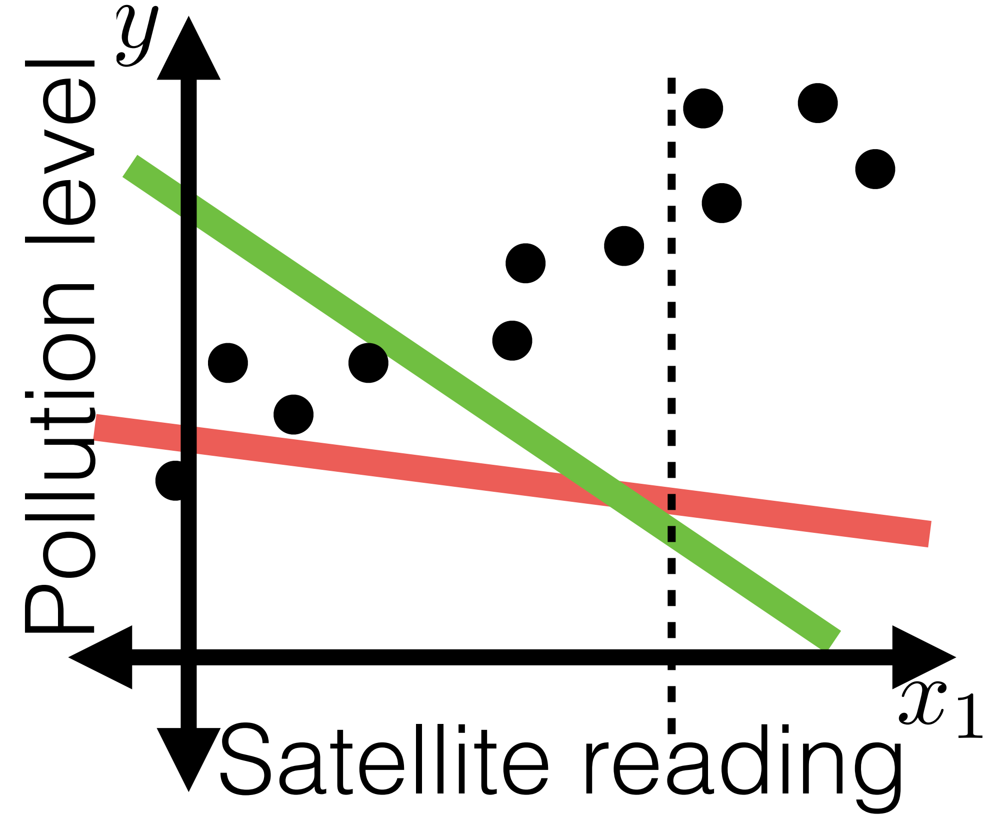

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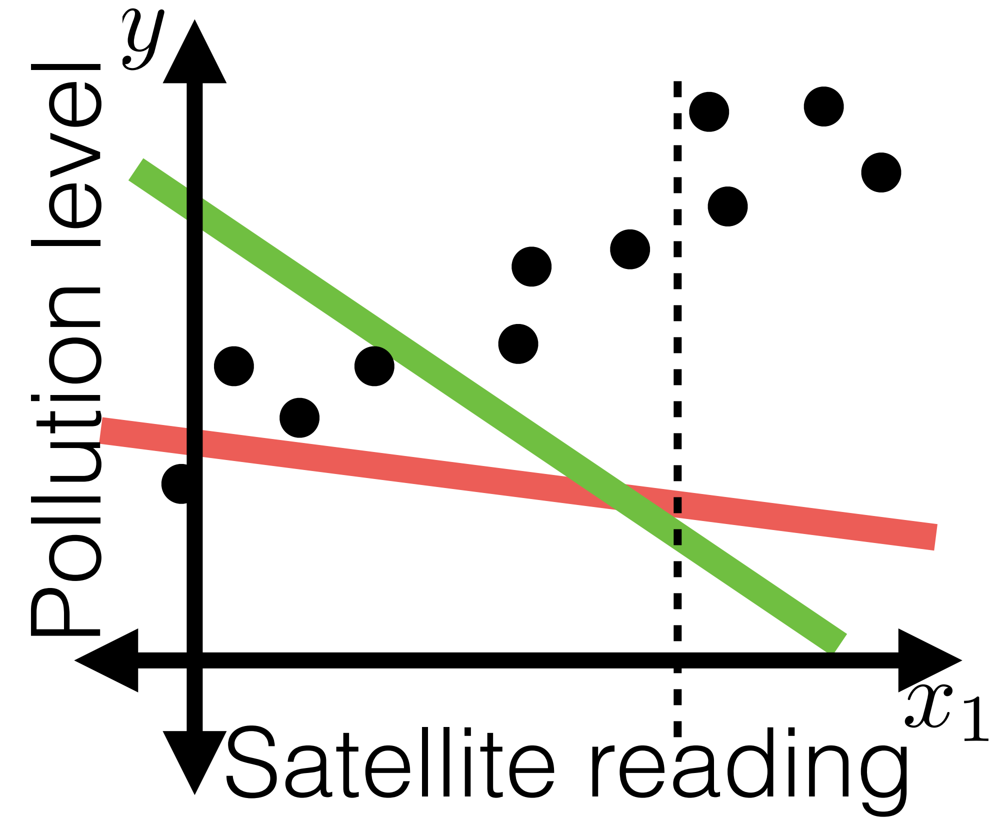

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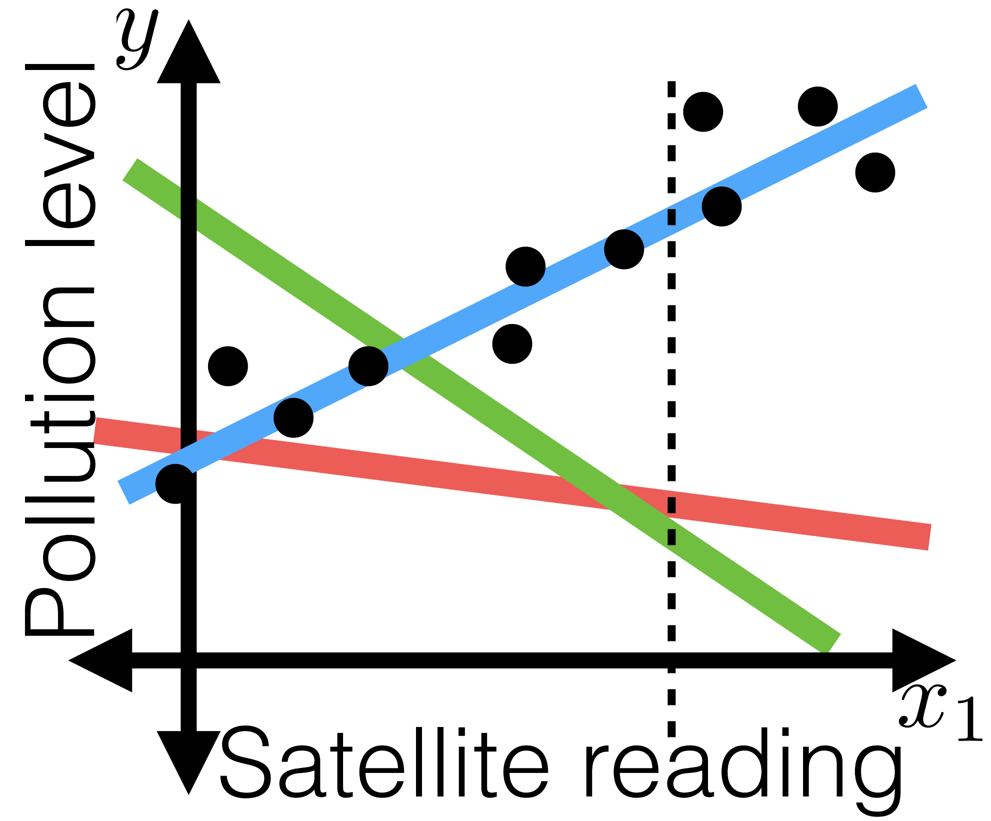

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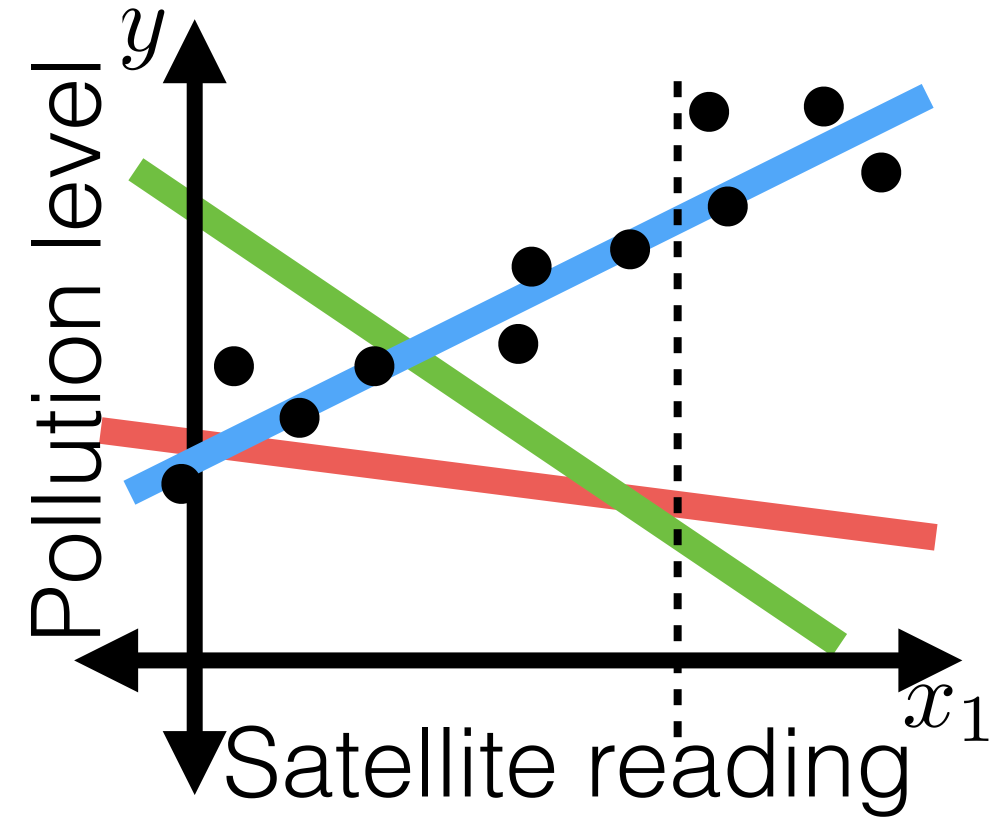

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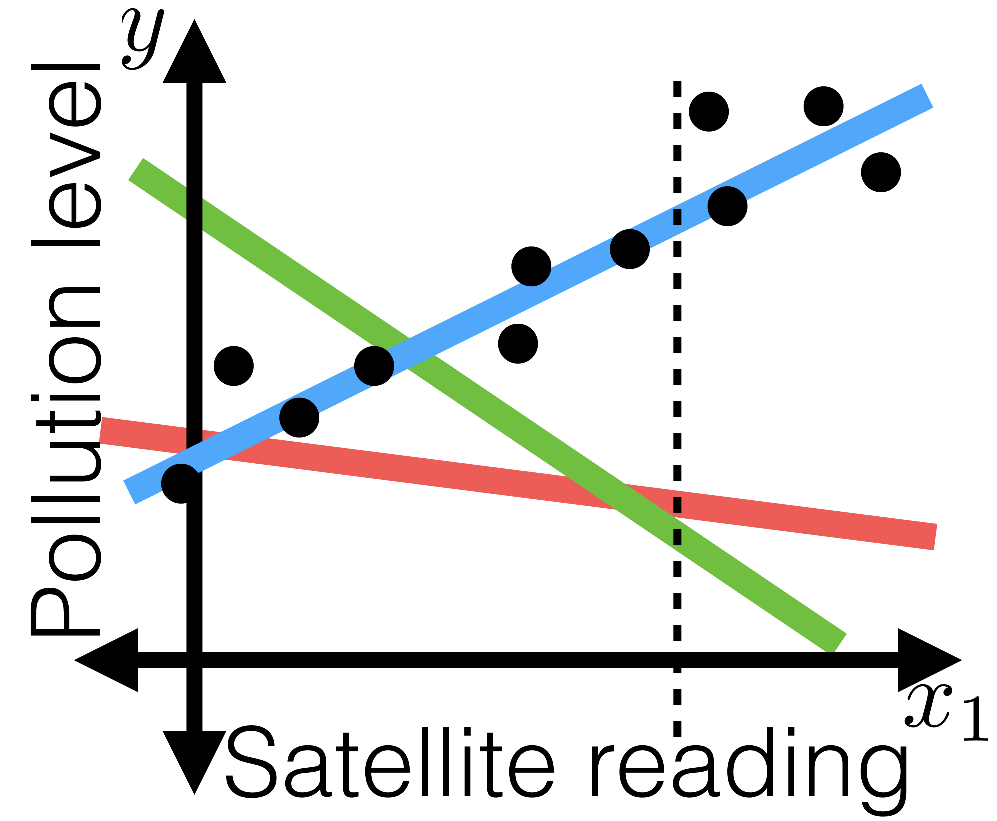

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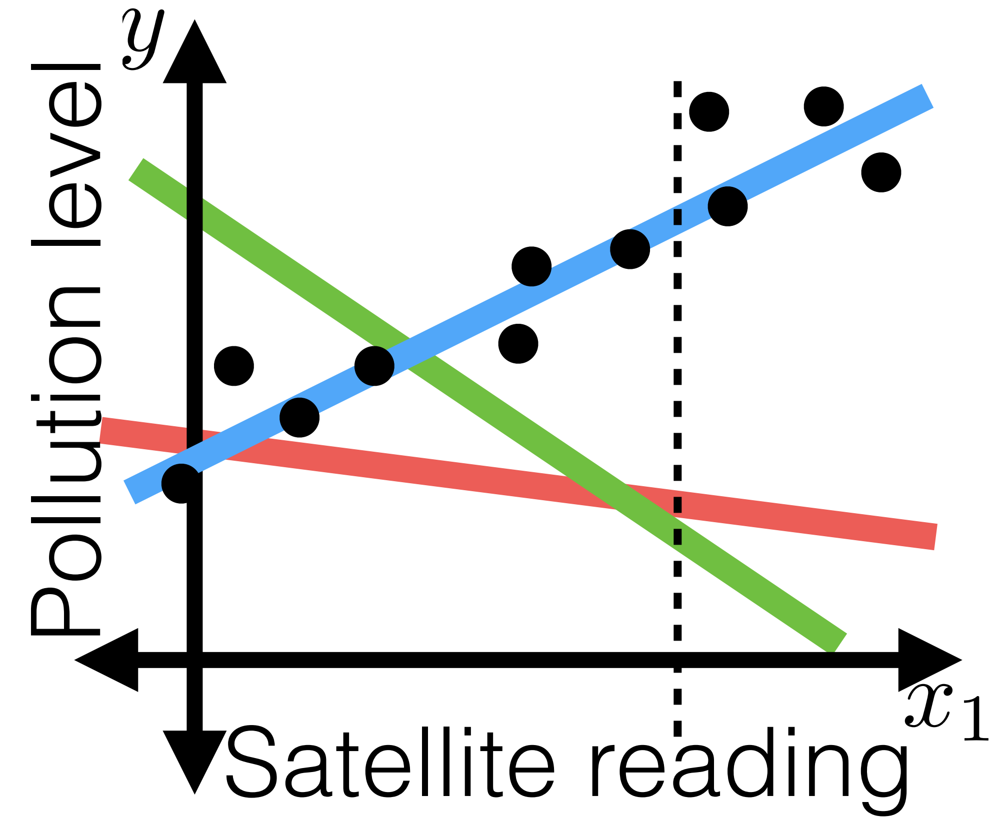

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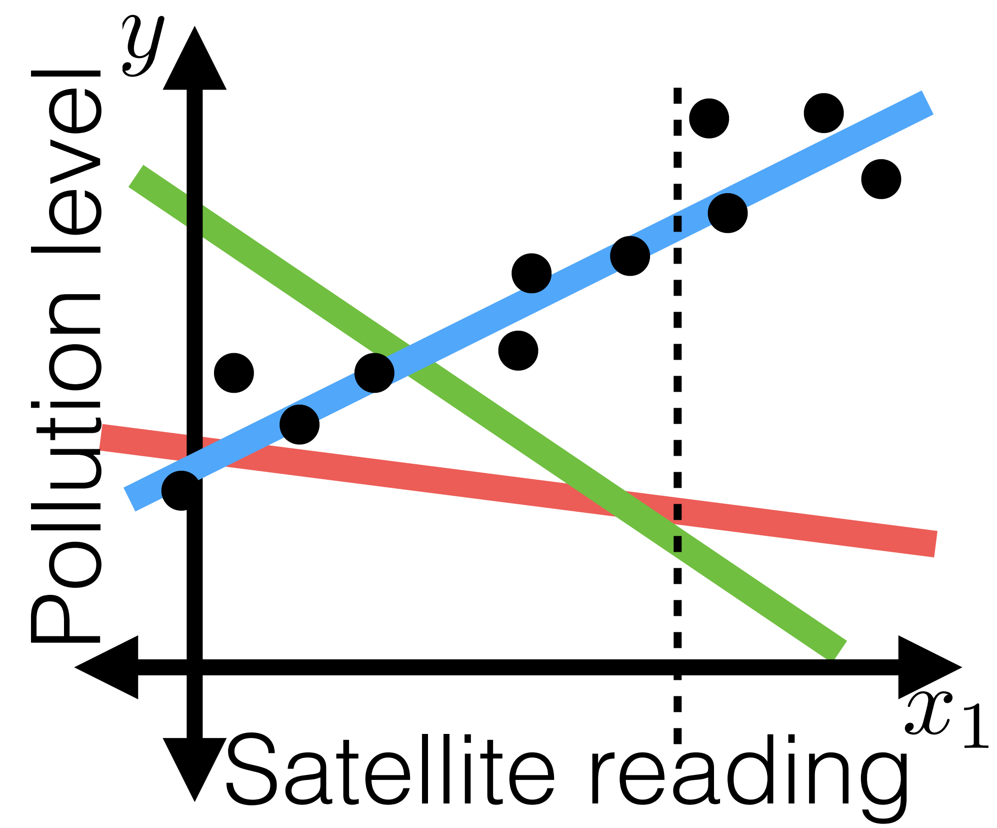

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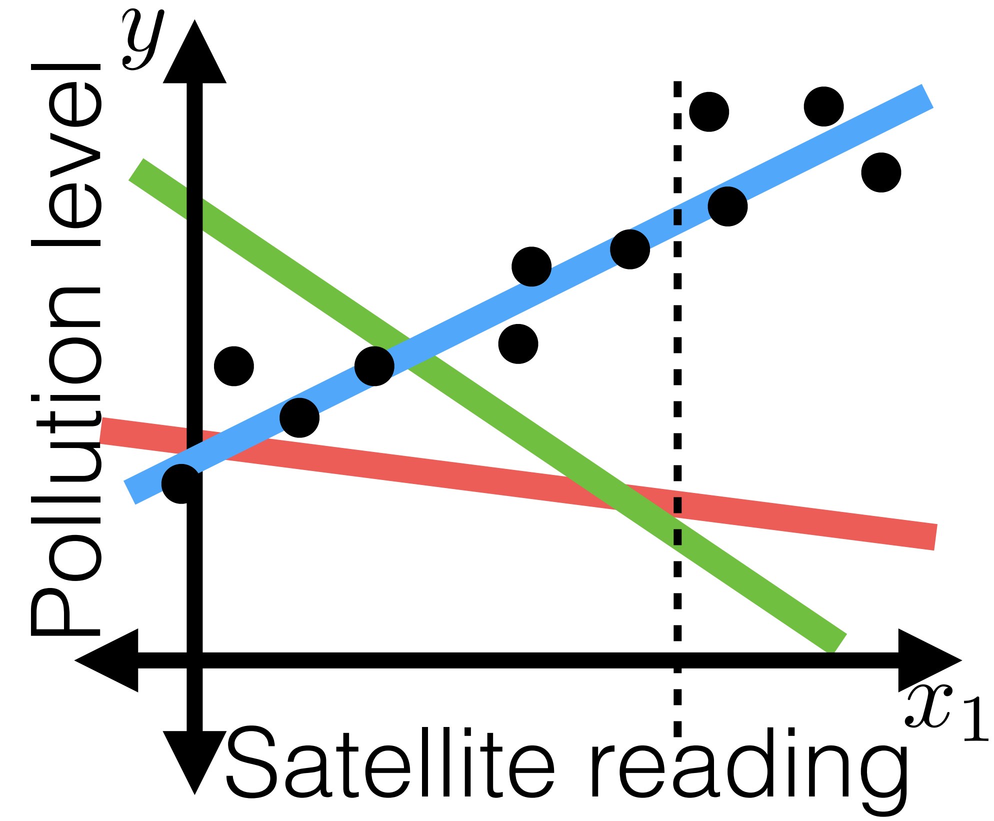

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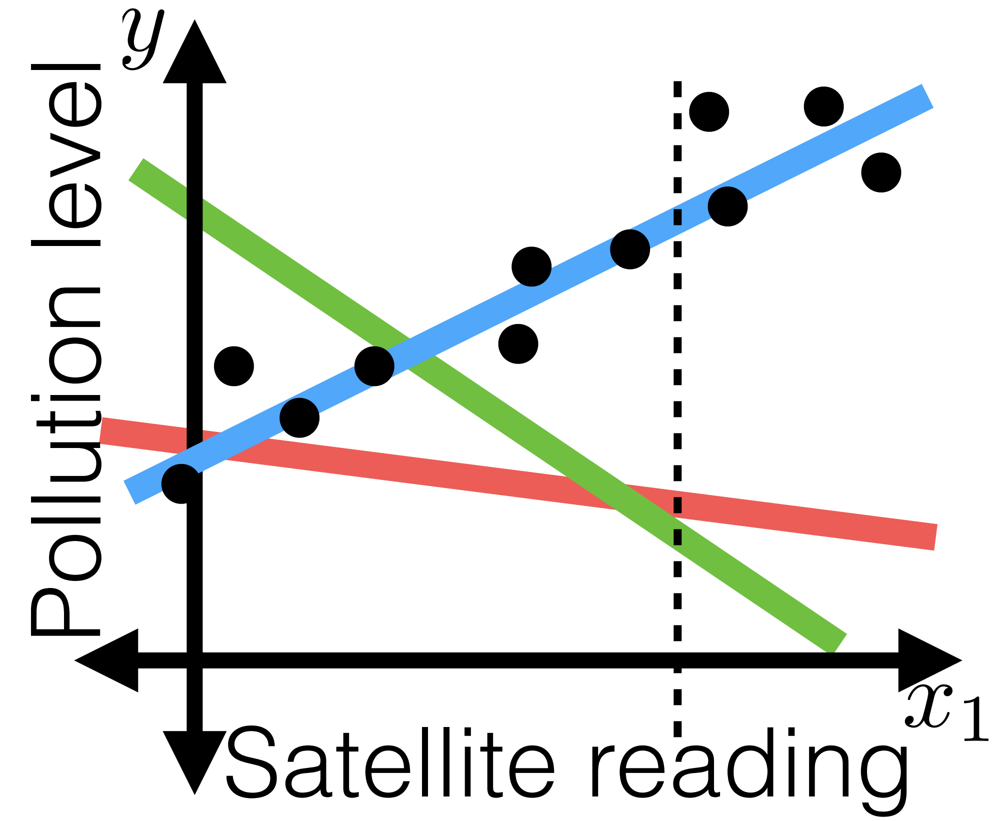

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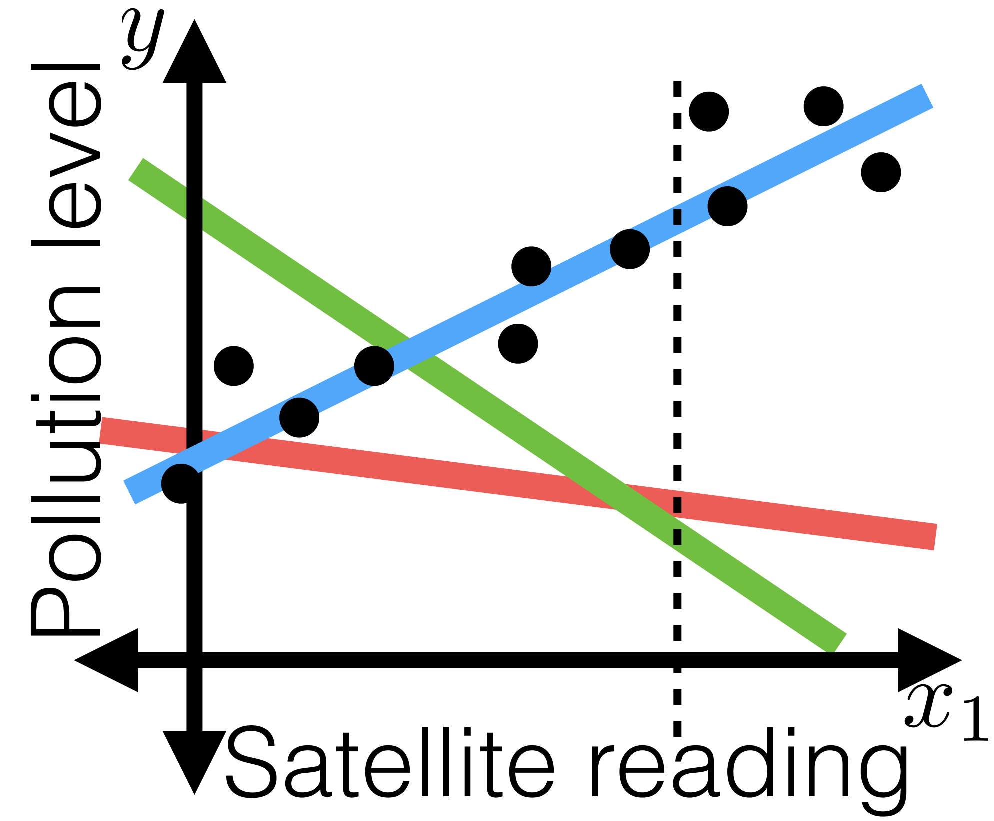

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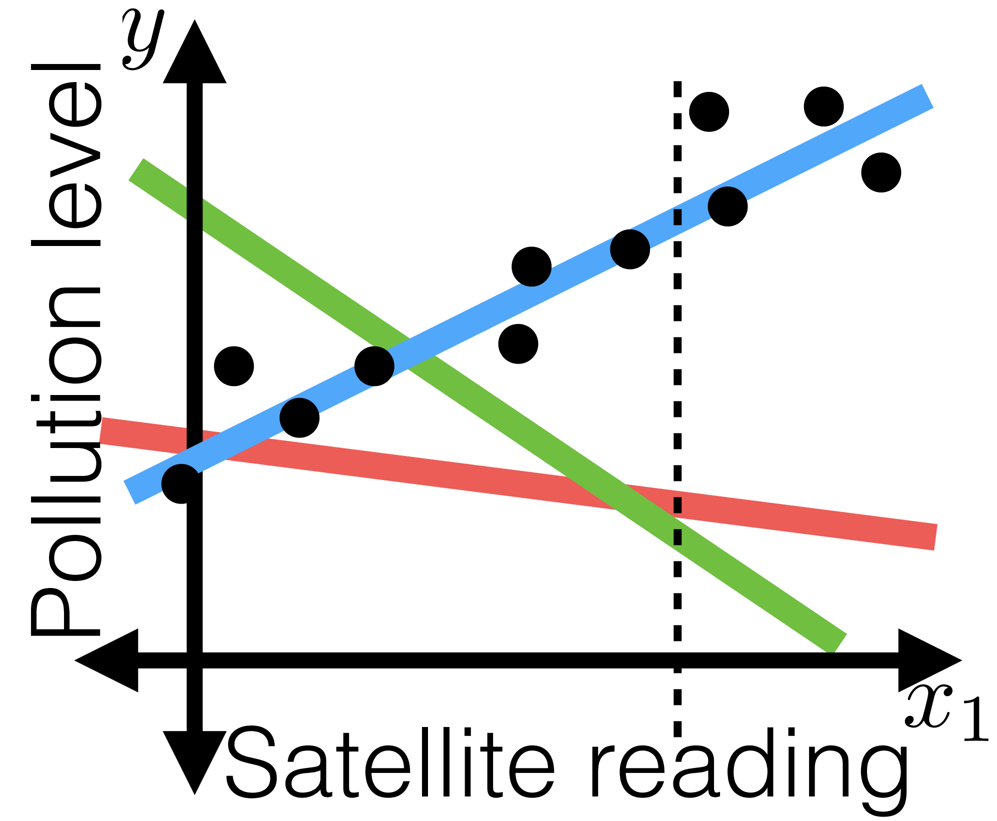

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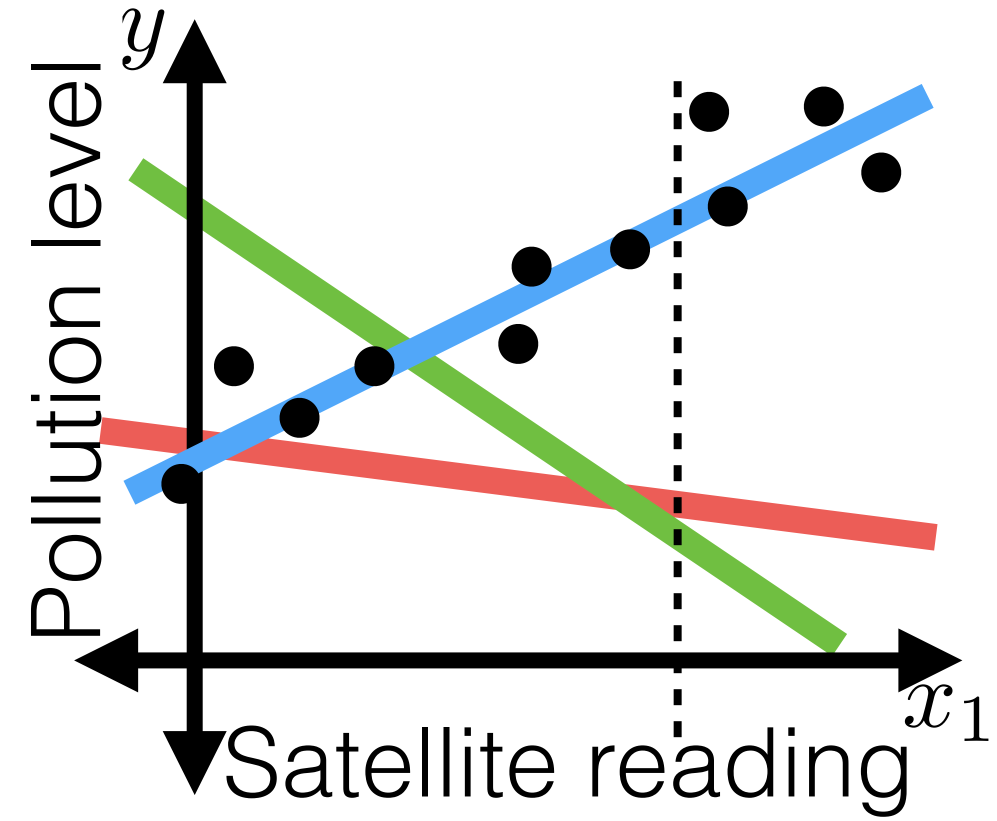

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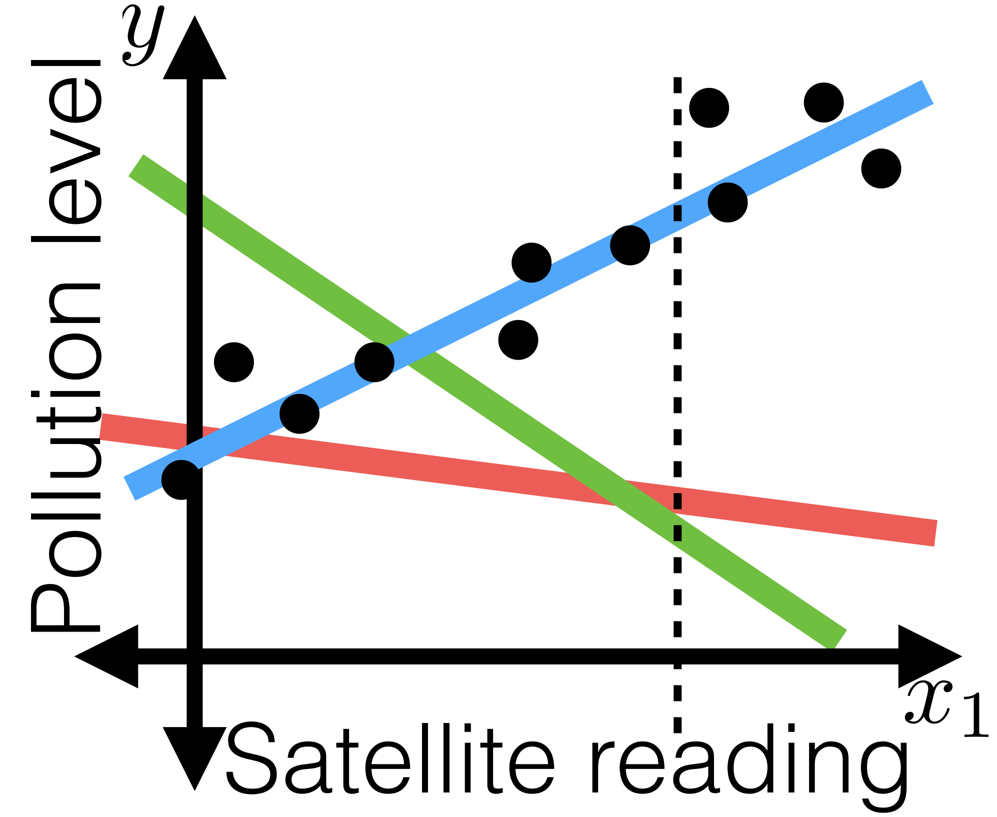

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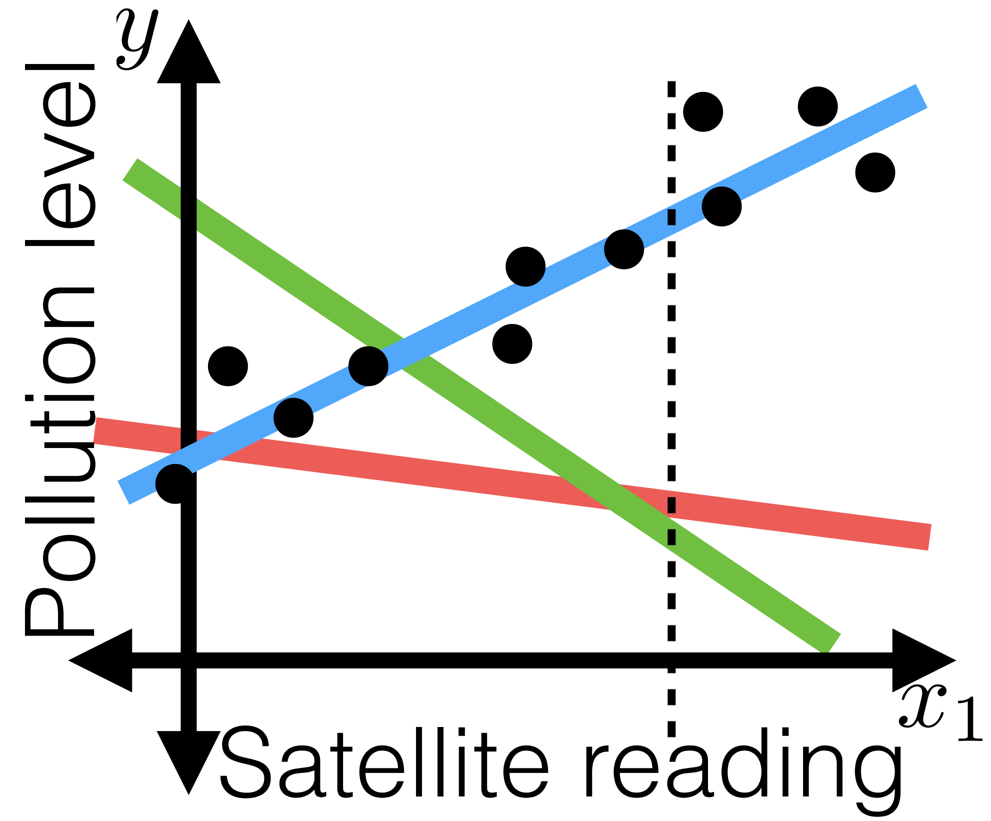

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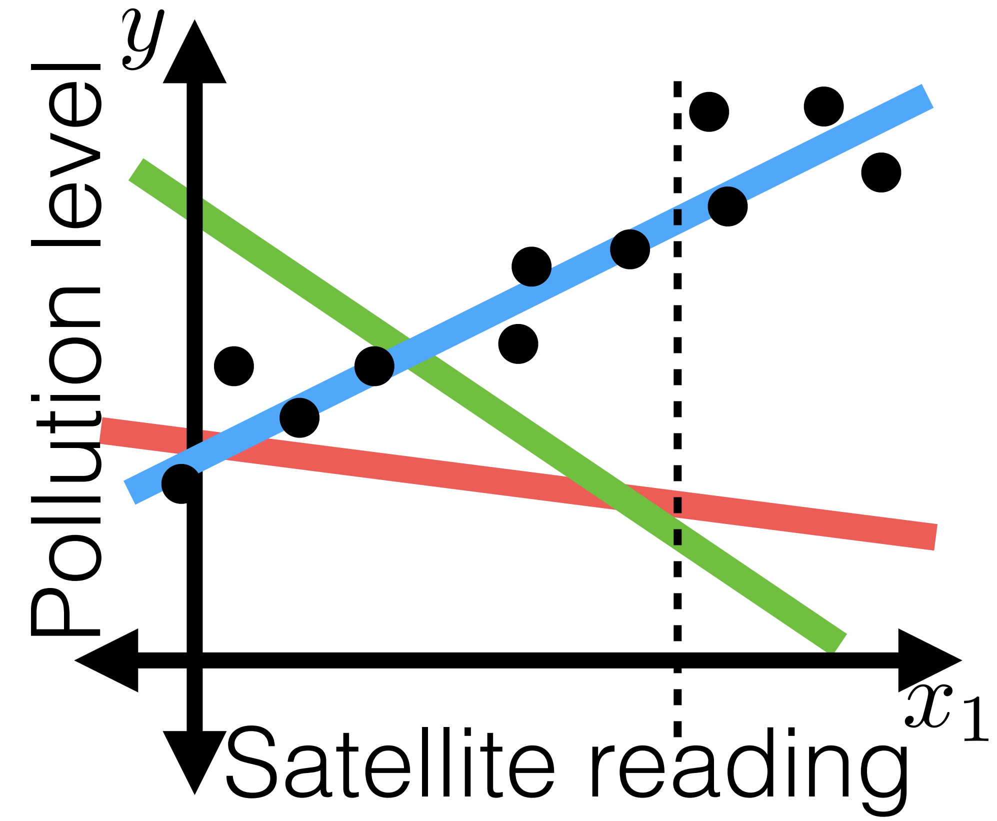

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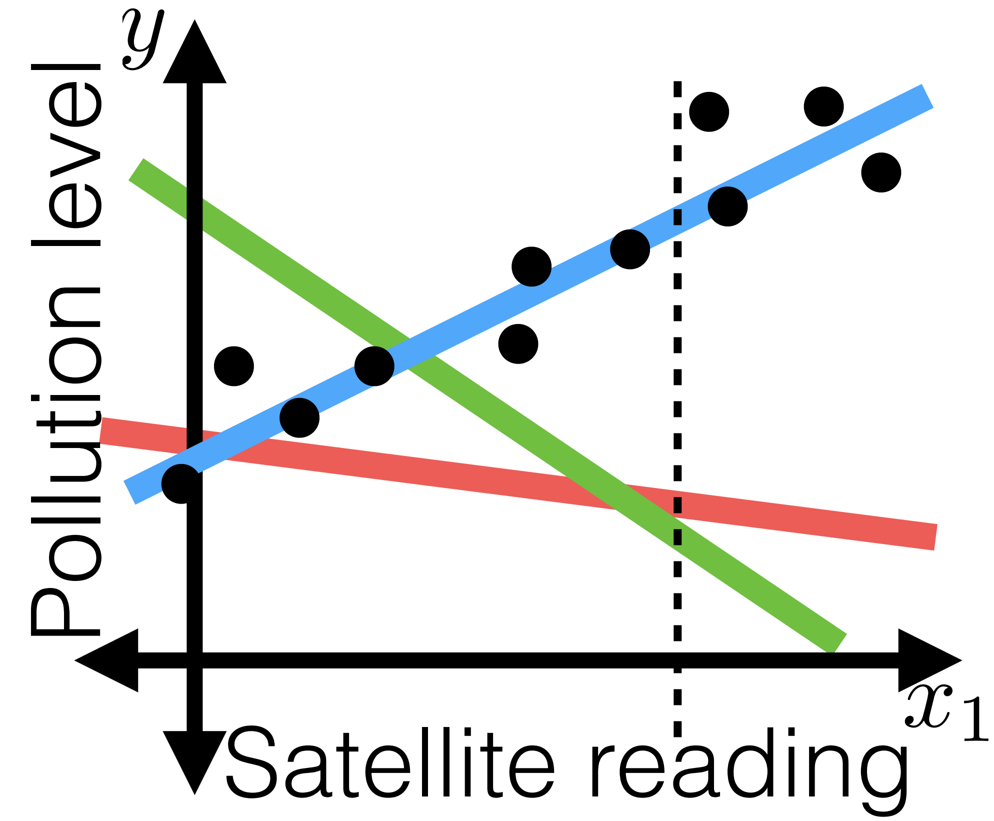

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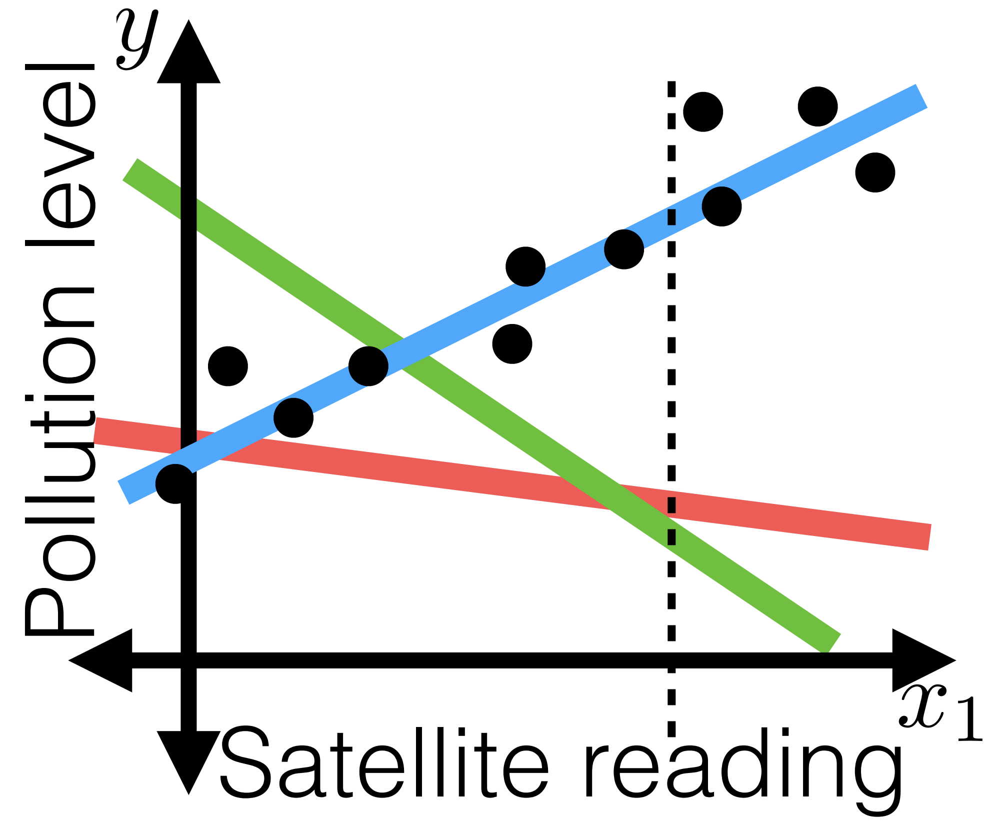

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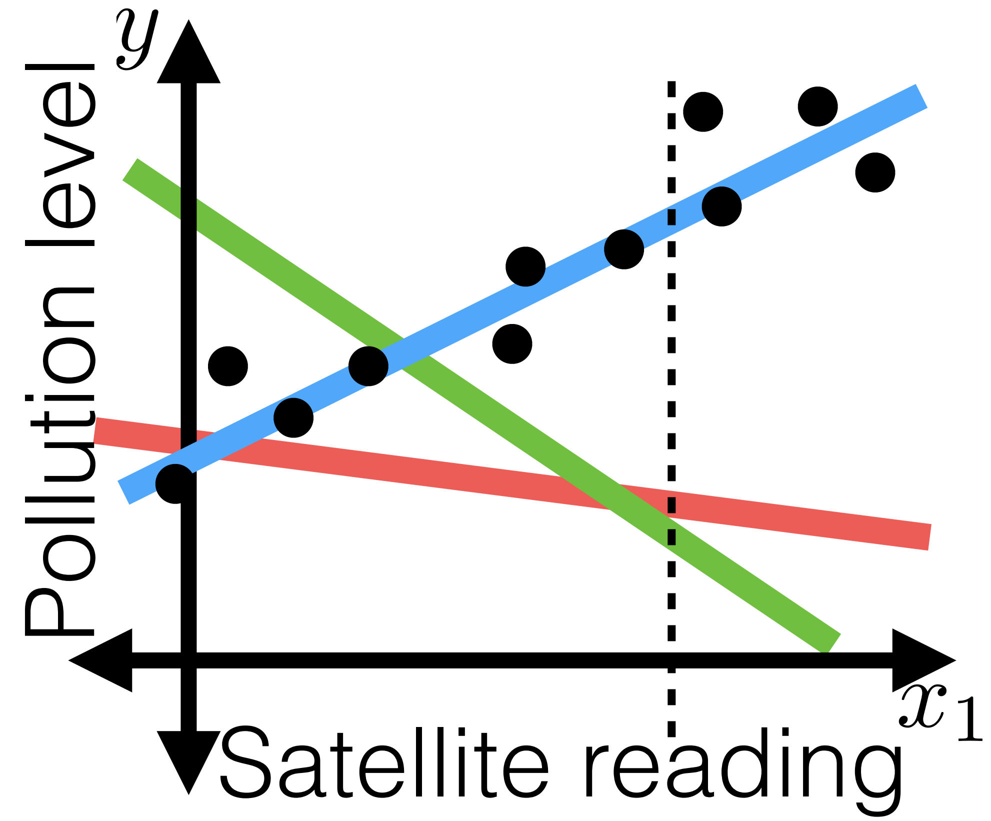

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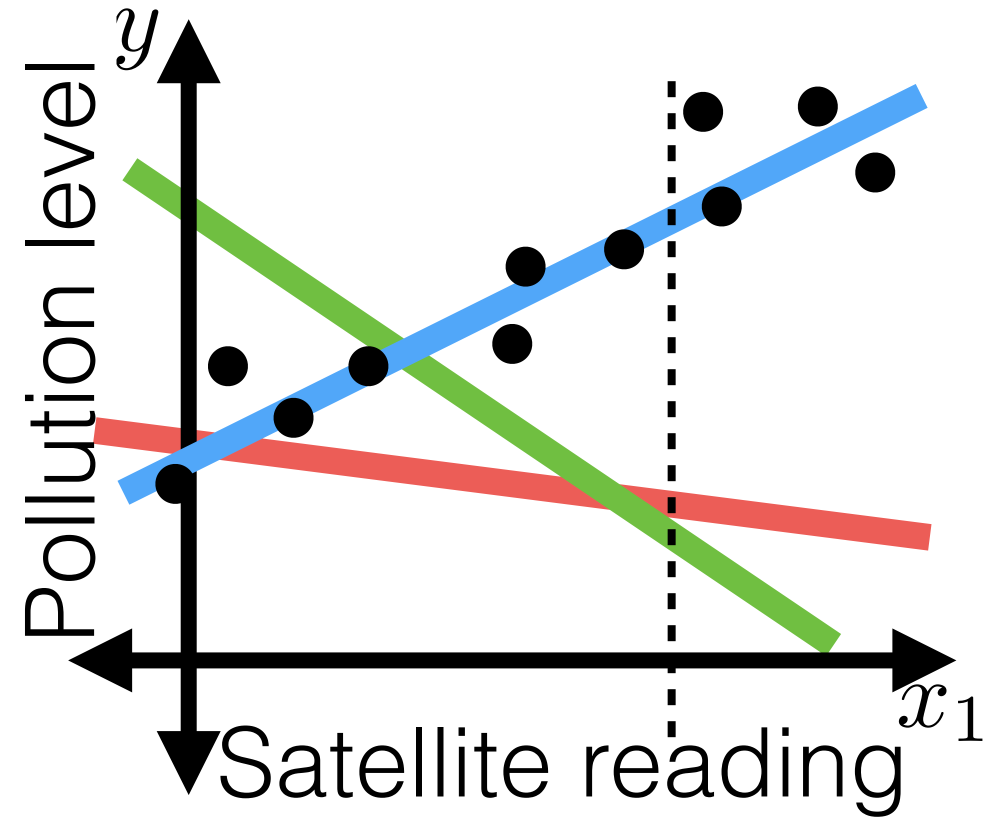

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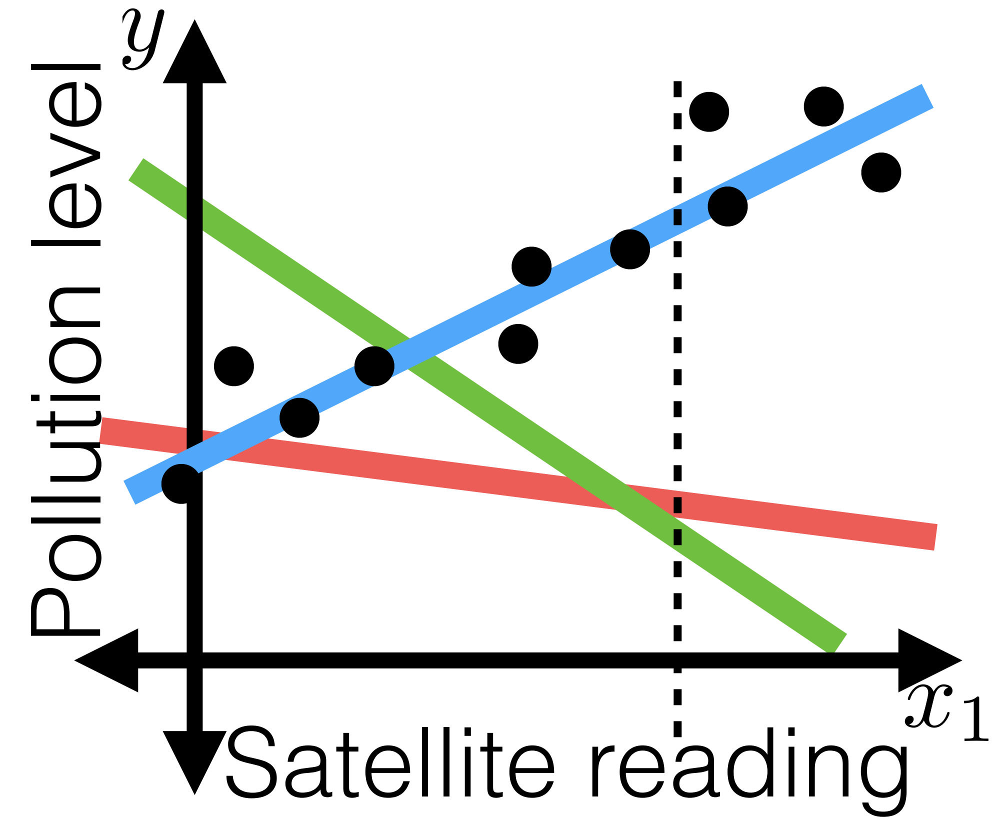

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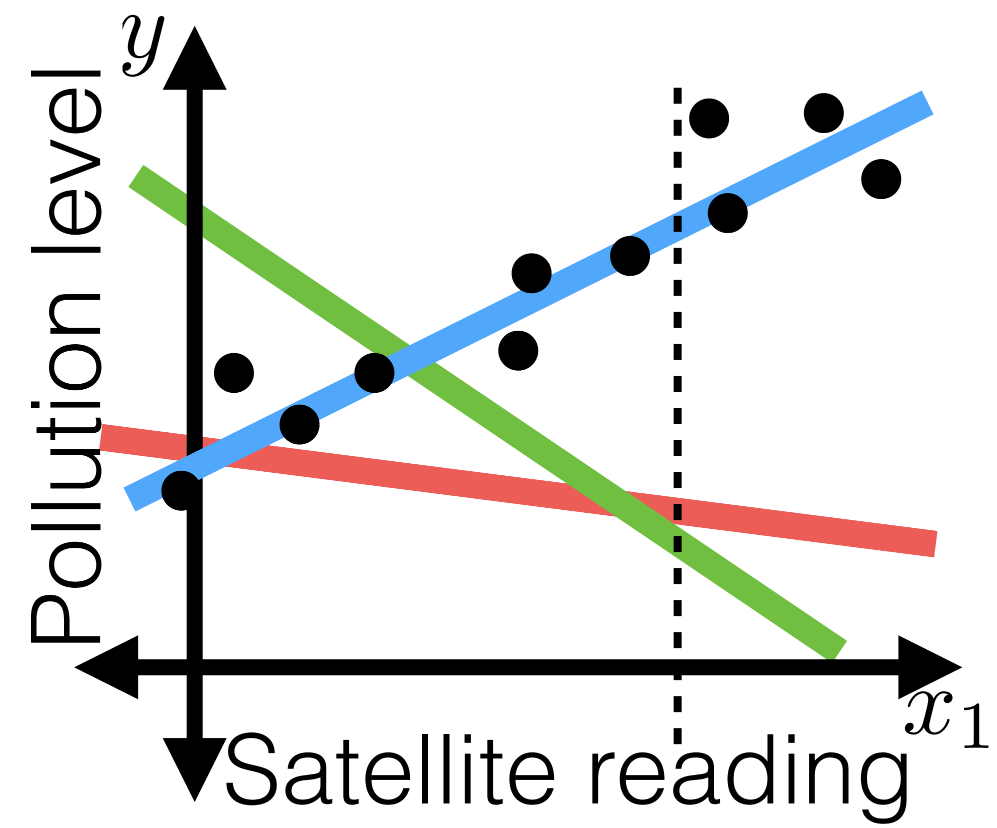

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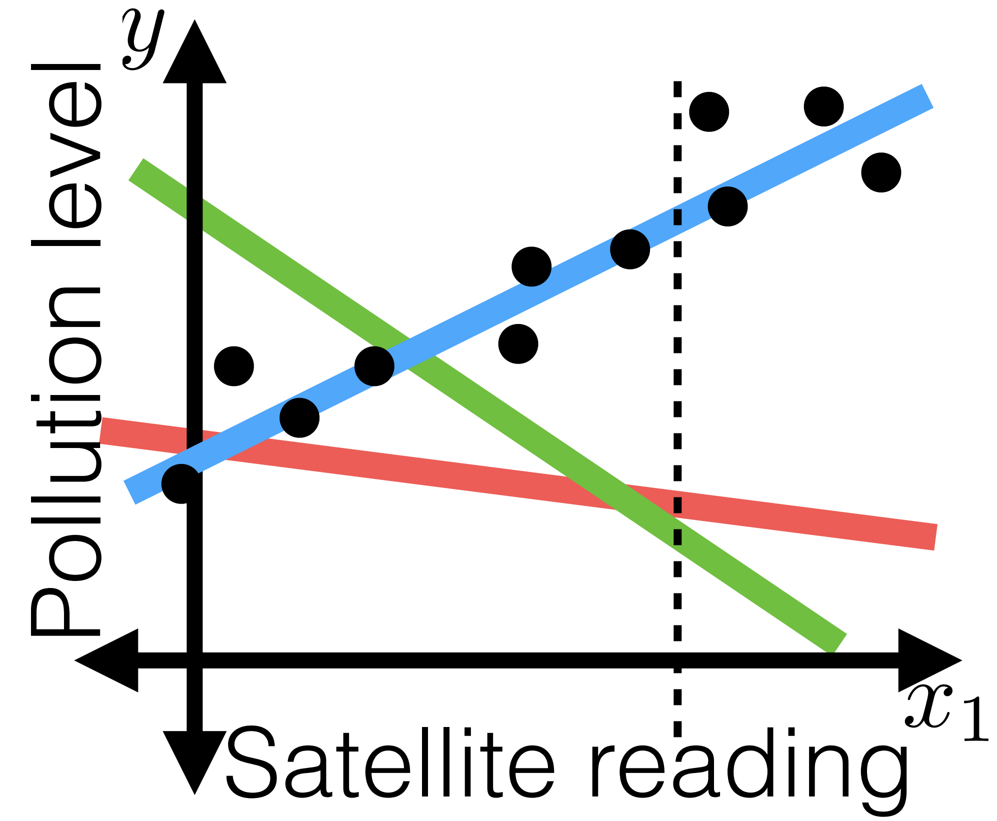

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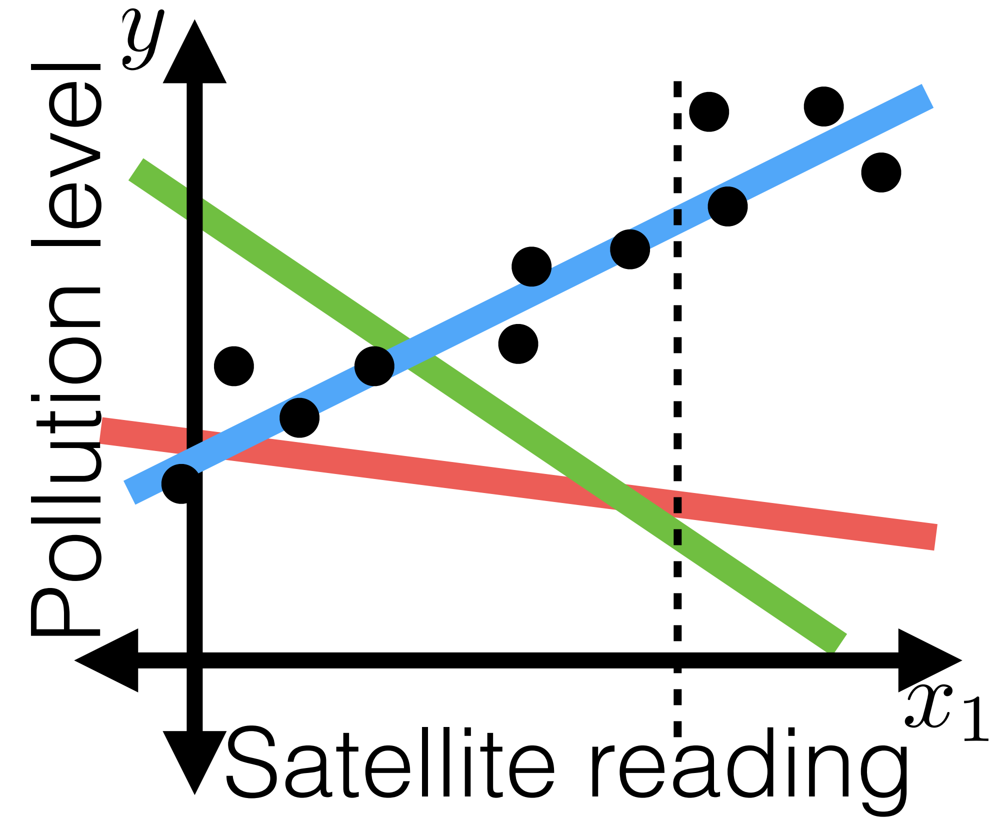

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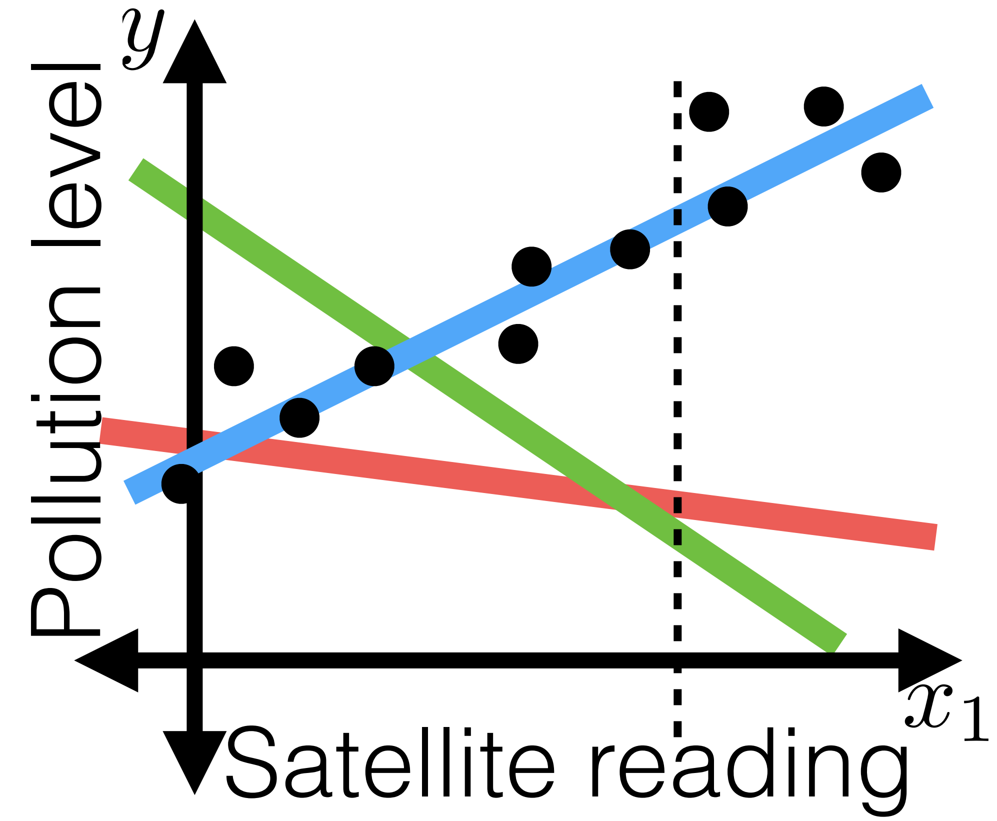

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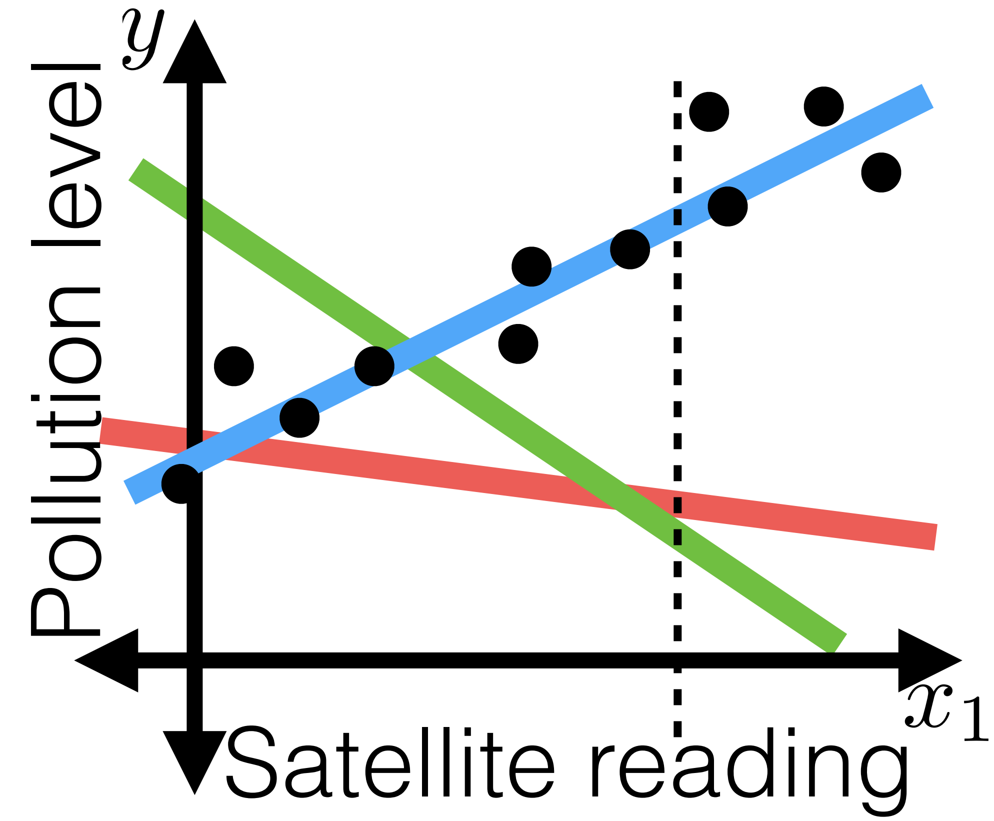

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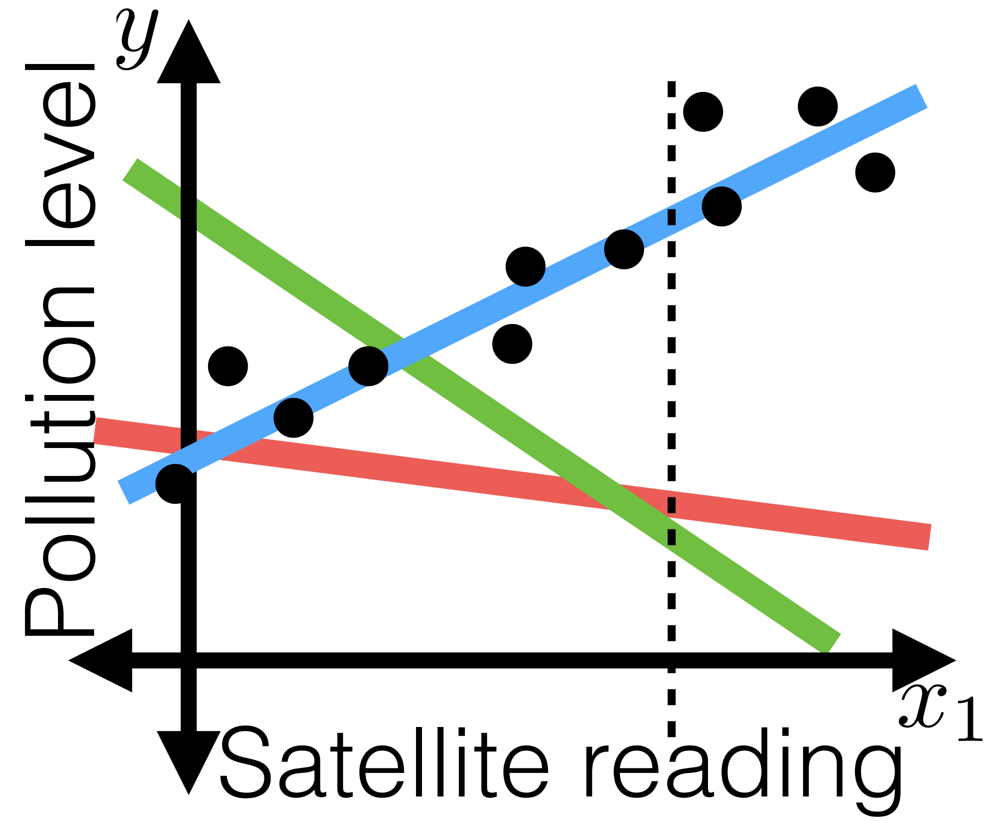

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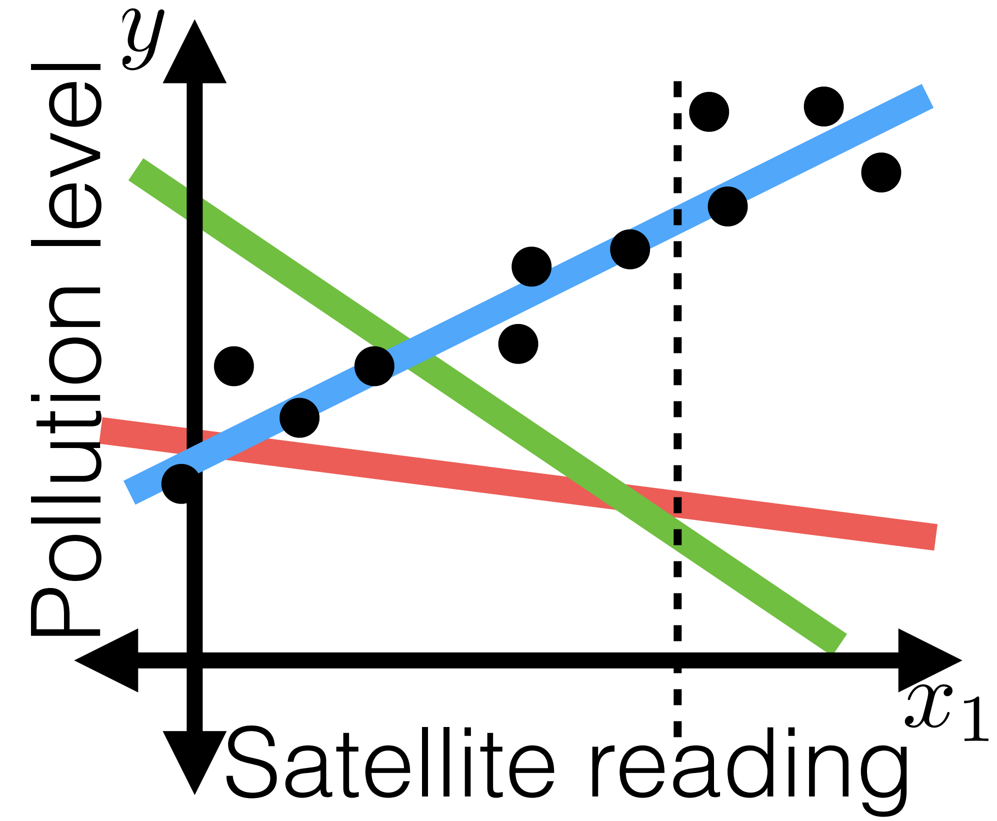

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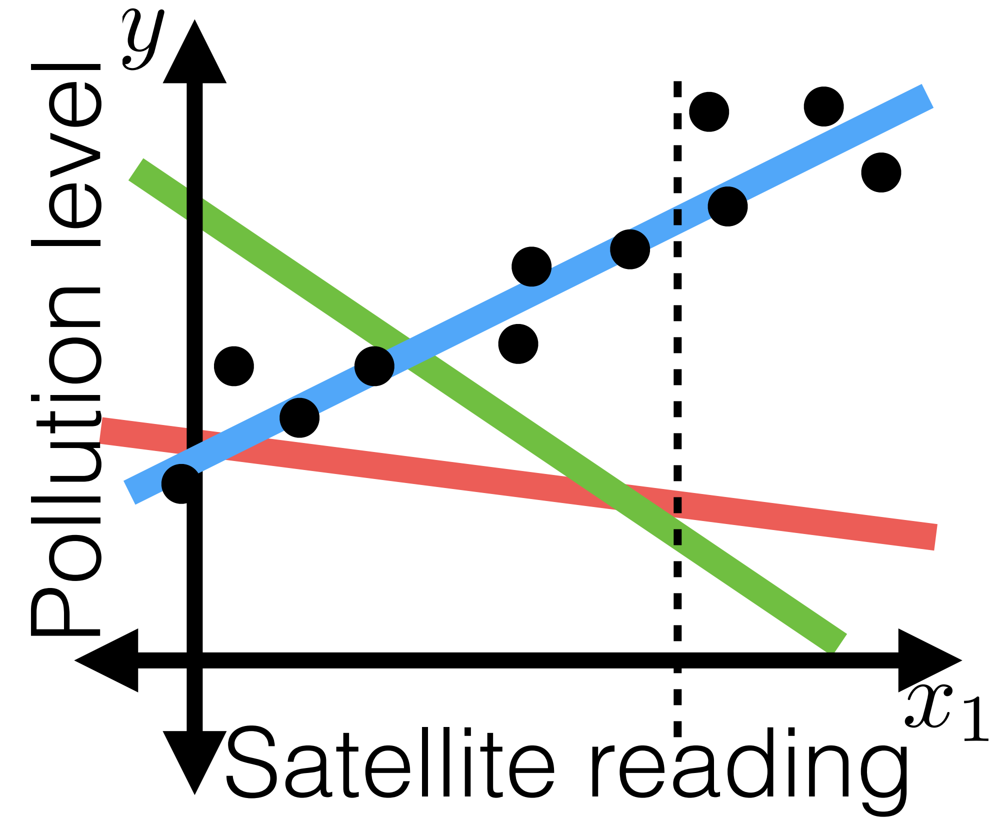

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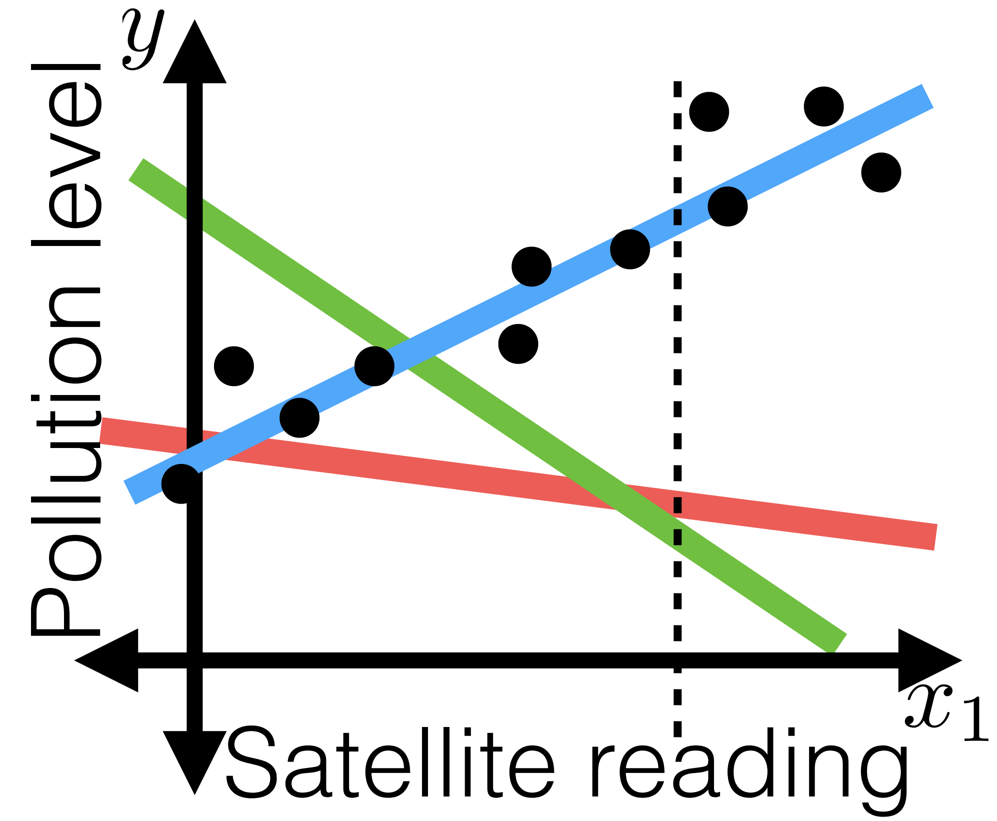

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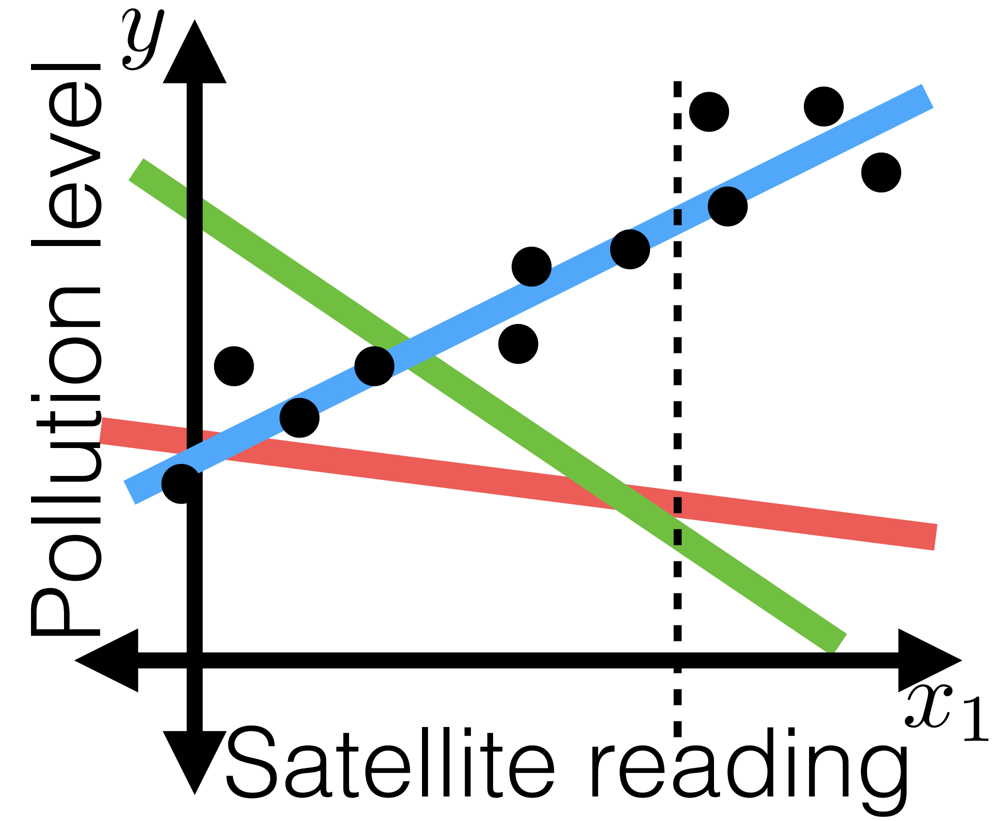

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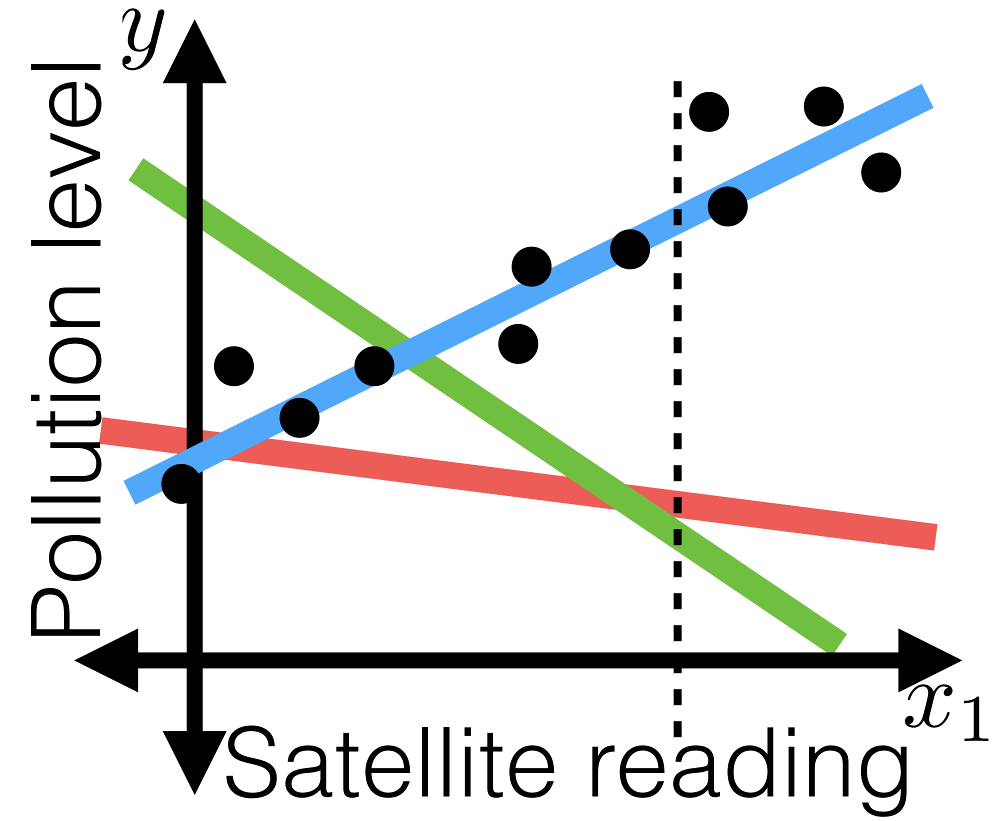

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e.g. $L(g,a) =$

$$\begin{cases} (g-a)^2 \text{ if } g > a \\ 5(g-a)^2 \text{ if } g \leq a \end{cases}$$

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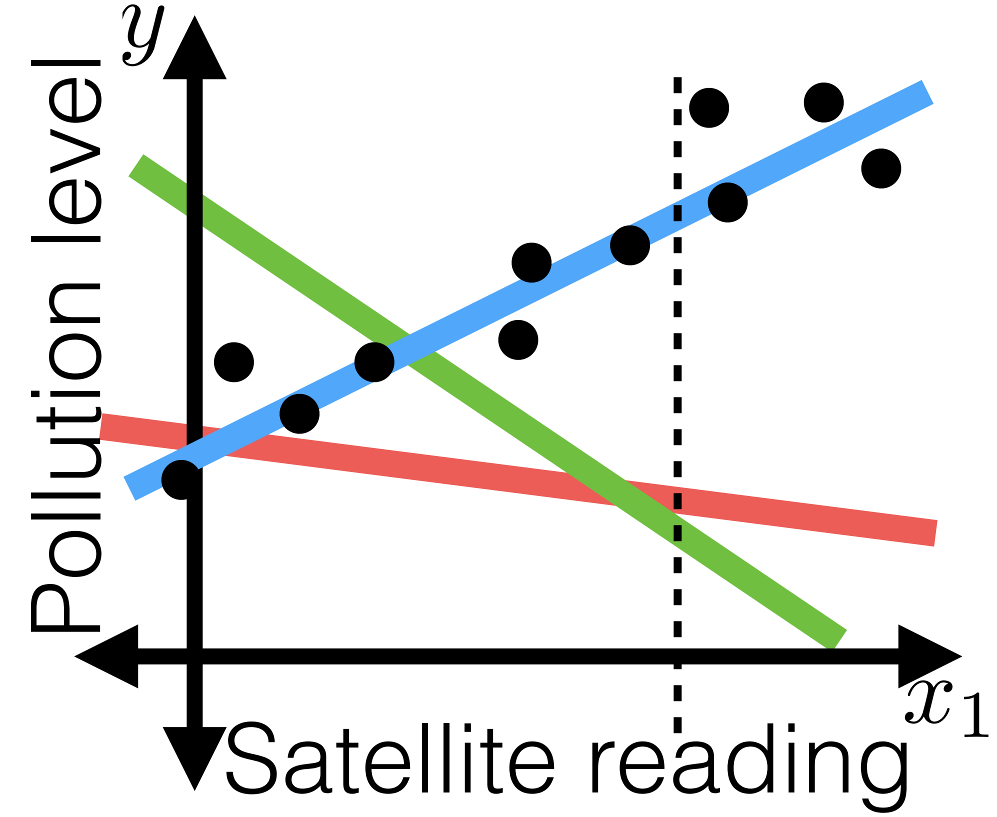

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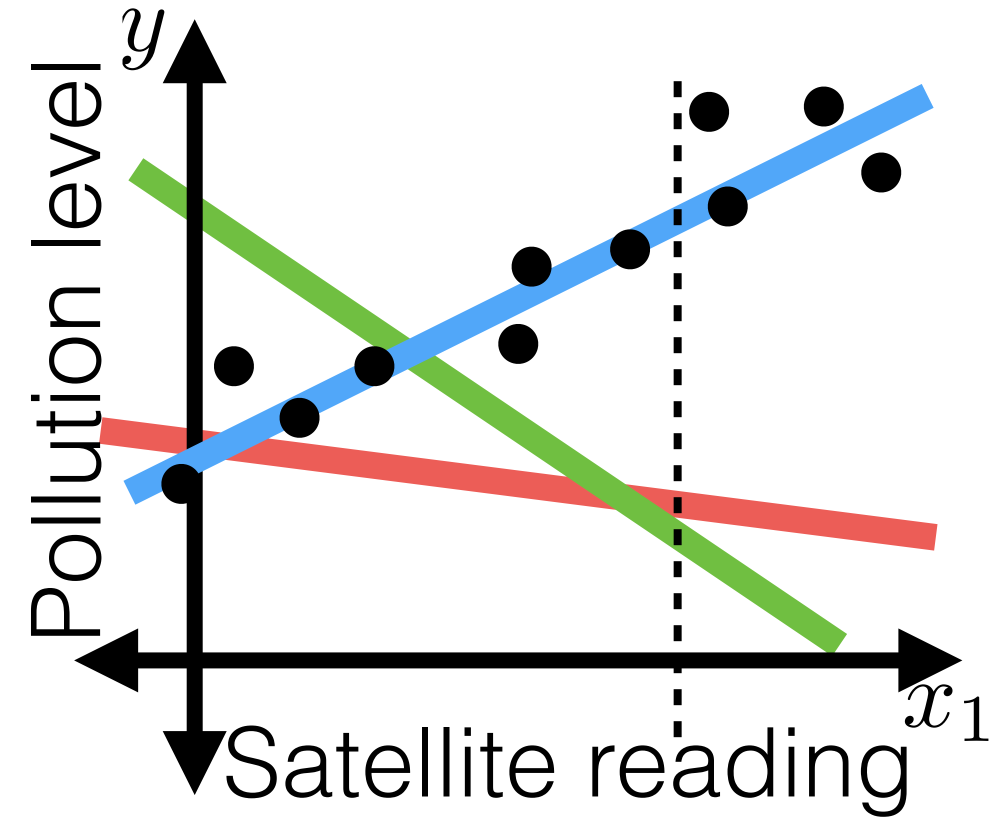
- Can be too slow to run, even in ridge regression

Recall



- A general ML approach:
  - Collect data
  - Choose hypothesis class
  - Choose "good" hypothesis by minimizing training loss + regularizer
- Example: ridge regression

linear regression hypothesis
squared loss $L(g,a) = (g-a)^2$

e.g.

$$f(\Theta) = J_{\text{ridge}}(\theta, \theta_0) = \frac{1}{n}\sum_{i=1}^{n}(\theta^\top x^{(i)} + \theta_0 - y^{(i)})^2 + \lambda\|\theta\|^2 \quad (\lambda > 0)$$

squared-norm as regularizer

- "All models are wrong, but some are useful" -George Box
- Limitations of a closed-form solution for objective minimizer
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Gradient descent

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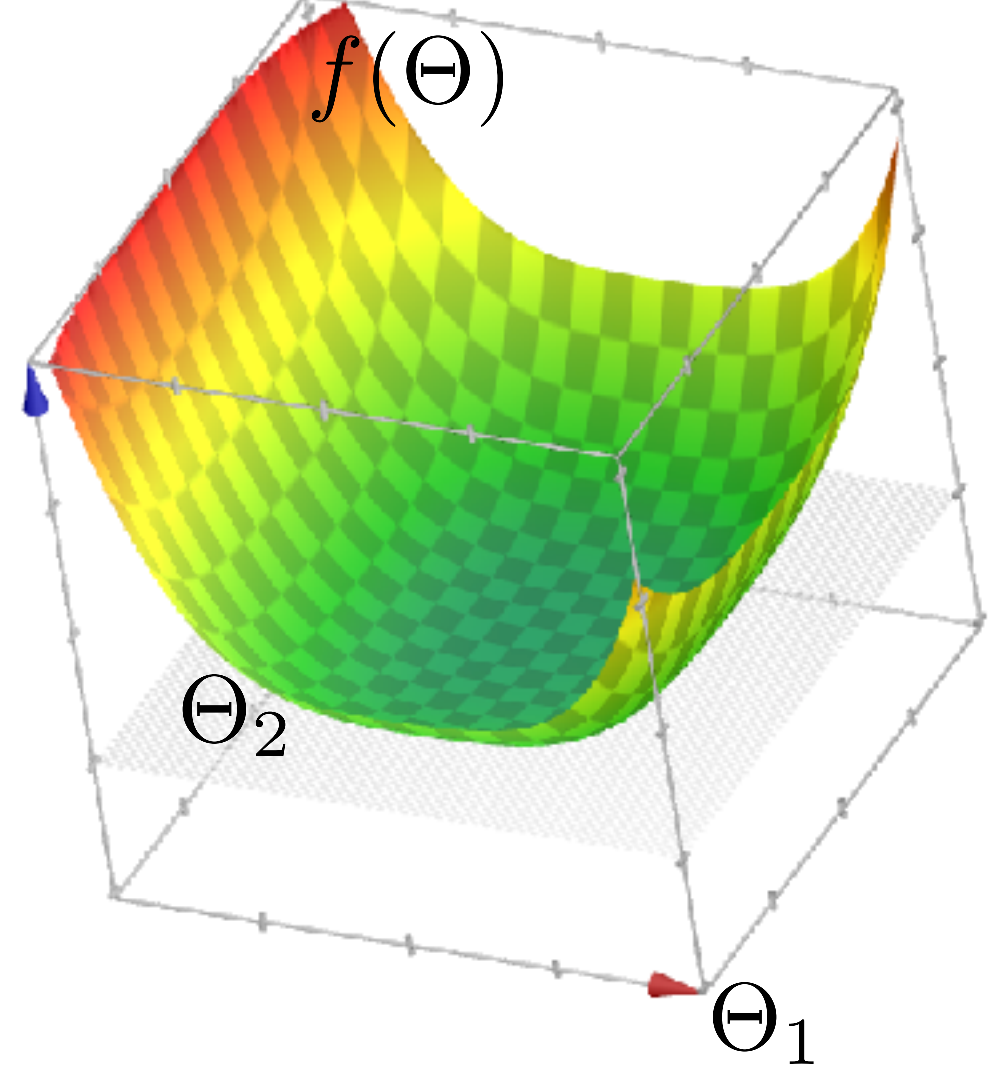

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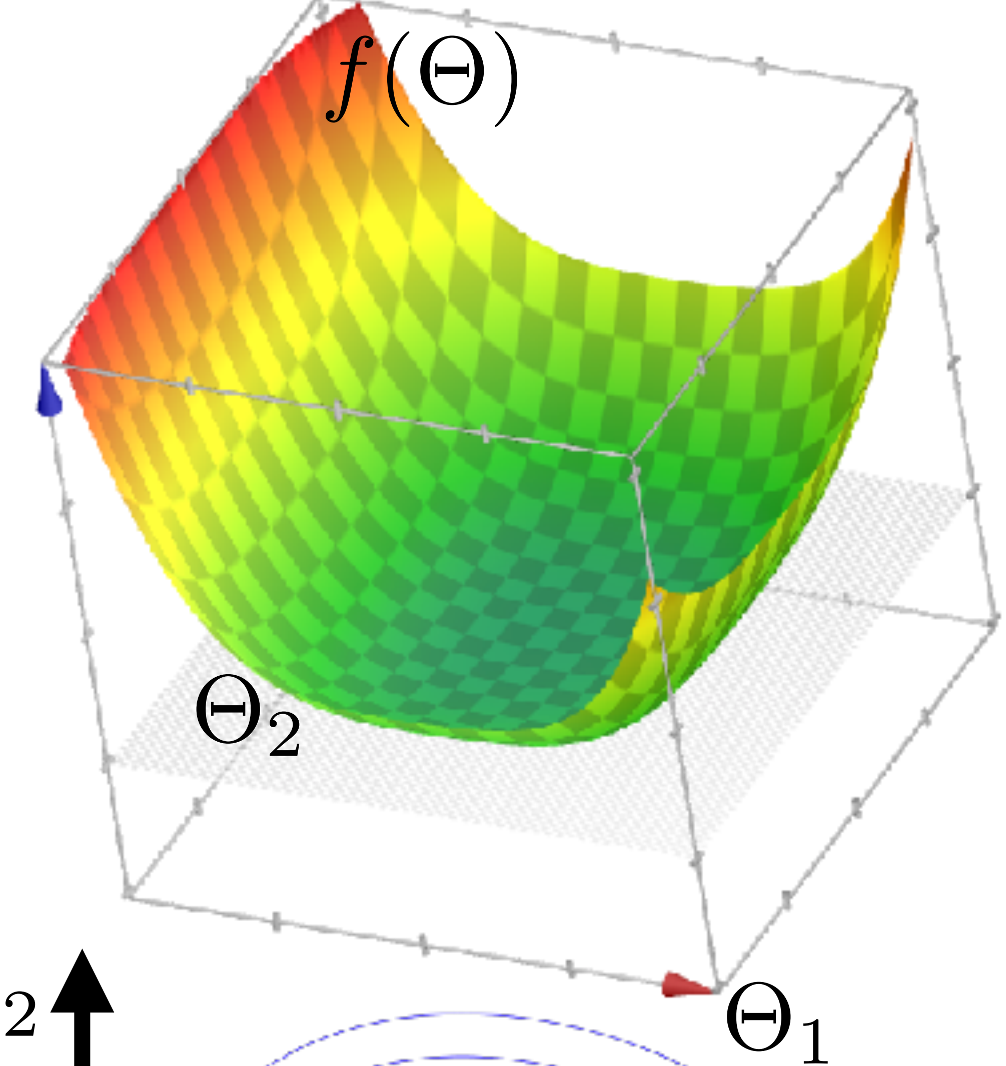

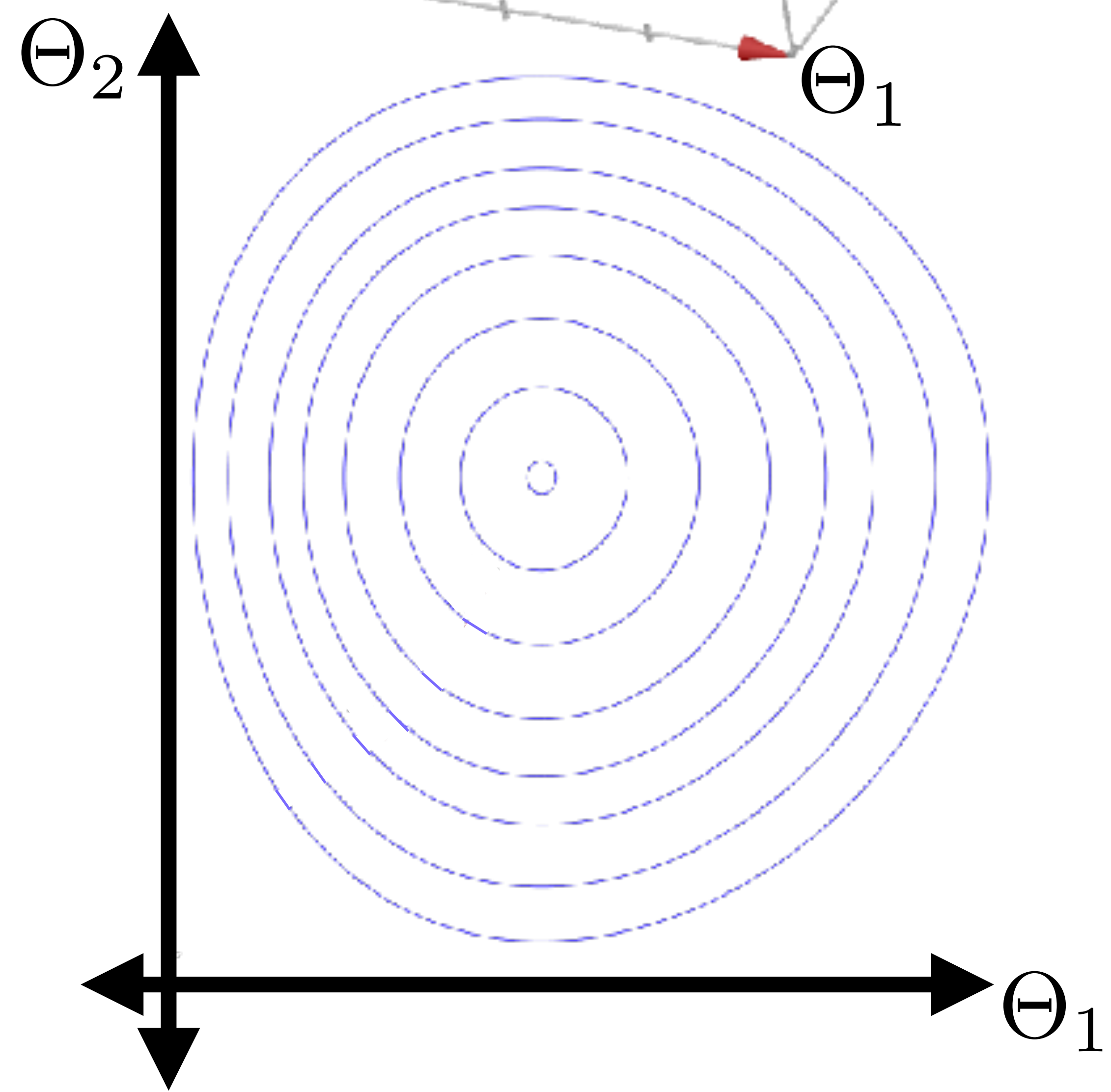

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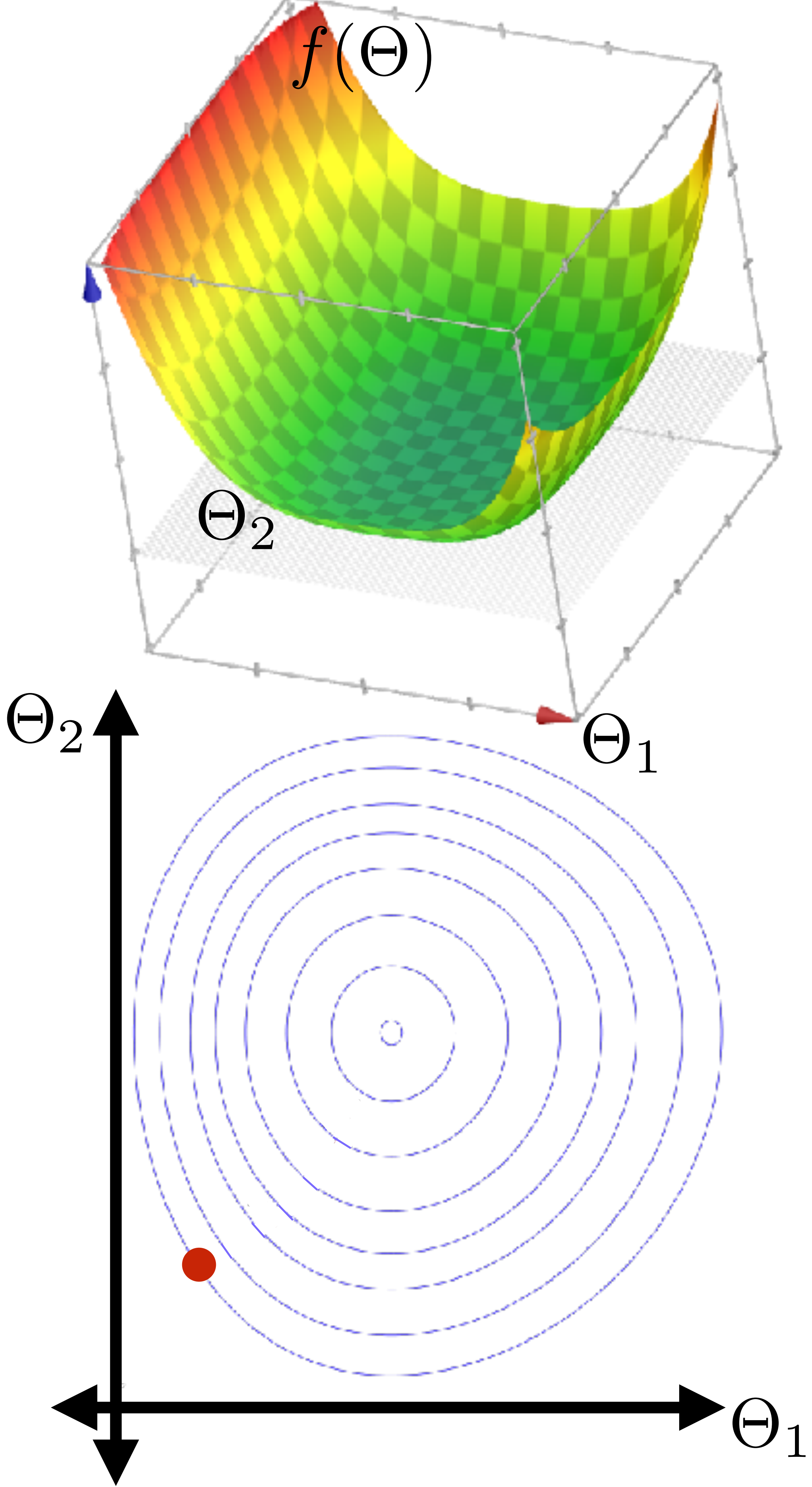

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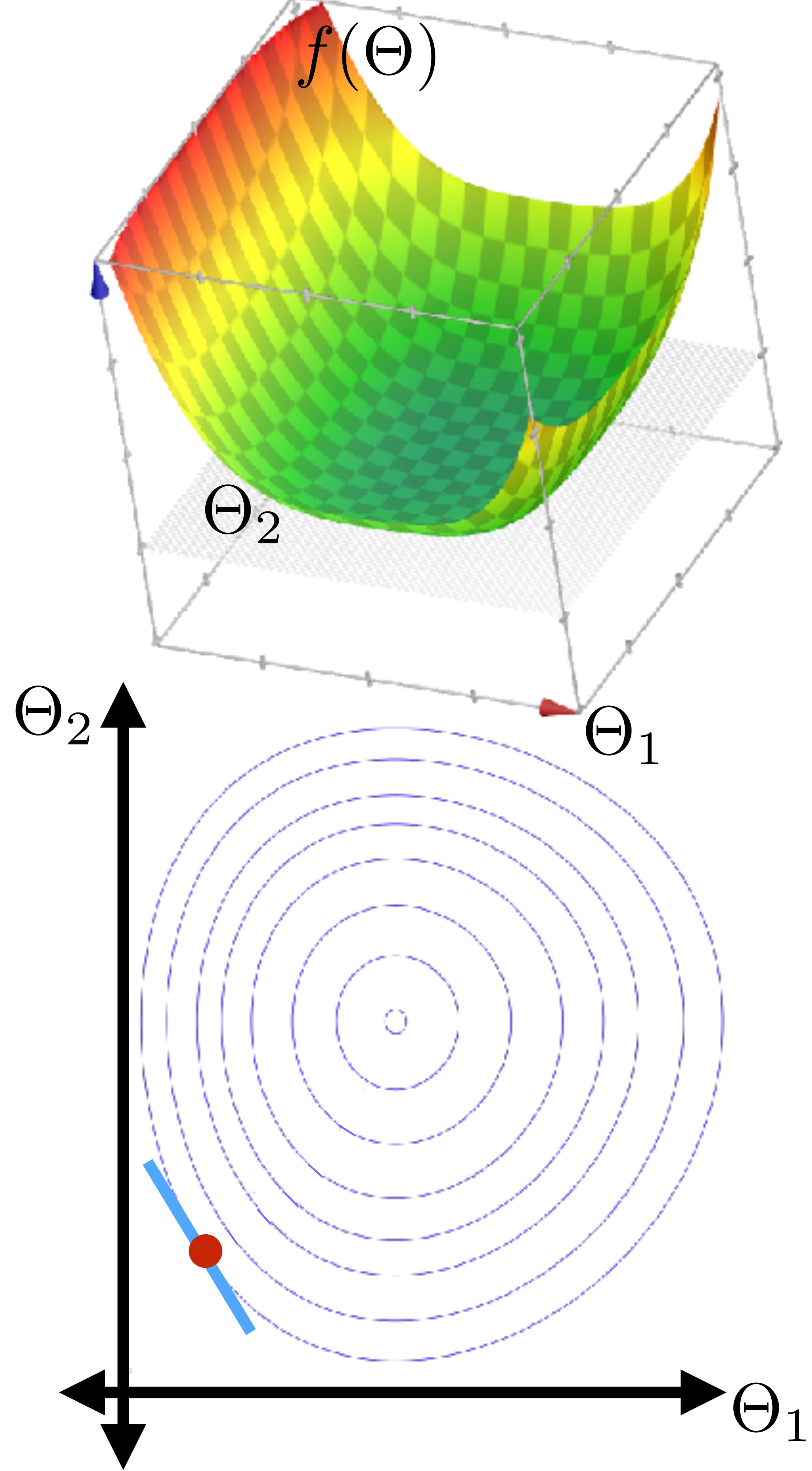

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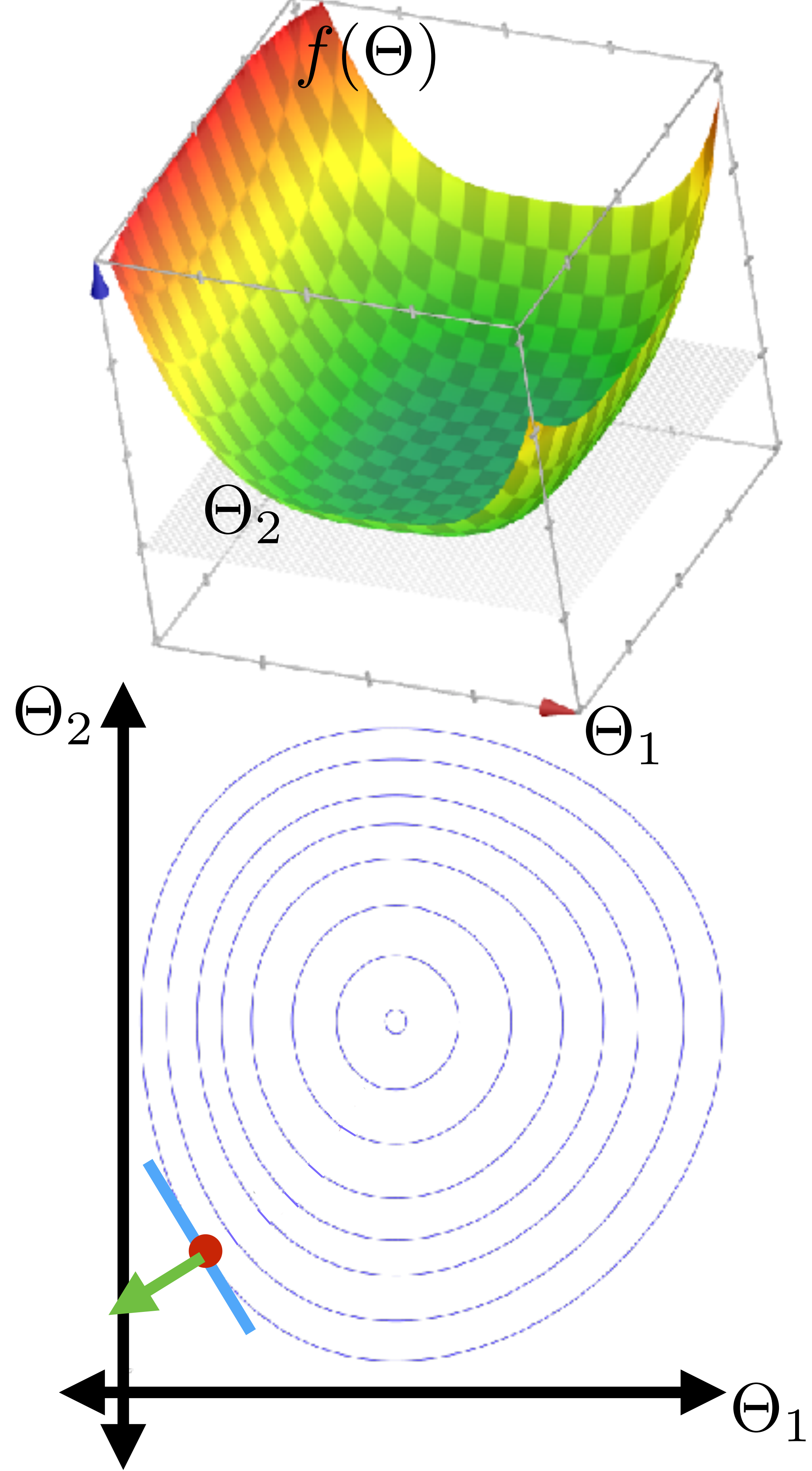

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- Gradient $\nabla_{\Theta} f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m} \right]^\top$
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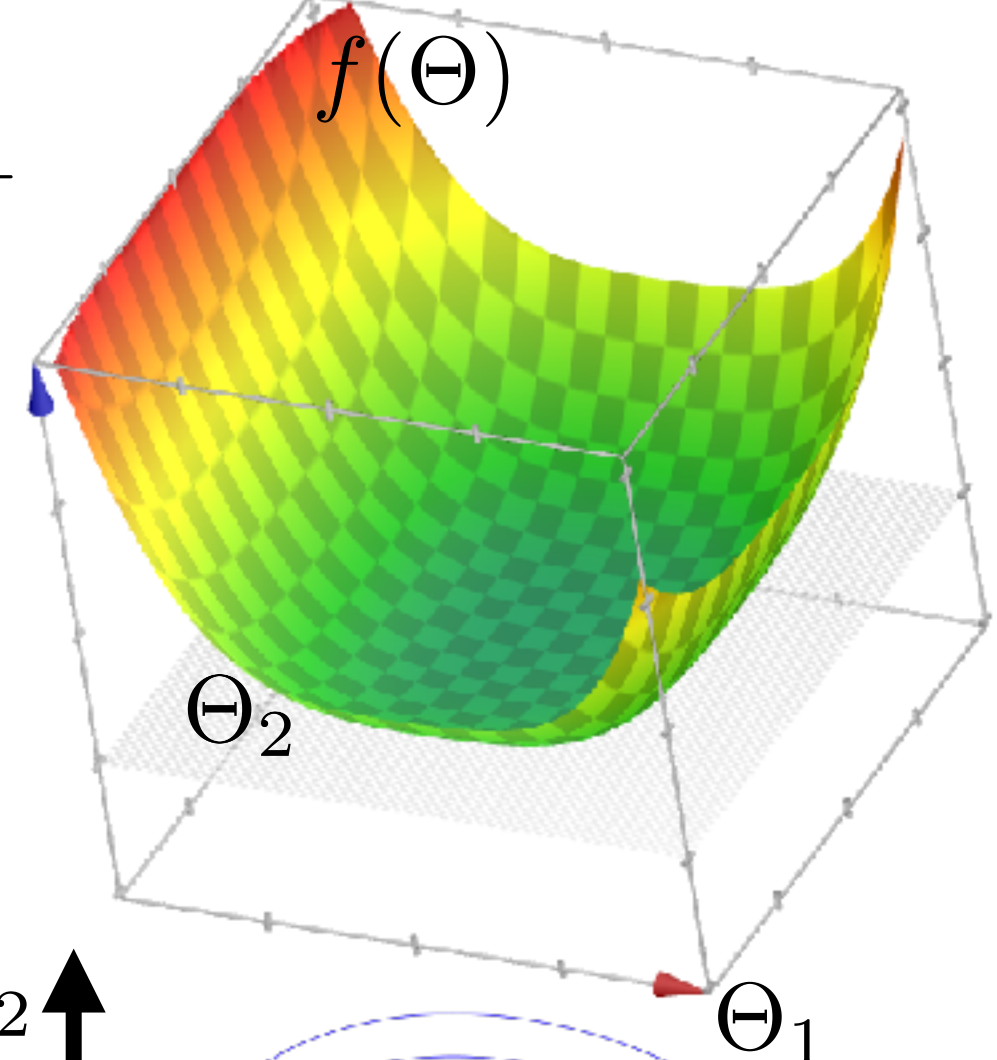

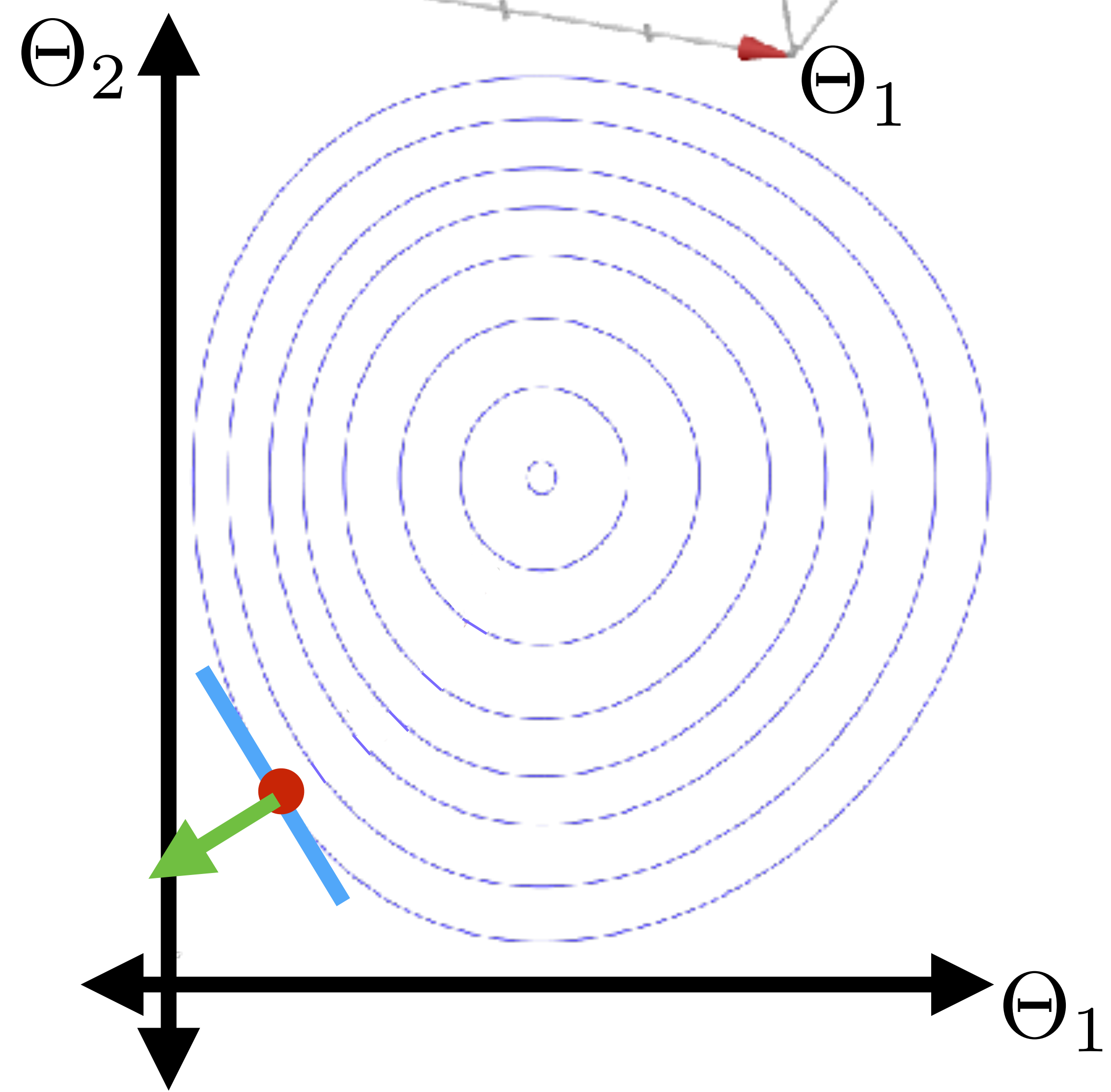

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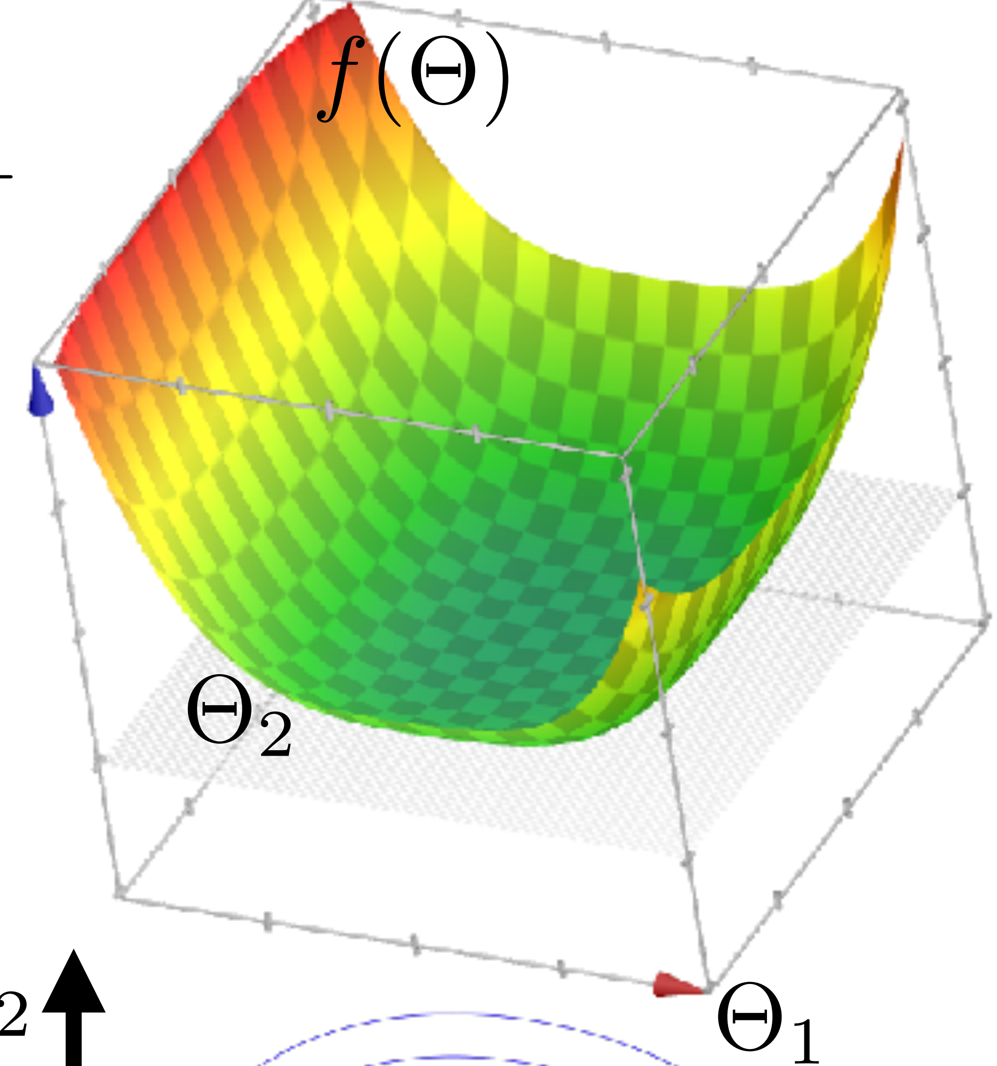

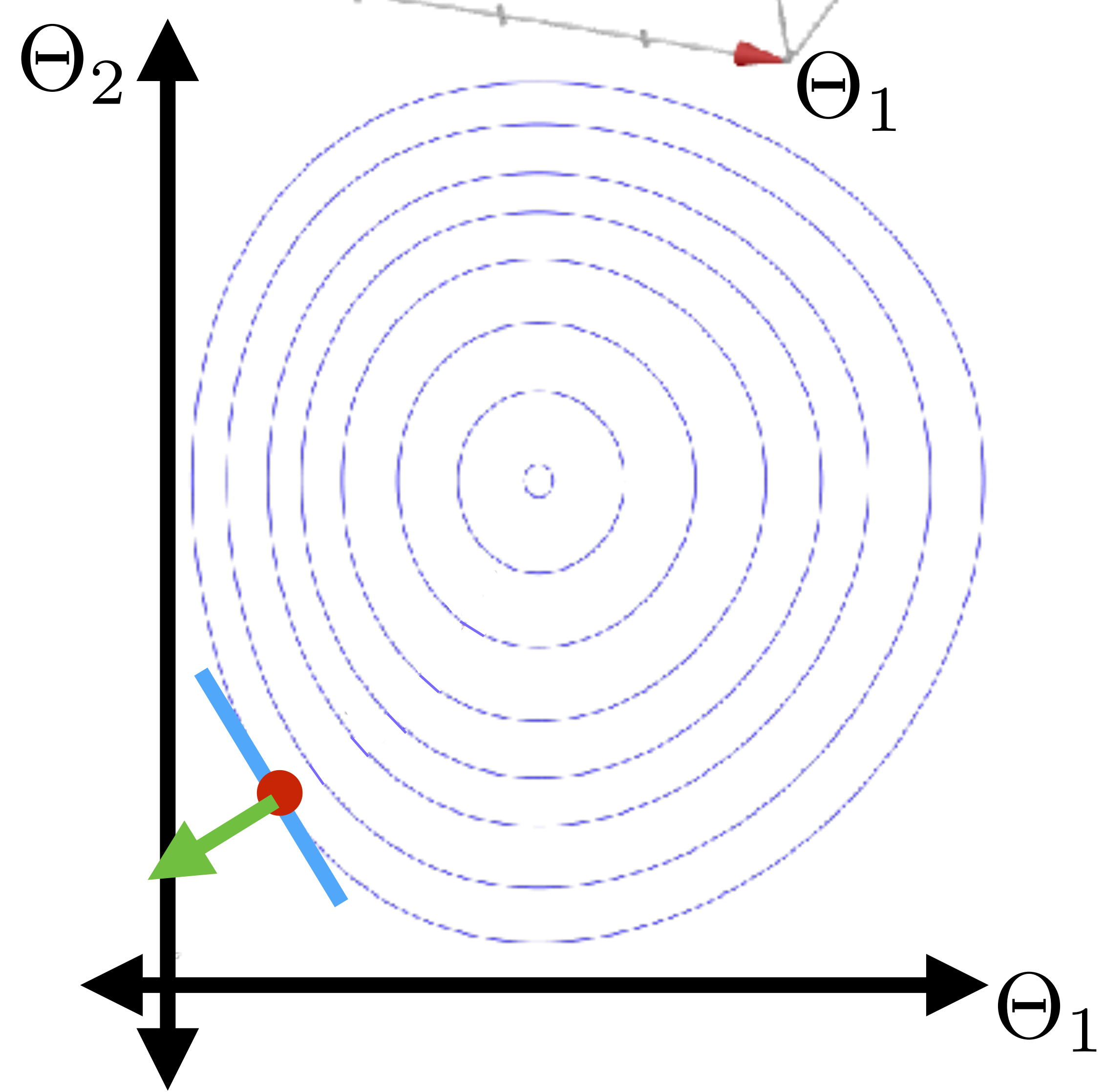

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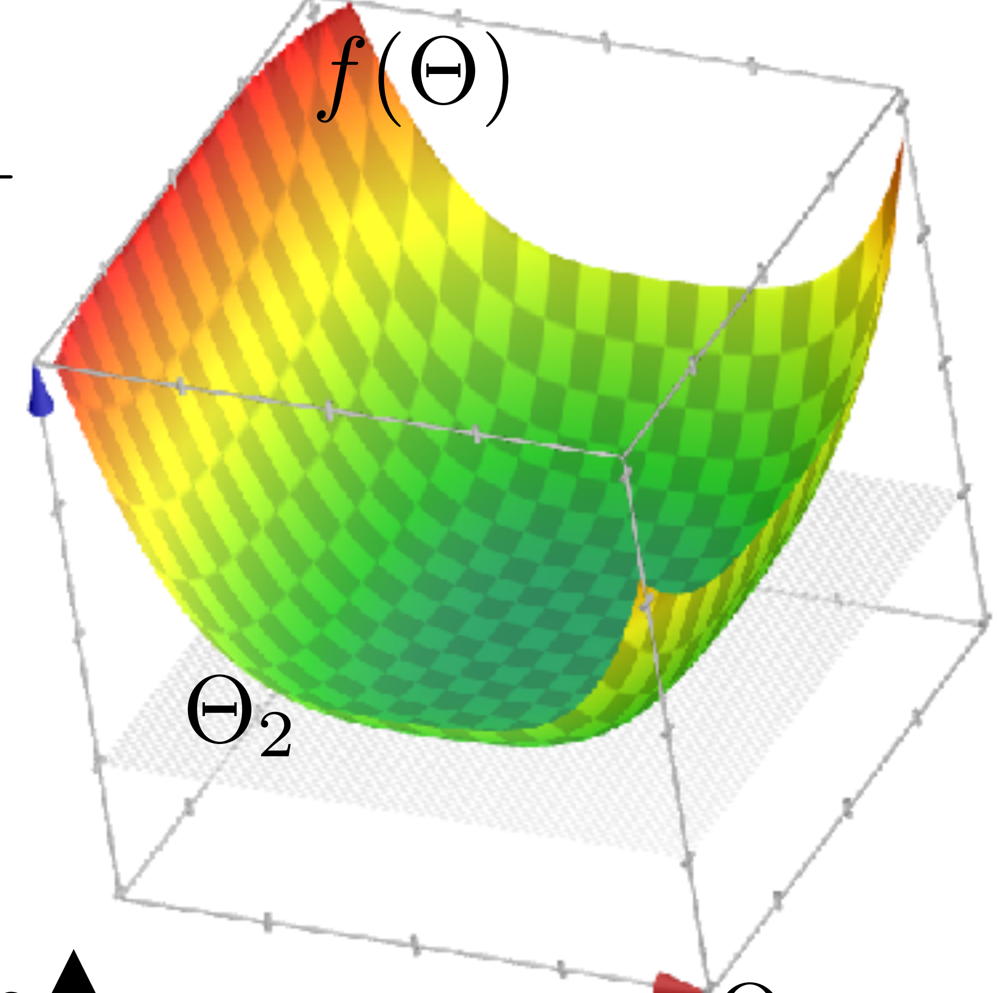

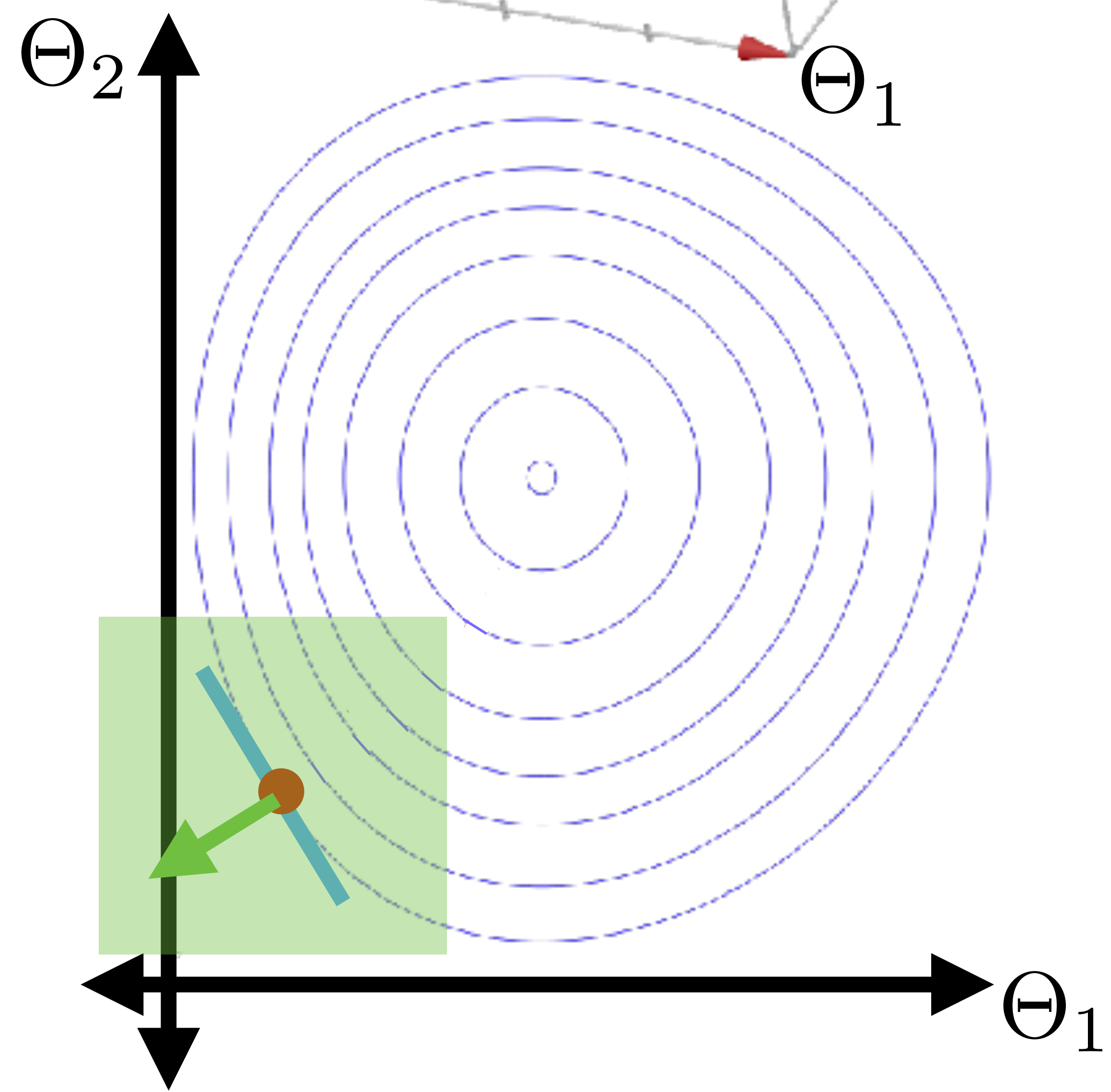

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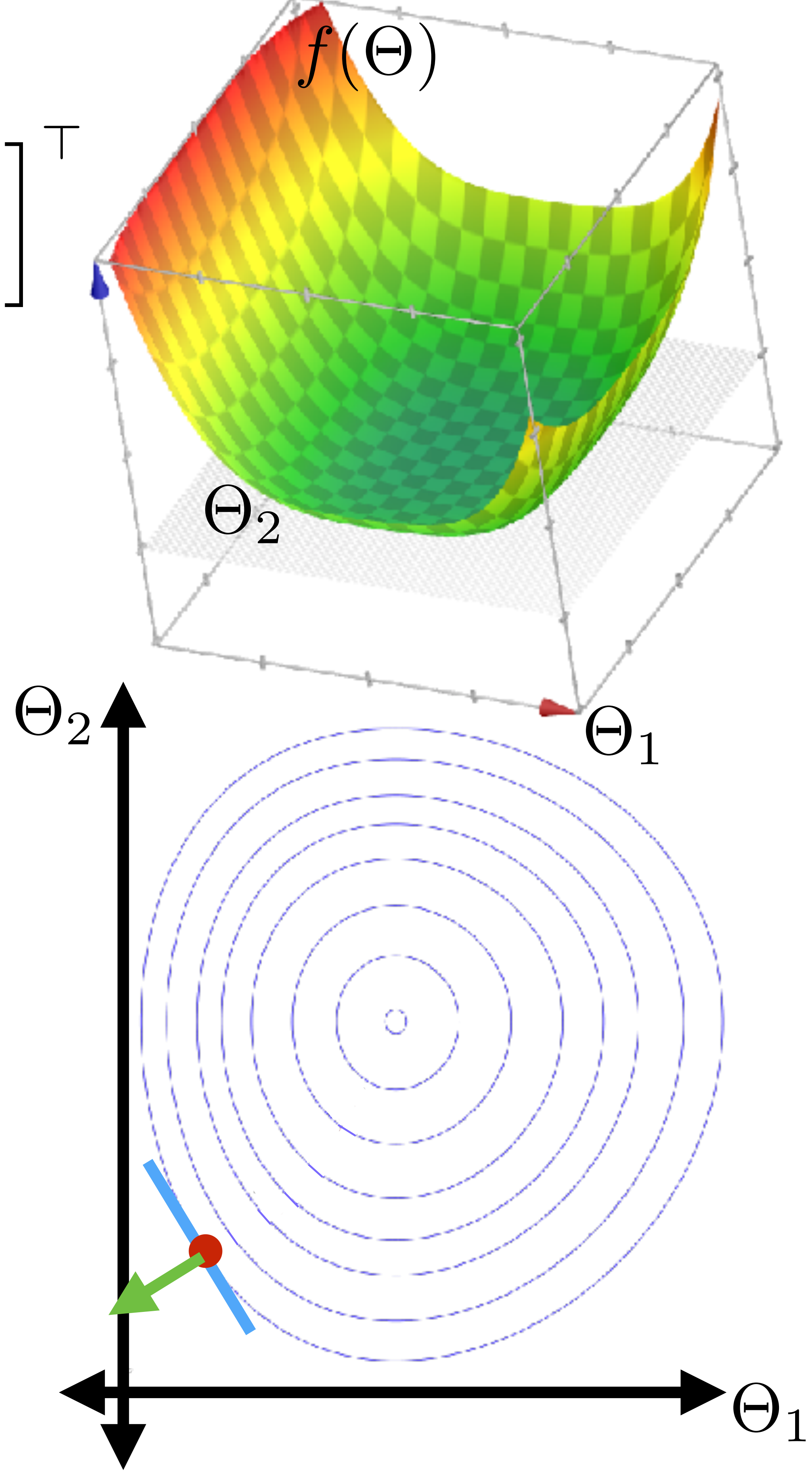

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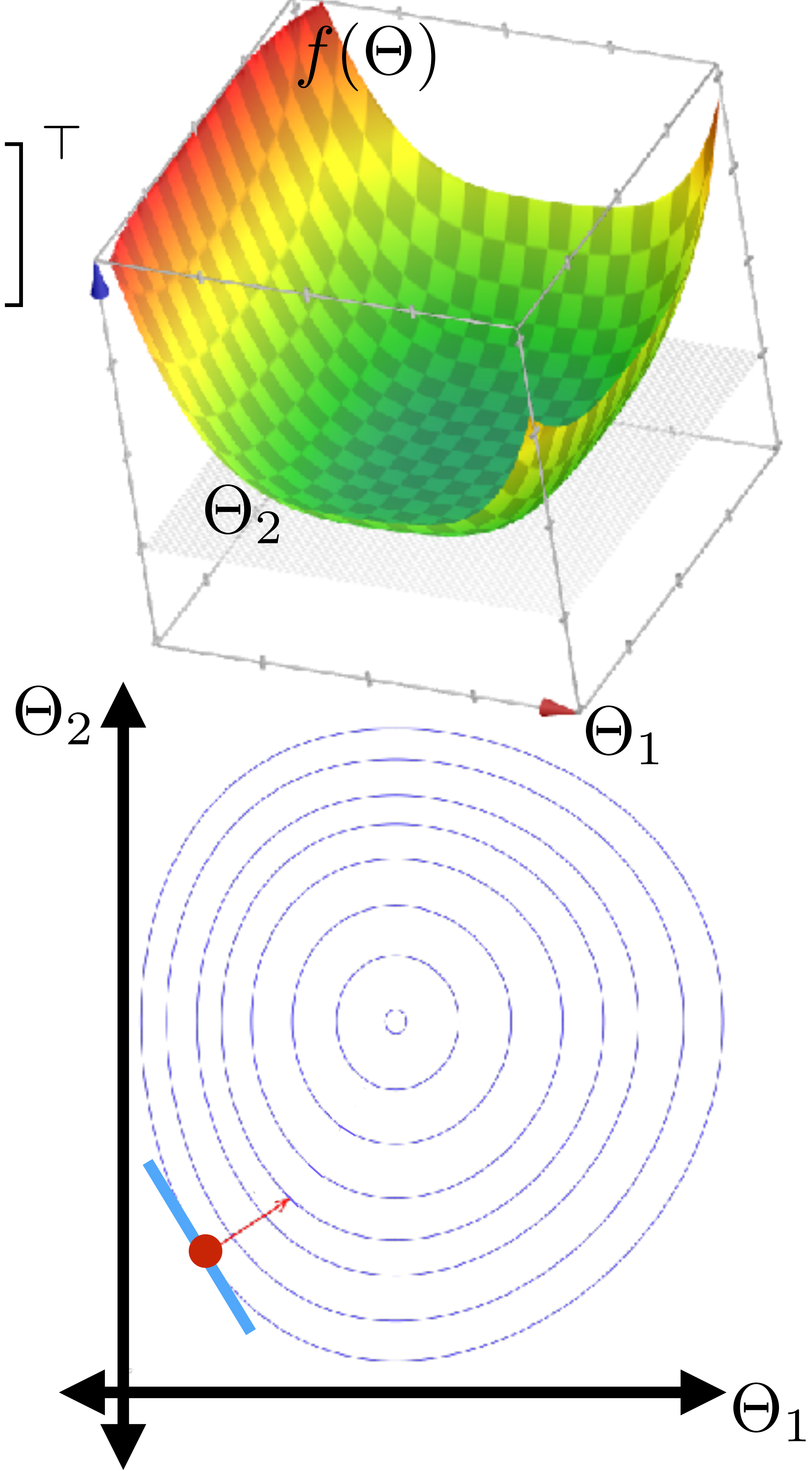

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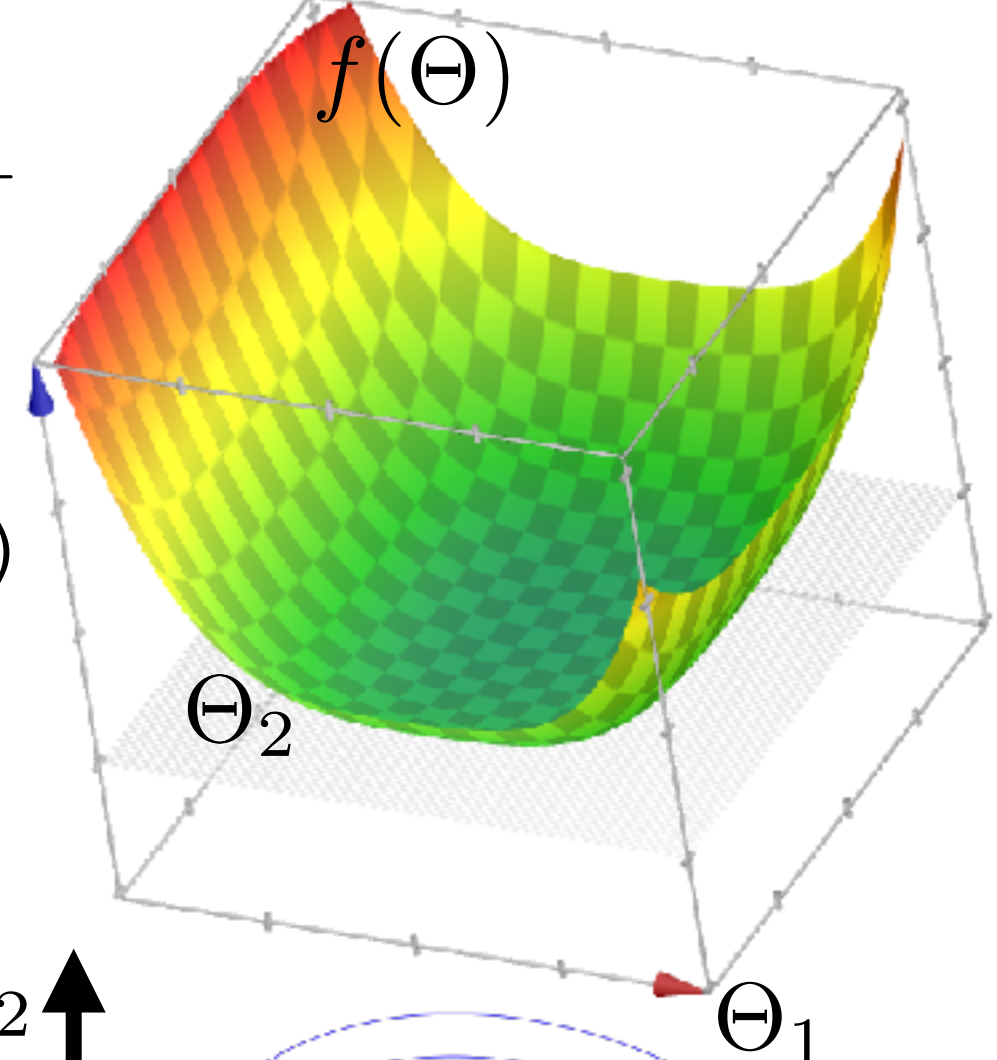

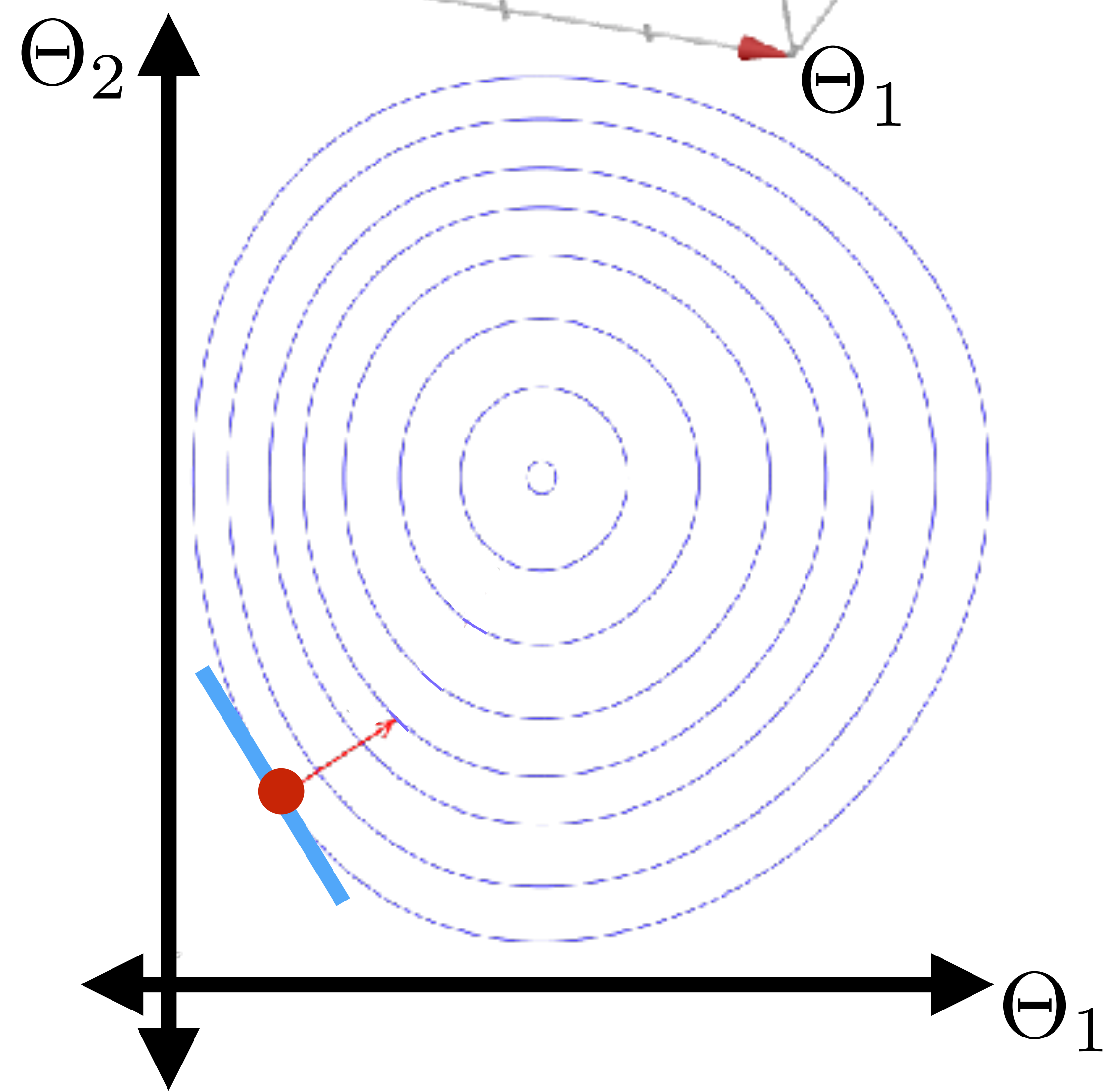

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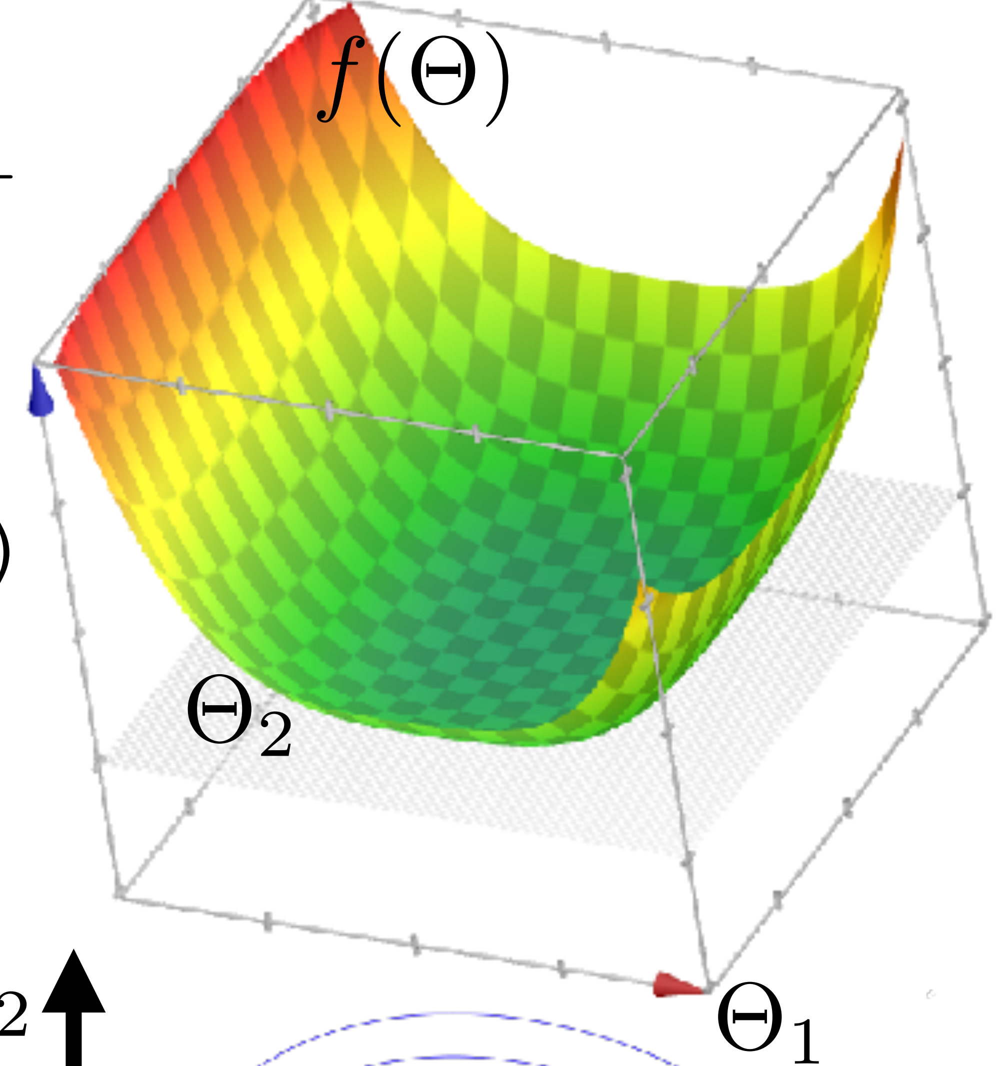

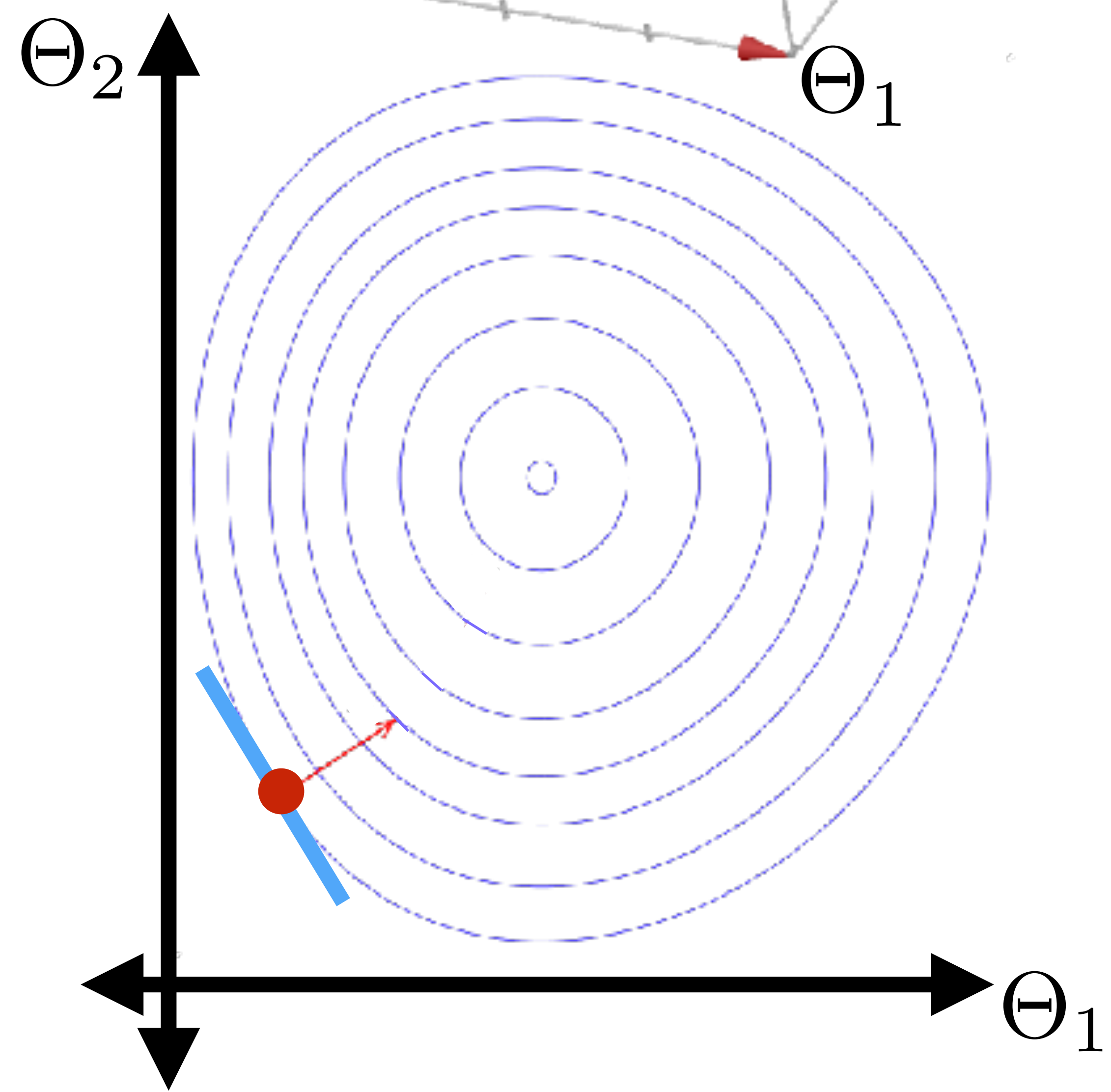

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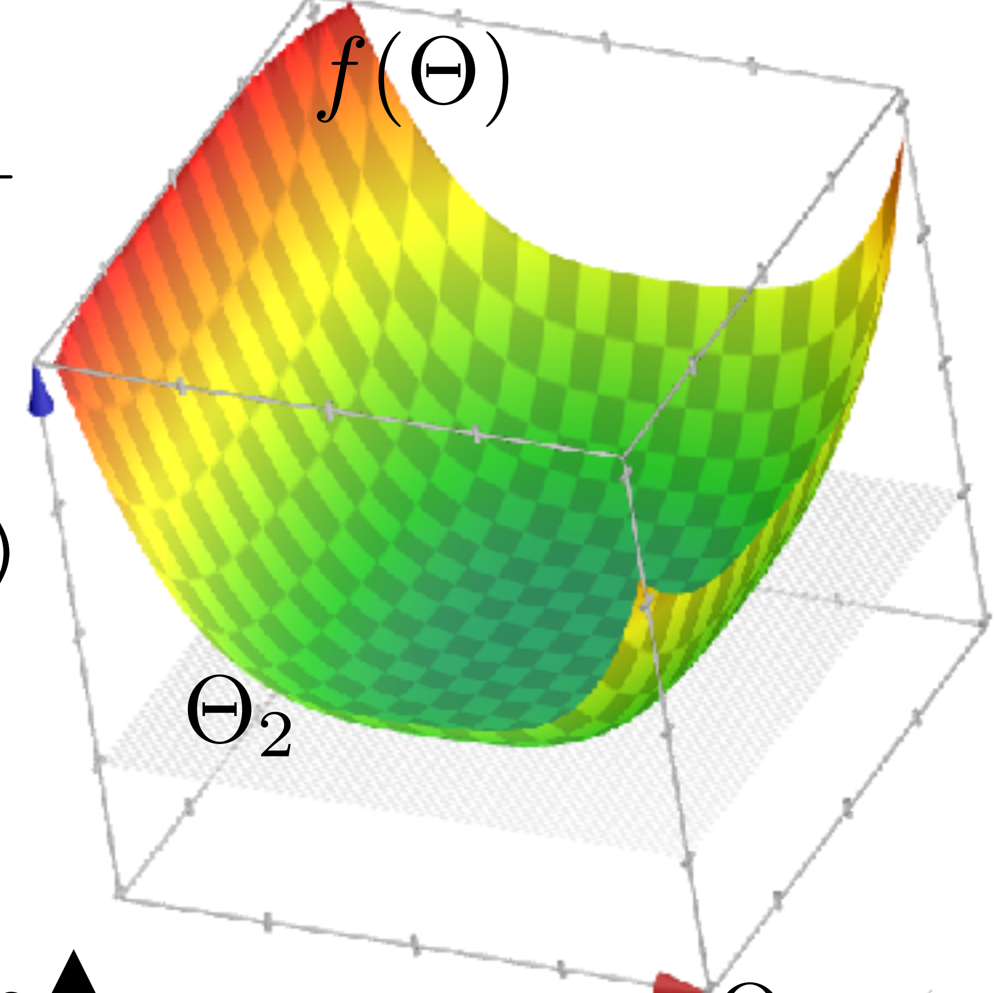

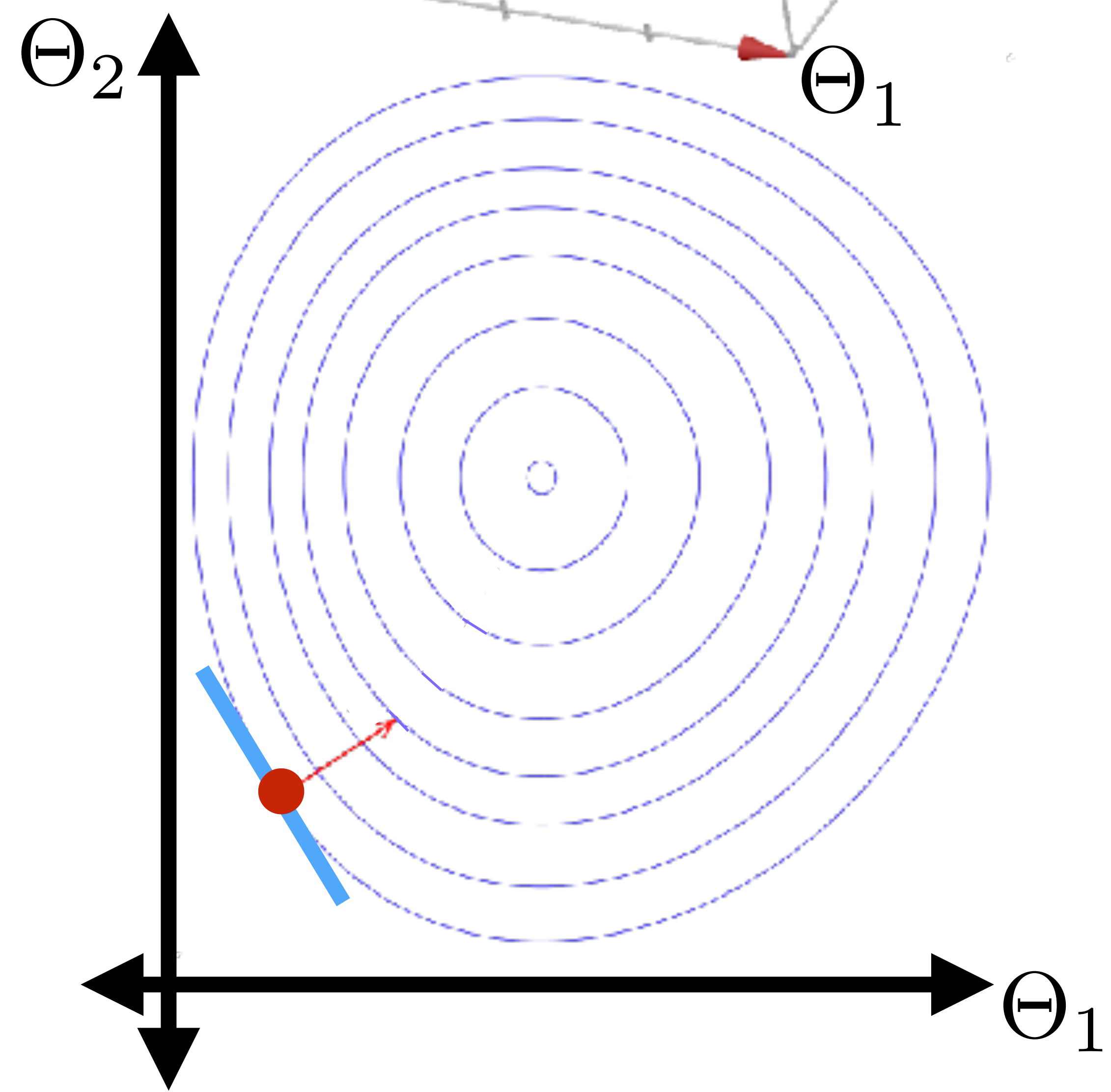

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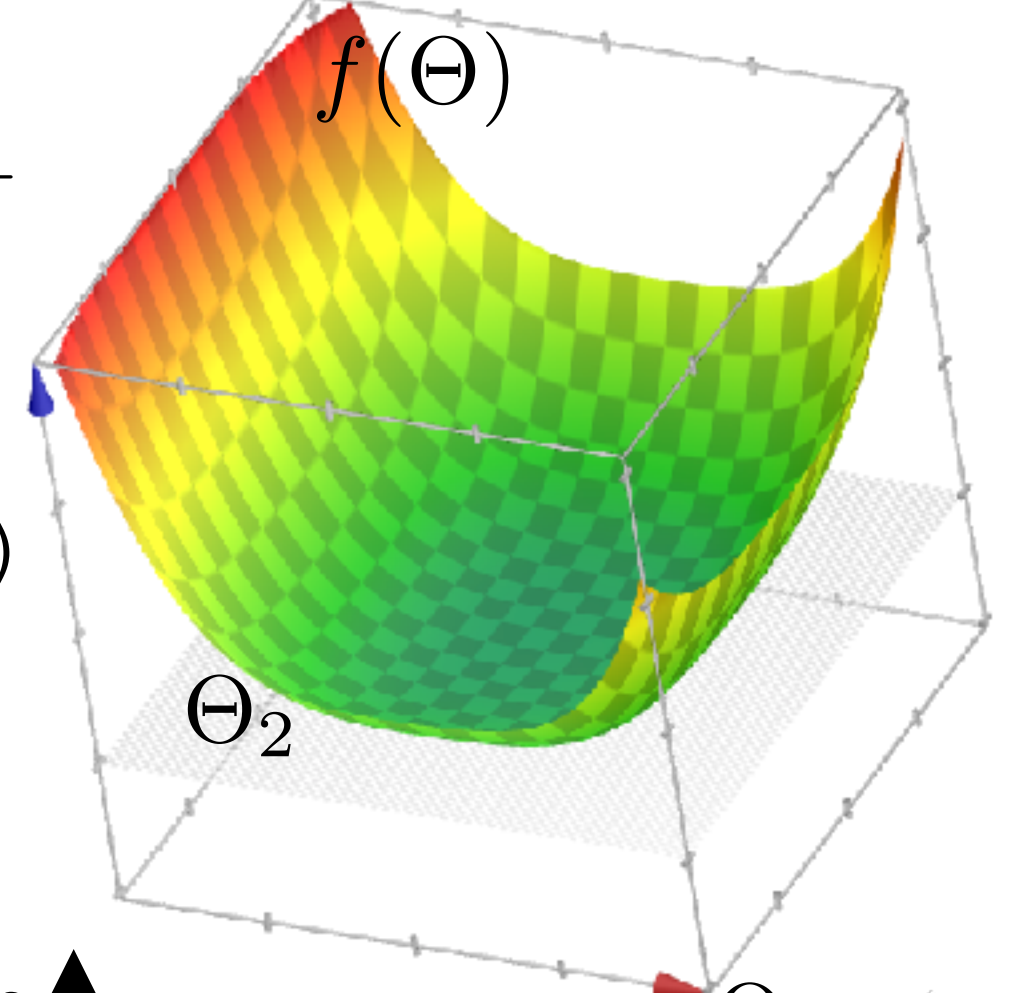

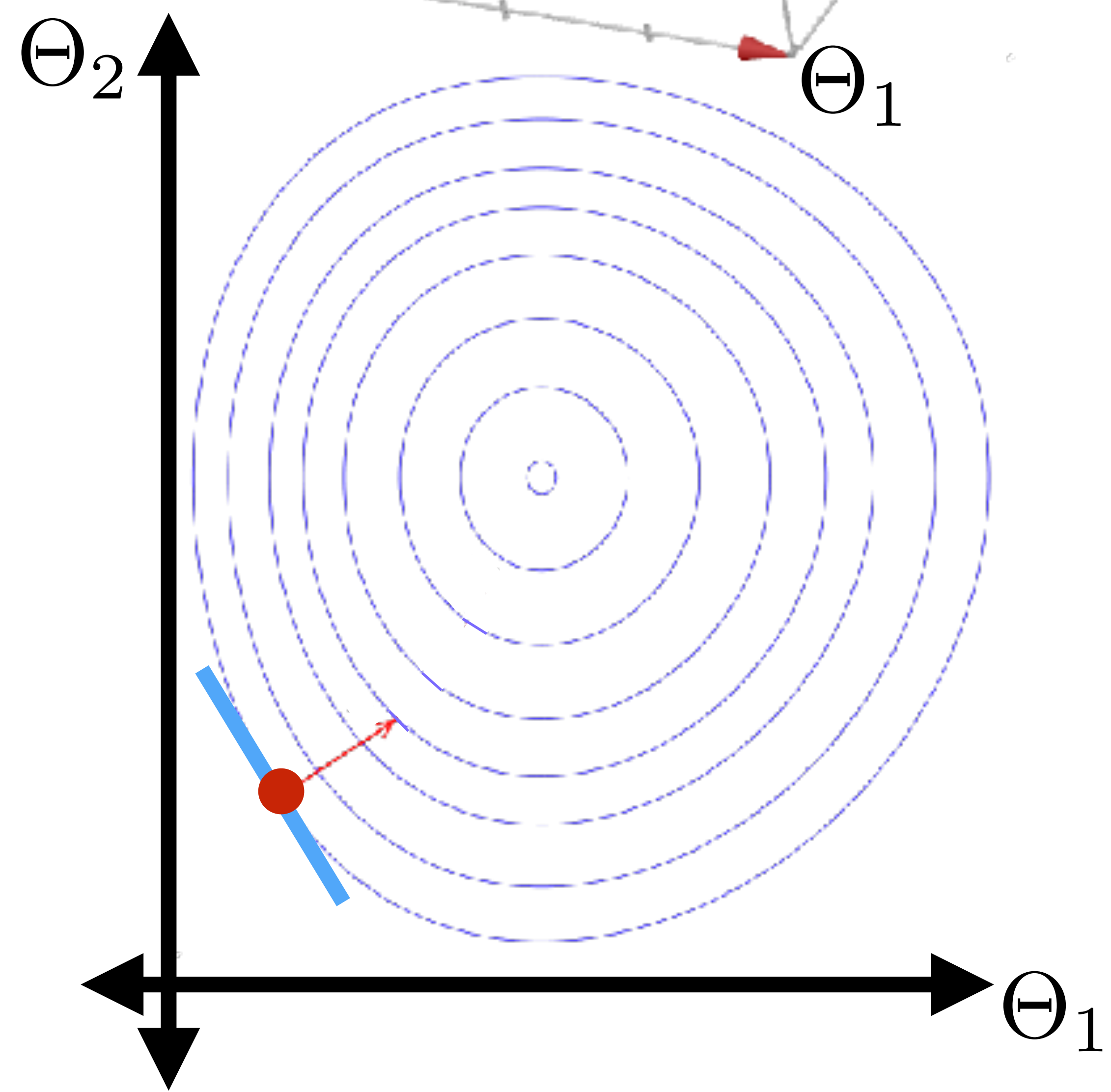

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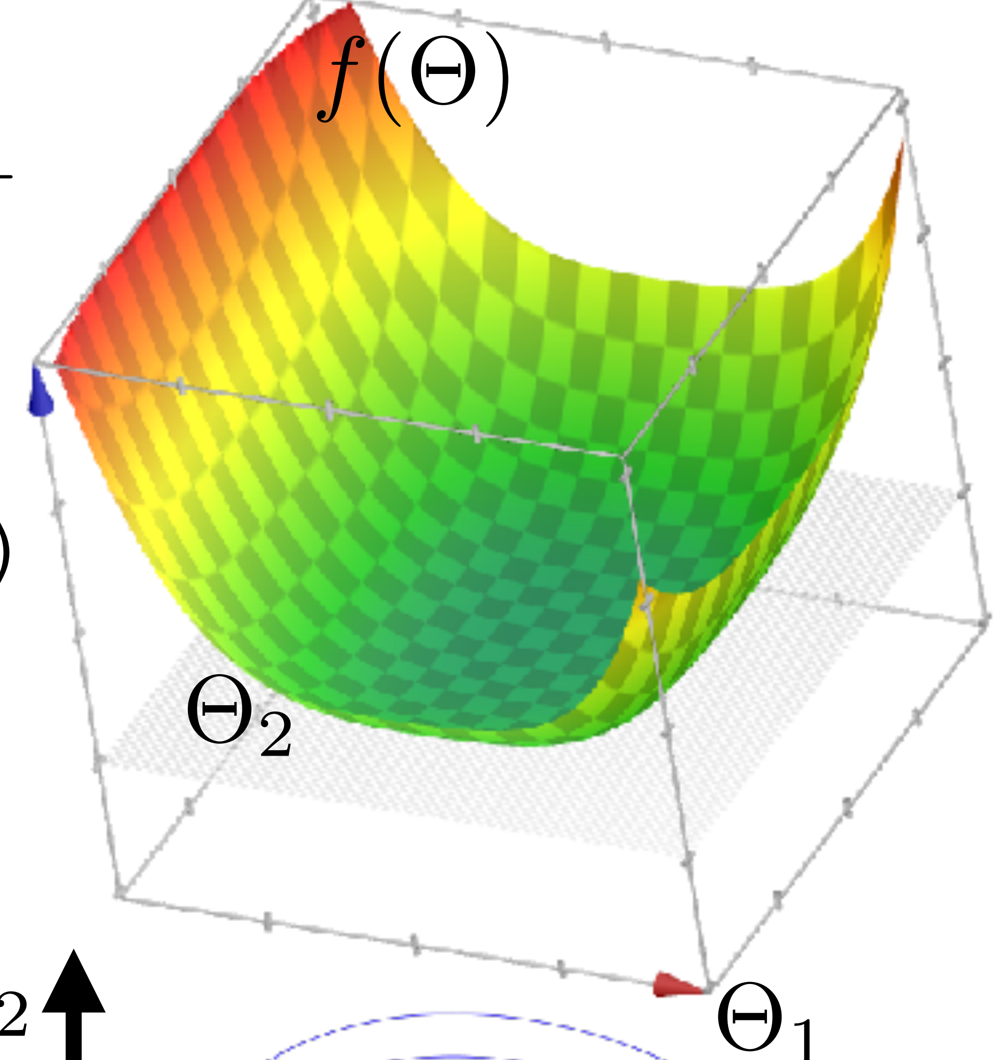

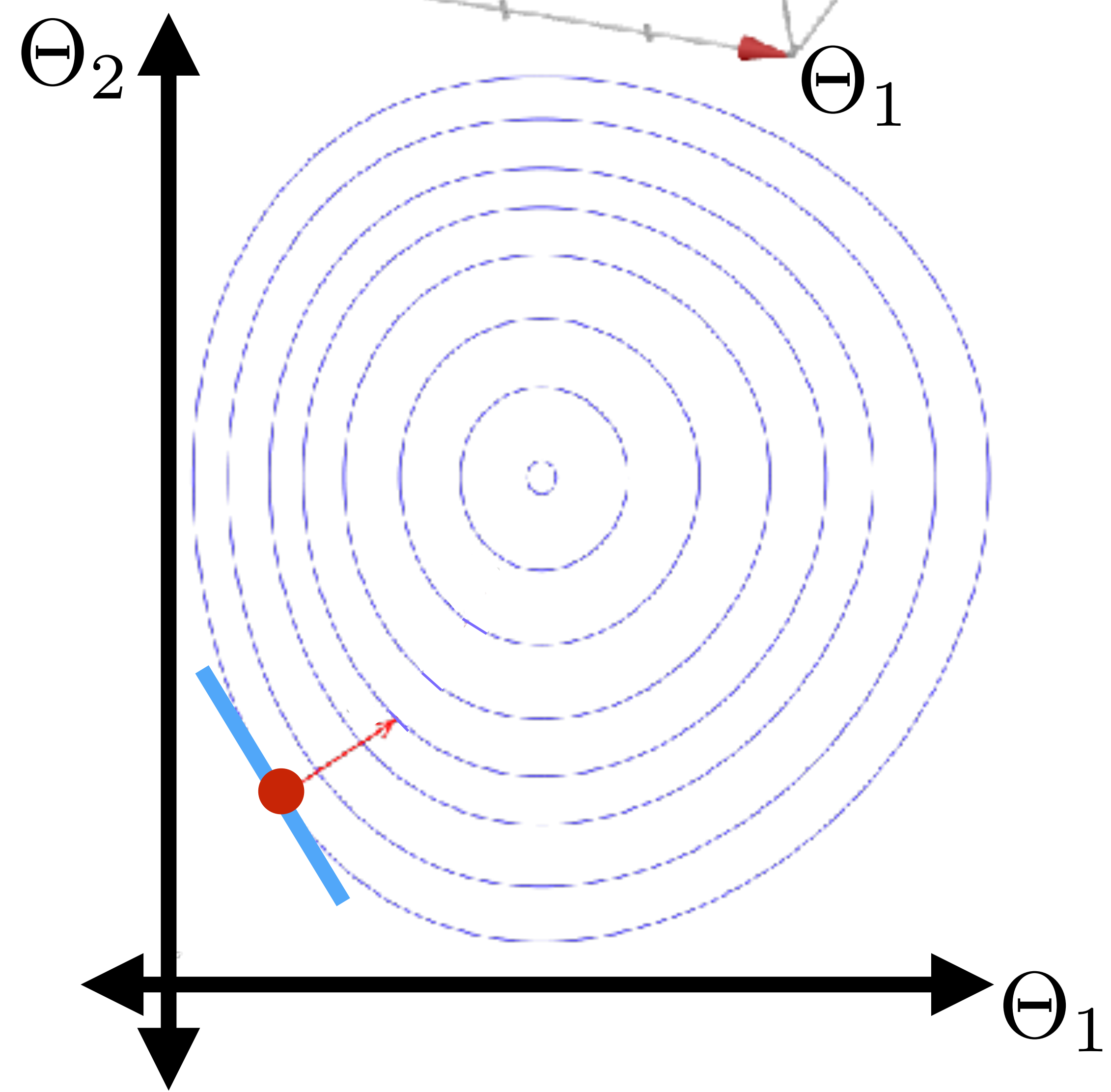

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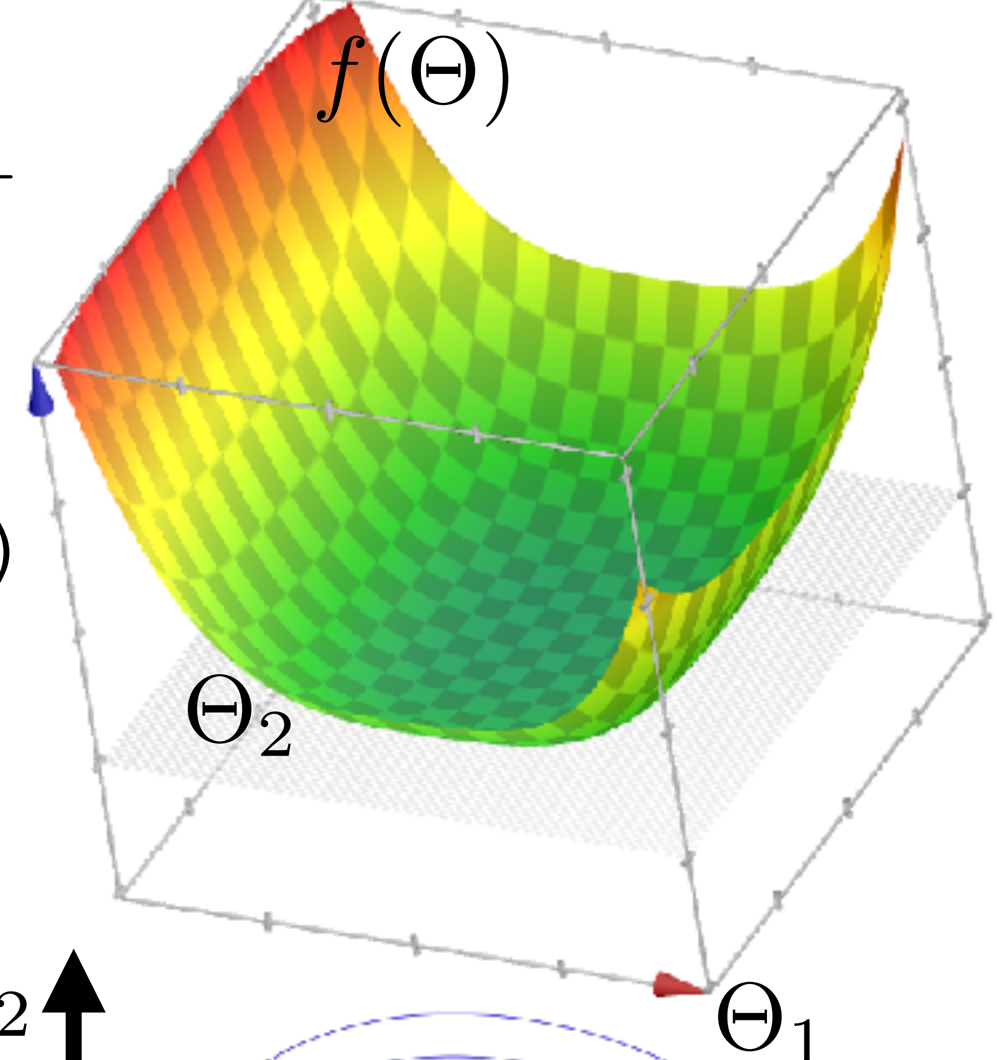

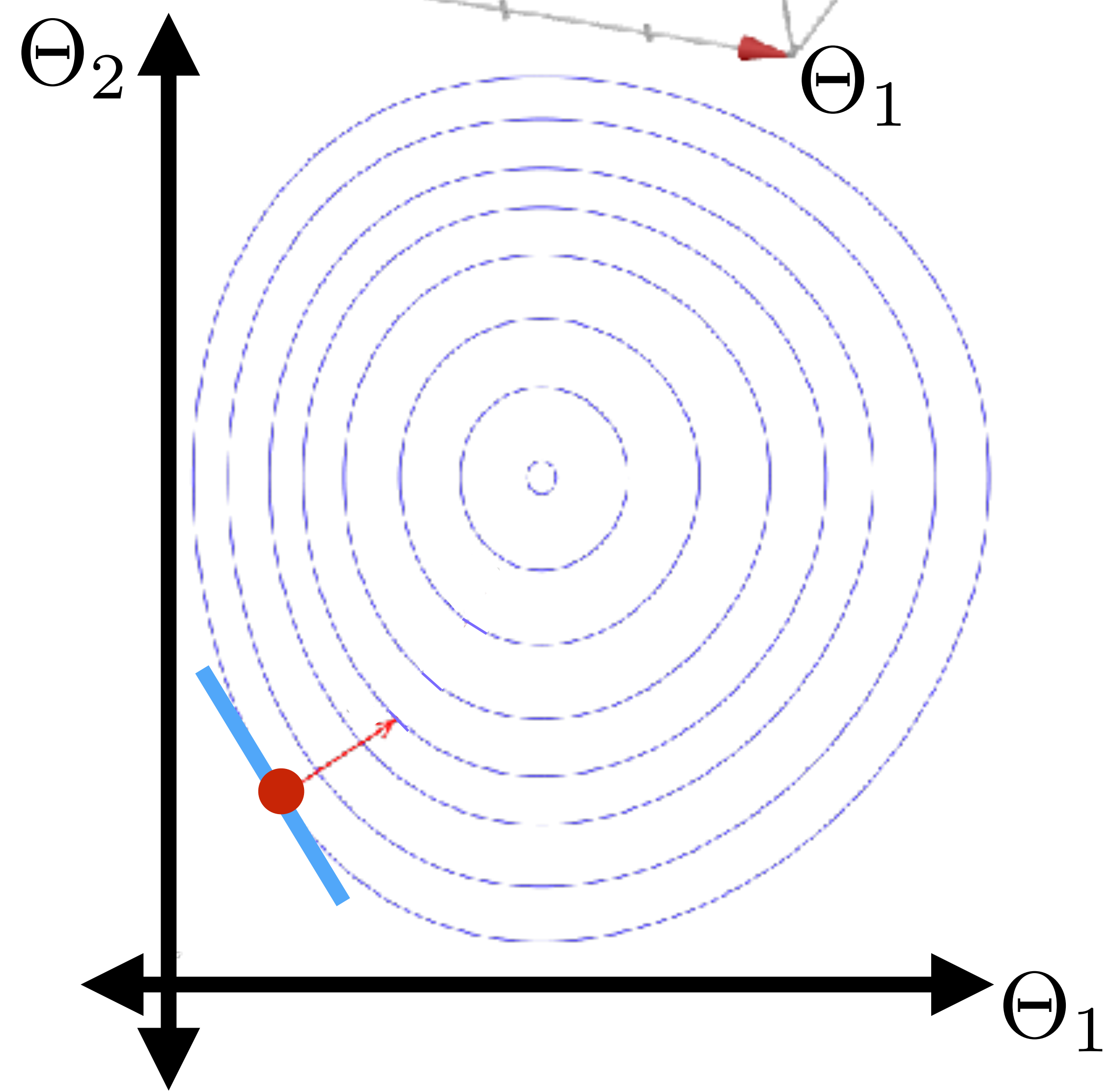

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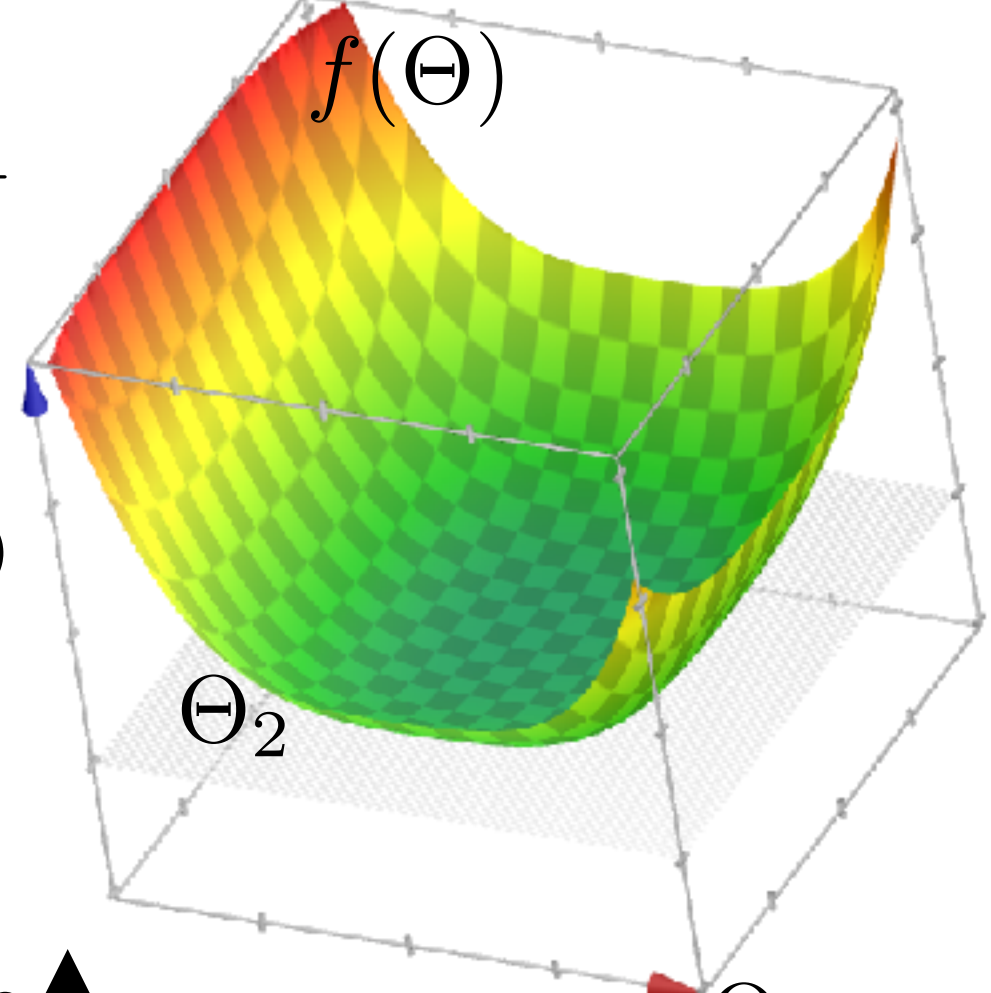

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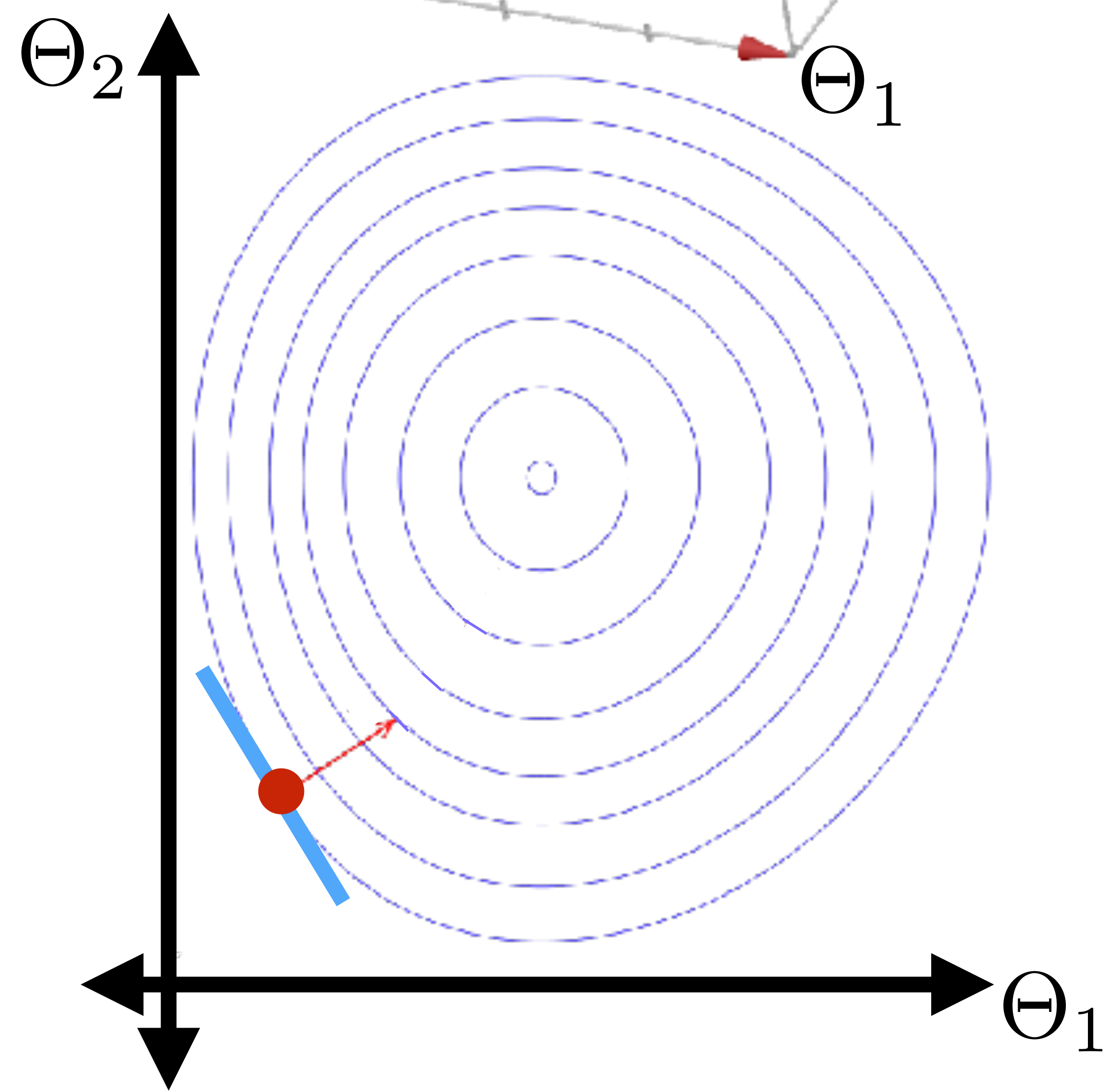

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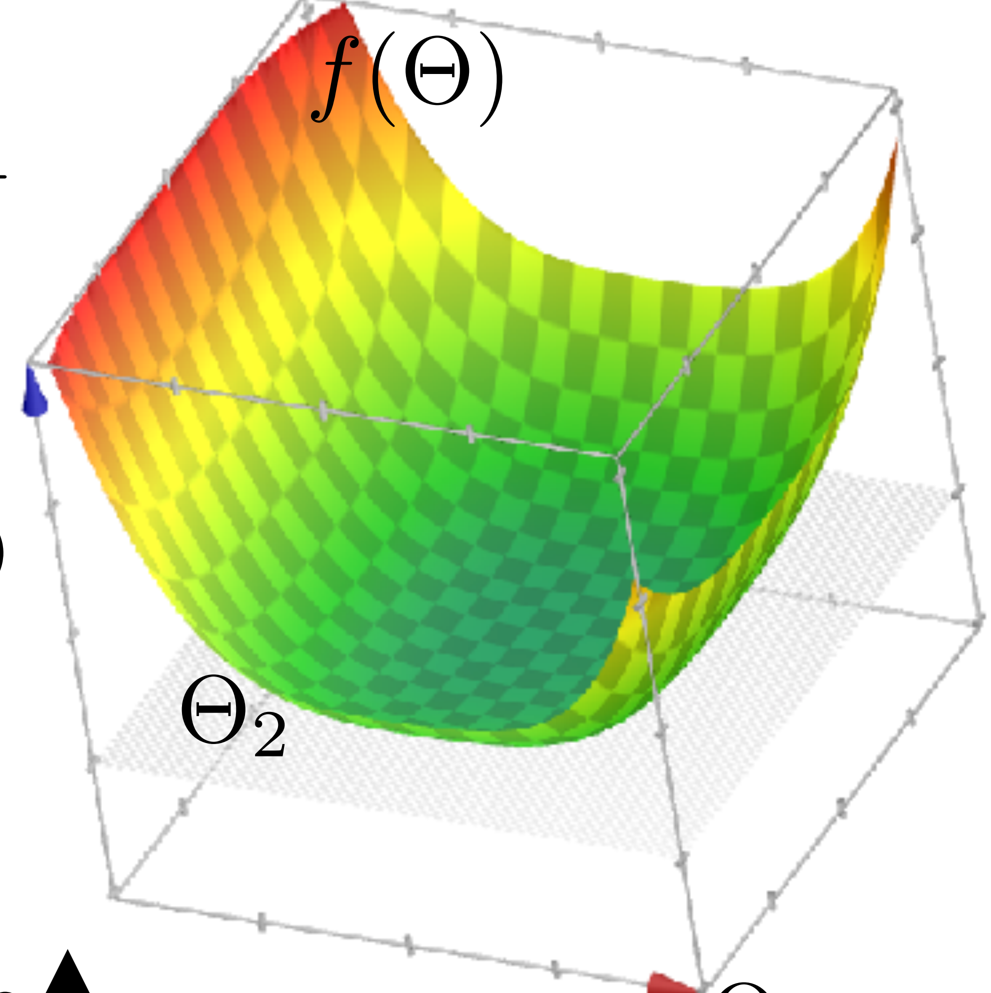

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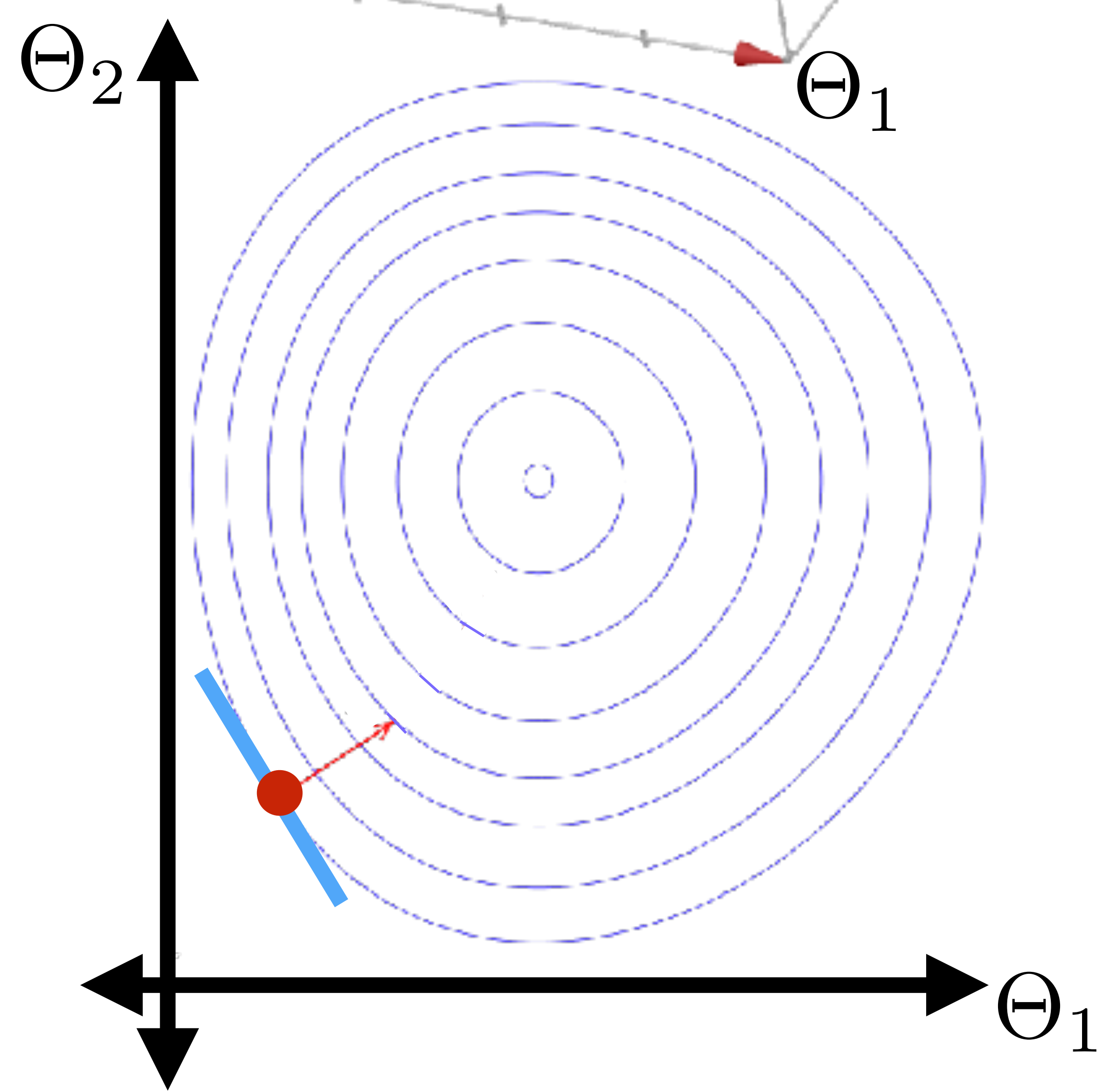

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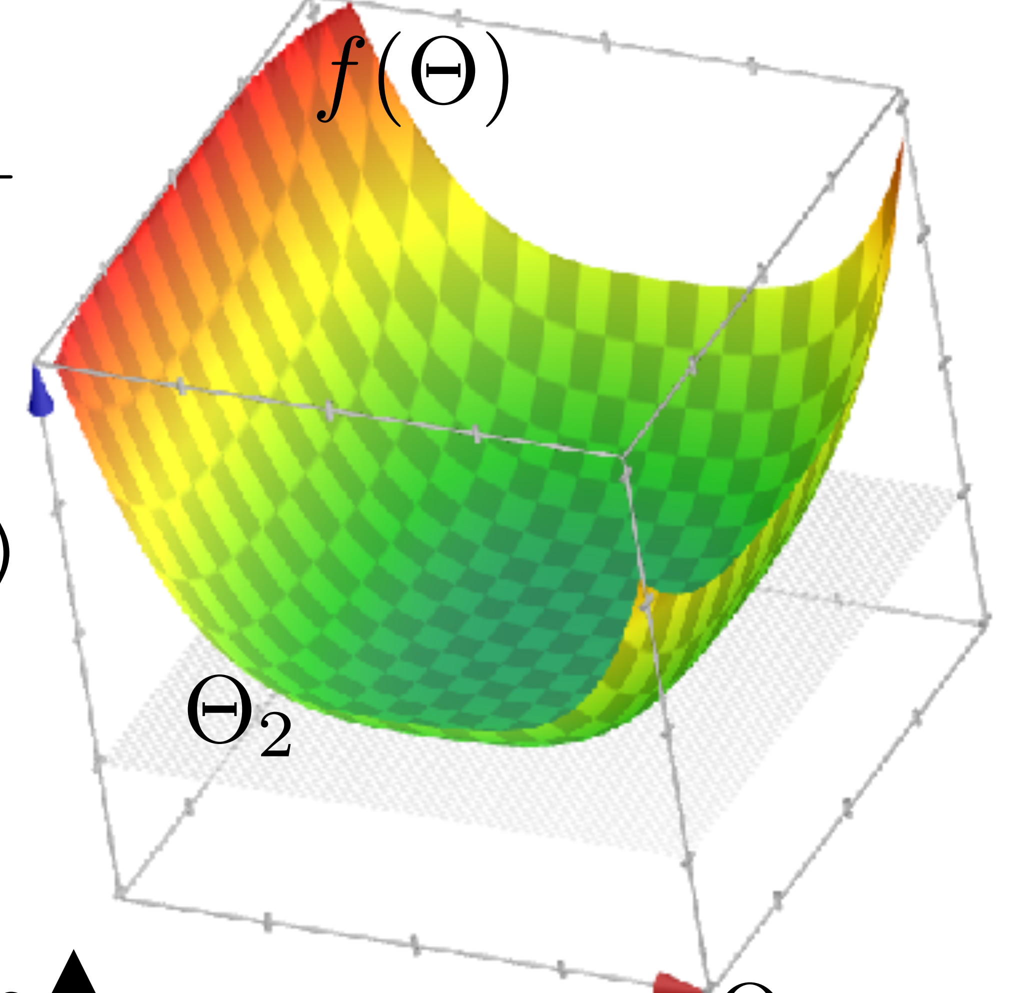

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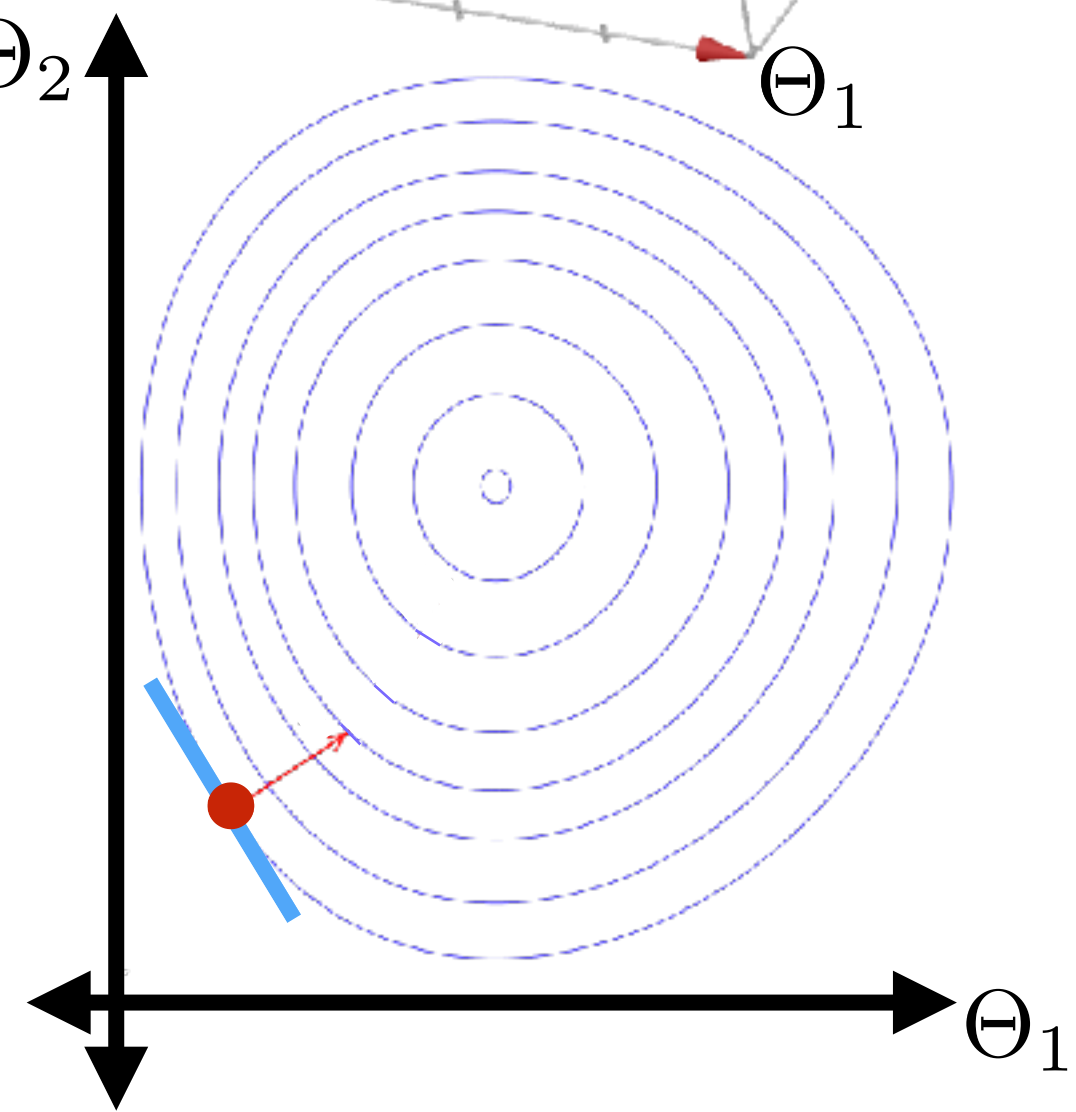

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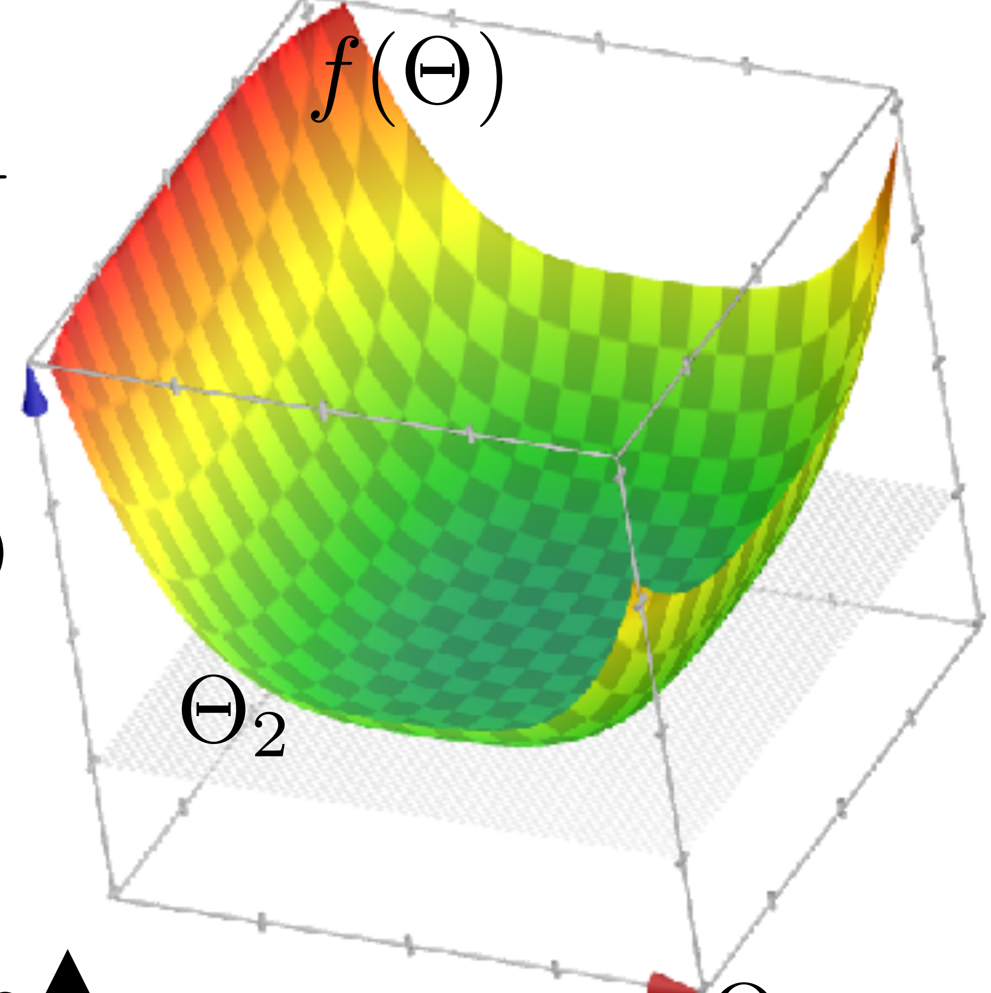


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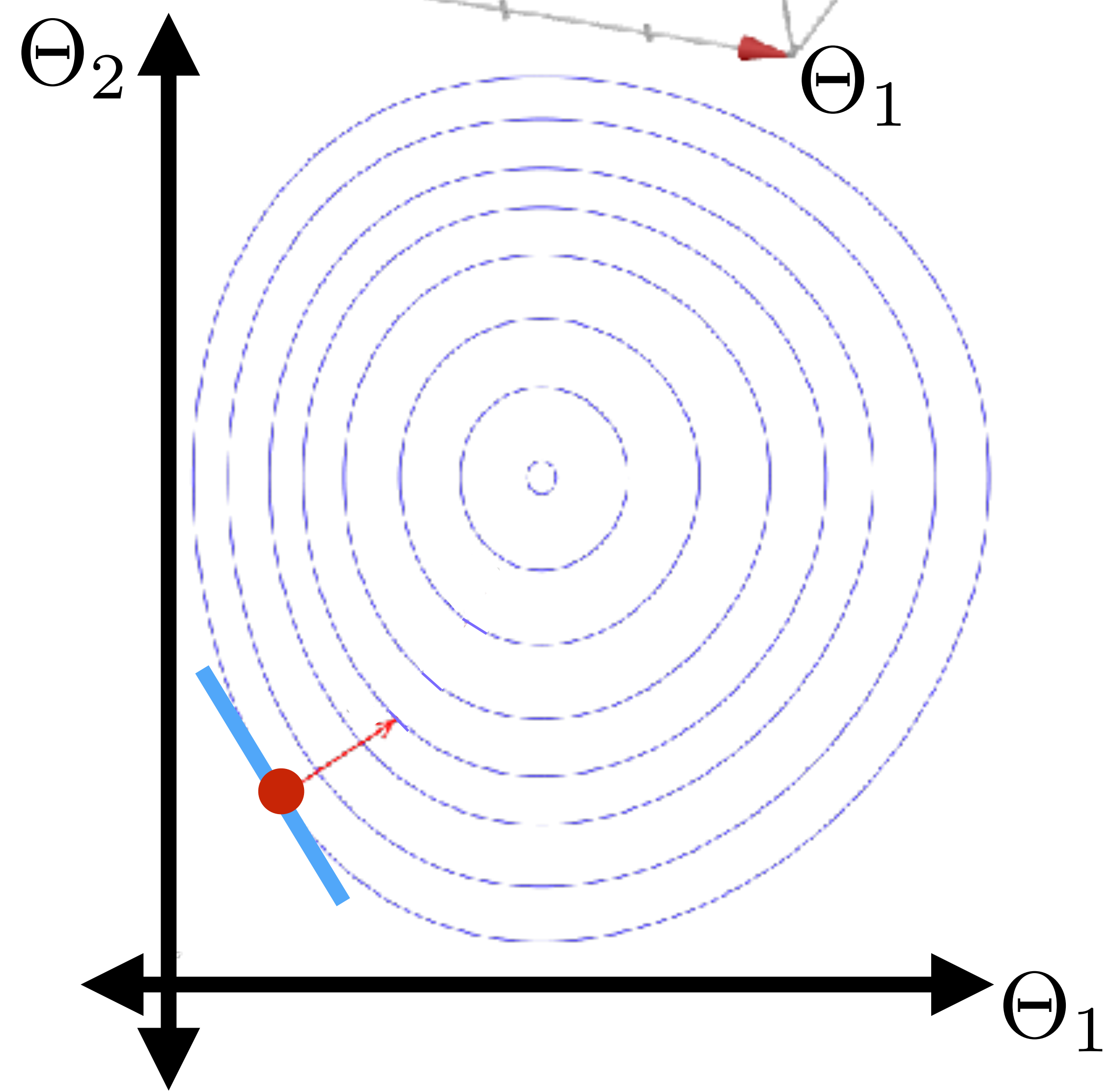

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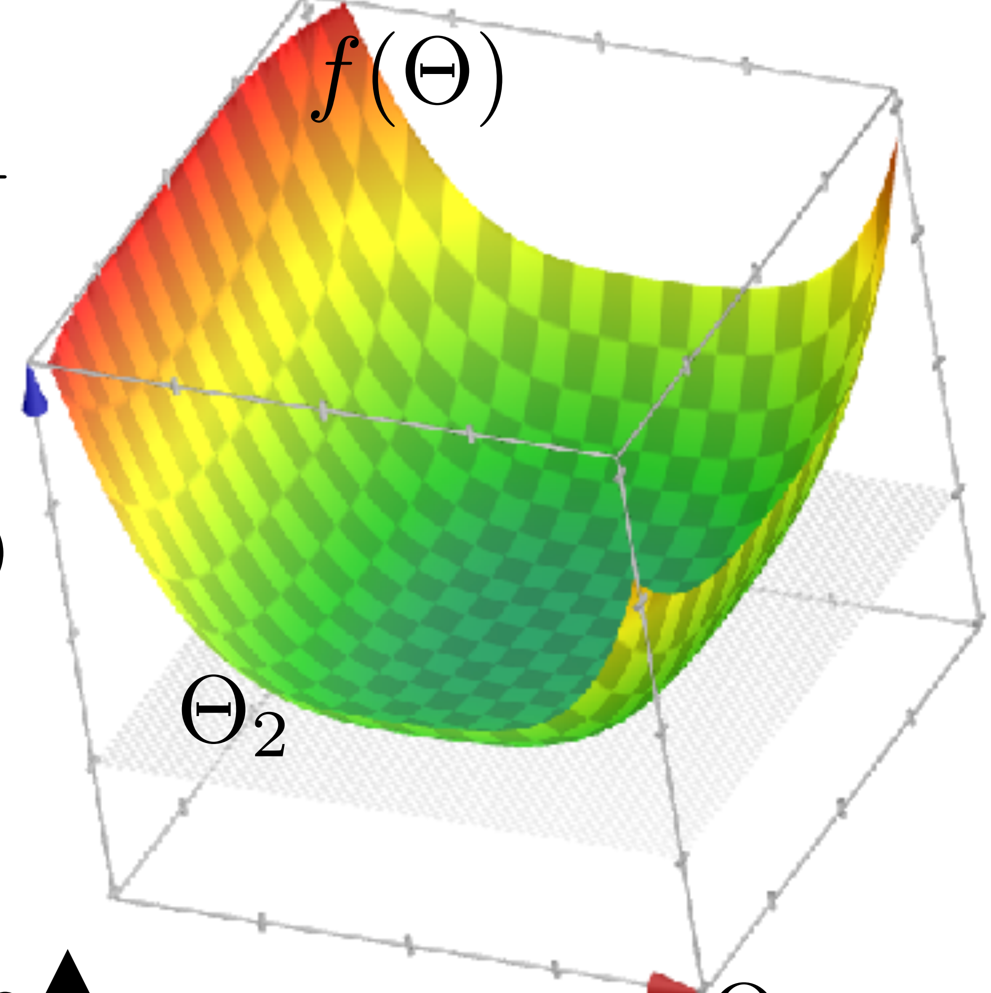

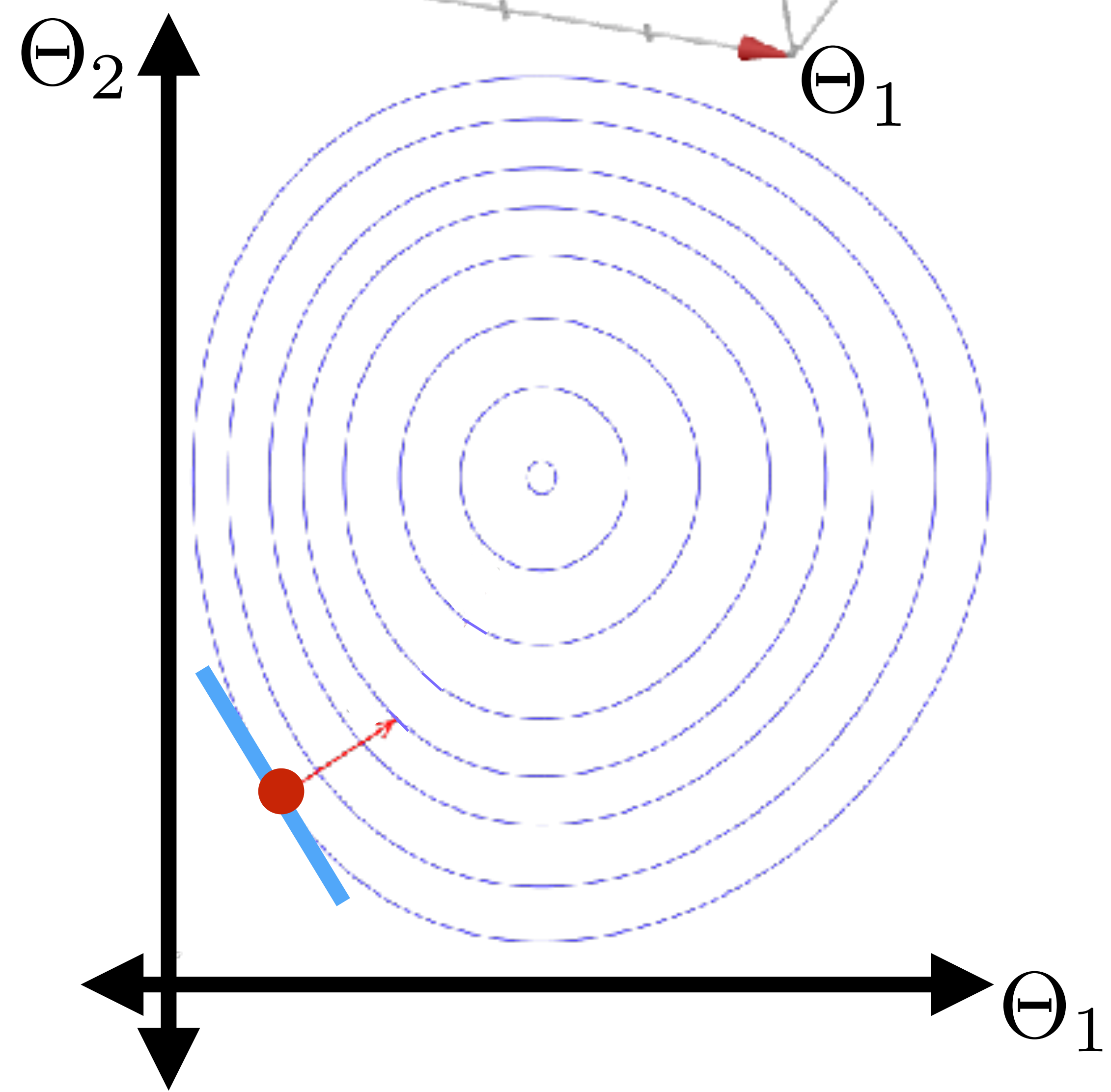

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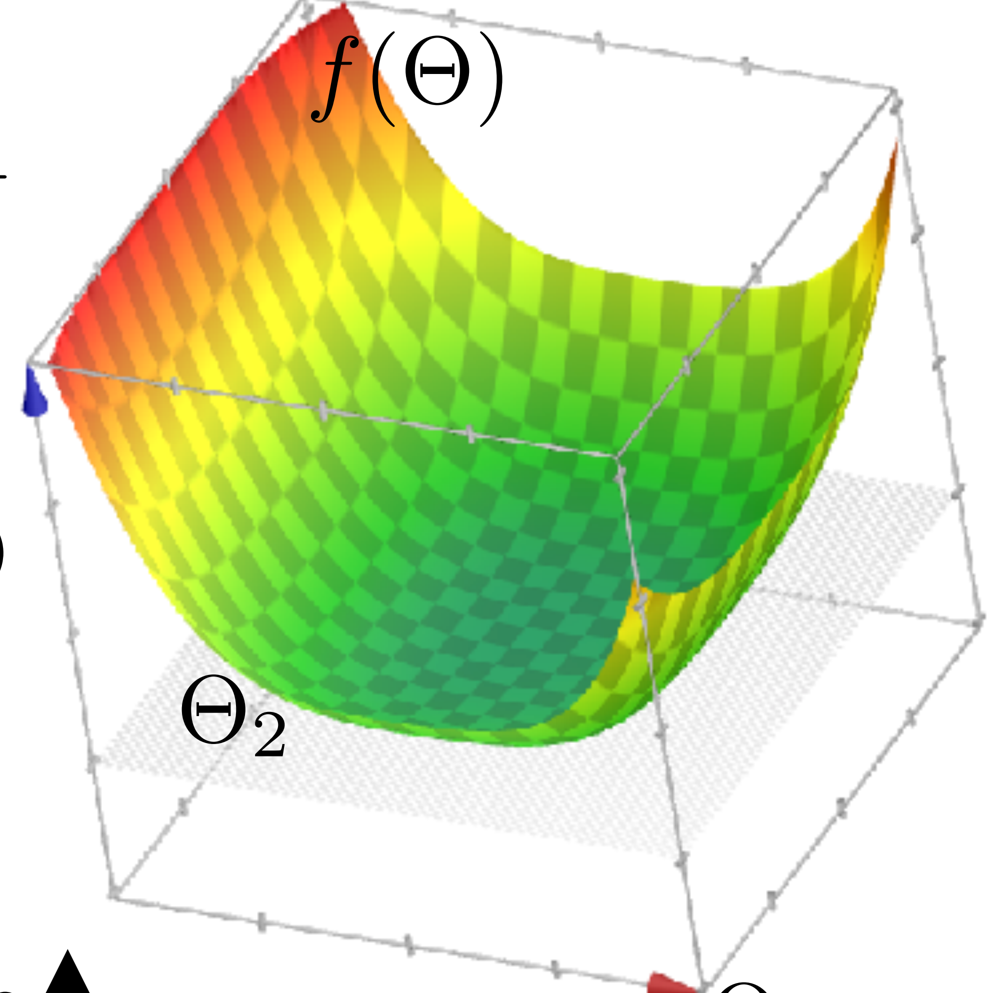


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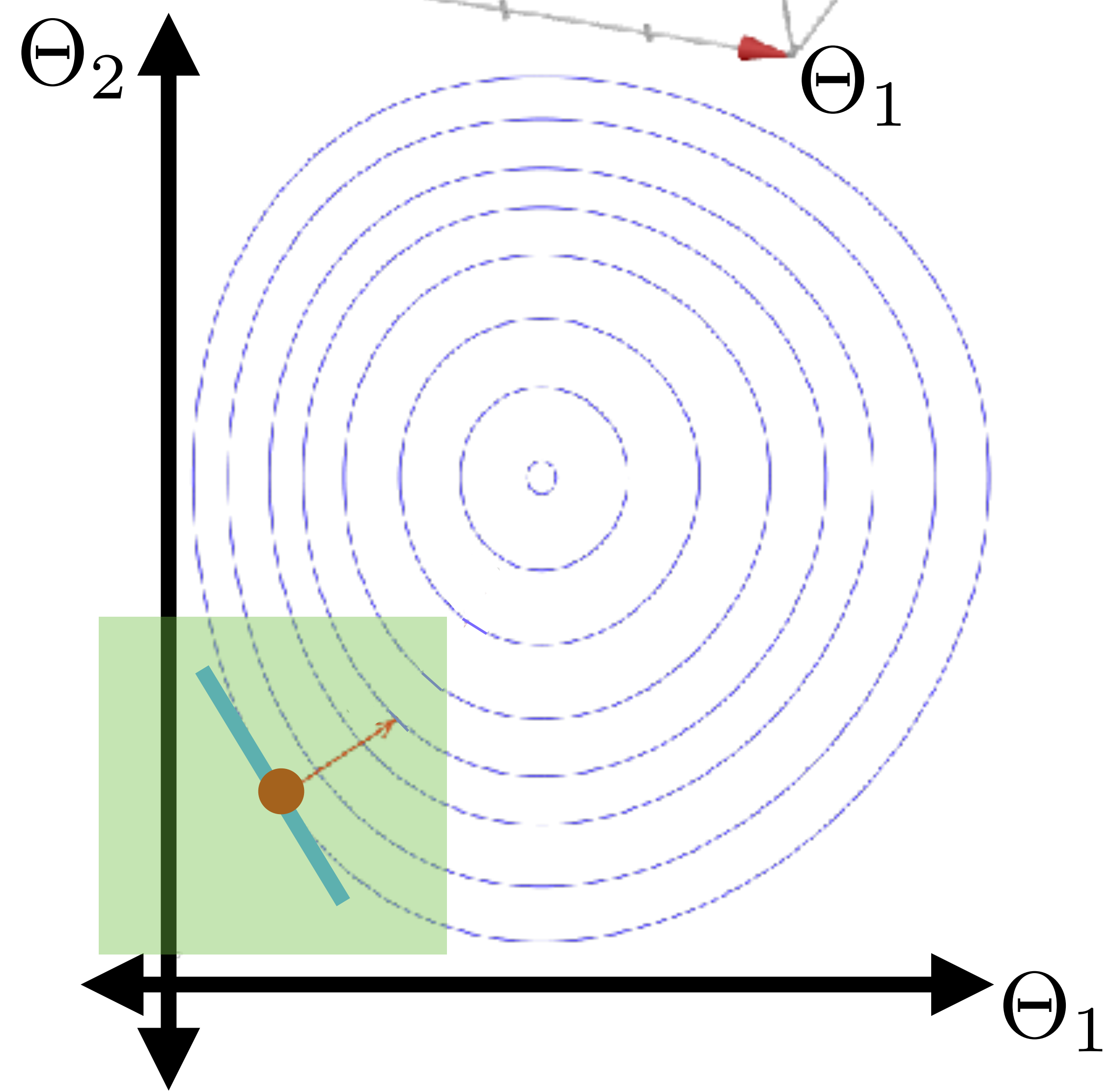

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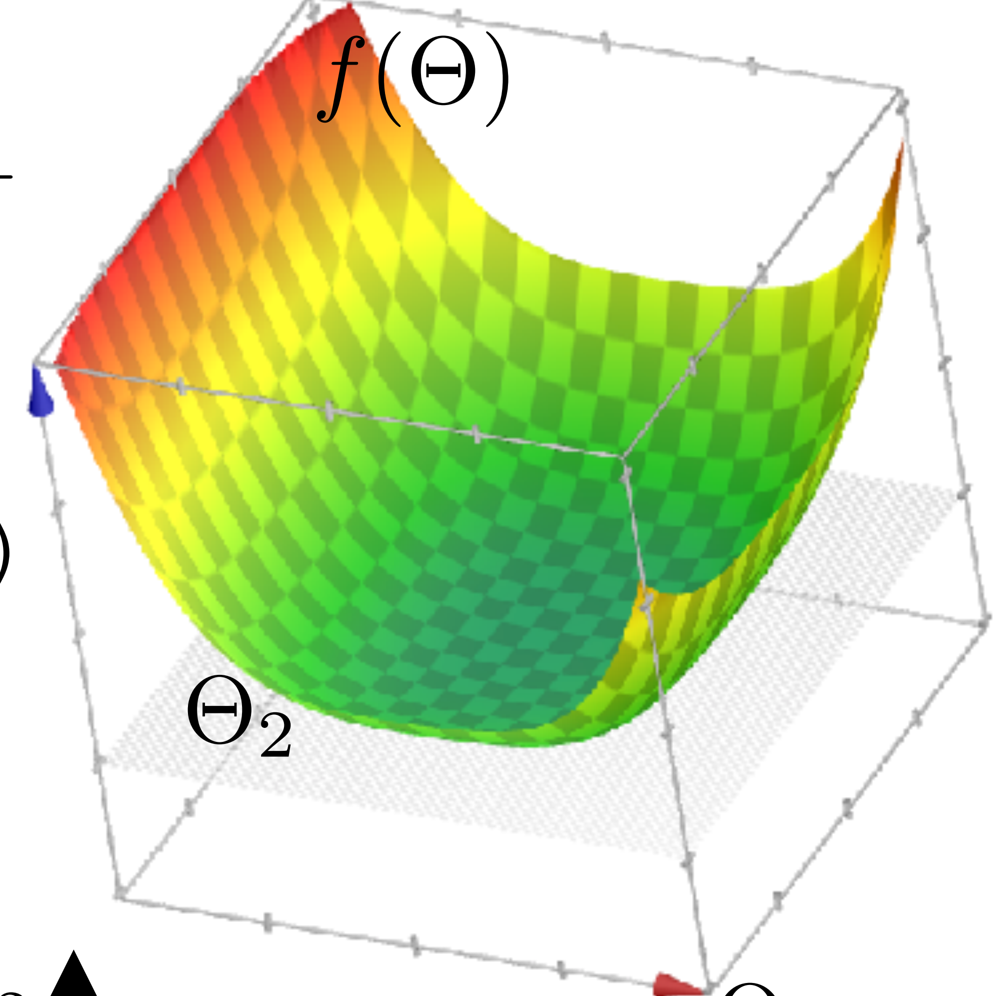

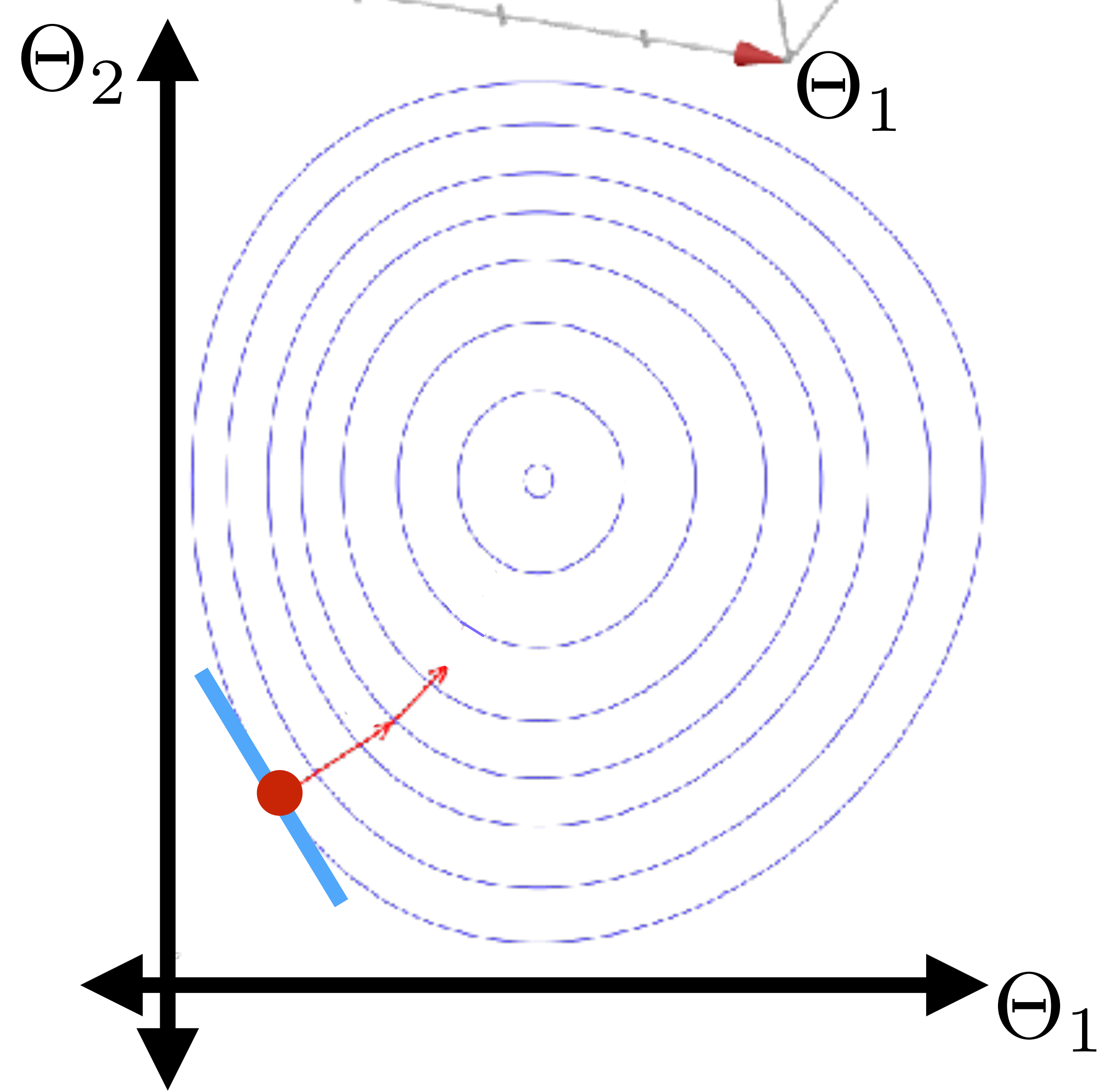

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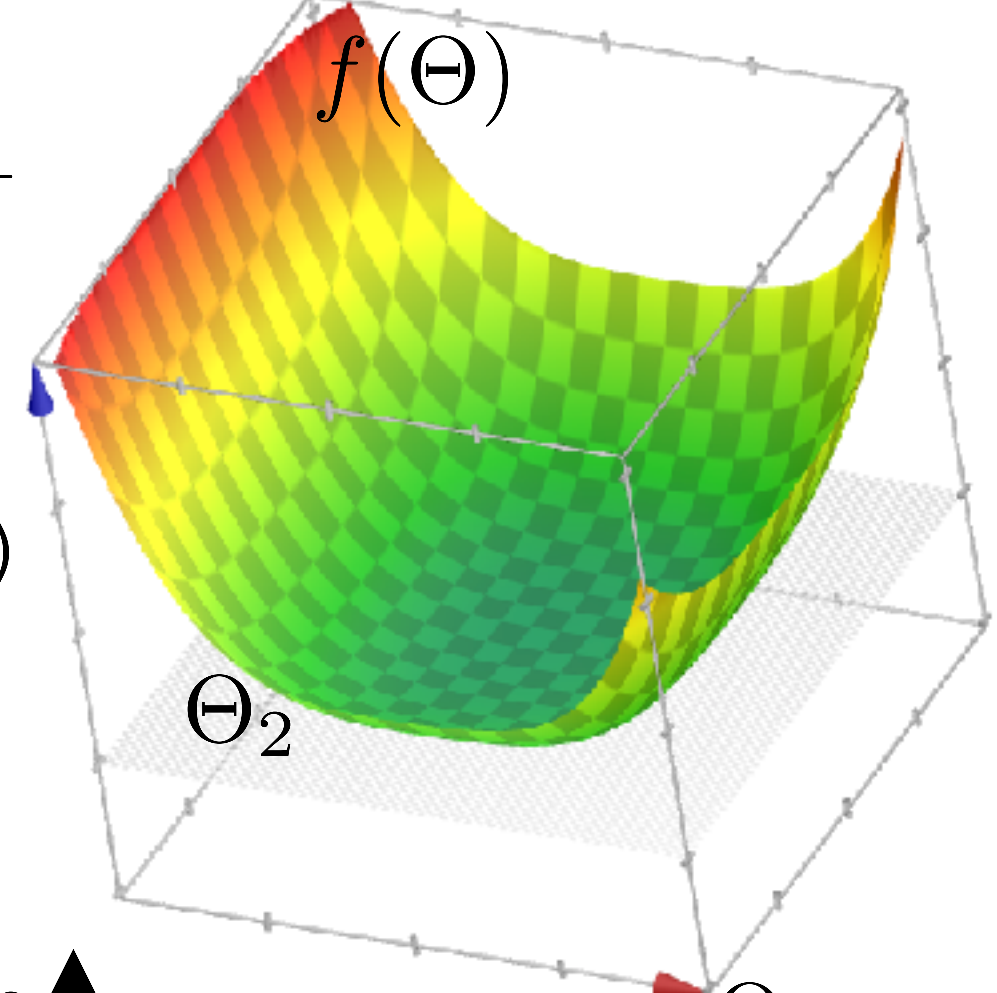

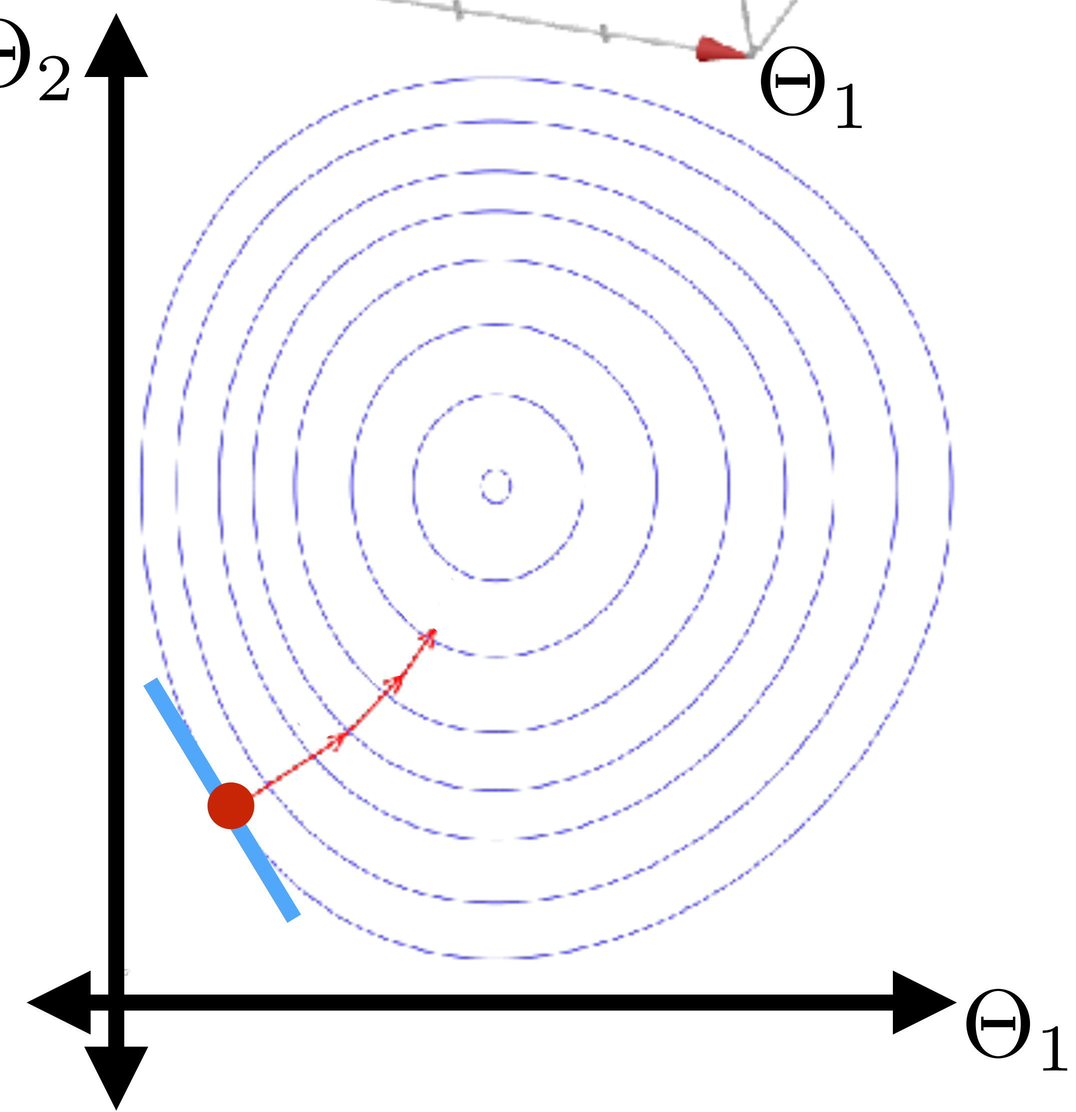

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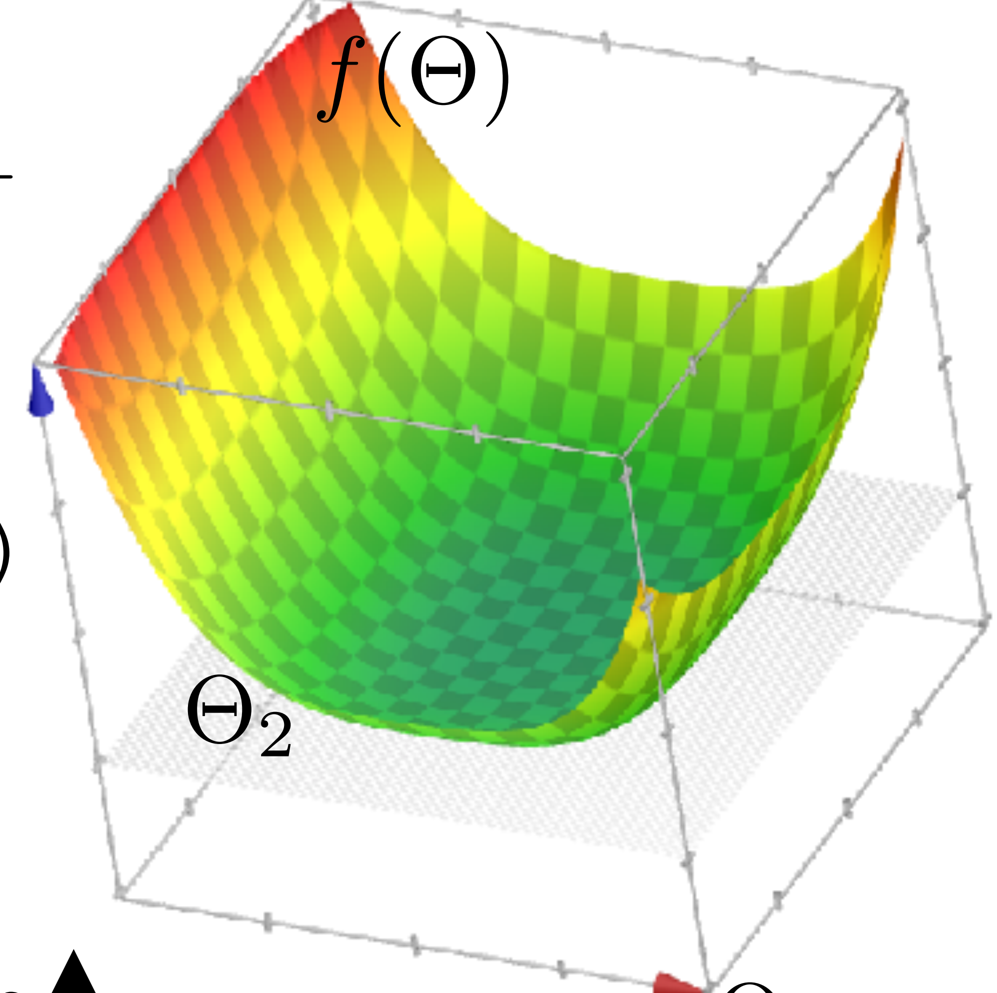

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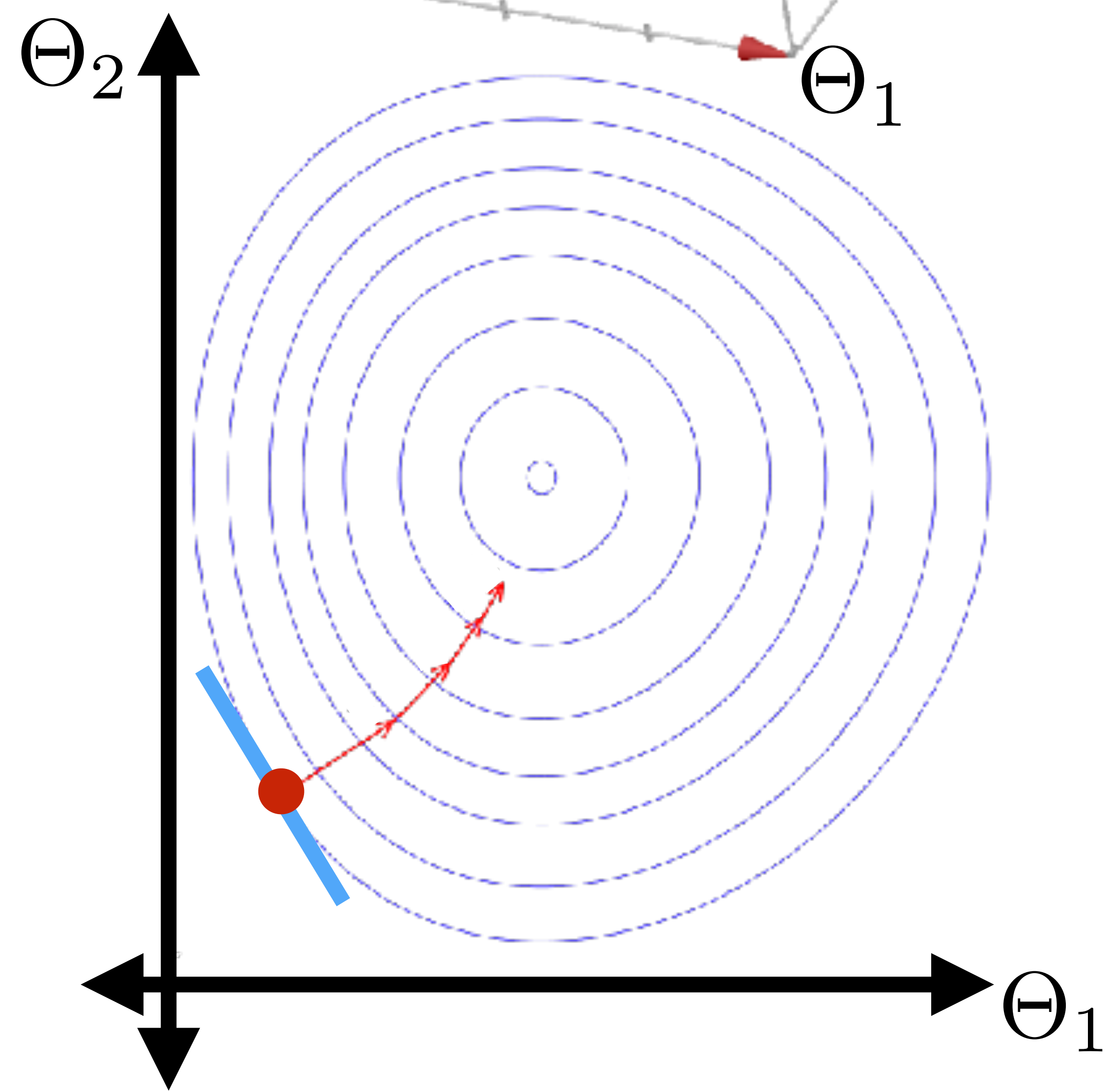

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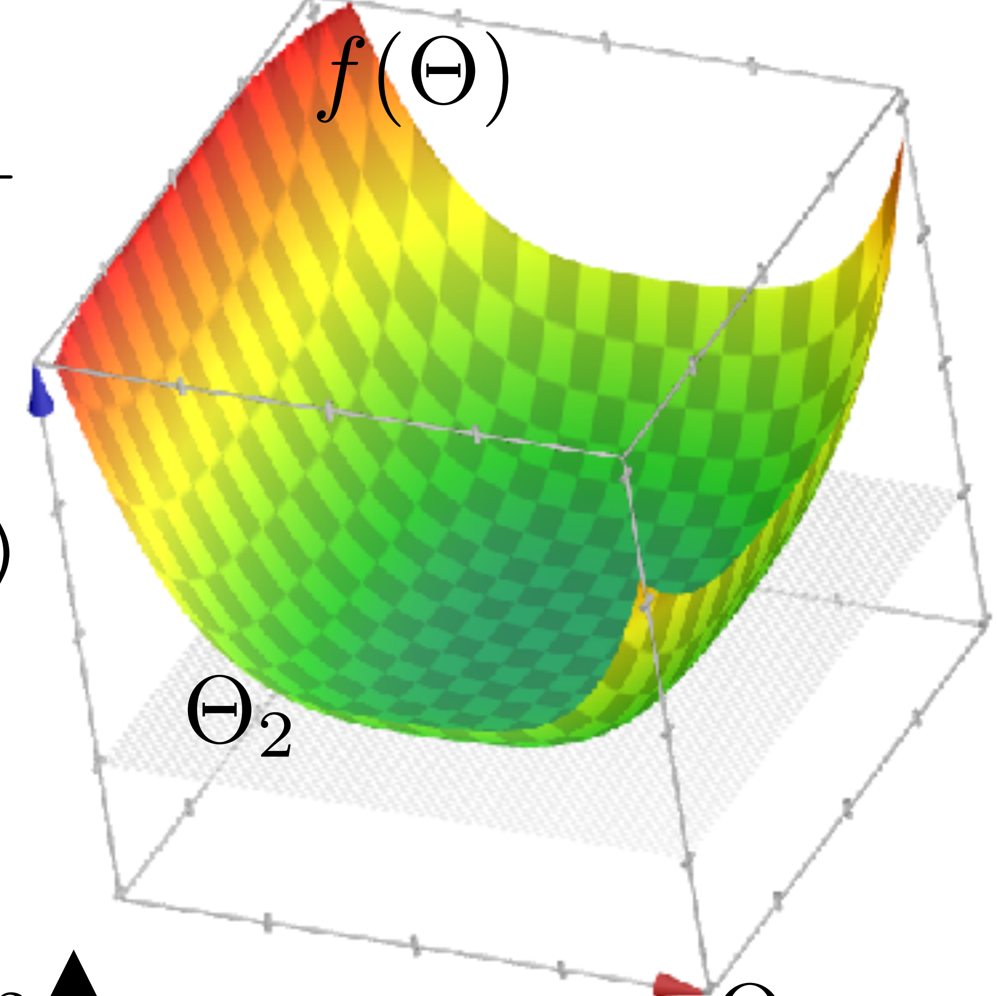

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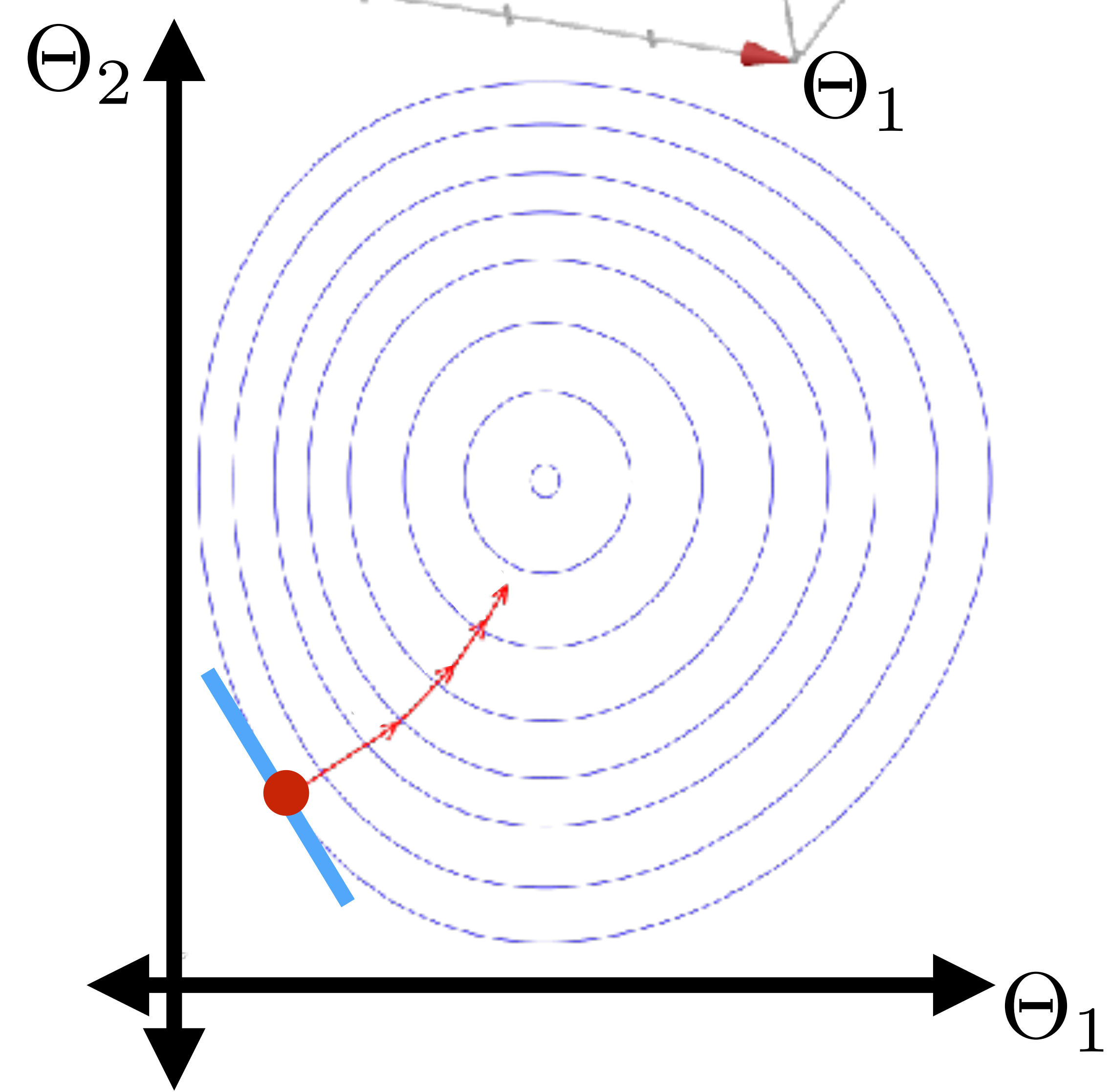
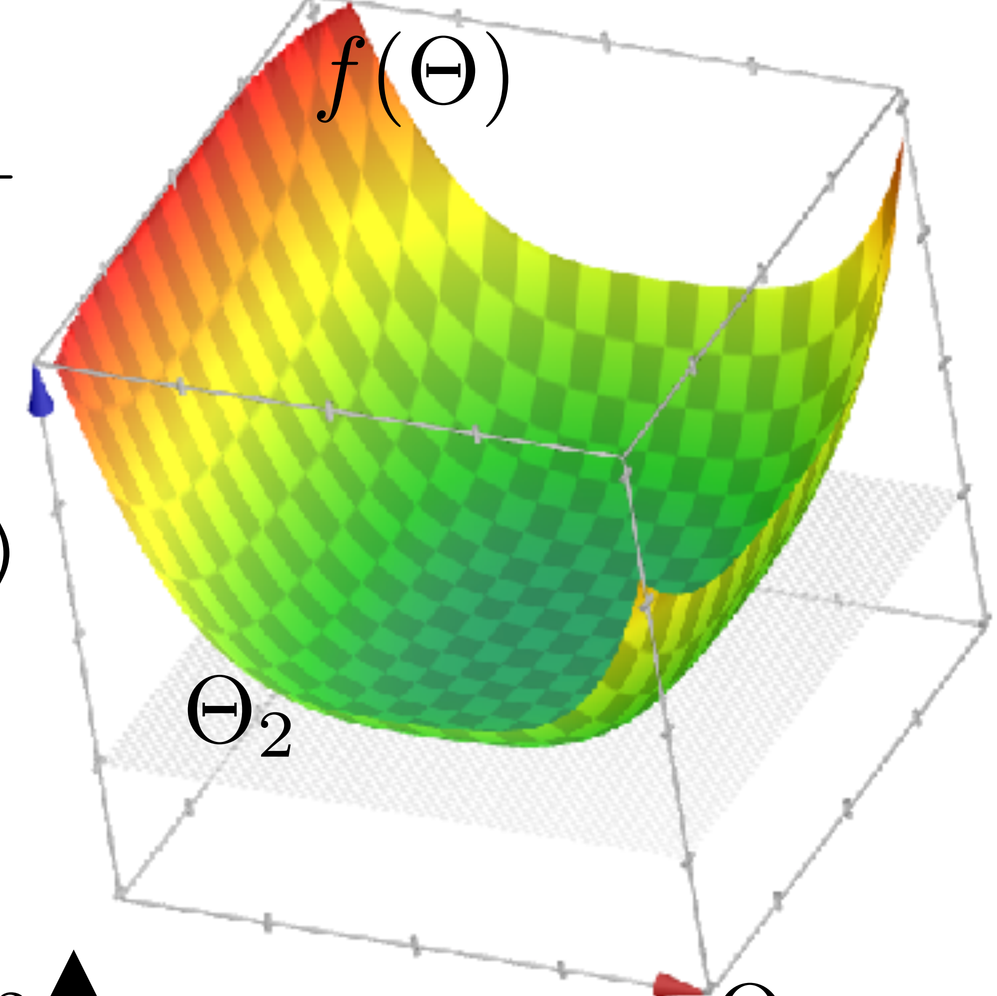
    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  `until`



Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$



`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  `Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

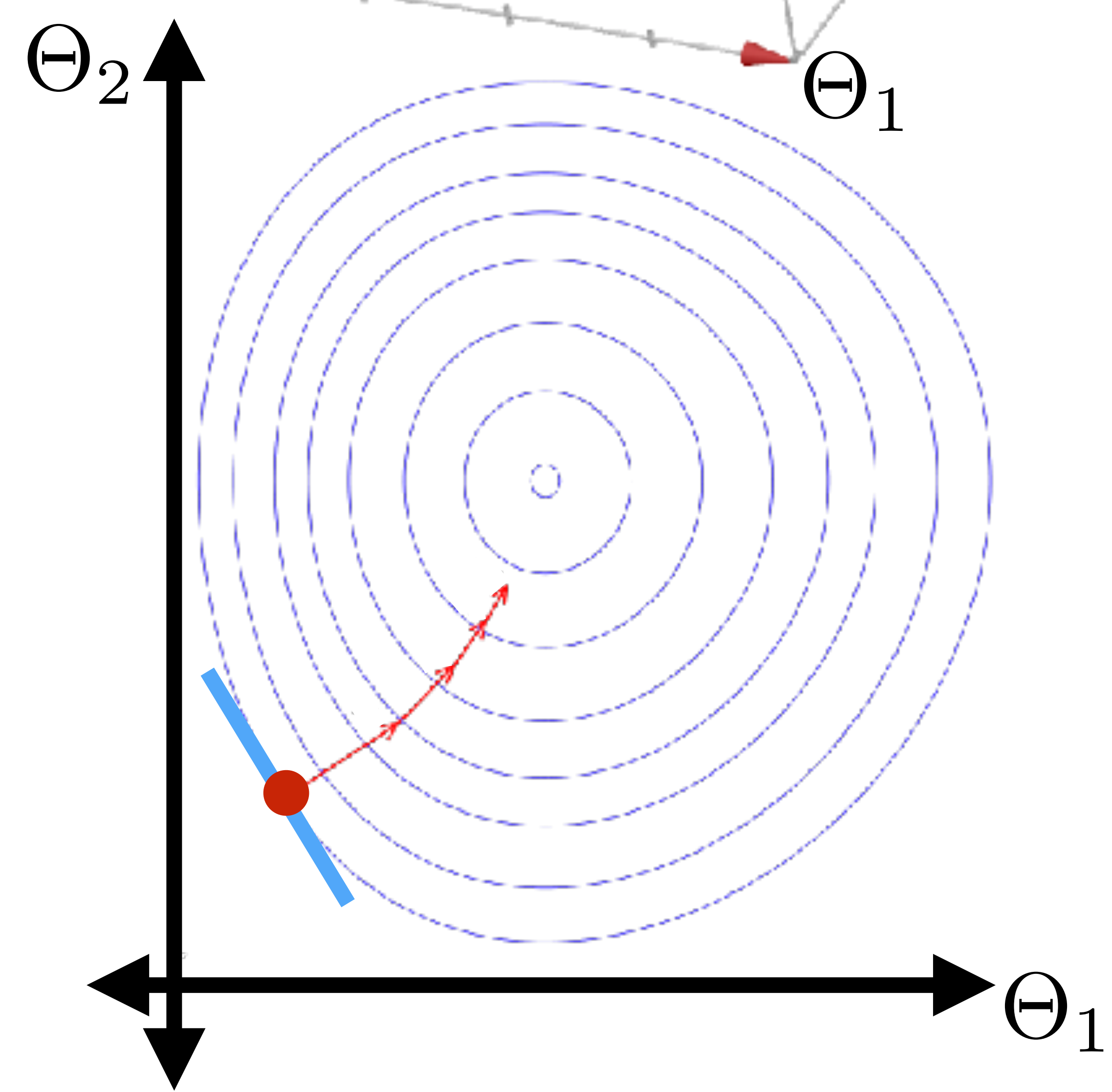
  `Initialize t = 0`

  `repeat`

    `t = t + 1`

    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  `until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$



Gradient descent

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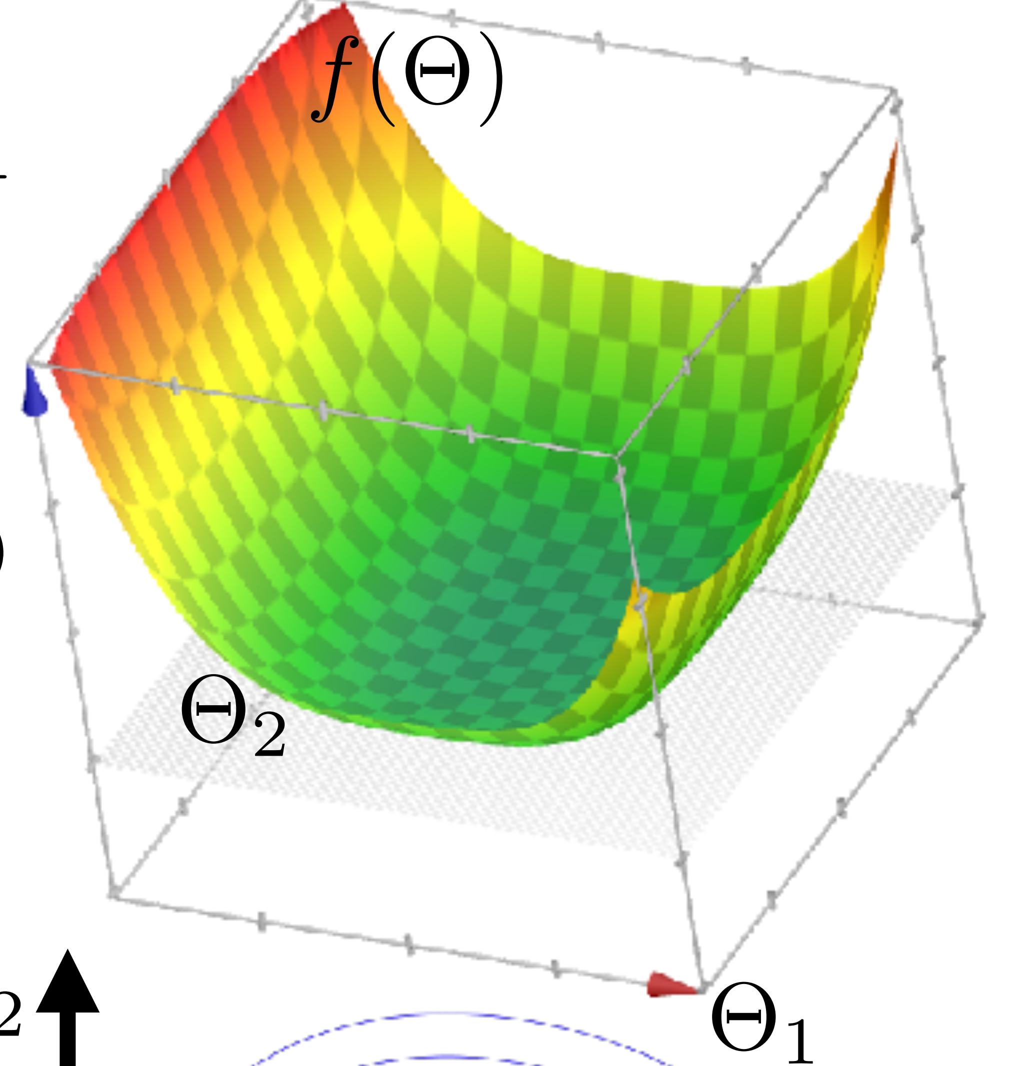
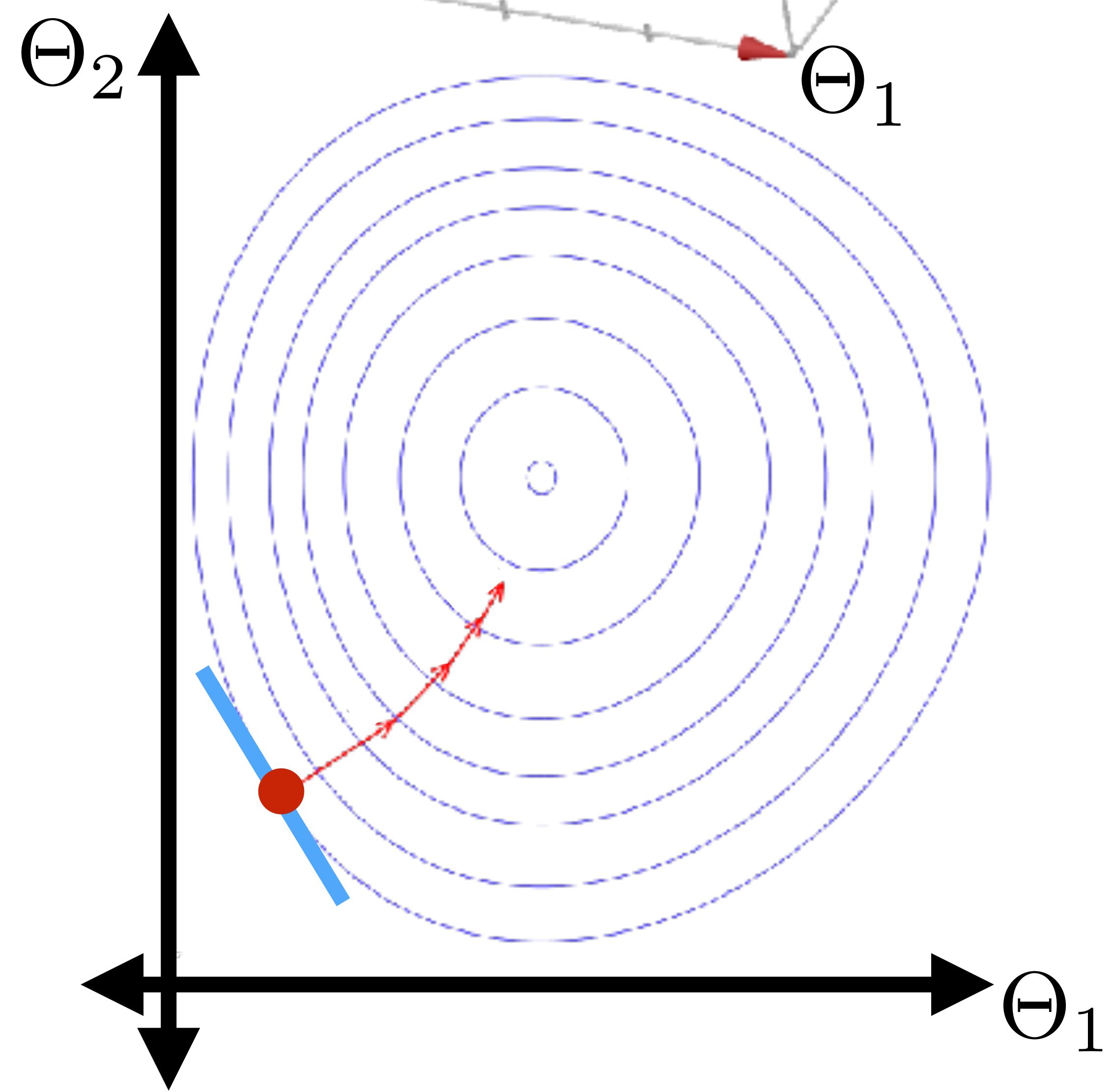
  - with $\Theta \in \mathbb{R}^m$




```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t−1) − η ∇_Θ f(Θ^(t−1))
  until |f(Θ^(t)) − f(Θ^(t−1))| < ε
  Return Θ^(t)
```



Gradient descent

- Gradient $\nabla_{\Theta} f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_{\Theta} f, \epsilon)$

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  Initialize t = 0

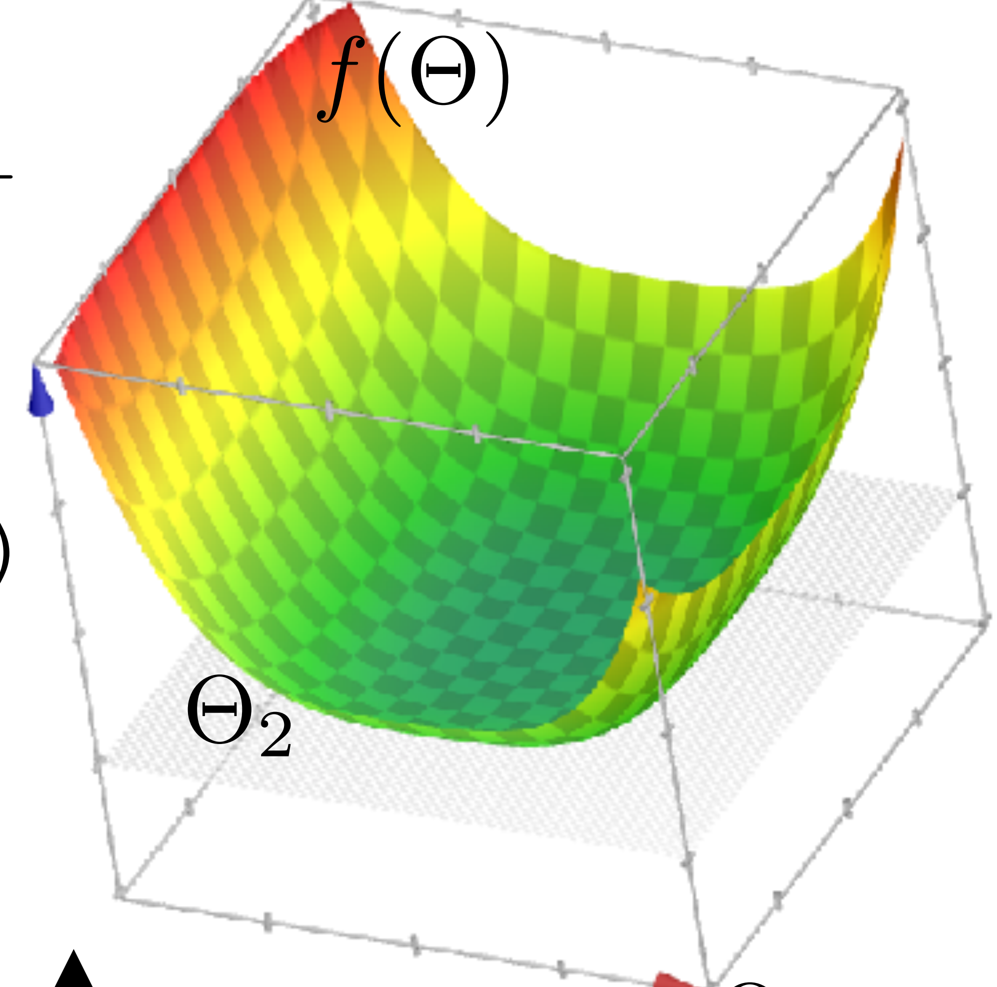
  repeat

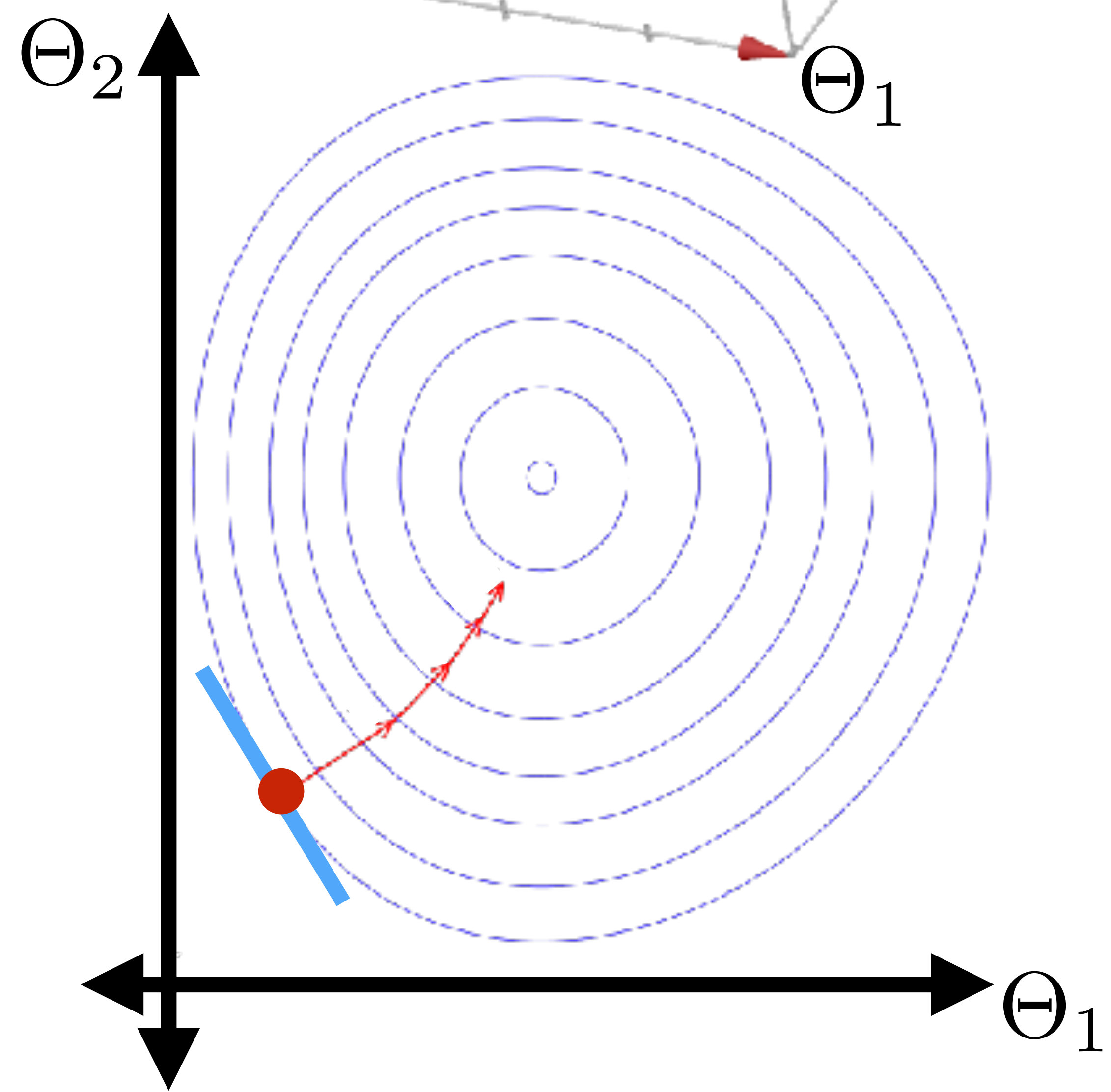
    t = t + 1

    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_{\Theta} f(\Theta^{(t-1)})$

  until $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

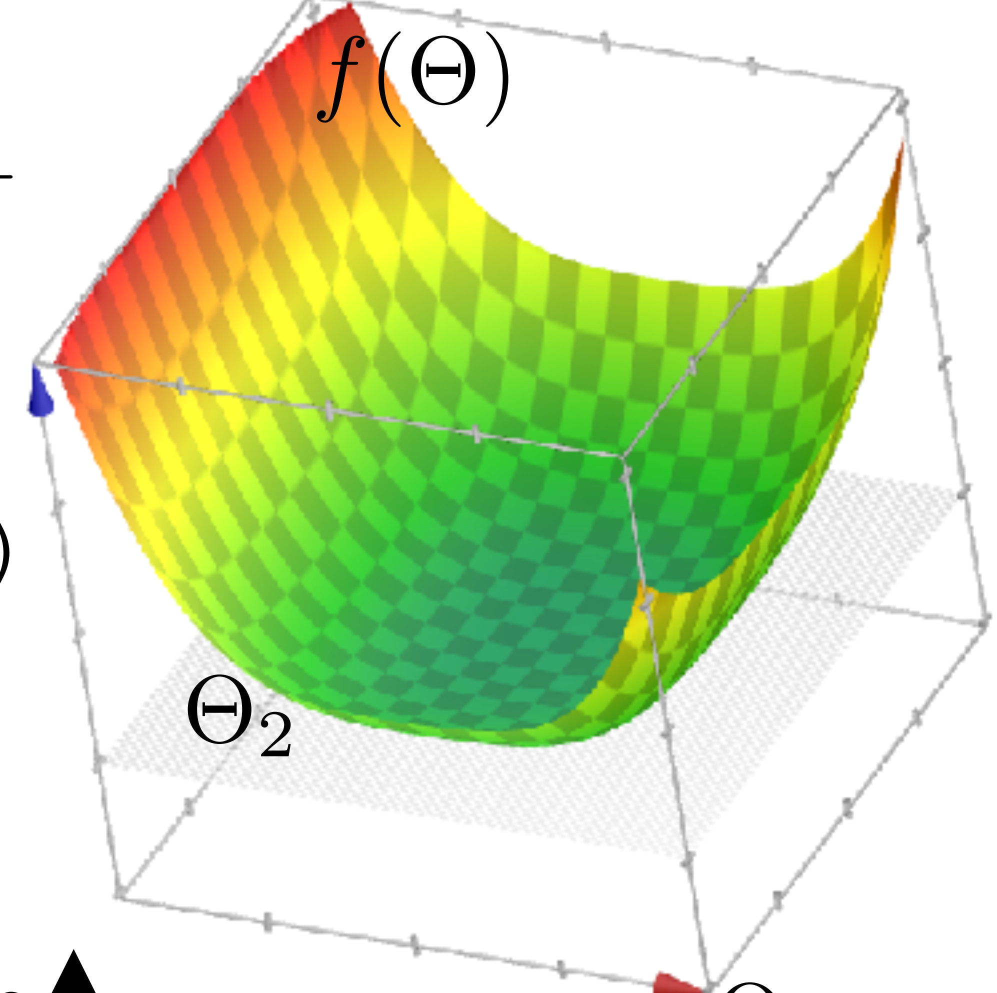
  Return $\Theta^{(t)}$



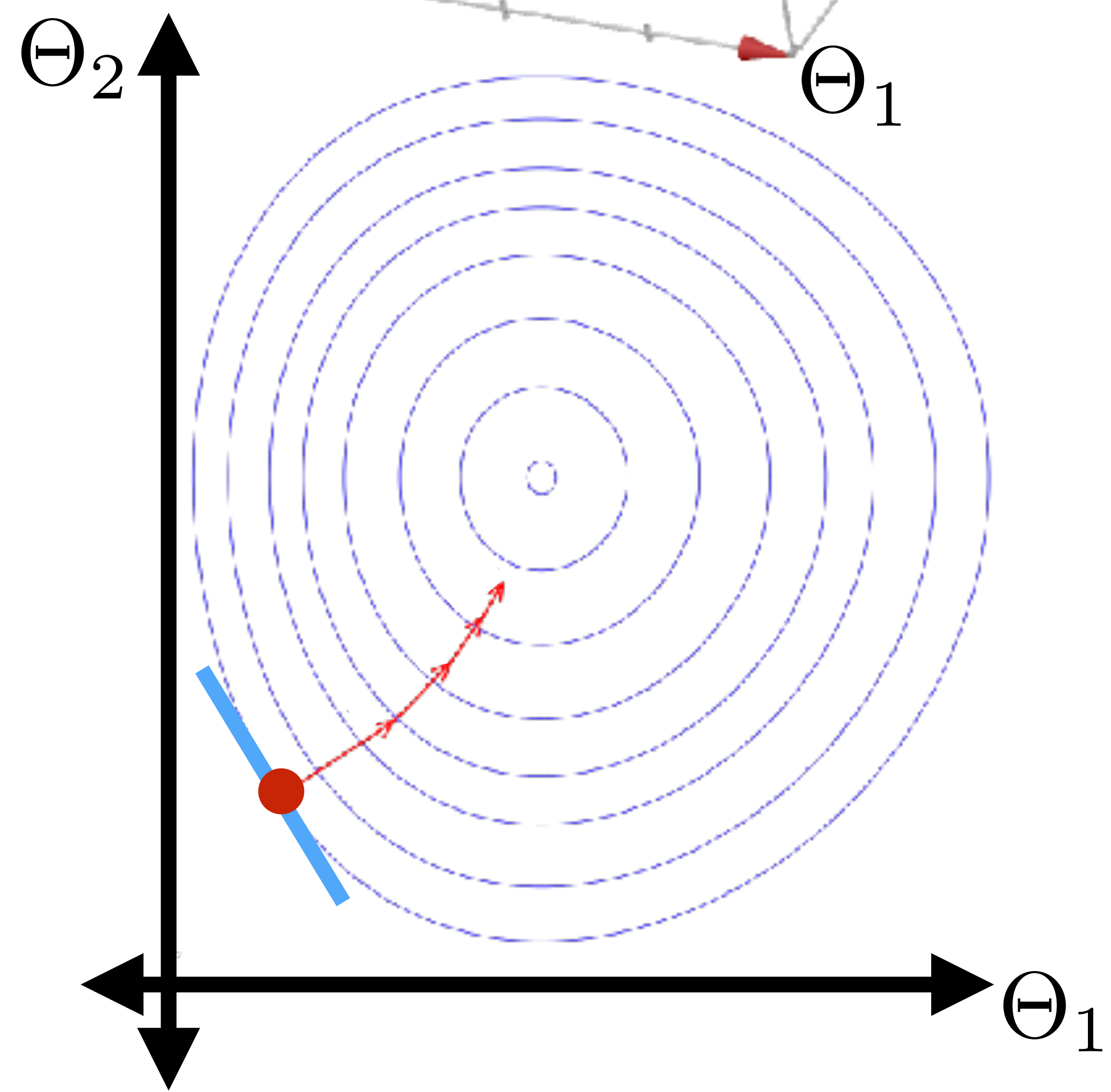


Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$




```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```



Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m} \right]^\top$
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`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  `Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

  `Initialize t = 0`

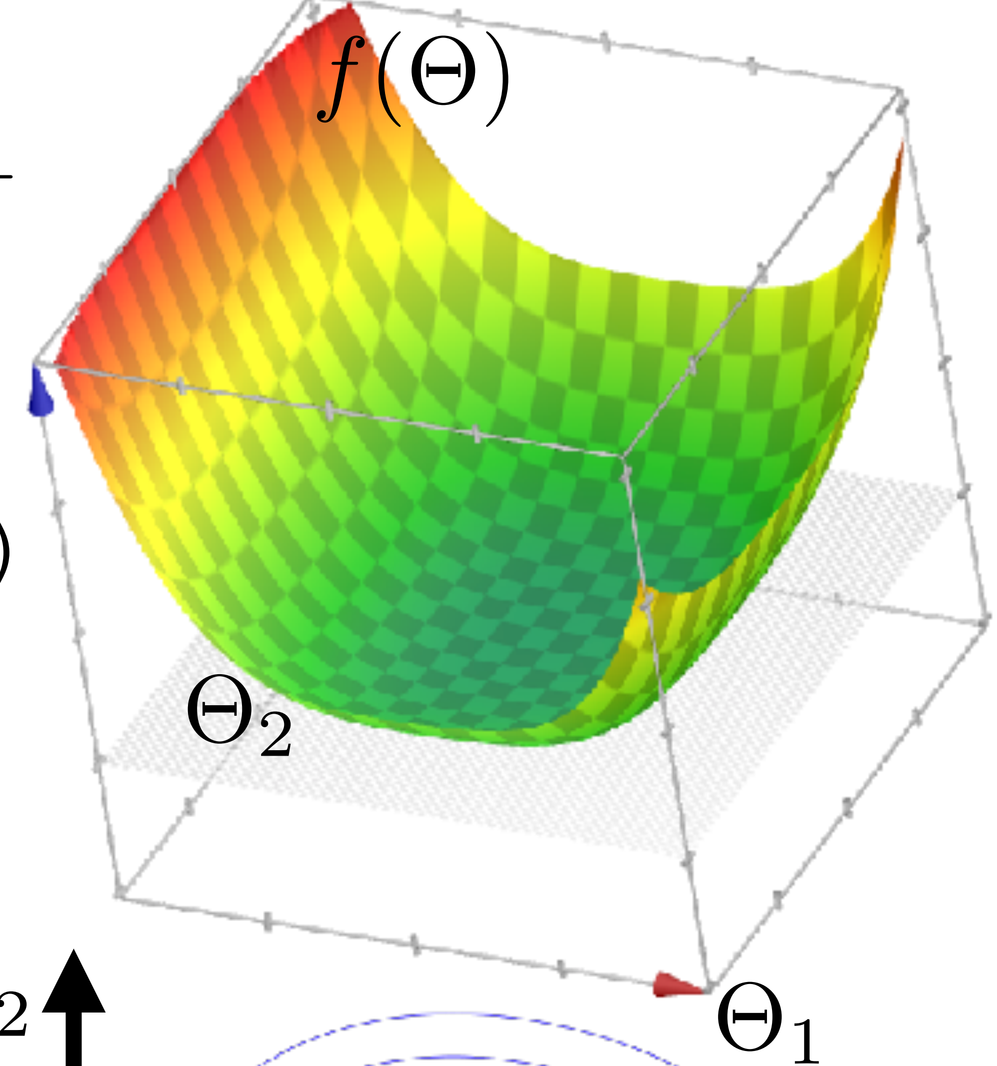
  `repeat`

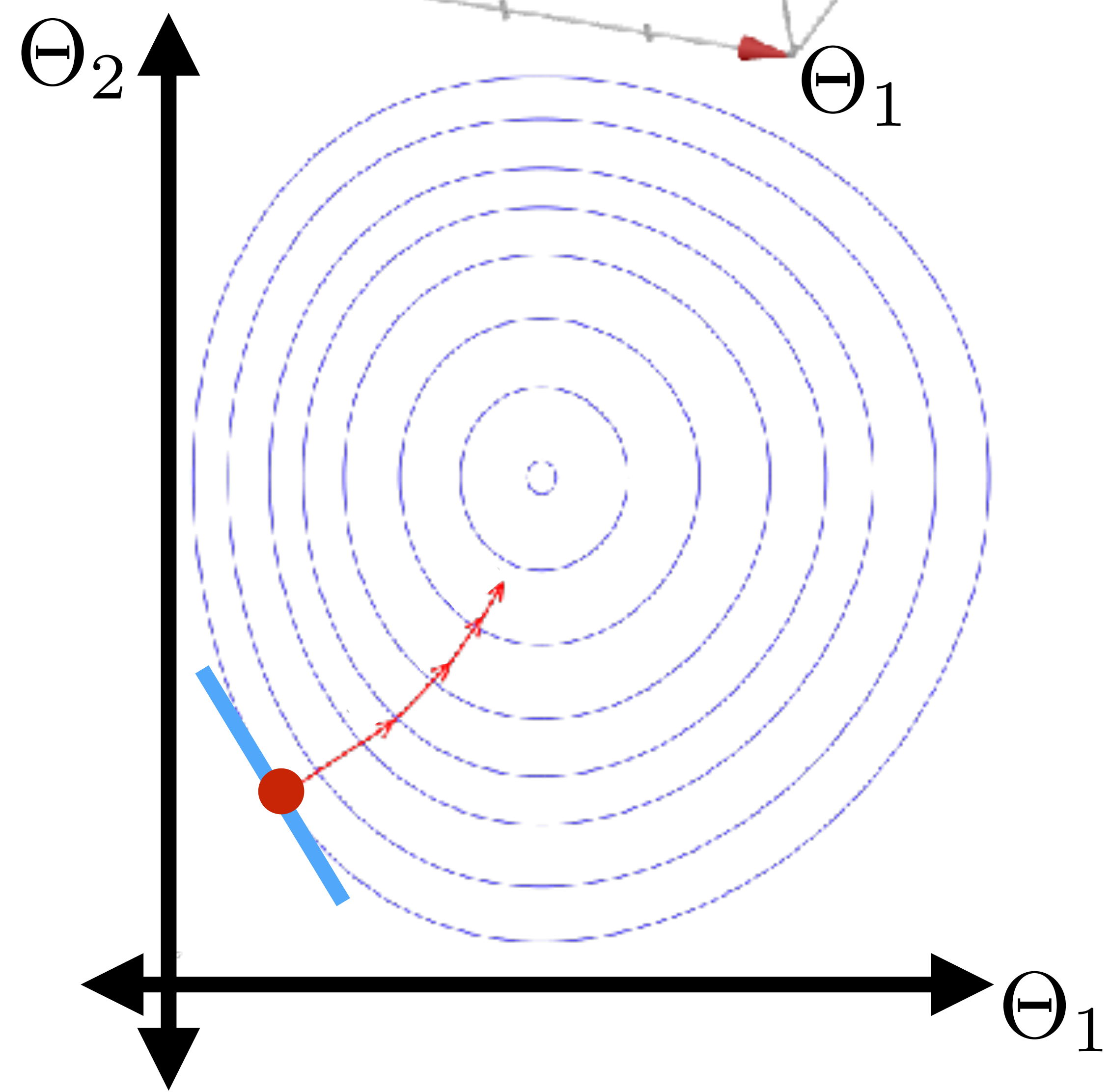
    `t = t + 1`

    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  `until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  `Return` $\Theta^{(t)}$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

Gradient-Descent $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  Initialize t = 0

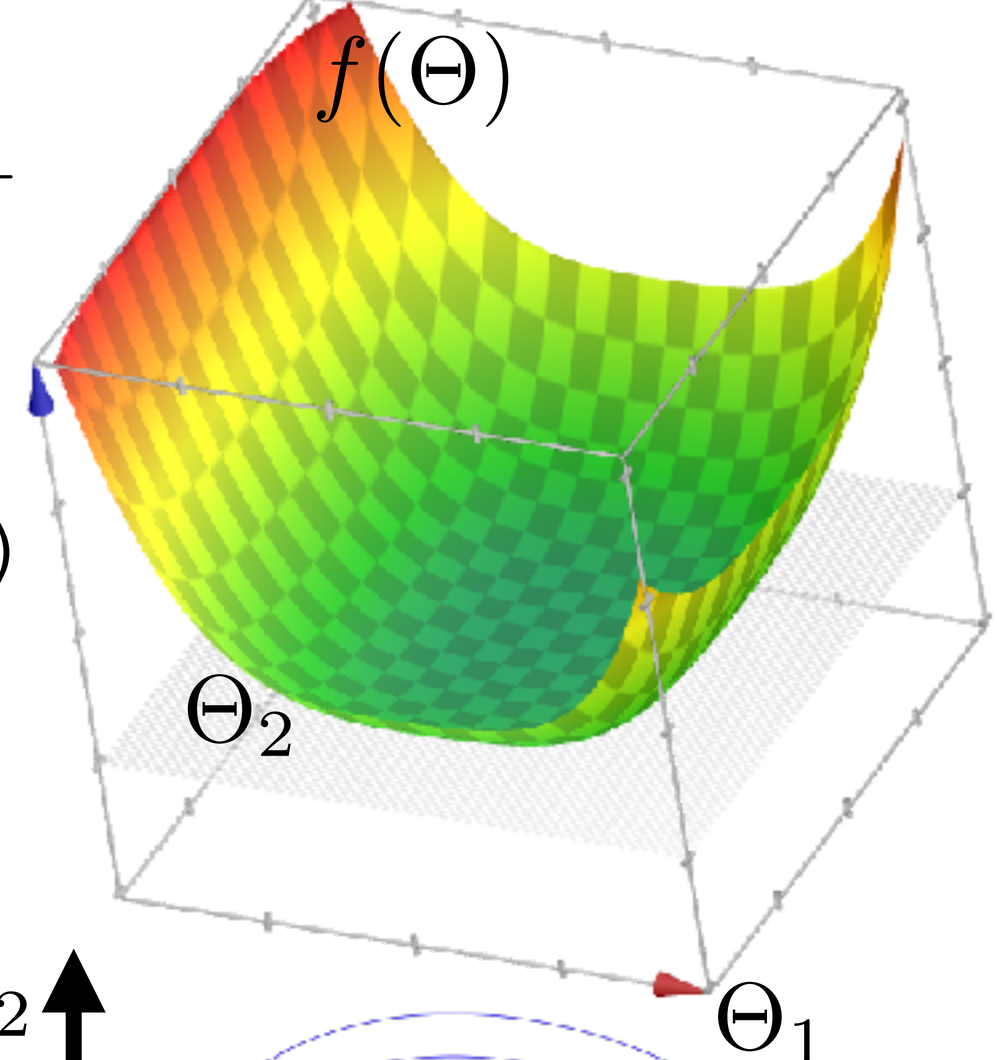
  repeat

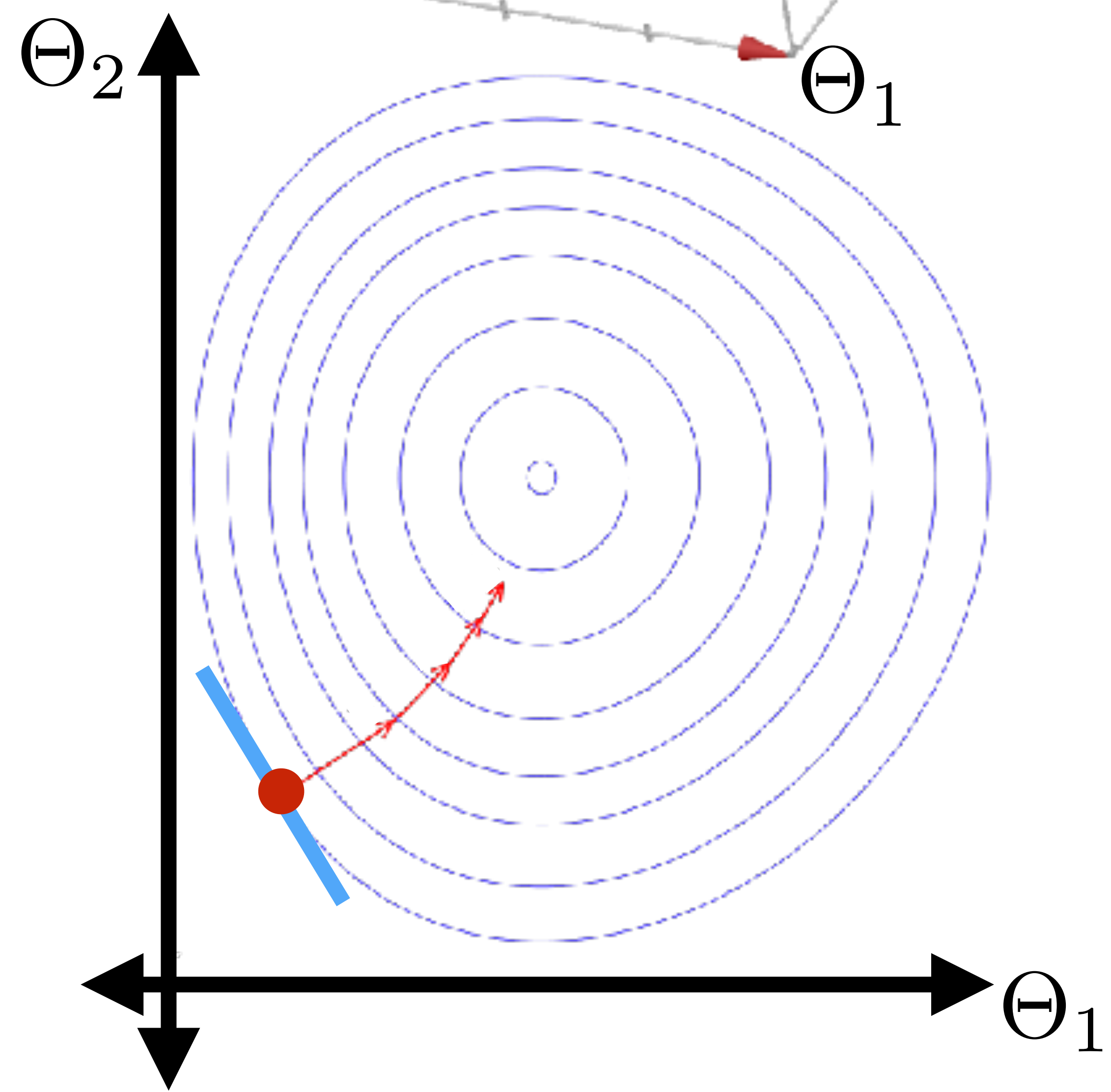
    t = t + 1

    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  until $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  Return $\Theta^{(t)}$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  `Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

  `Initialize t = 0`

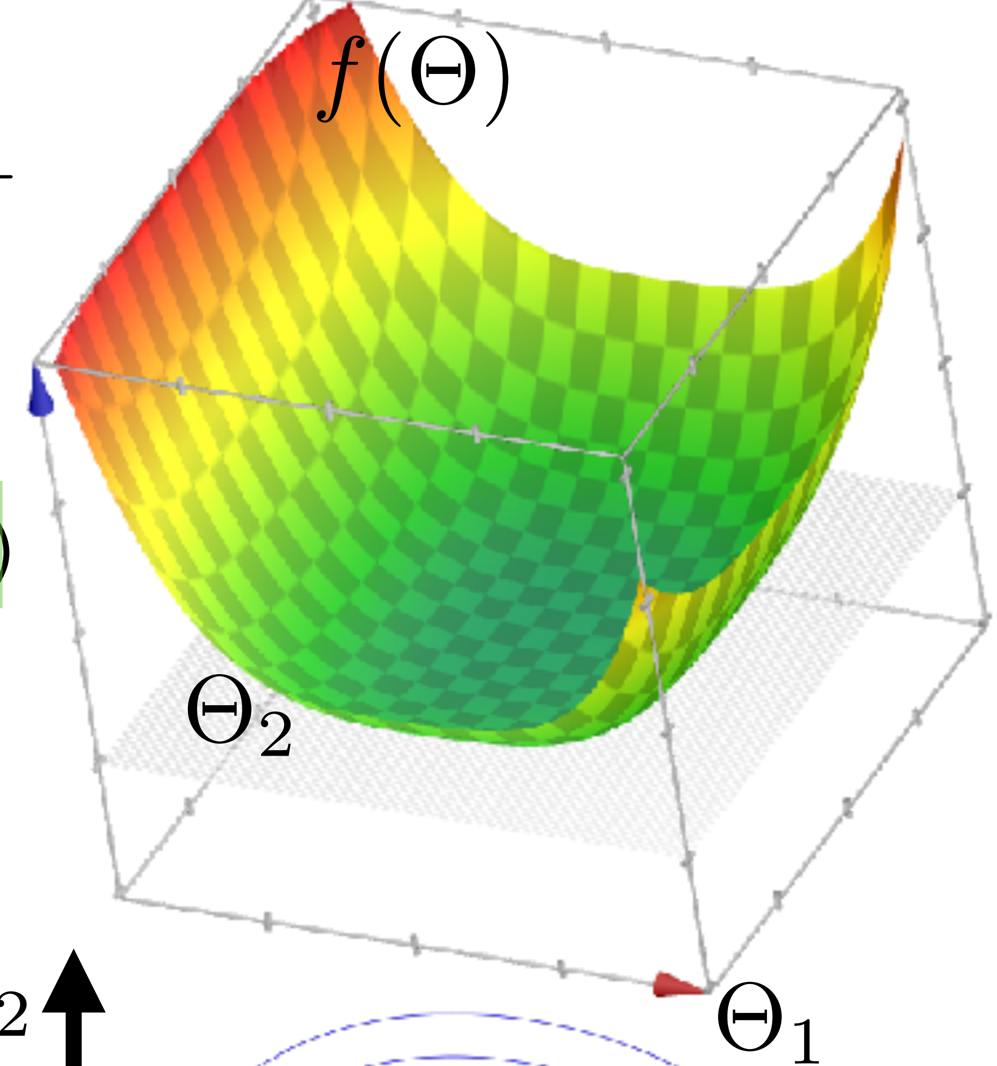
  `repeat`

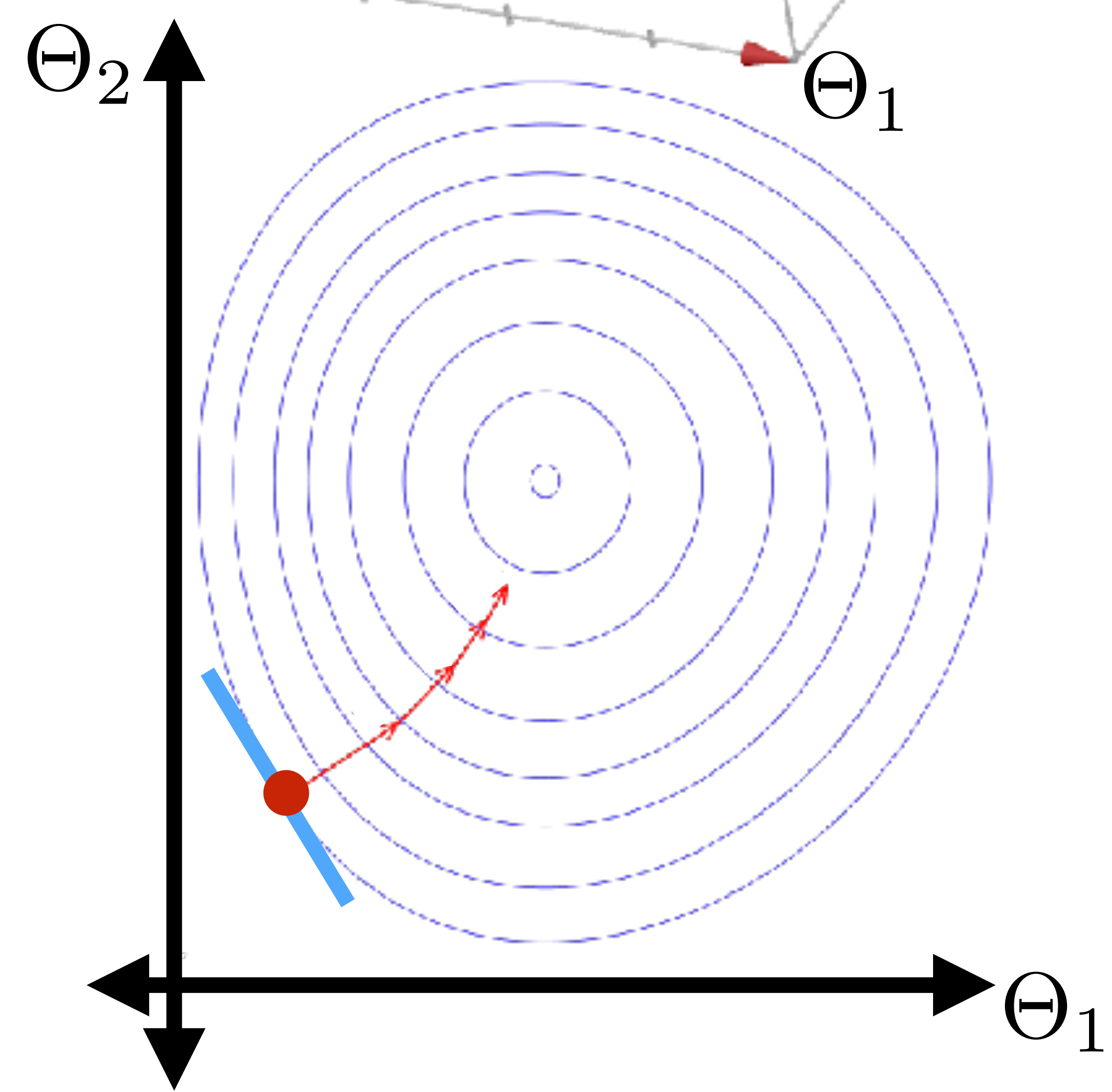
    `t = t + 1`

    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  `until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  `Return` $\Theta^{(t)}$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

Gradient-Descent $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  Initialize t = 0

  repeat

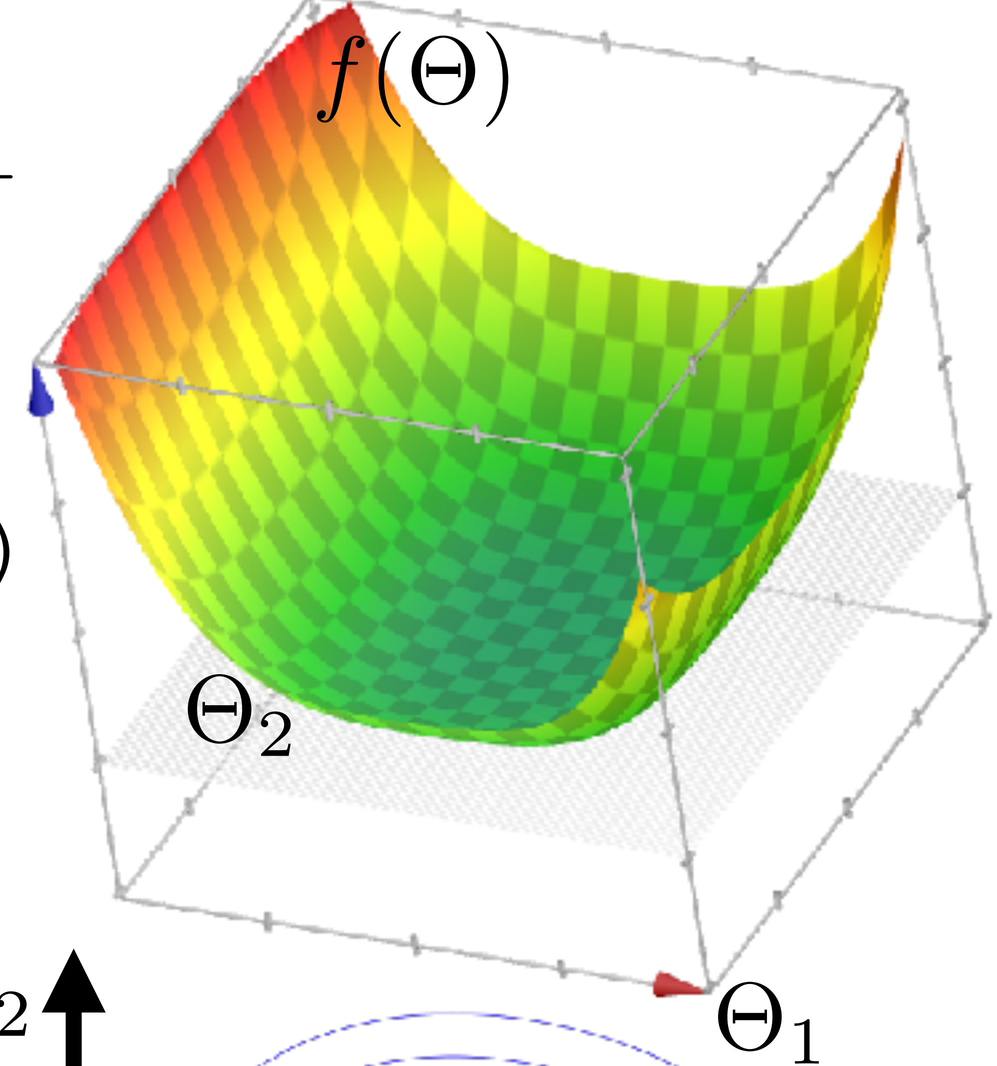
    t = t + 1

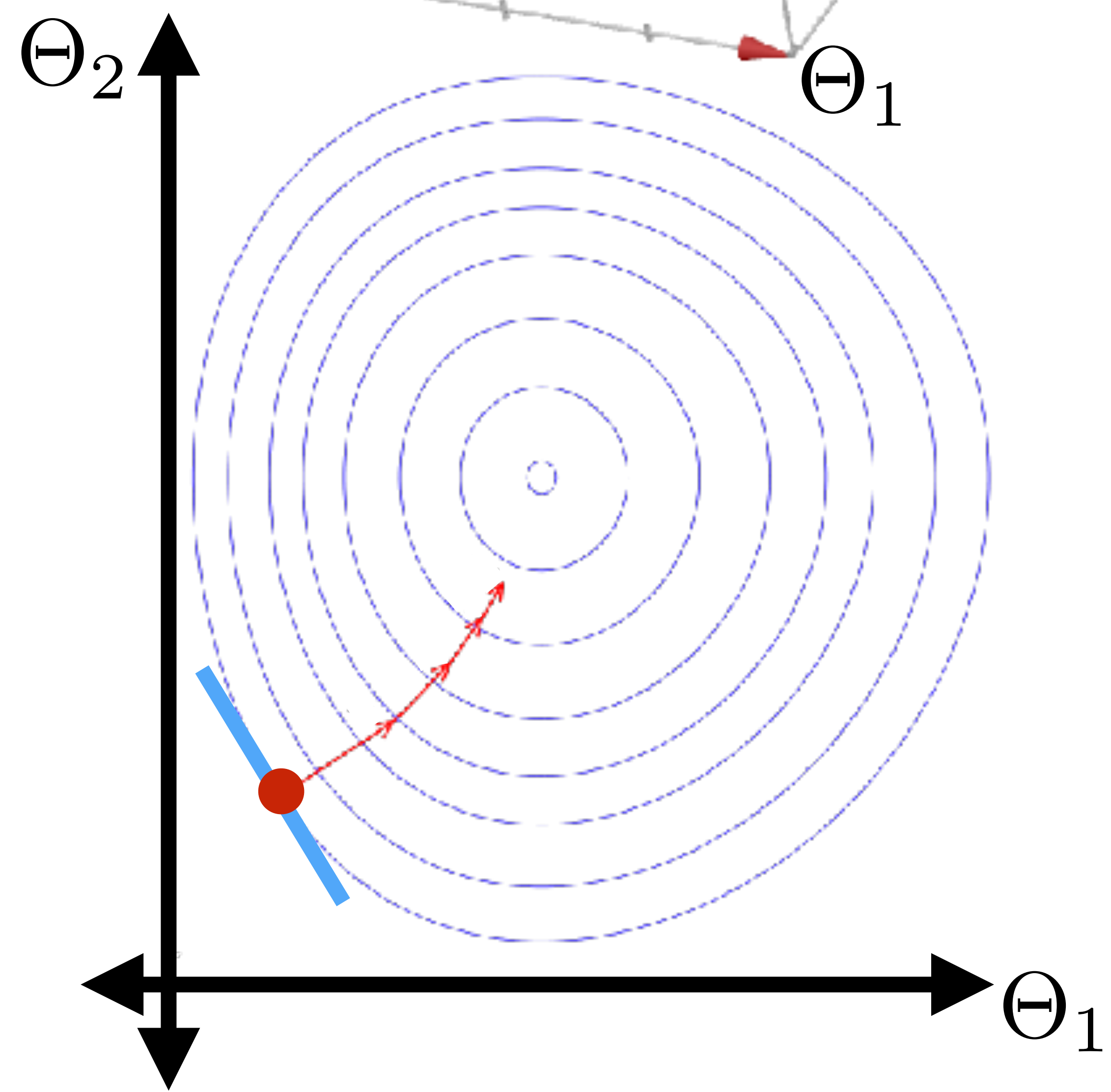
    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  until $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  Return $\Theta^{(t)}$

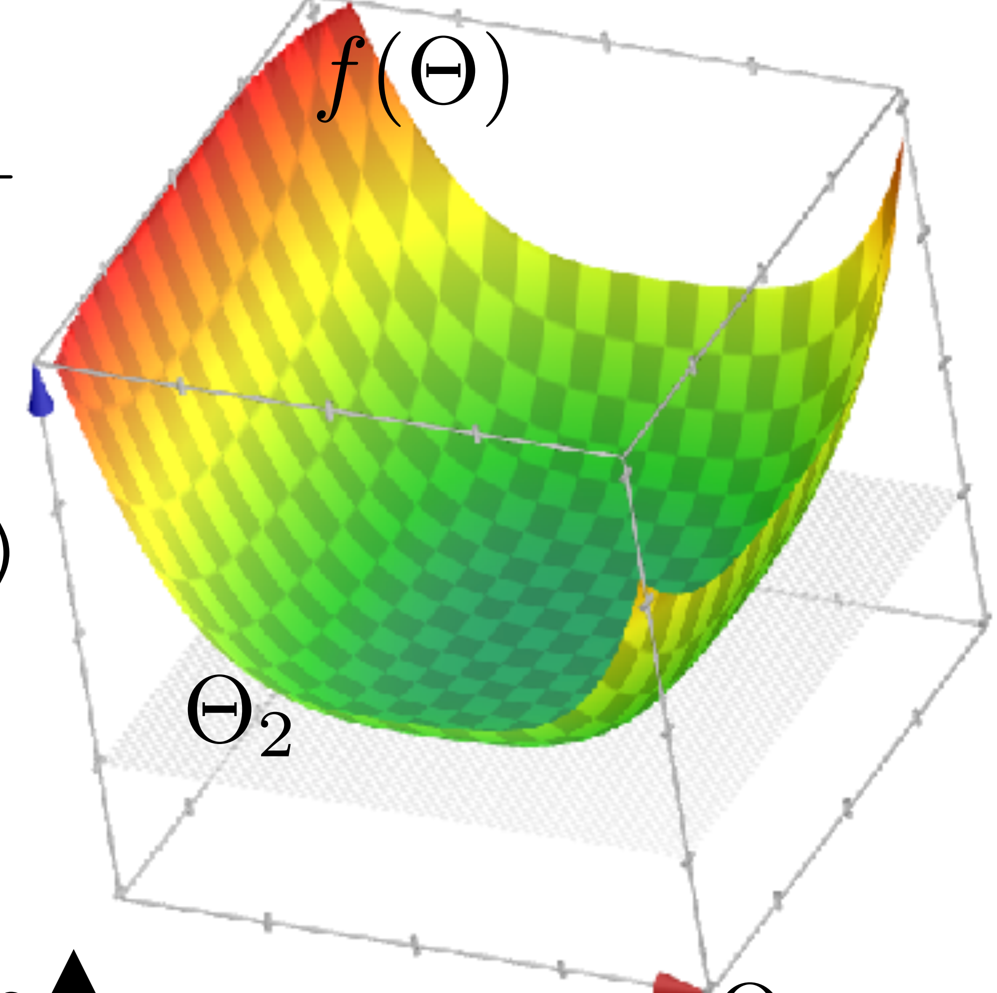
- Other possible stopping criteria:





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
    - with $\Theta \in \mathbb{R}^m$



`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

`Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

`Initialize t = 0`

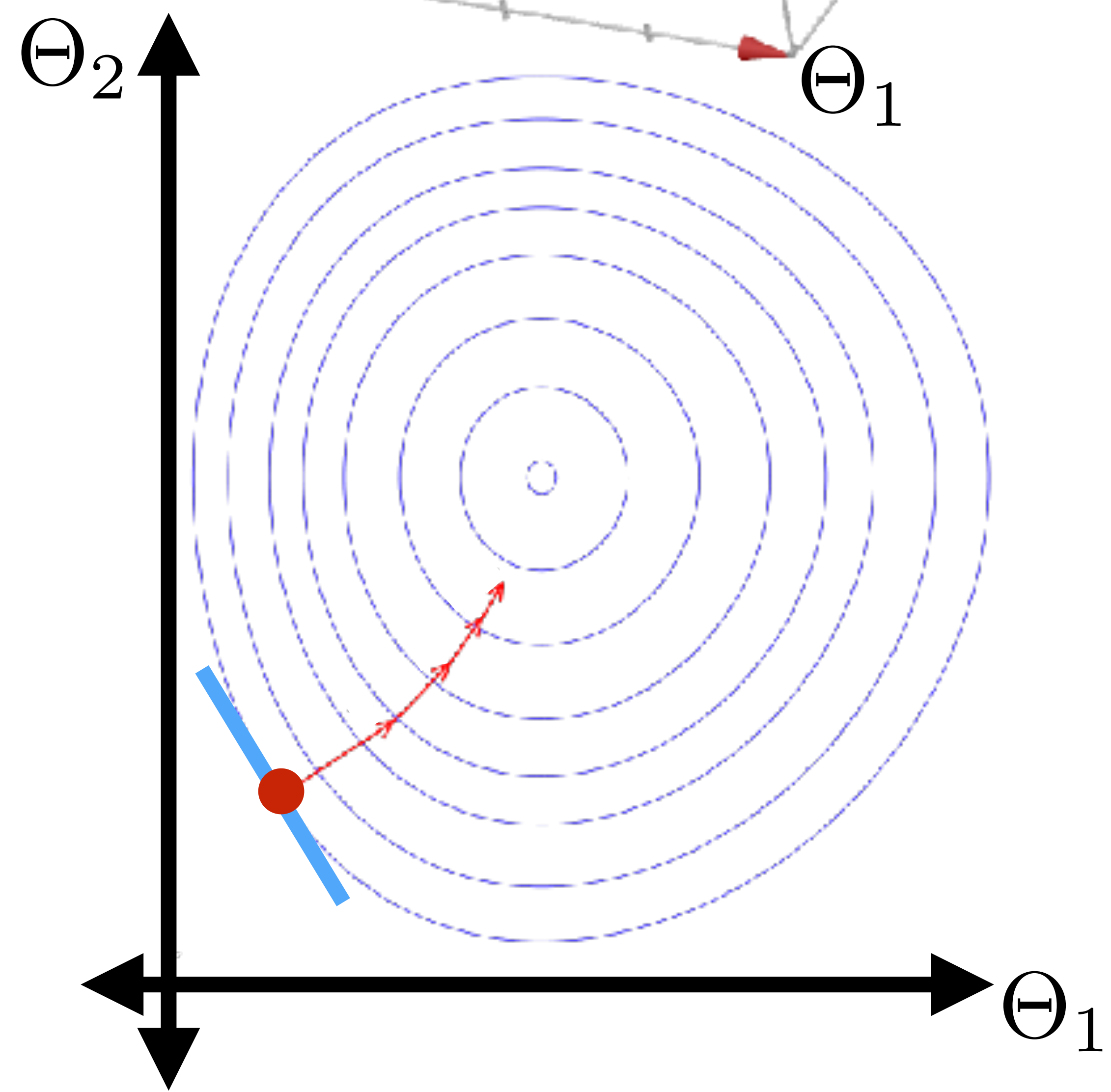
`repeat`

`t = t + 1`

$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

`until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

`Return` $\Theta^{(t)}$



- Other possible stopping criteria:
    - Max number of iterations T

Gradient descent

- Gradient $\nabla_{\Theta} f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m} \right]^{\top}$
    - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_{\Theta} f, \epsilon)$

  `Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

  `Initialize t = 0`

  `repeat`

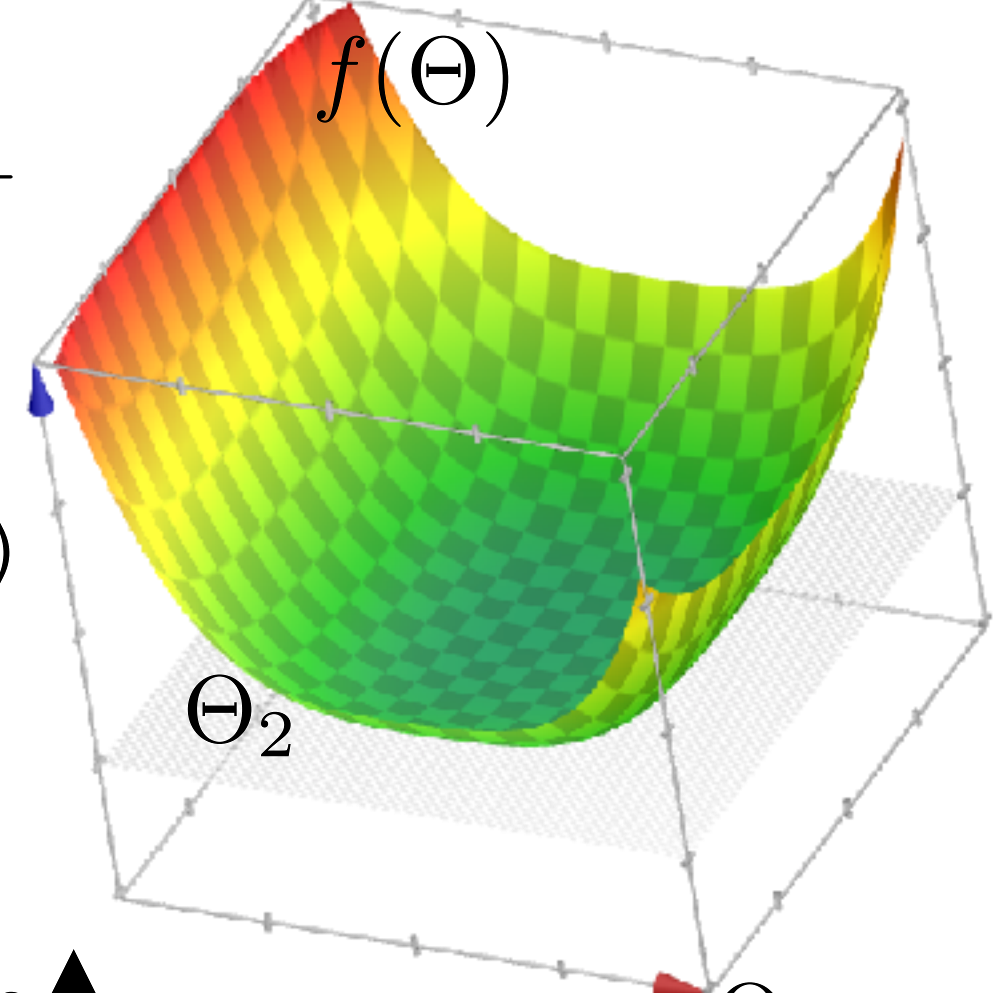
    `t = t + 1`

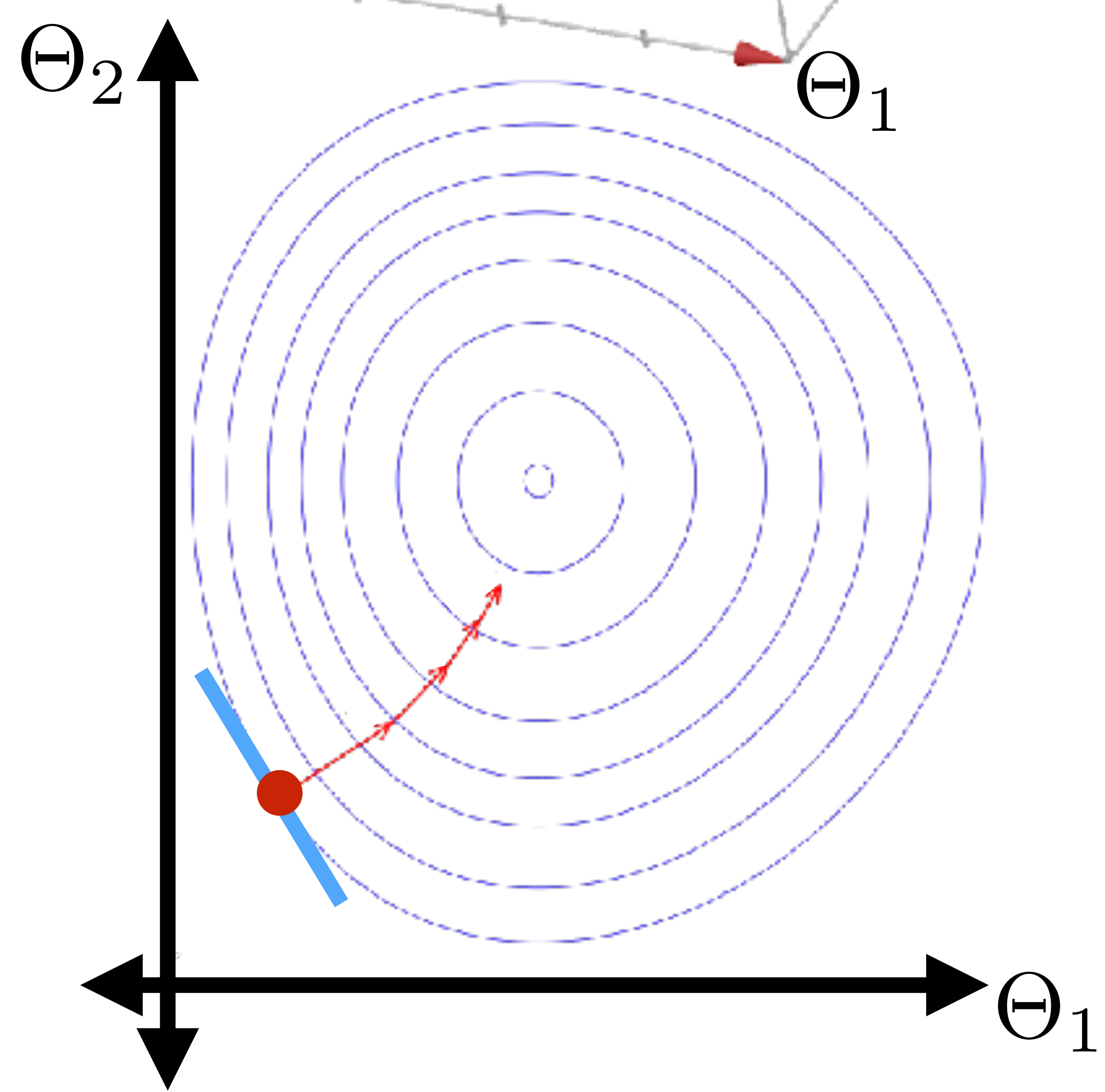
    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_{\Theta} f(\Theta^{(t-1)})$

  `until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  `Return` $\Theta^{(t)}$

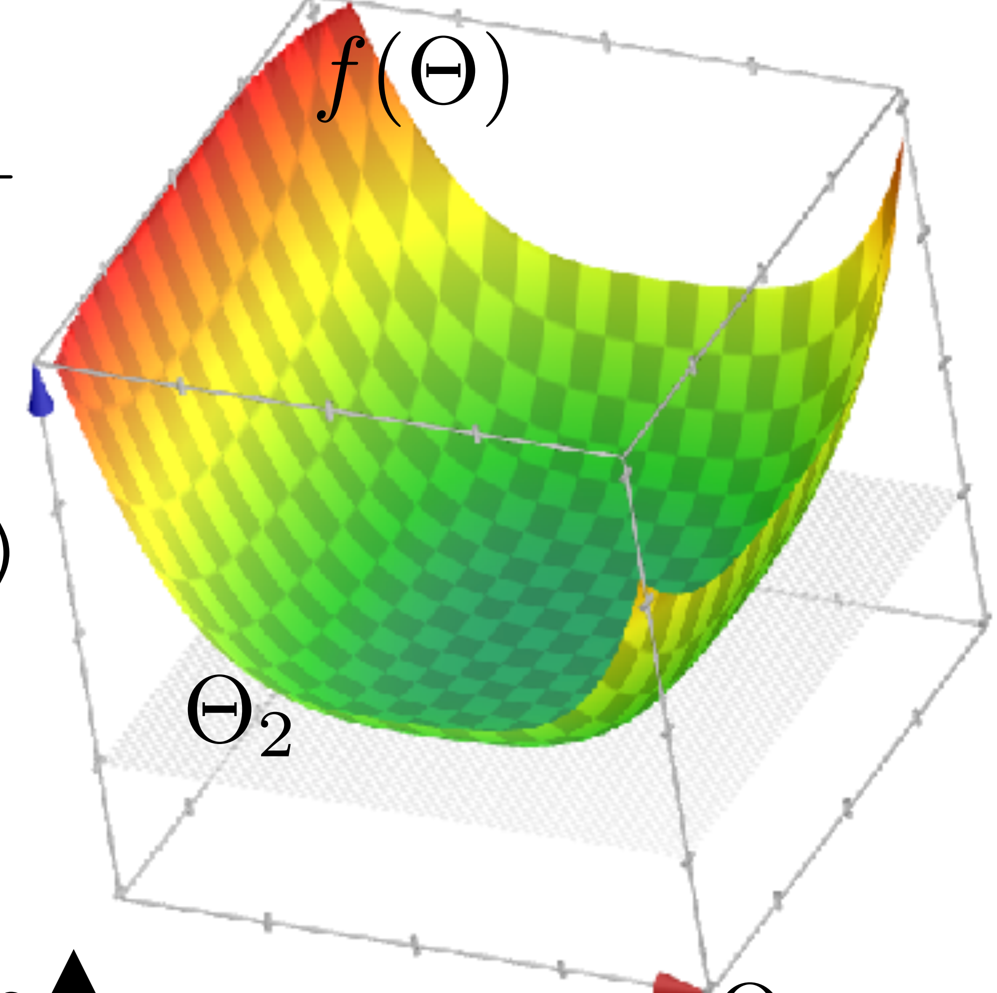
- Other possible stopping criteria:
    - Max number of iterations T
    - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
    - with $\Theta \in \mathbb{R}^m$

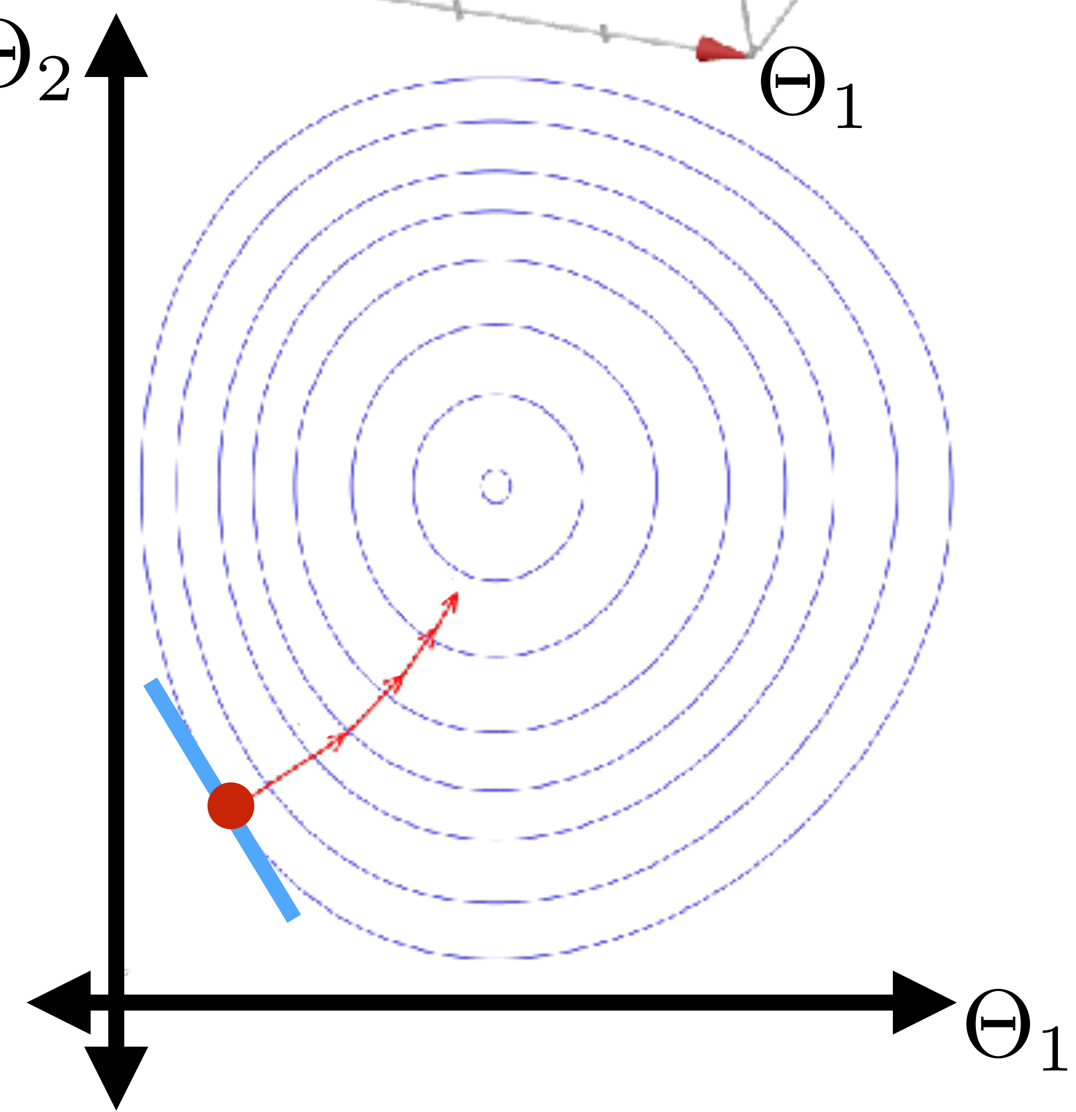


Gradient-Descent $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

```
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

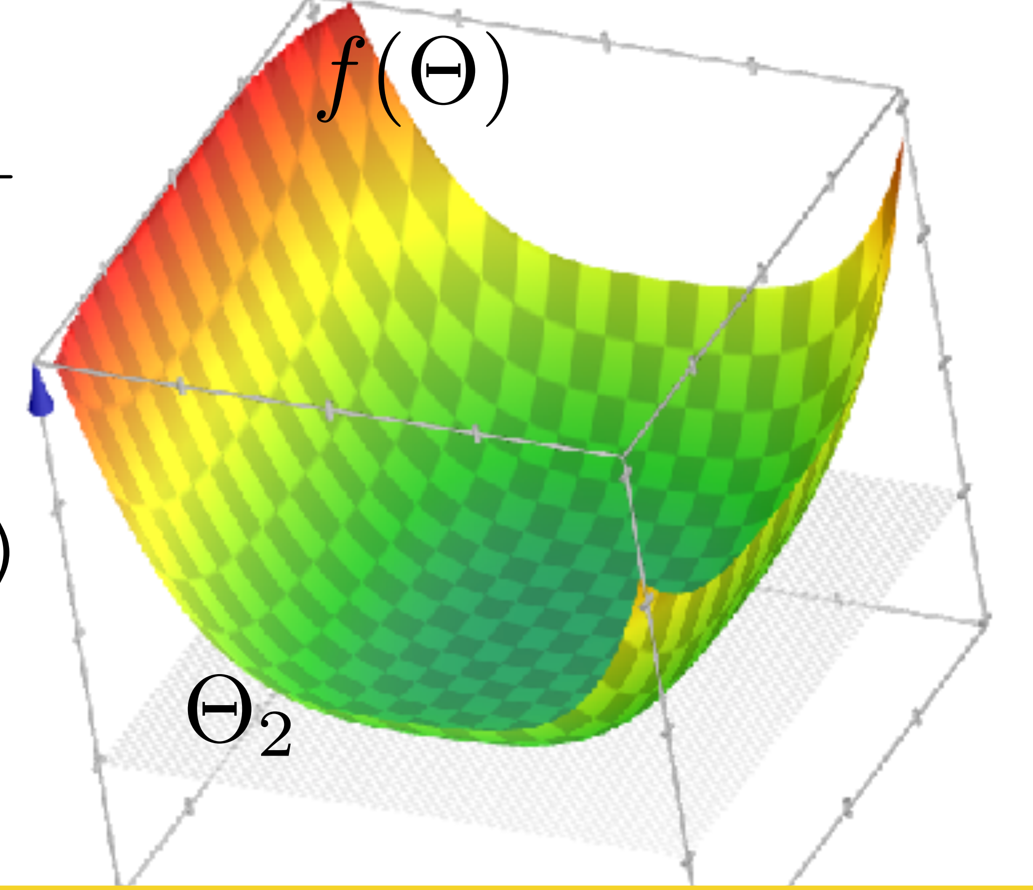
$$\Theta^{(0)} = \Theta_{\text{init}}$$
$$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$$
until $\left|f(\Theta^{(t)}) - f(\Theta^{(t-1)})\right| < \epsilon$

Return $\Theta^{(t)}$



- Other possible stopping criteria:
    - Max number of iterations T
    - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
    - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$

Gradient descent



- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
    - with $\Theta \in \mathbb{R}^m$

```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

$\texttt{Gradient-Descent}\,(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

$\quad \texttt{Initialize}\ \Theta^{(0)} = \Theta_{\text{init}}$

$\quad \texttt{Initialize t = 0}$

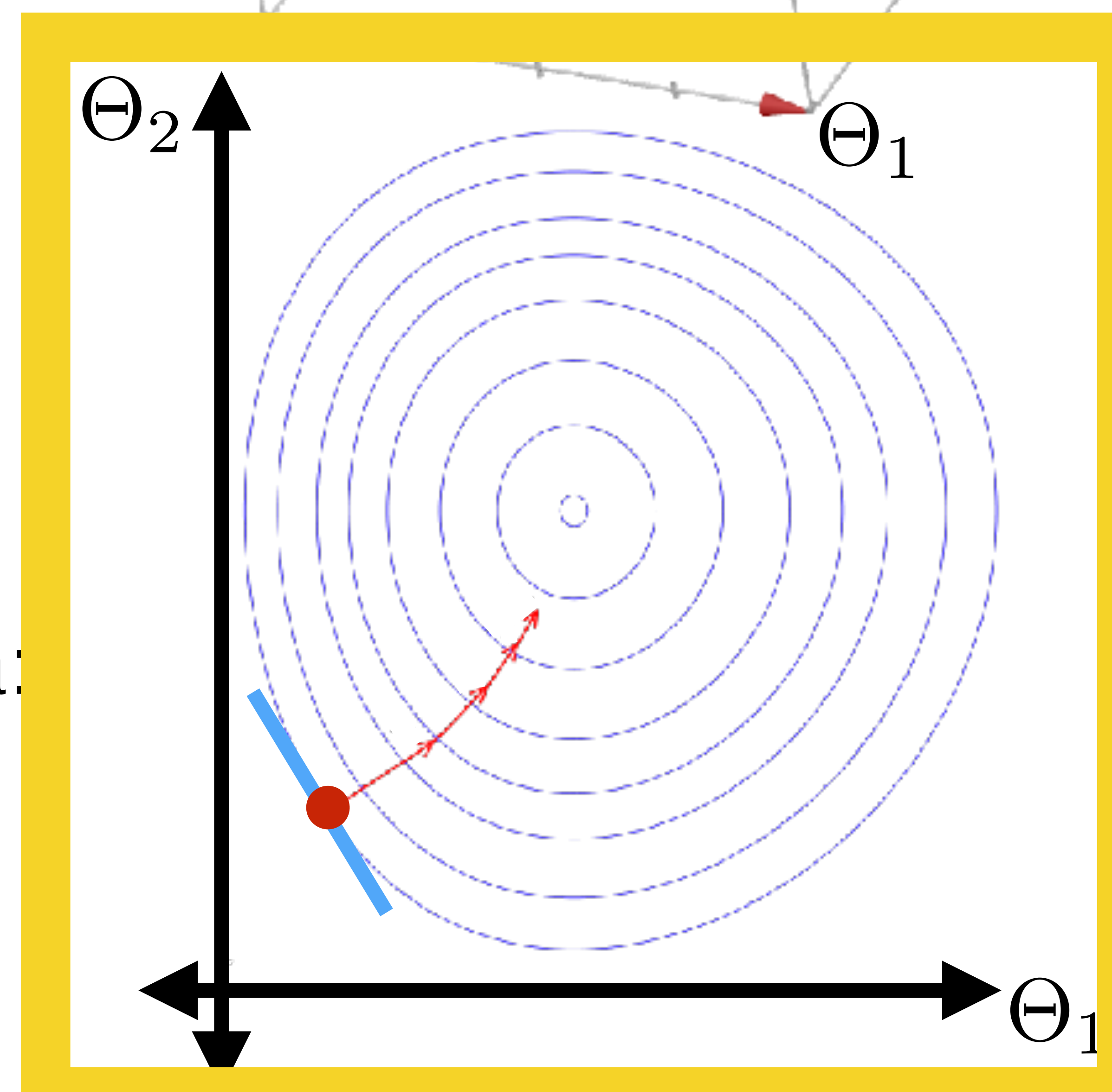
$\quad$ **repeat**

$\qquad \texttt{t = t + 1}$

$\qquad \Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

$\quad$ **until** $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

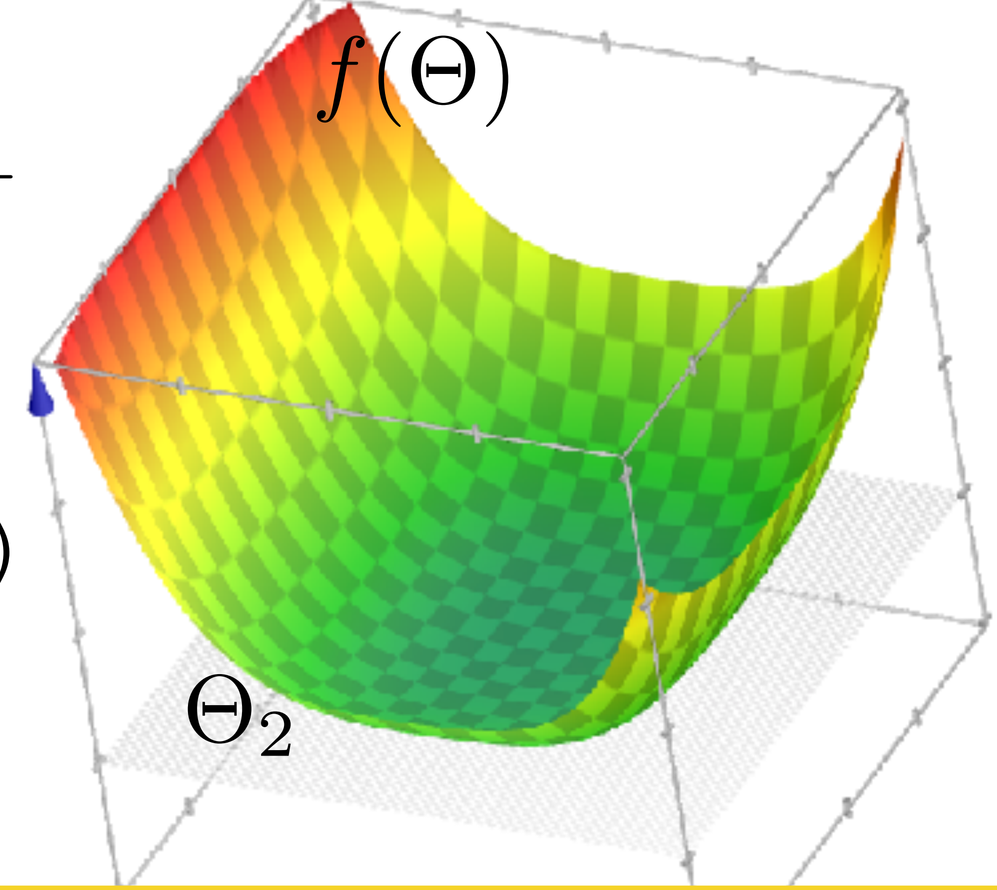
$\quad$ **Return** $\Theta^{(t)}$



- Other possible stopping criteria:
    - Max number of iterations T
    - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
    - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$

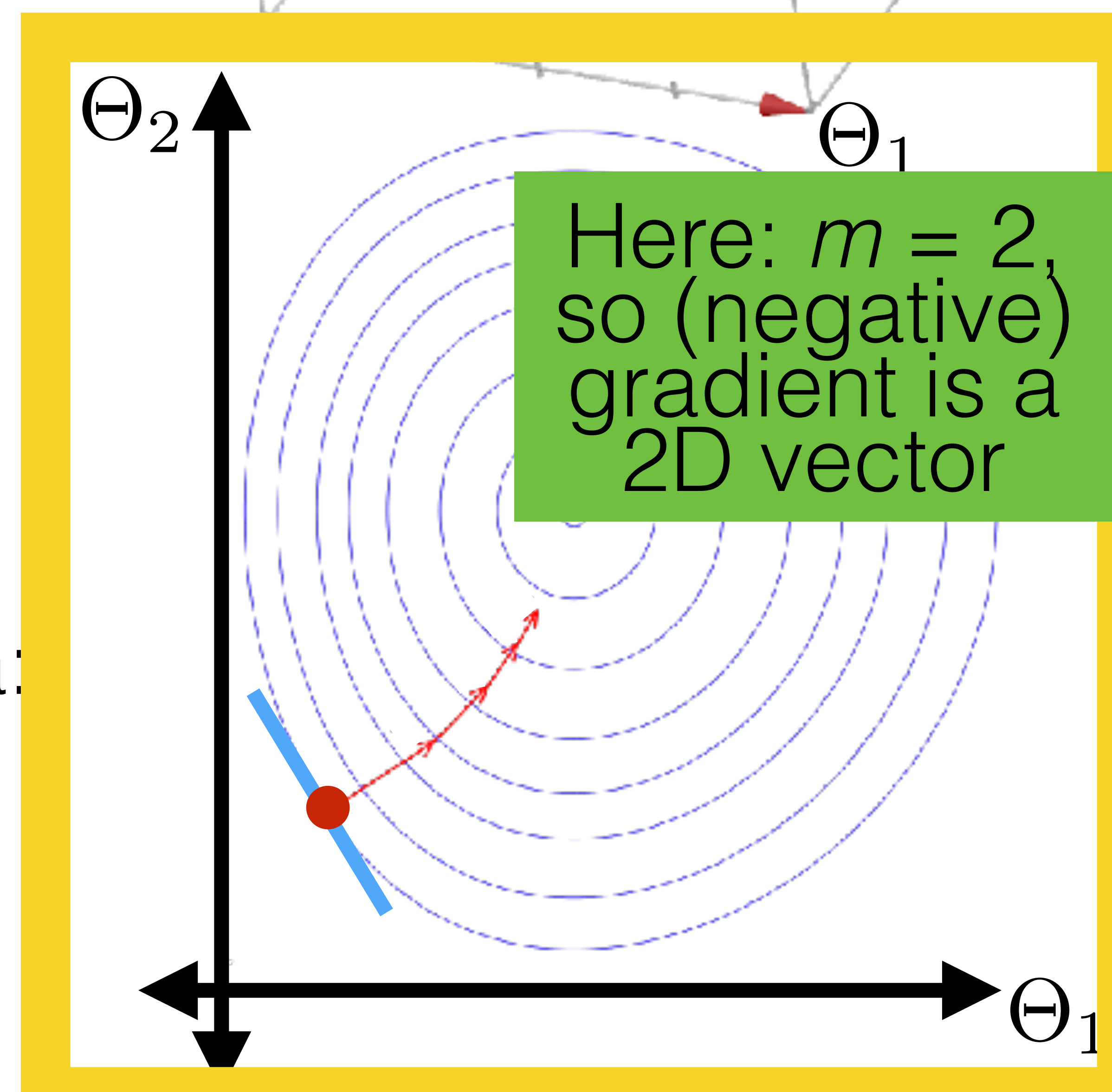
Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
    - with $\Theta \in \mathbb{R}^m$



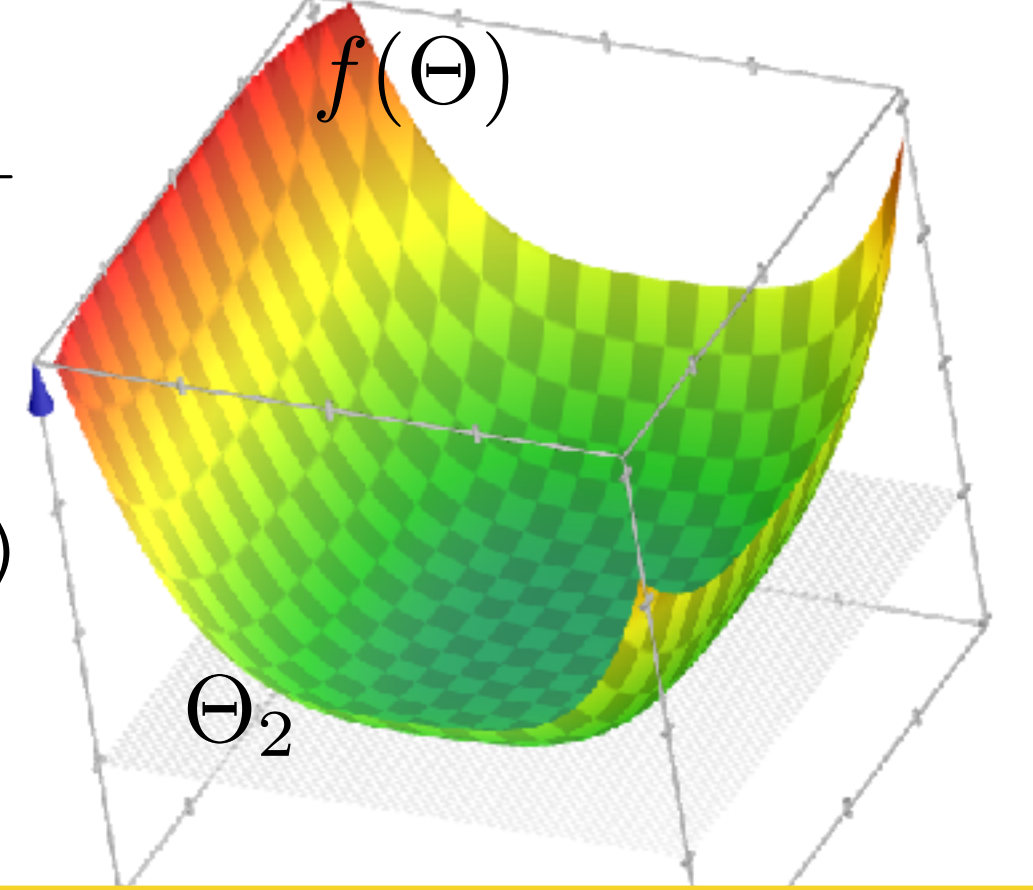

```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

- Other possible stopping criteria:
    - Max number of iterations T
    - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
    - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$



Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$




```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
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  repeat
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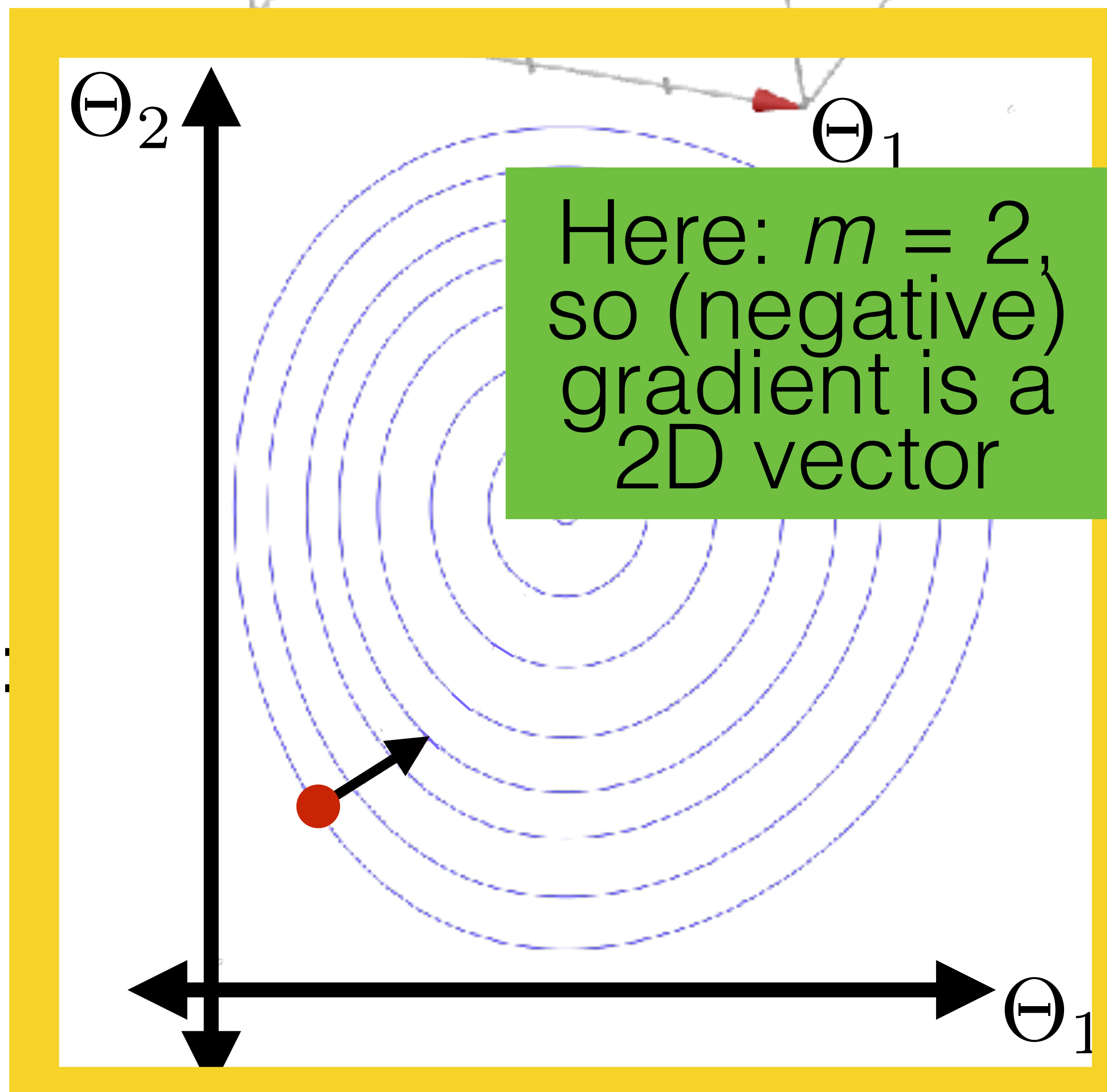
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

$$\texttt{Gradient-Descent}\,(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$$
$$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$$
$$\textbf{until}\ \left|f(\Theta^{(t)}) - f(\Theta^{(t-1)})\right| < \epsilon$$

- Other possible stopping criteria:
  - Max number of iterations T
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$

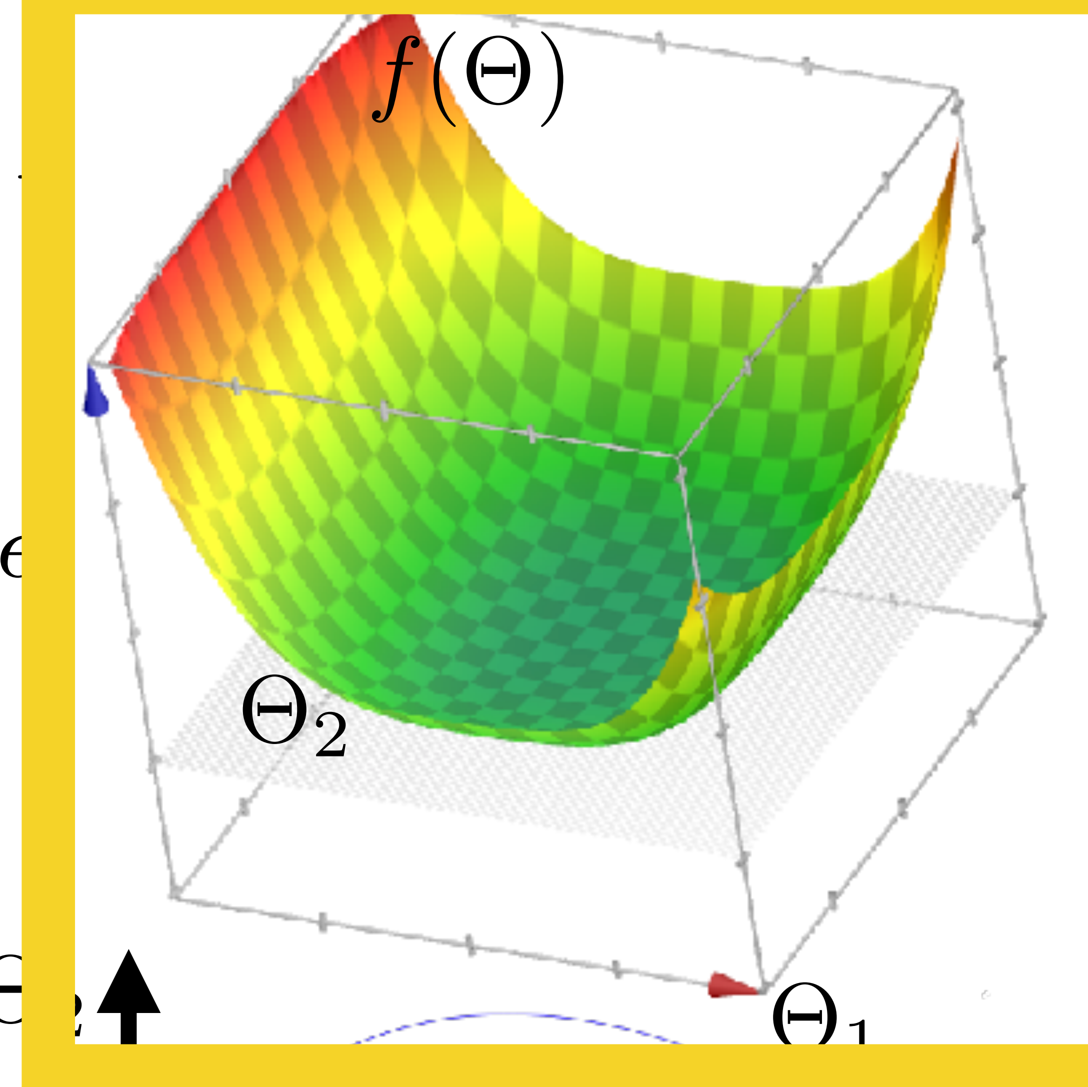


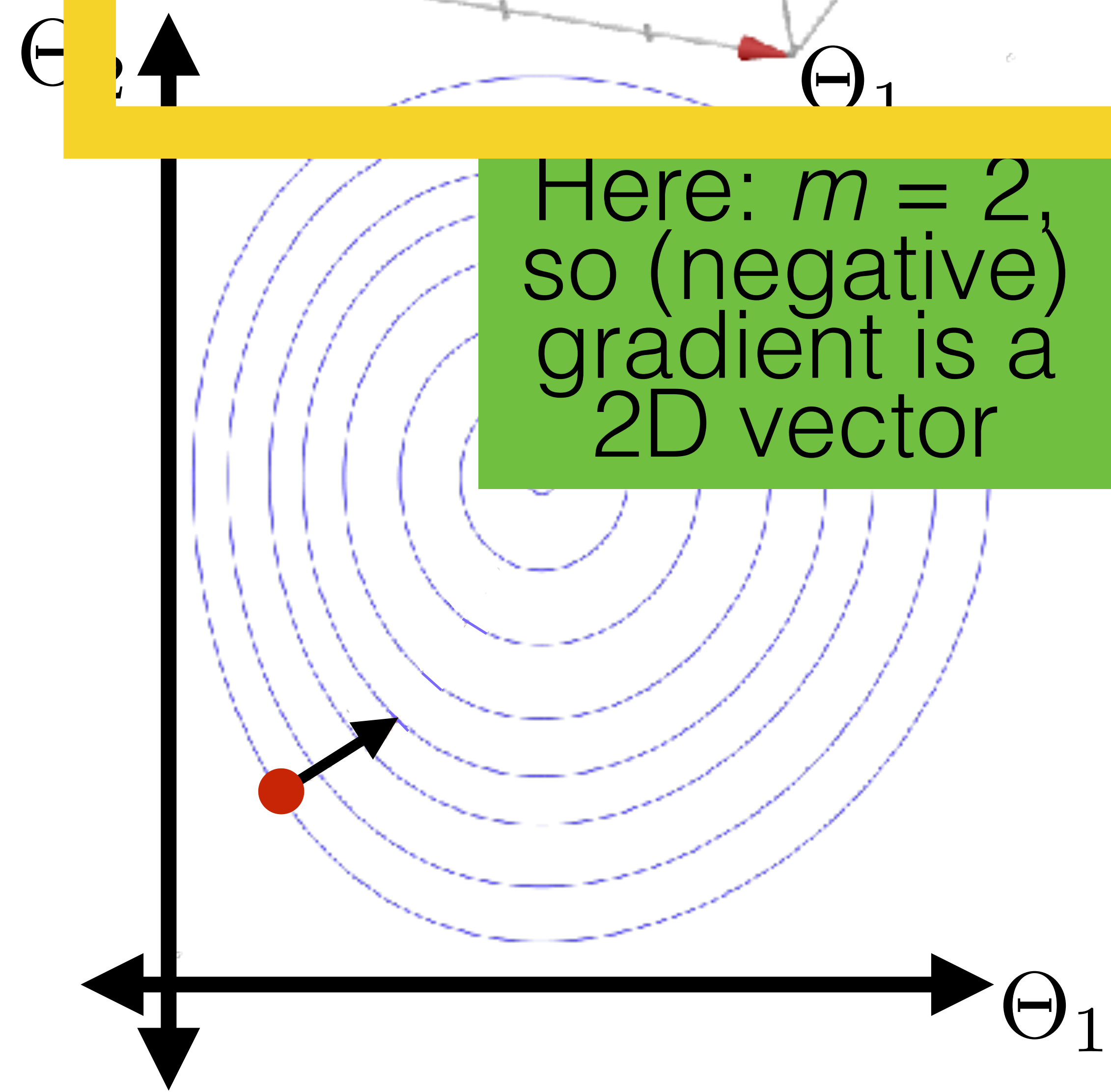
Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]$
  - with $\Theta \in \mathbb{R}^m$

```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

- Other possible stopping criteria:
  - Max number of iterations T
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^T$
  - with $\Theta \in \mathbb{R}^m$

```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
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  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) - η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) - f(Θ^(t-1))| < ε
  Return Θ^(t)
```

$$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$$

$$\textbf{until} \left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$$

- Other possible stopping criteria:
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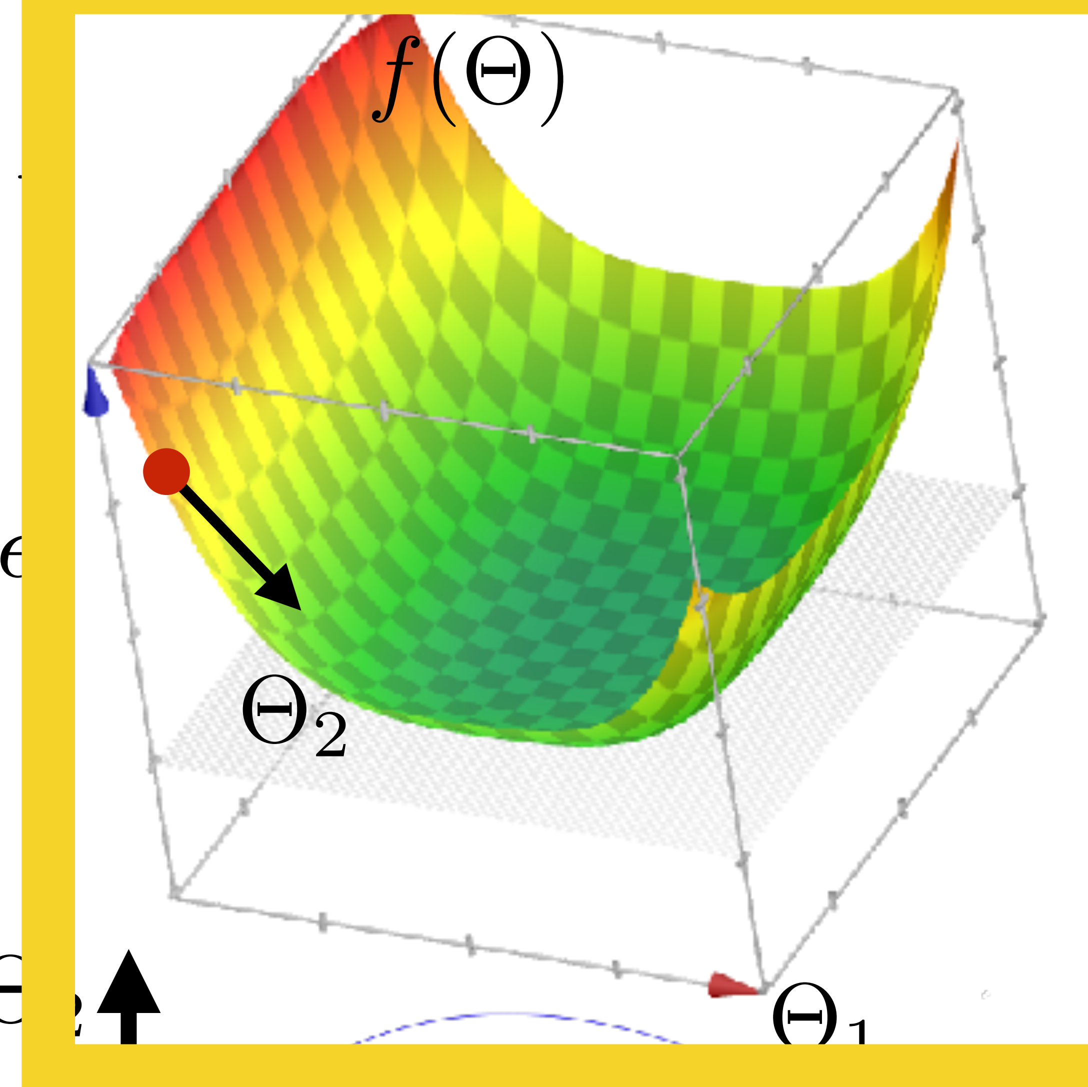
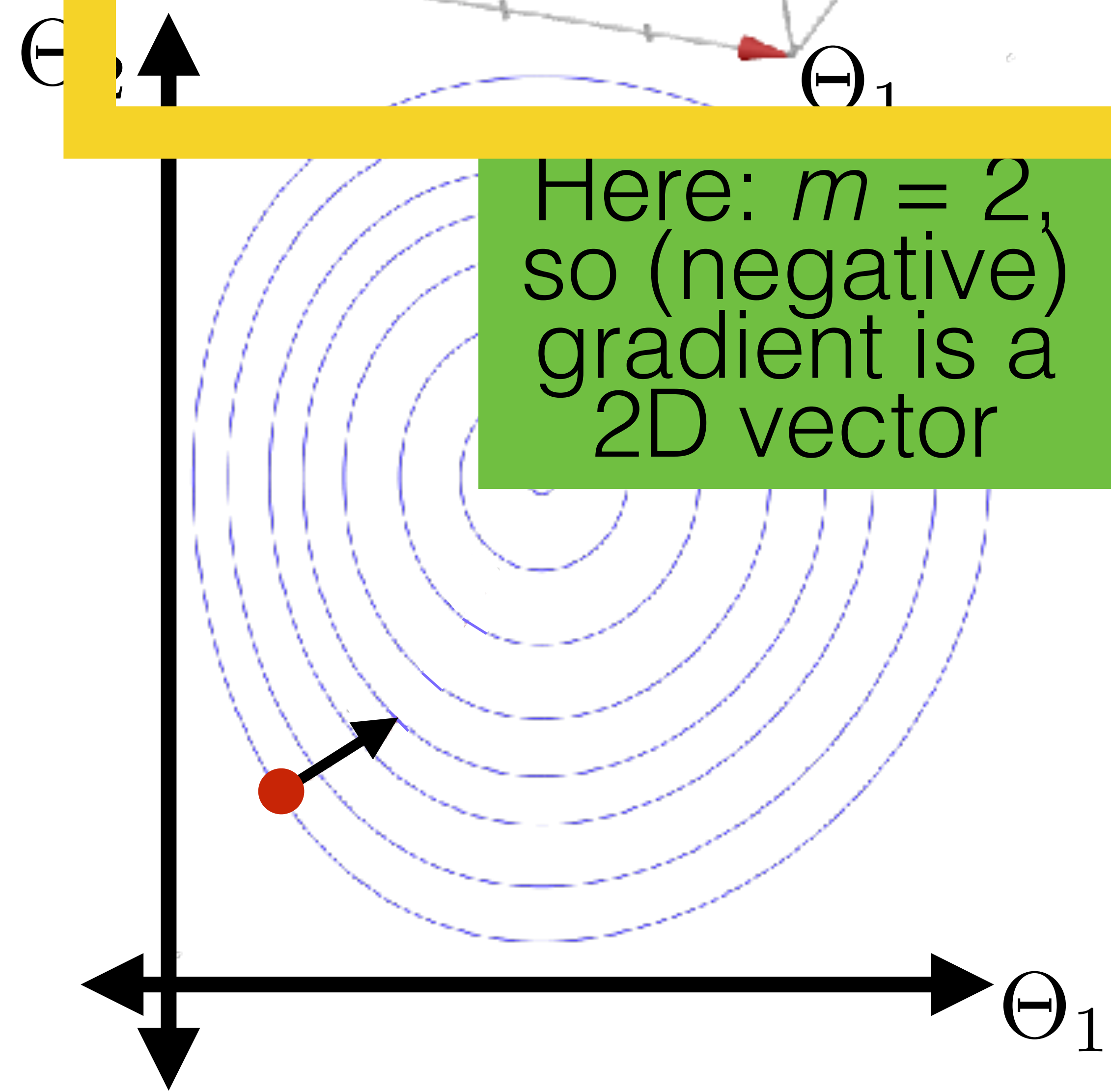
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

```
Gradient-Descent (Θinit, η, f, ∇Θ f, ϵ
  Initialize Θ(0) = Θinit
  Initialize t = 0
  repeat
    t = t + 1
    Θ(t) = Θ(t−1) − η∇Θ f(Θ(t−1))
  until |f(Θ(t)) − f(Θ(t−1))| < ϵ
  Return Θ(t)
```

$\texttt{Gradient-Descent}\,(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon$

$\quad\texttt{Initialize}\ \Theta^{(0)} = \Theta_{\text{init}}$

$\quad\texttt{Initialize t = 0}$

$\quad\textbf{repeat}$

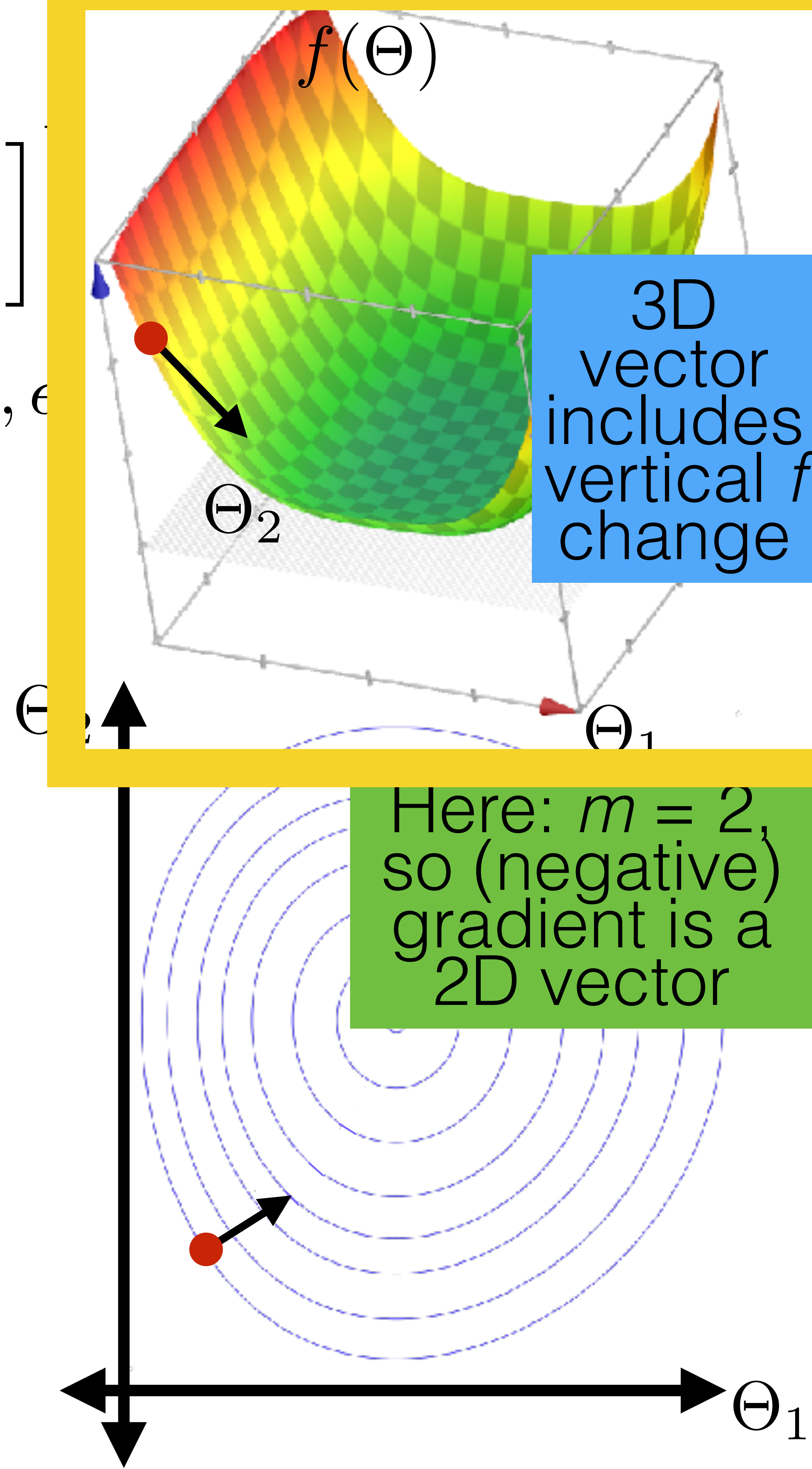
$\qquad\texttt{t = t + 1}$

$\qquad\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

$\quad\textbf{until}\ \left|f(\Theta^{(t)}) - f(\Theta^{(t-1)})\right| < \epsilon$

$\quad\textbf{Return}\ \Theta^{(t)}$

- Other possible stopping criteria:
  - Max number of iterations T
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$



Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
  - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

`Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

`Initialize t = 0`

`repeat`

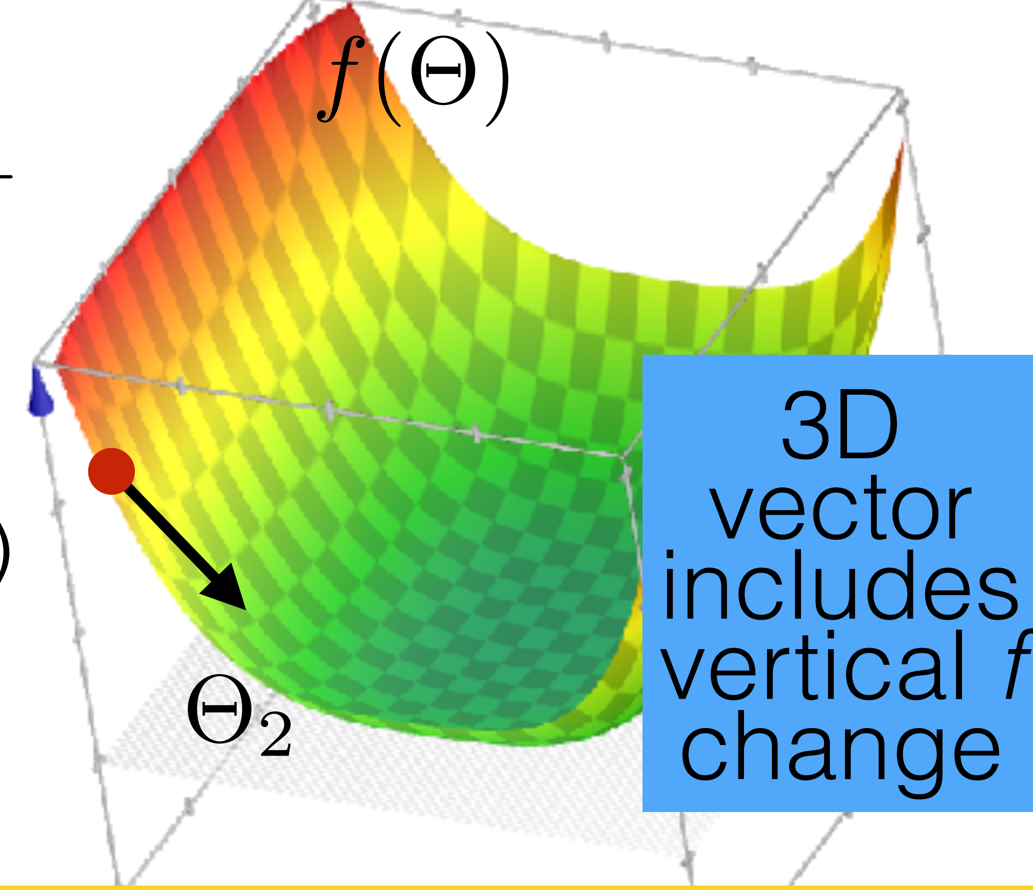
`t = t + 1`

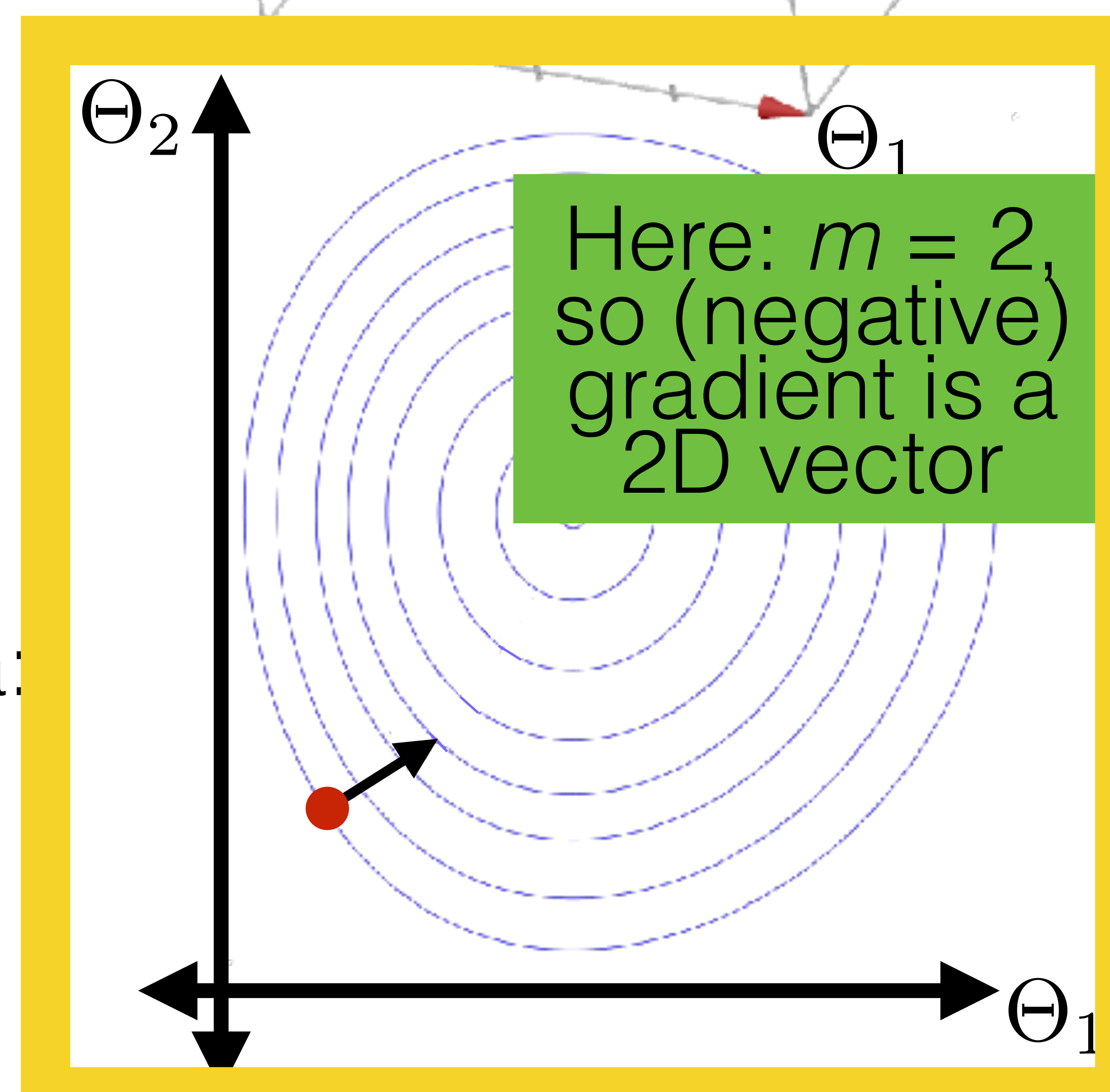
$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

`until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

`Return` $\Theta^{(t)}$

- Other possible stopping criteria:
  - Max number of iterations T
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$





Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m}\right]^\top$
    - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon)$

  `Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

  `Initialize t = 0`

  `repeat`

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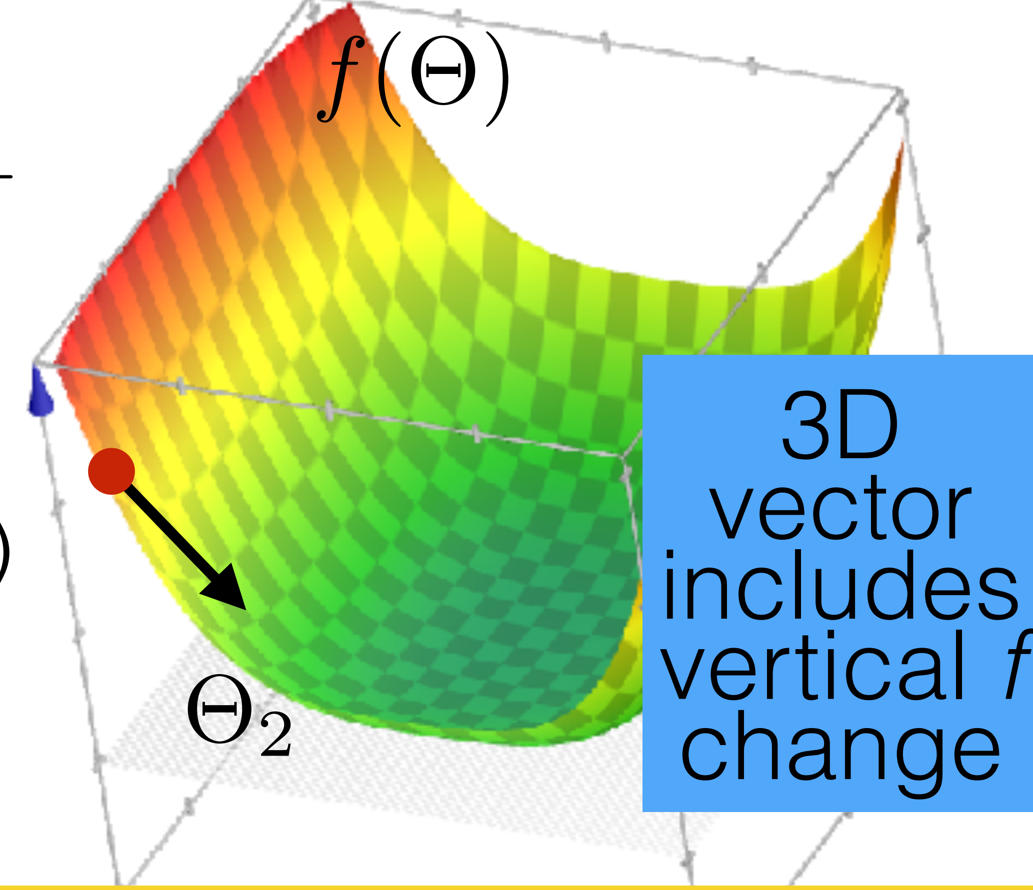
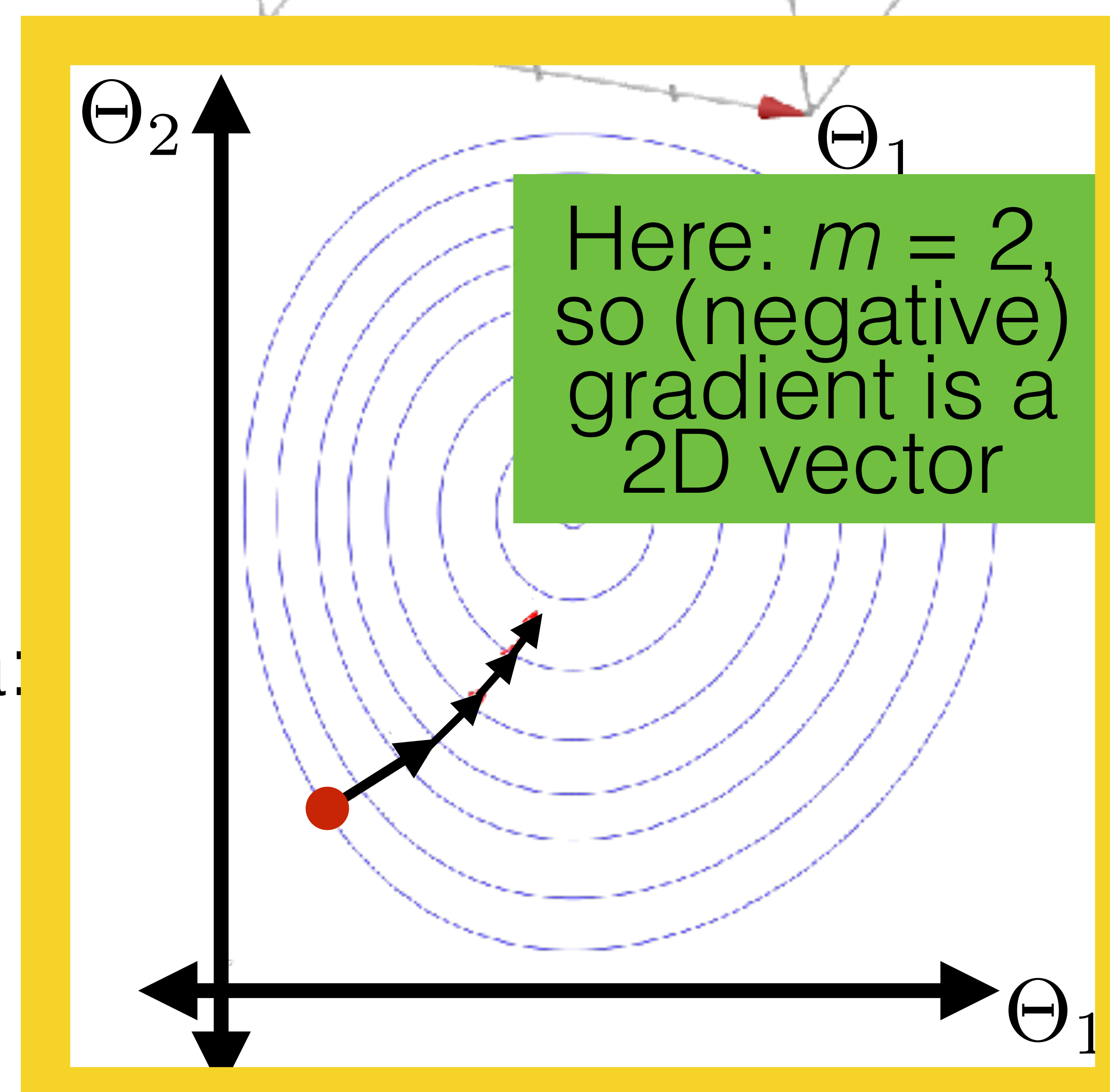
    $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  `until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

  `Return` $\Theta^{(t)}$





- Other possible stopping criteria:
    - Max number of iterations T
    - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
    - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$

Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m} \right]^\top$
  - with $\Theta \in \mathbb{R}^m$

`Gradient-Descent` $(\Theta_{\text{init}}, \eta, f, \nabla_\Theta f, \epsilon$

`Initialize` $\Theta^{(0)} = \Theta_{\text{init}}$

`Initialize t = 0`

`repeat`

`t = t + 1`

$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

`until` $\left| f(\Theta^{(t)}) - f(\Theta^{(t-1)}) \right| < \epsilon$

`Return` $\Theta^{(t)}$

- Other possible stopping criteria:
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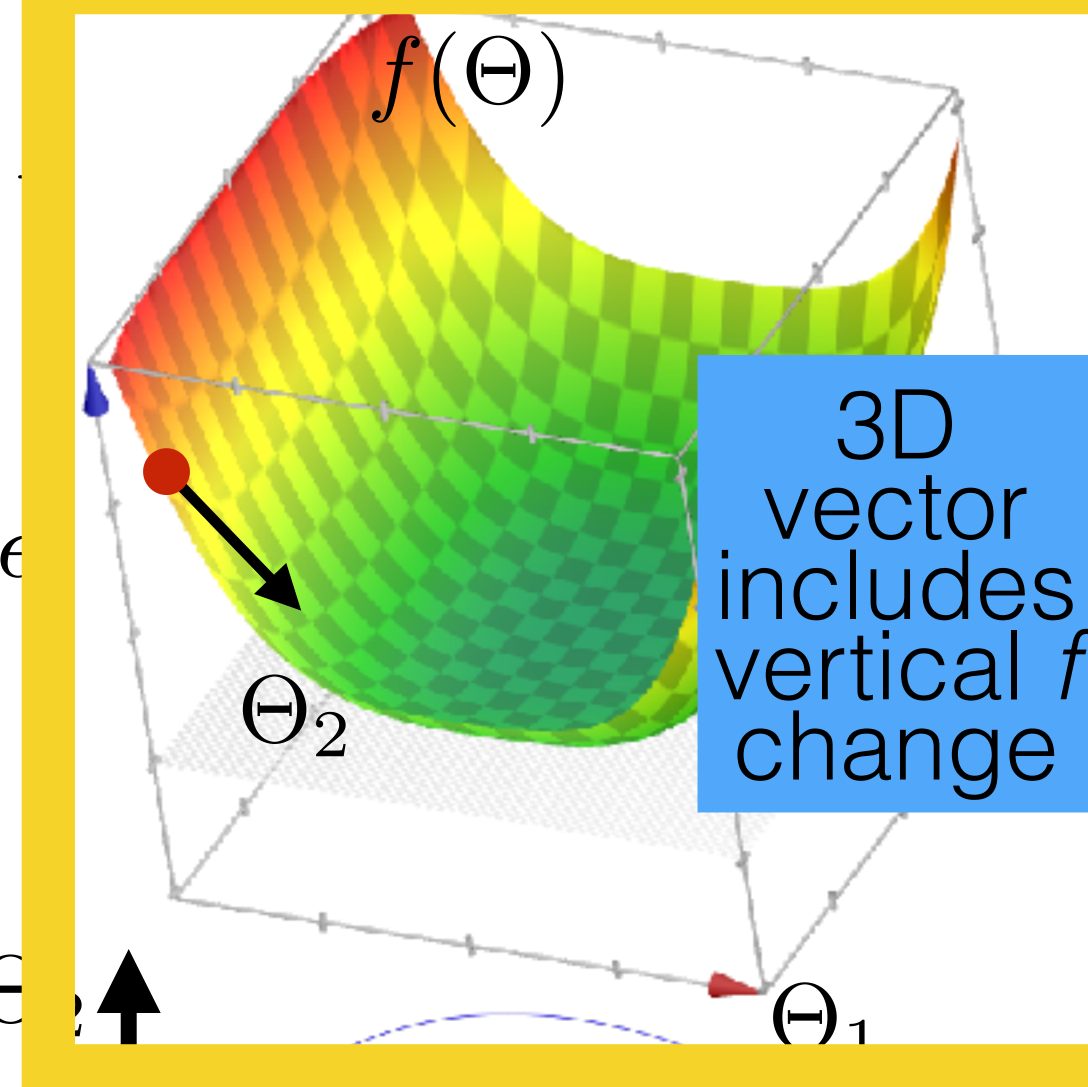
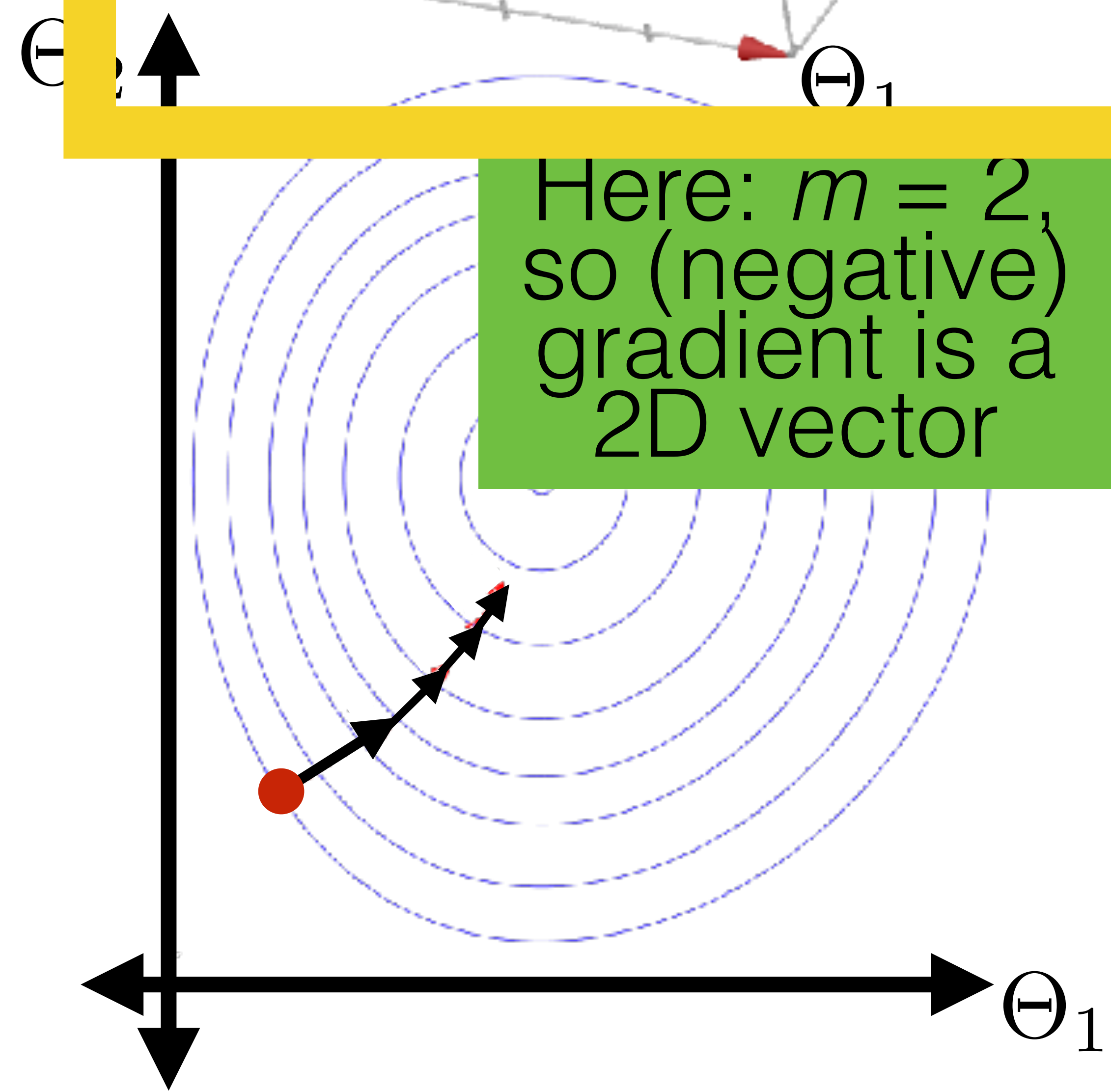


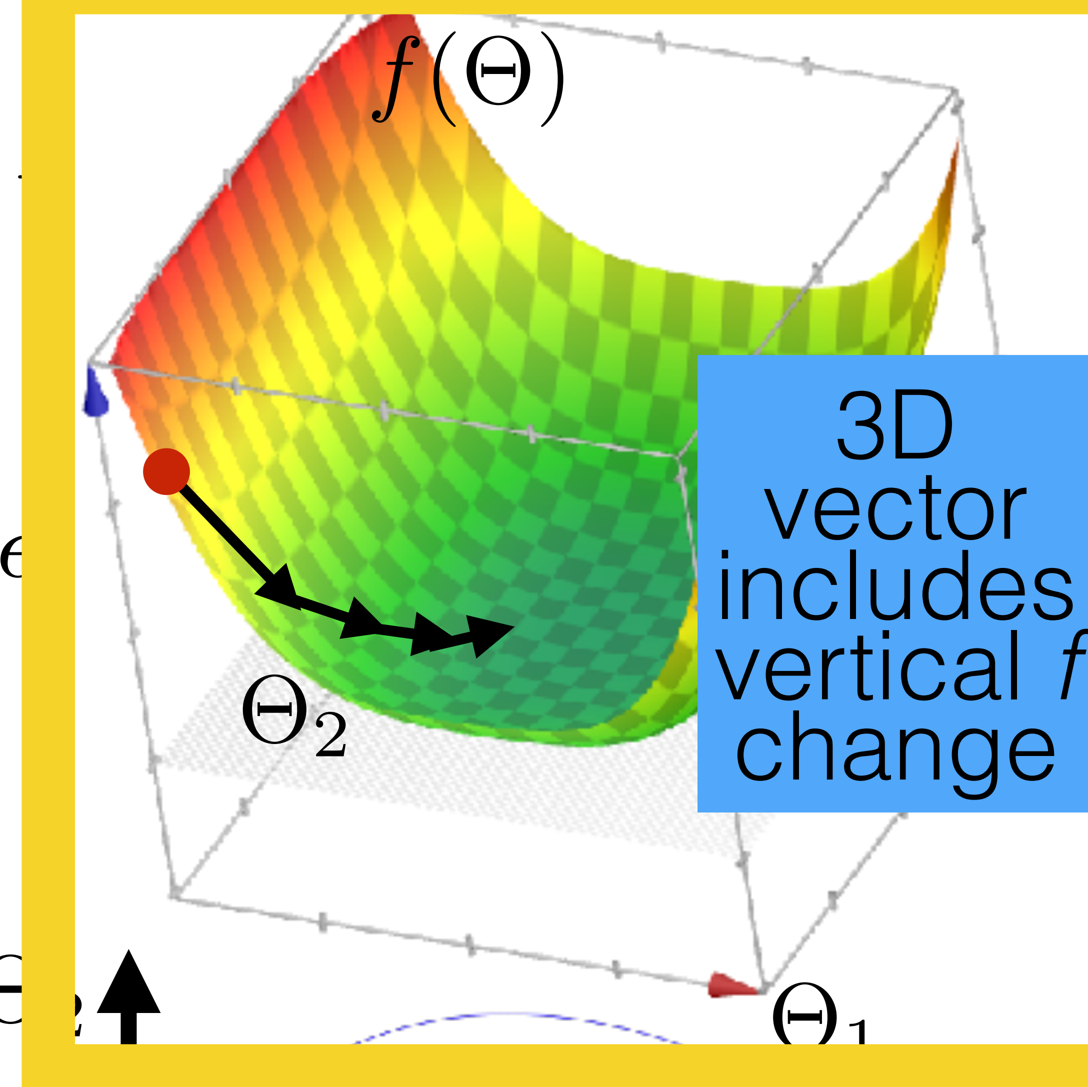


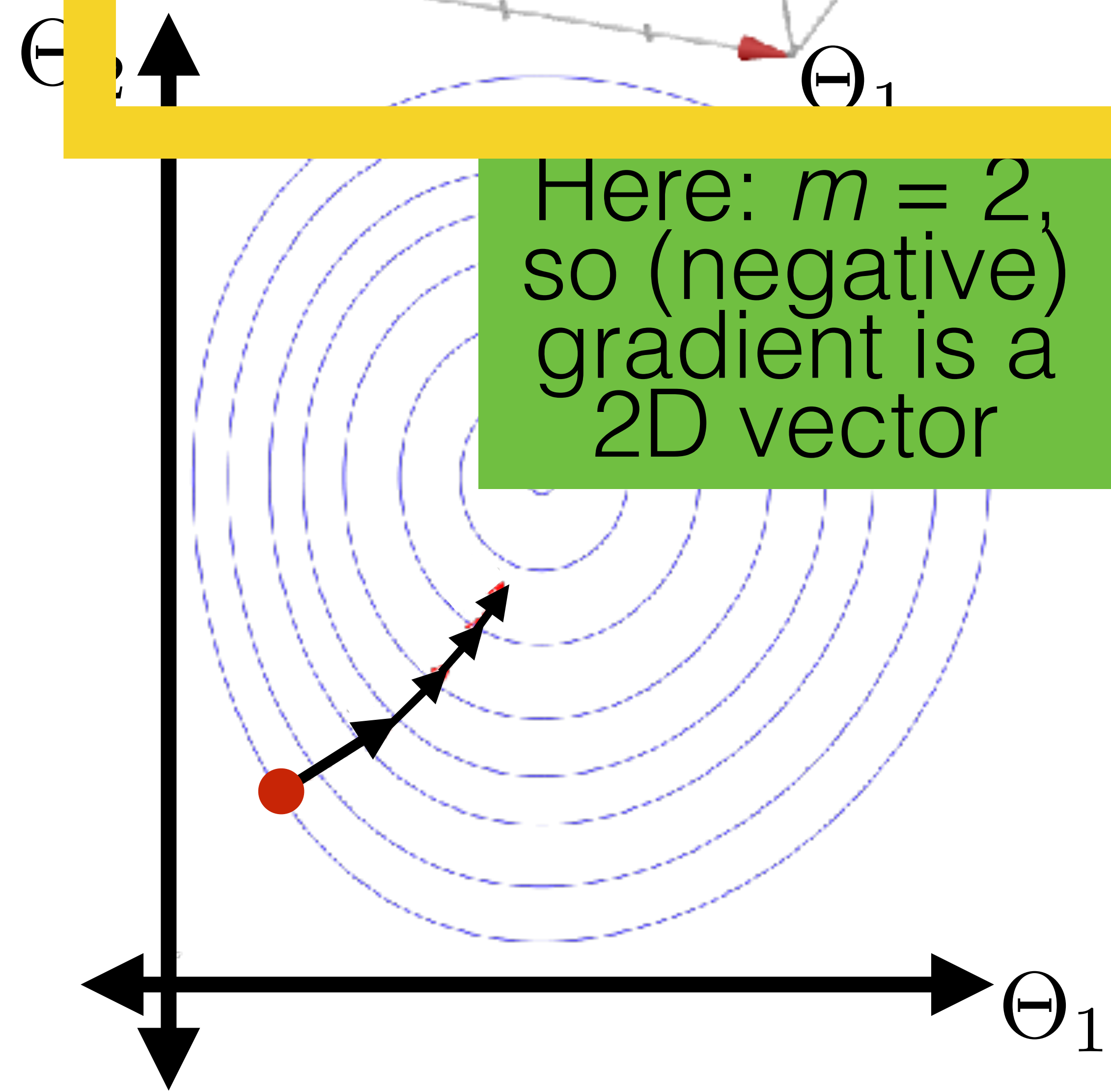
Gradient descent

- Gradient $\nabla_\Theta f = \left[\frac{\partial f}{\partial \Theta_1}, \ldots, \frac{\partial f}{\partial \Theta_m} \right]$
  - with $\Theta \in \mathbb{R}^m$

```
Gradient-Descent (Θ_init, η, f, ∇_Θ f, ε)
  Initialize Θ^(0) = Θ_init
  Initialize t = 0
  repeat
    t = t + 1
    Θ^(t) = Θ^(t-1) − η ∇_Θ f(Θ^(t-1))
  until |f(Θ^(t)) − f(Θ^(t-1))| < ε
  Return Θ^(t)
```

- Other possible stopping criteria:
  - Max number of iterations T
  - $\|\Theta^{(t)} - \Theta^{(t-1)}\| < \epsilon$
  - $\|\nabla_\Theta f(\Theta^{(t)})\| < \epsilon$





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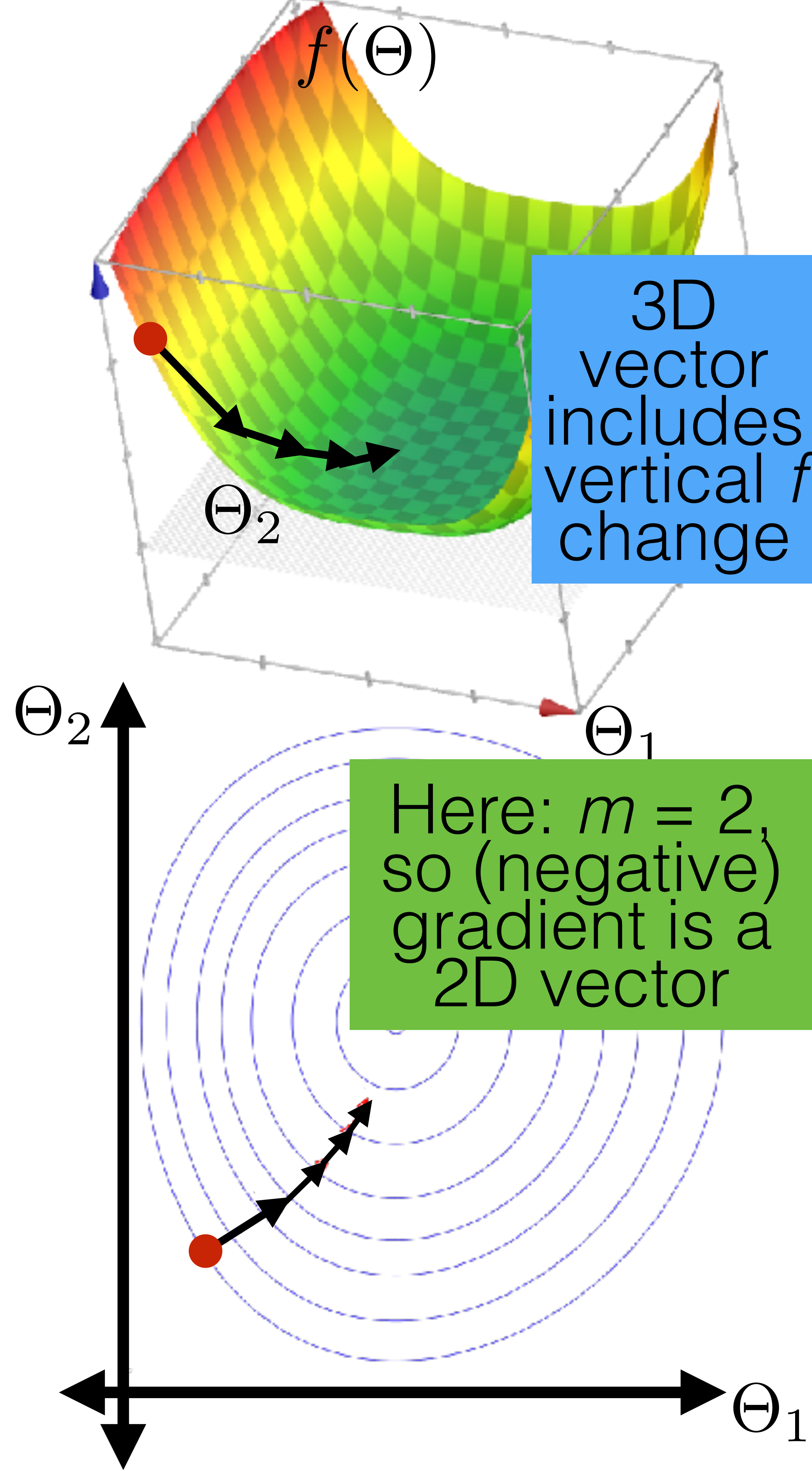
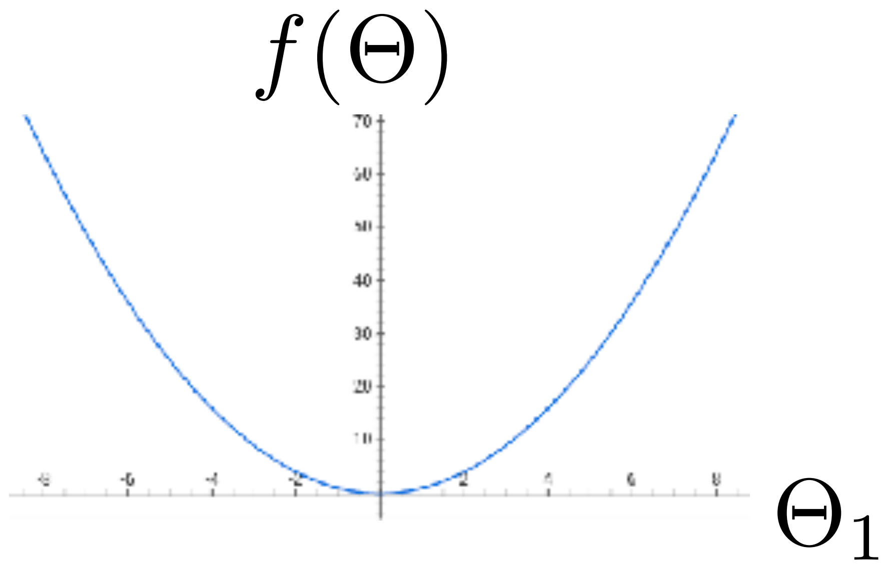
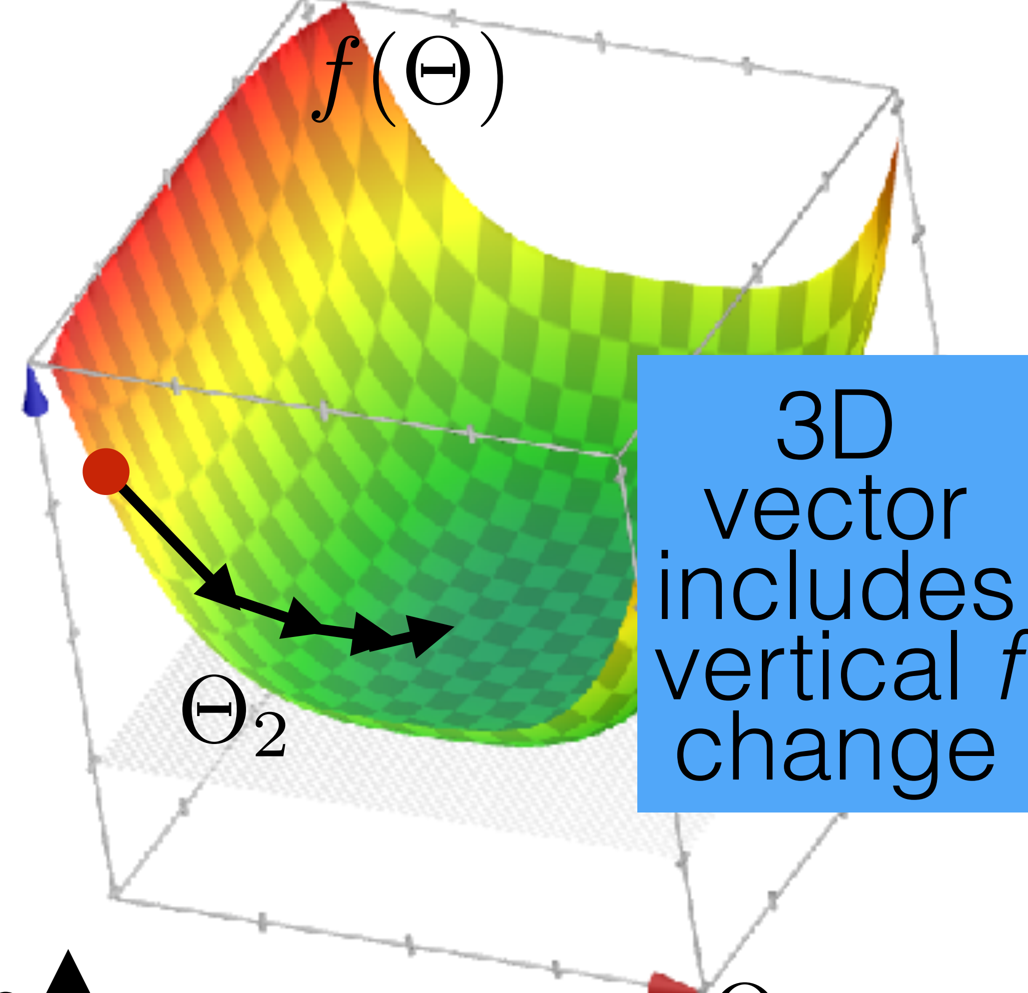
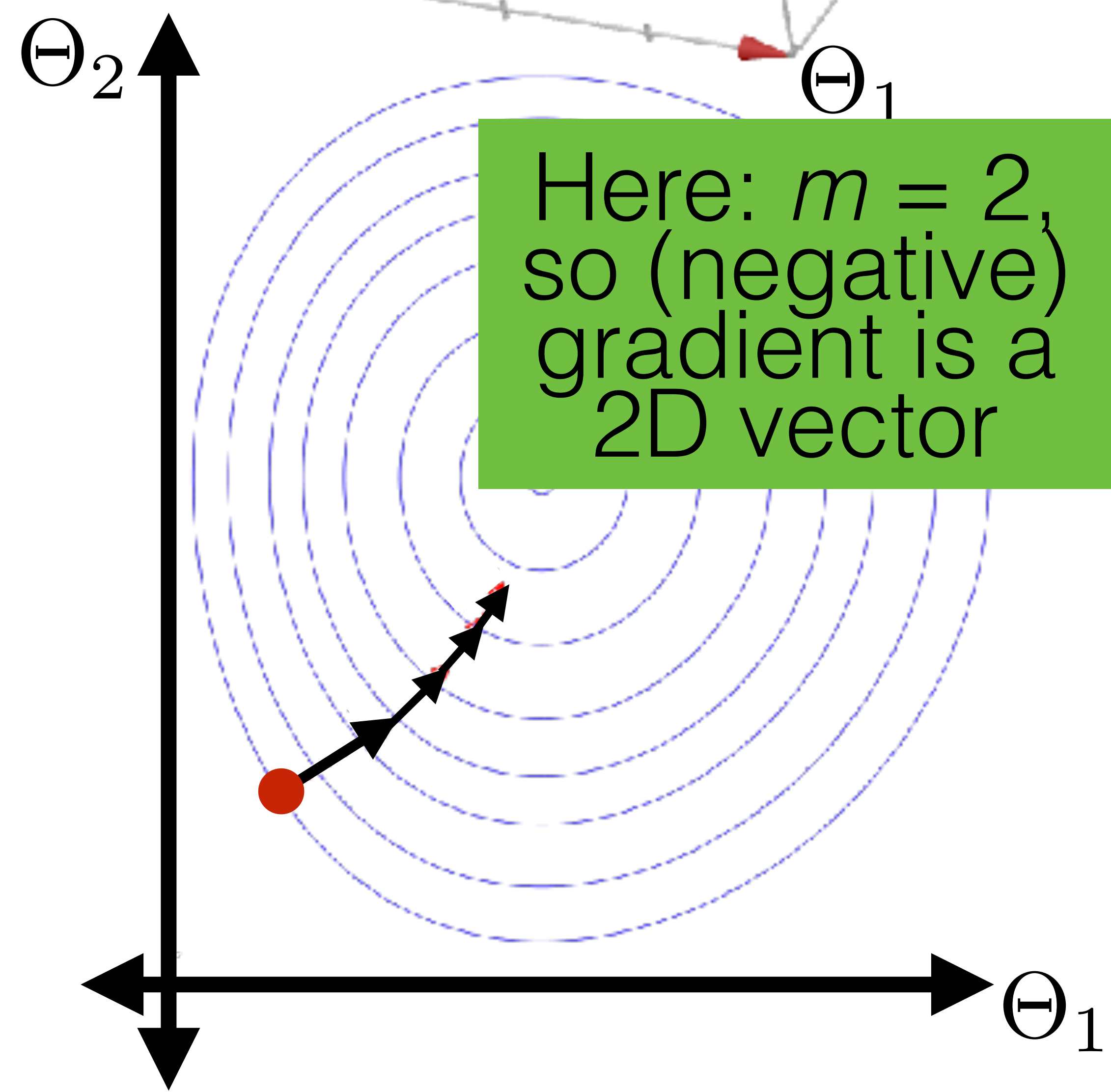


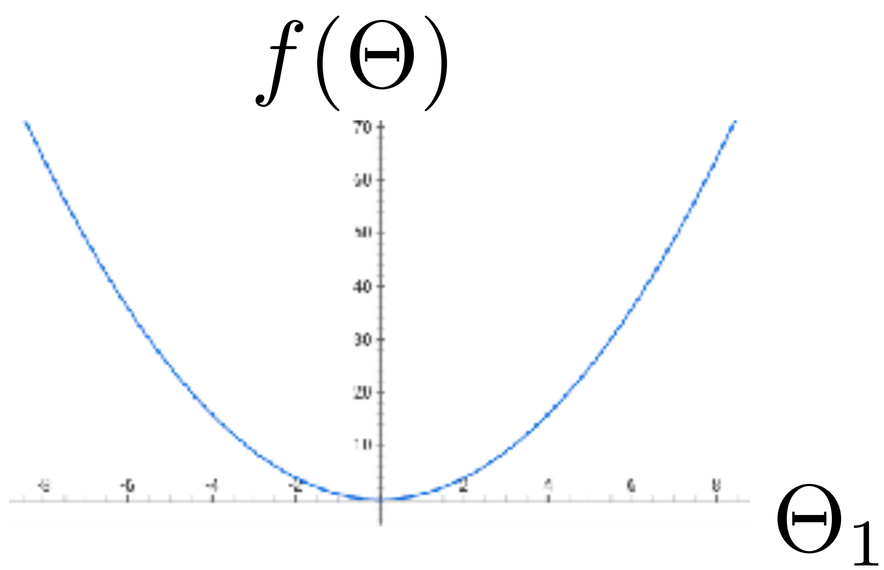
Gradient descent

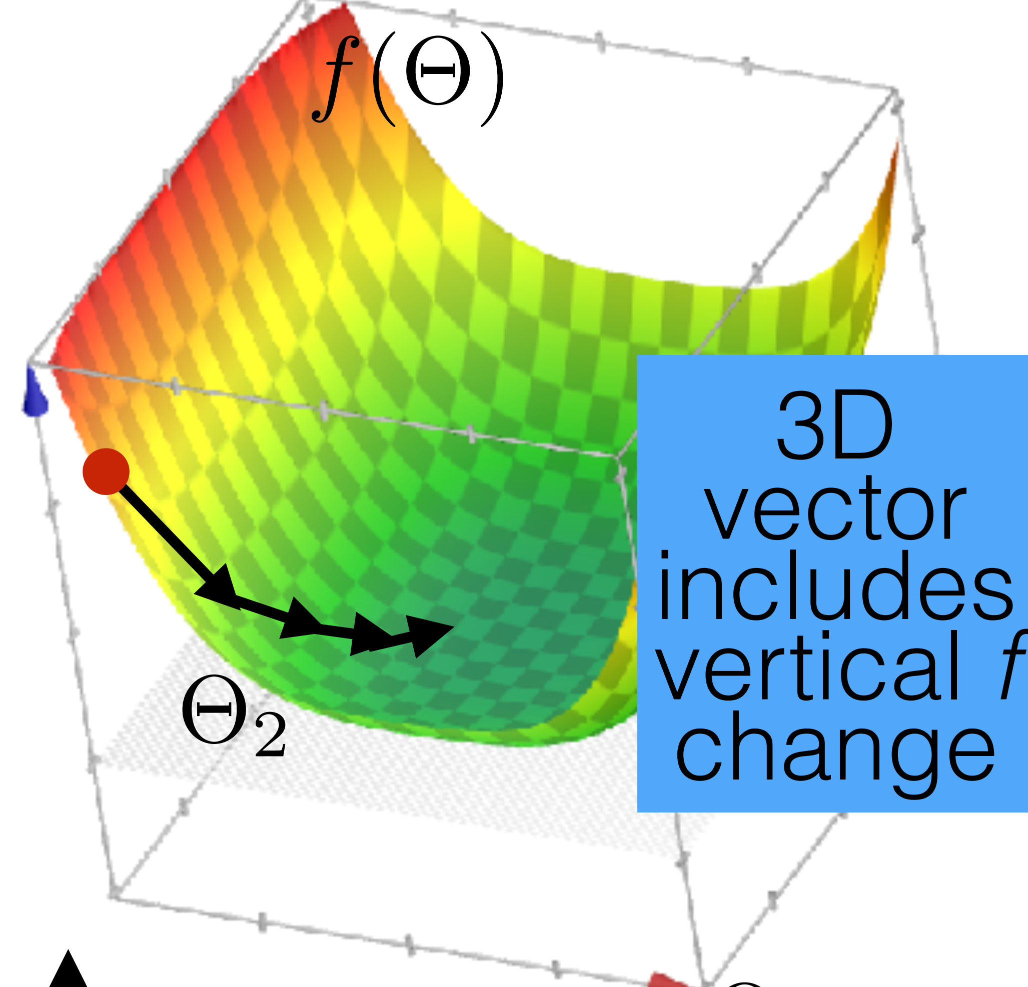


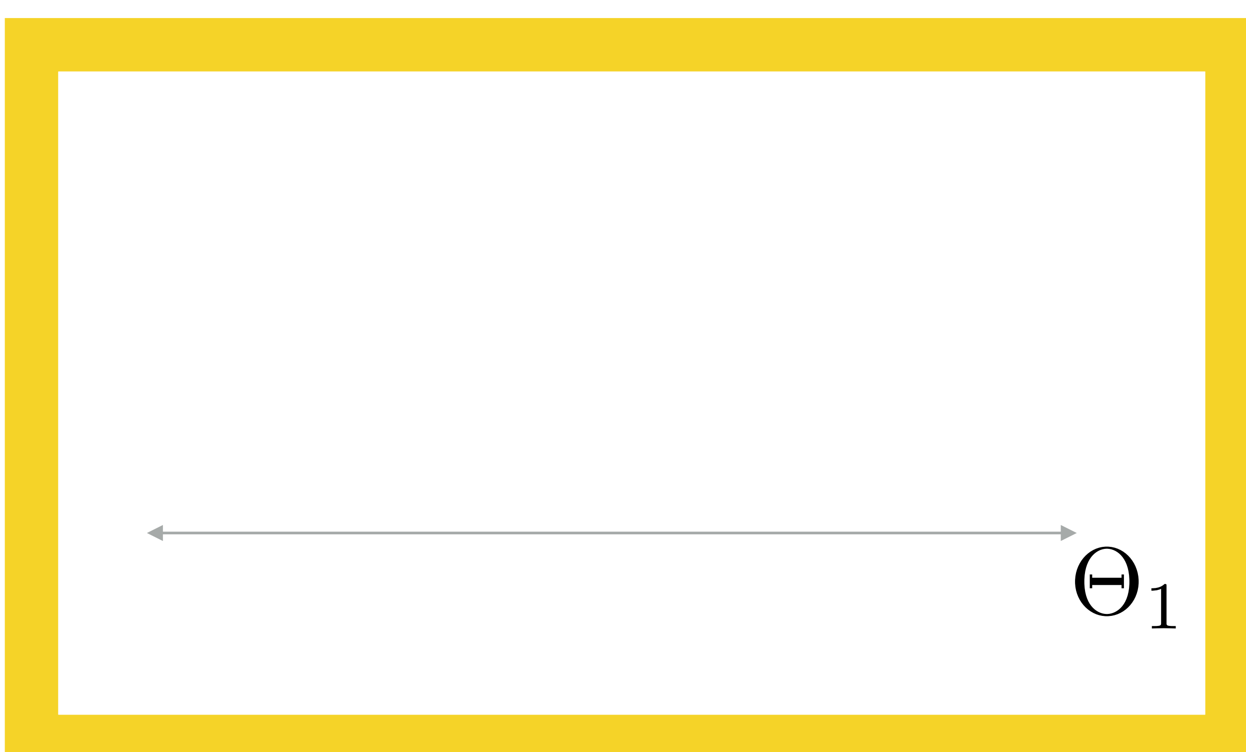


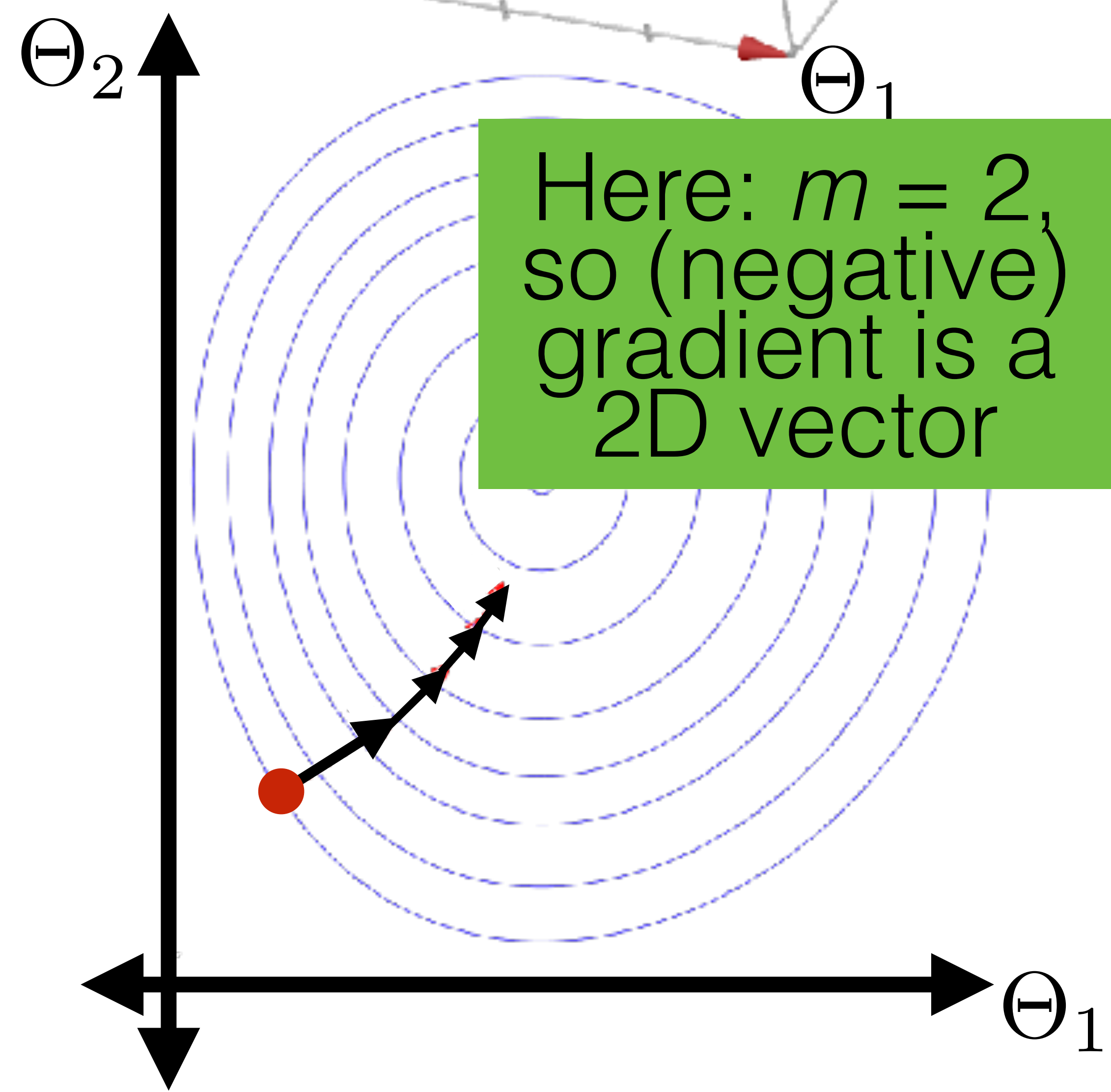


Gradient descent









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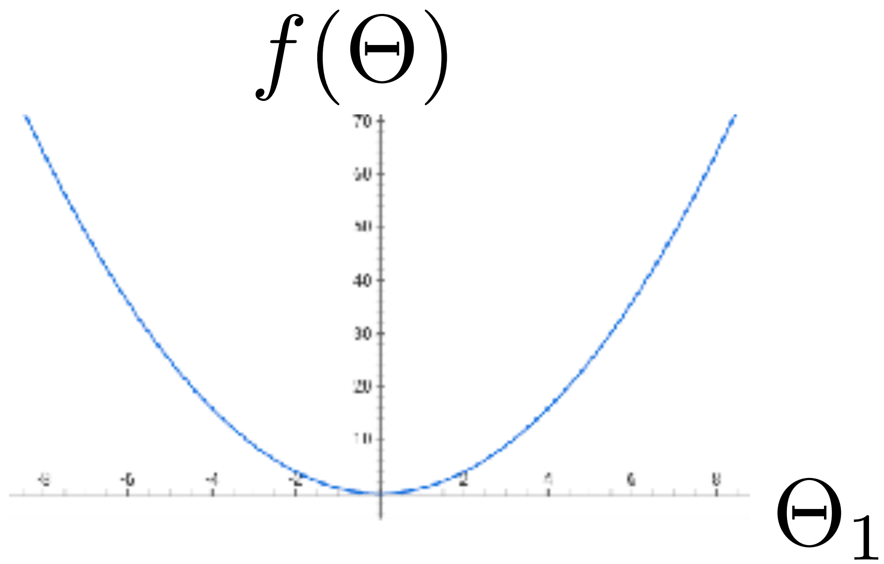

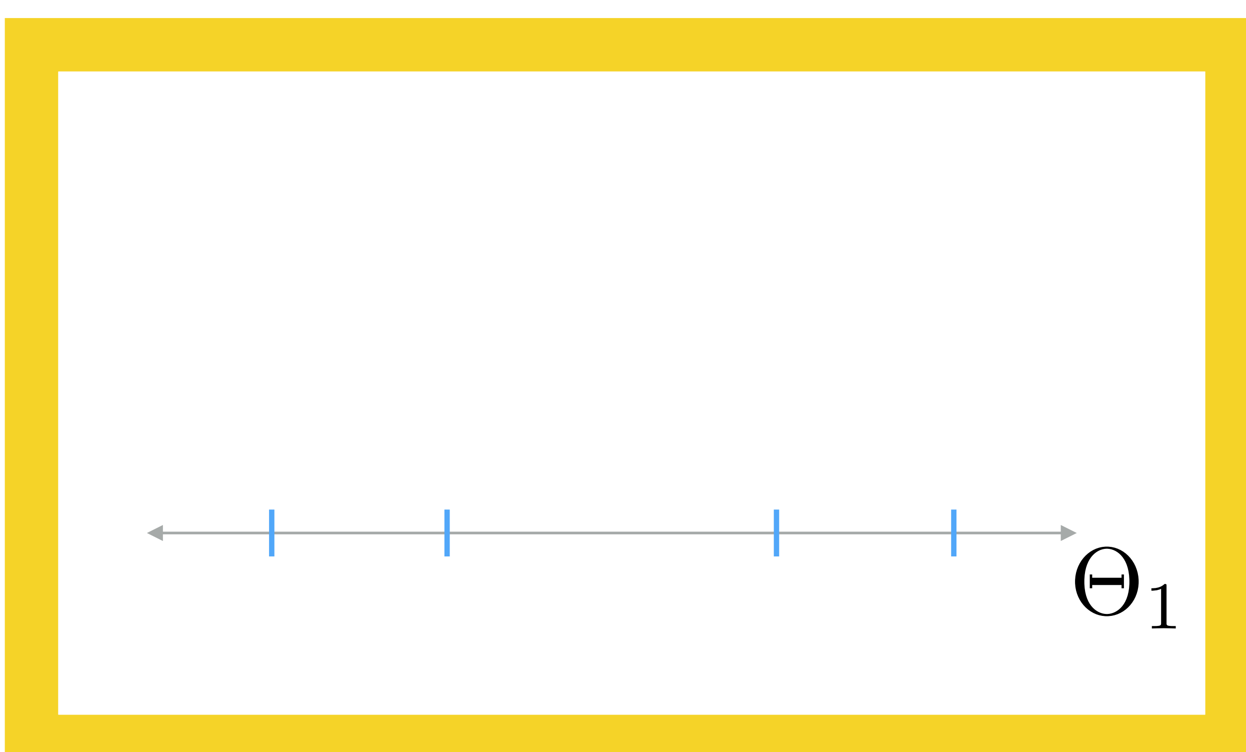

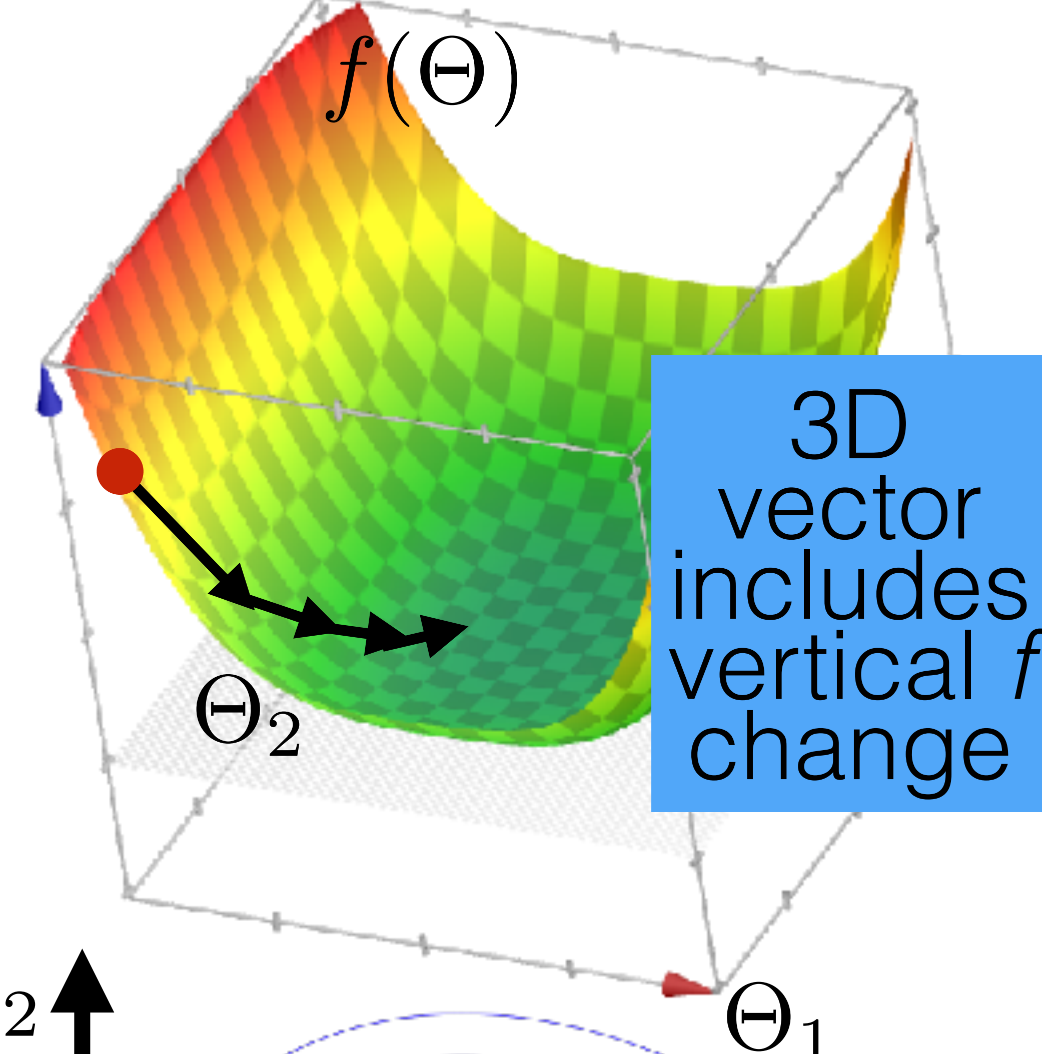

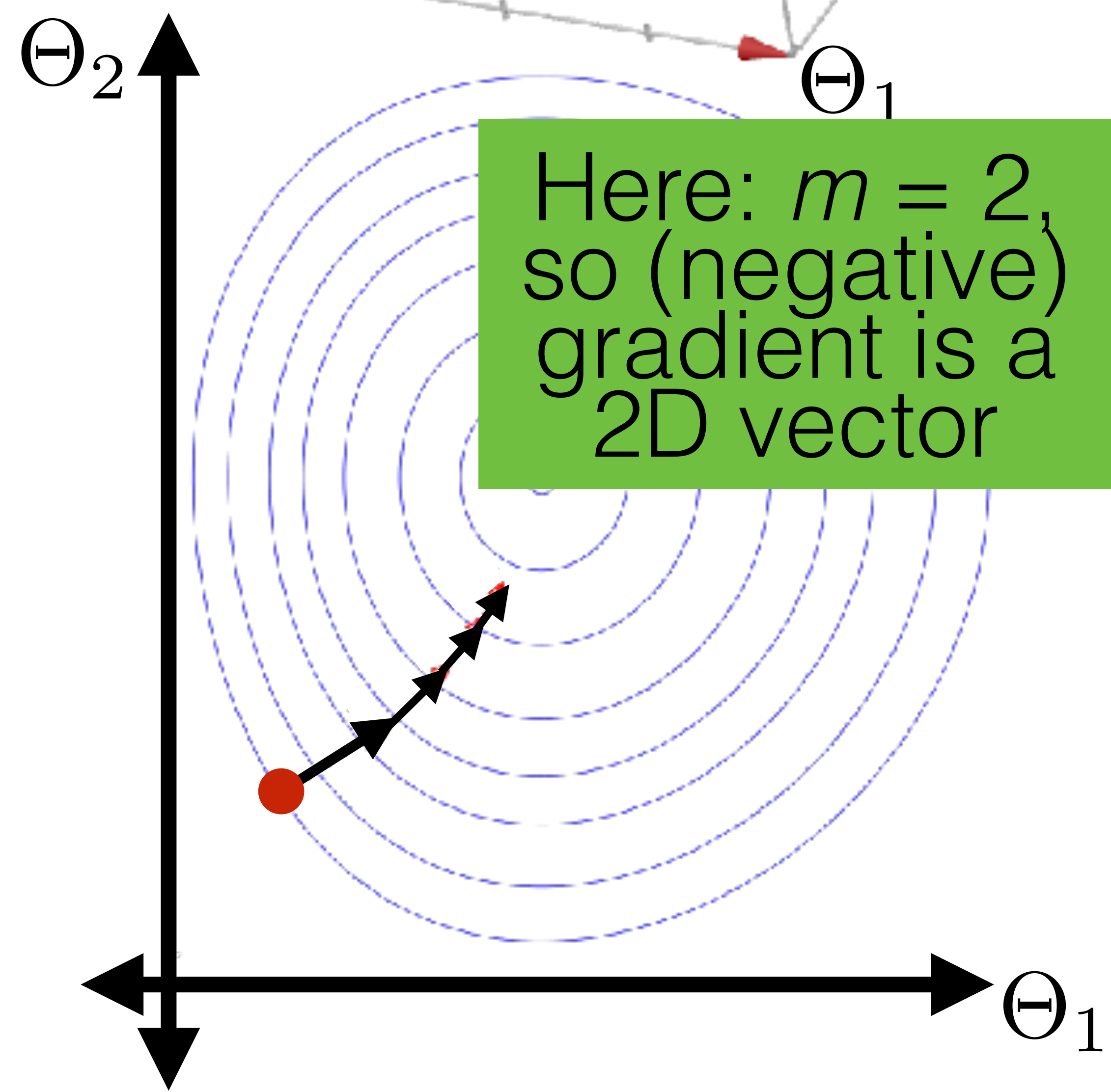

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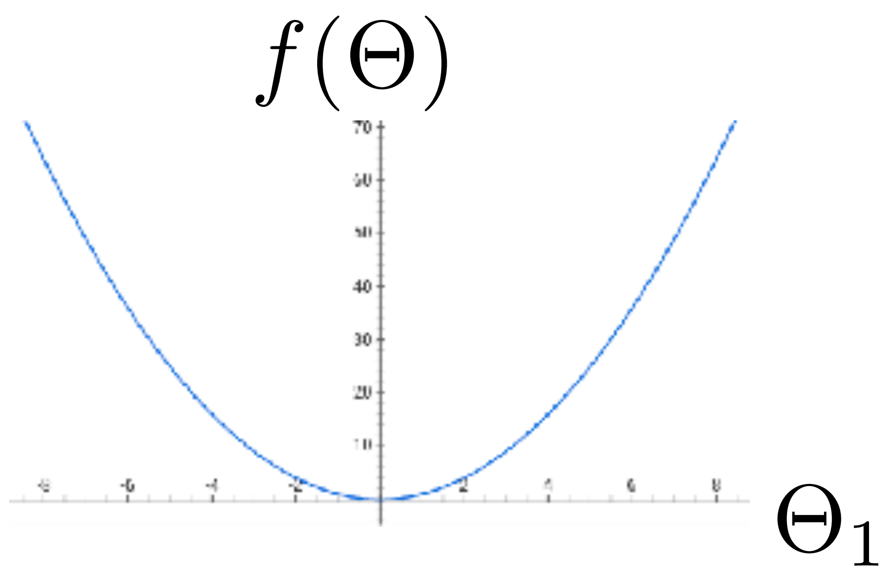

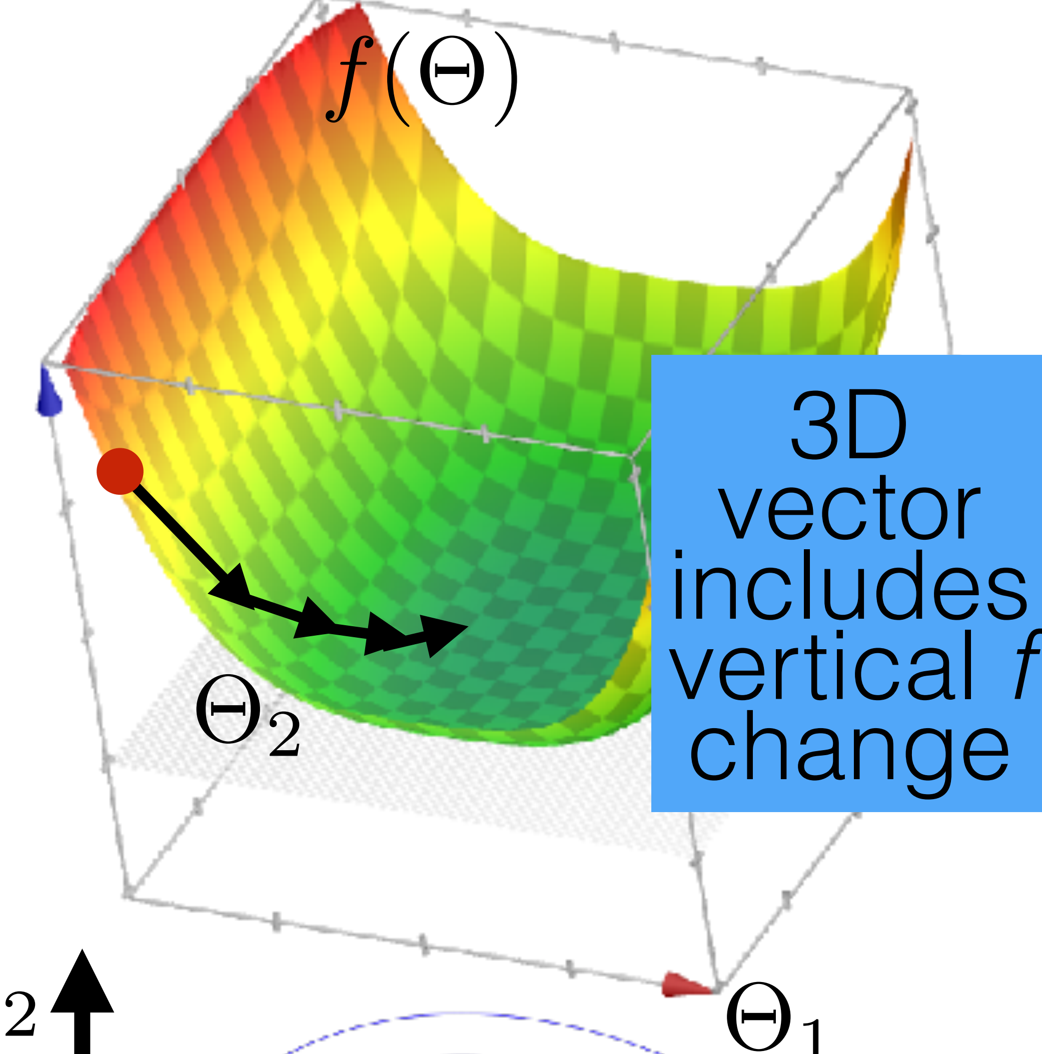

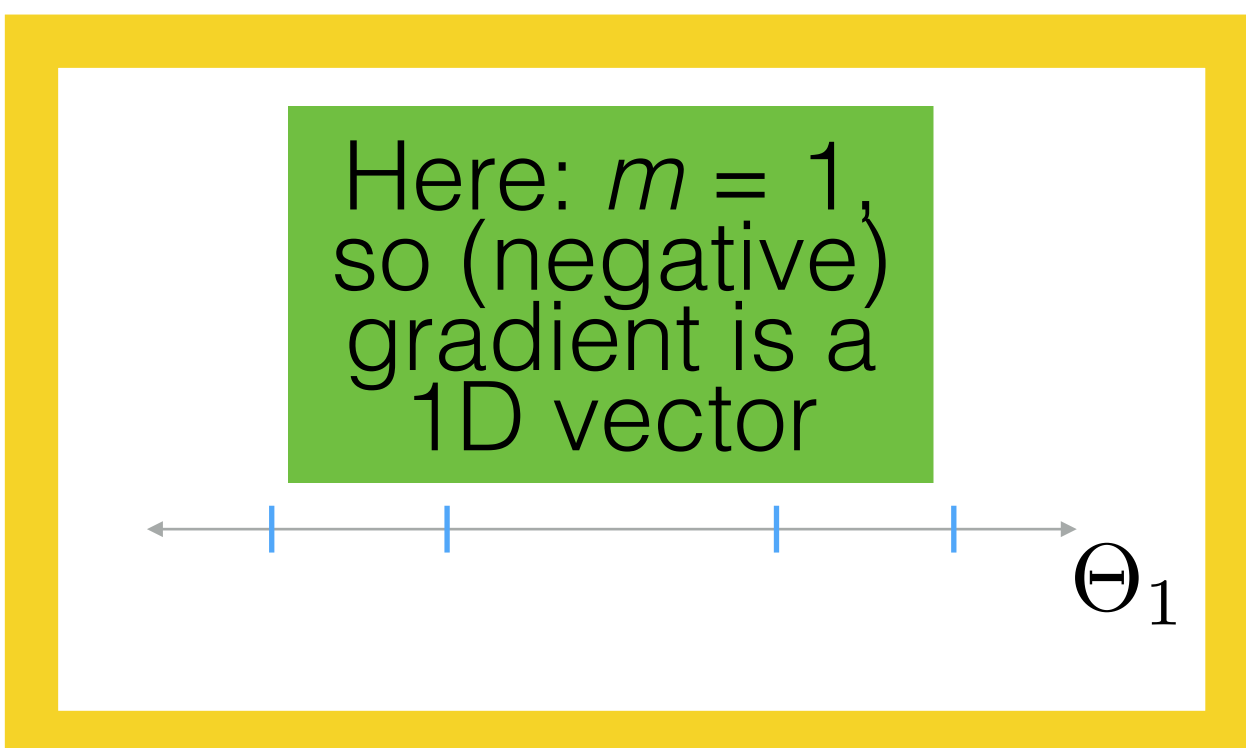

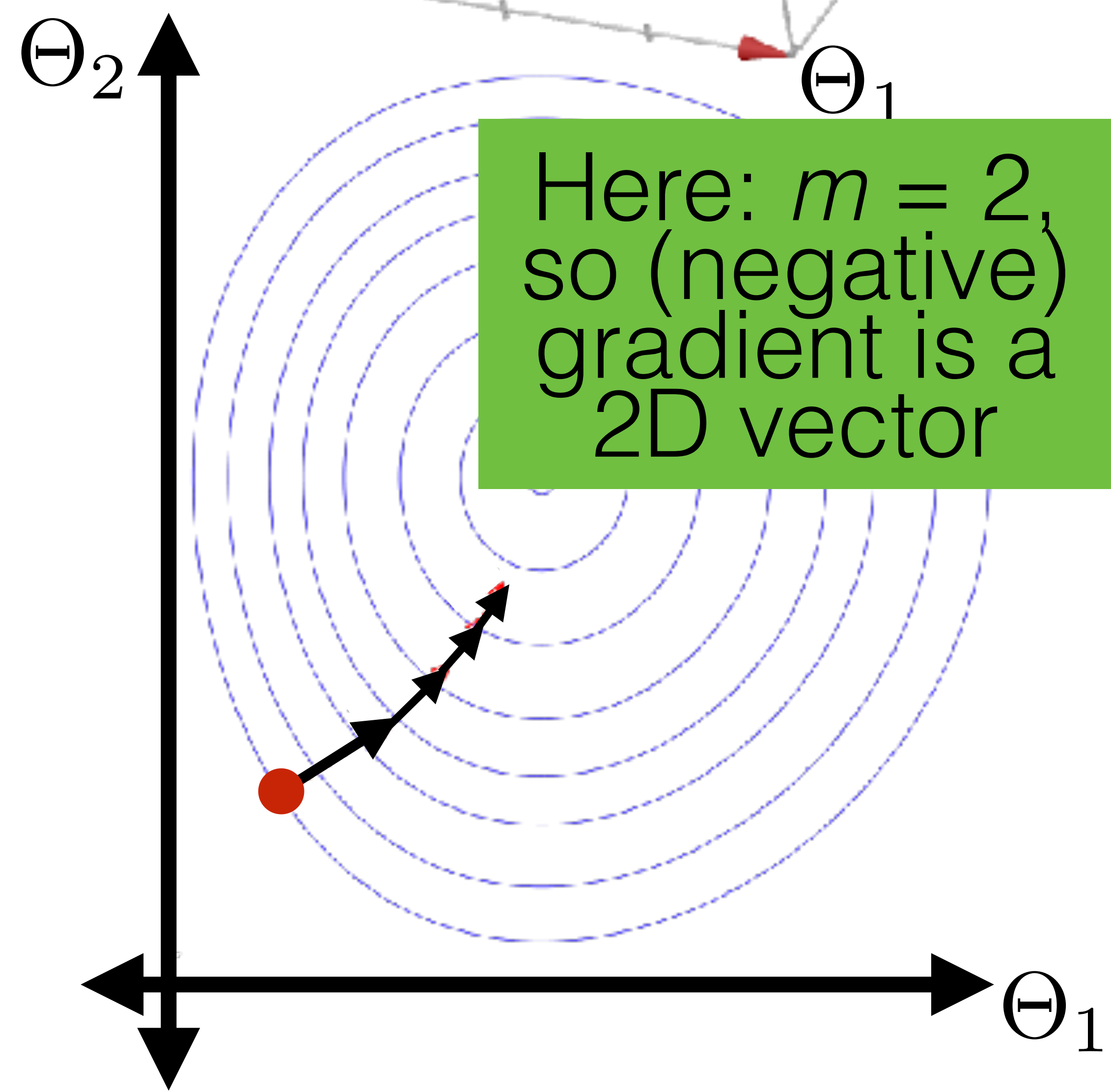

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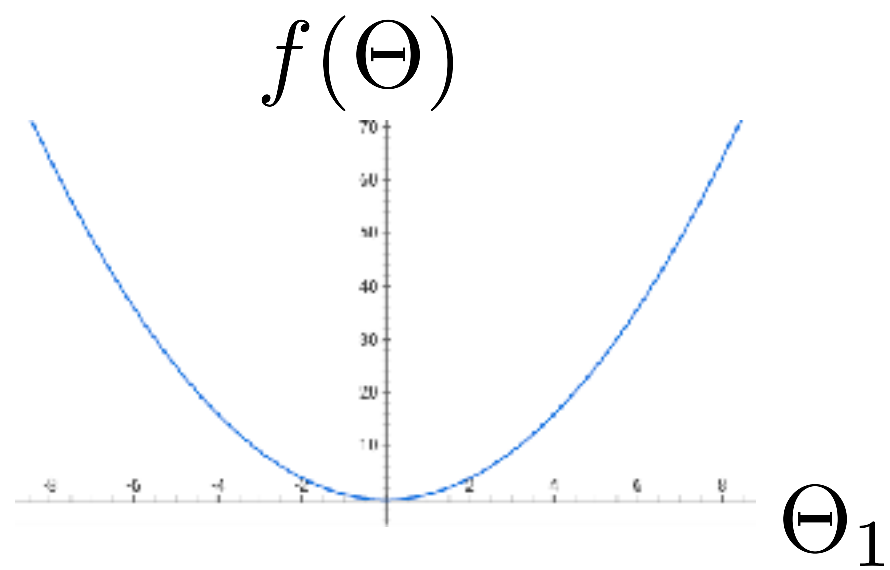

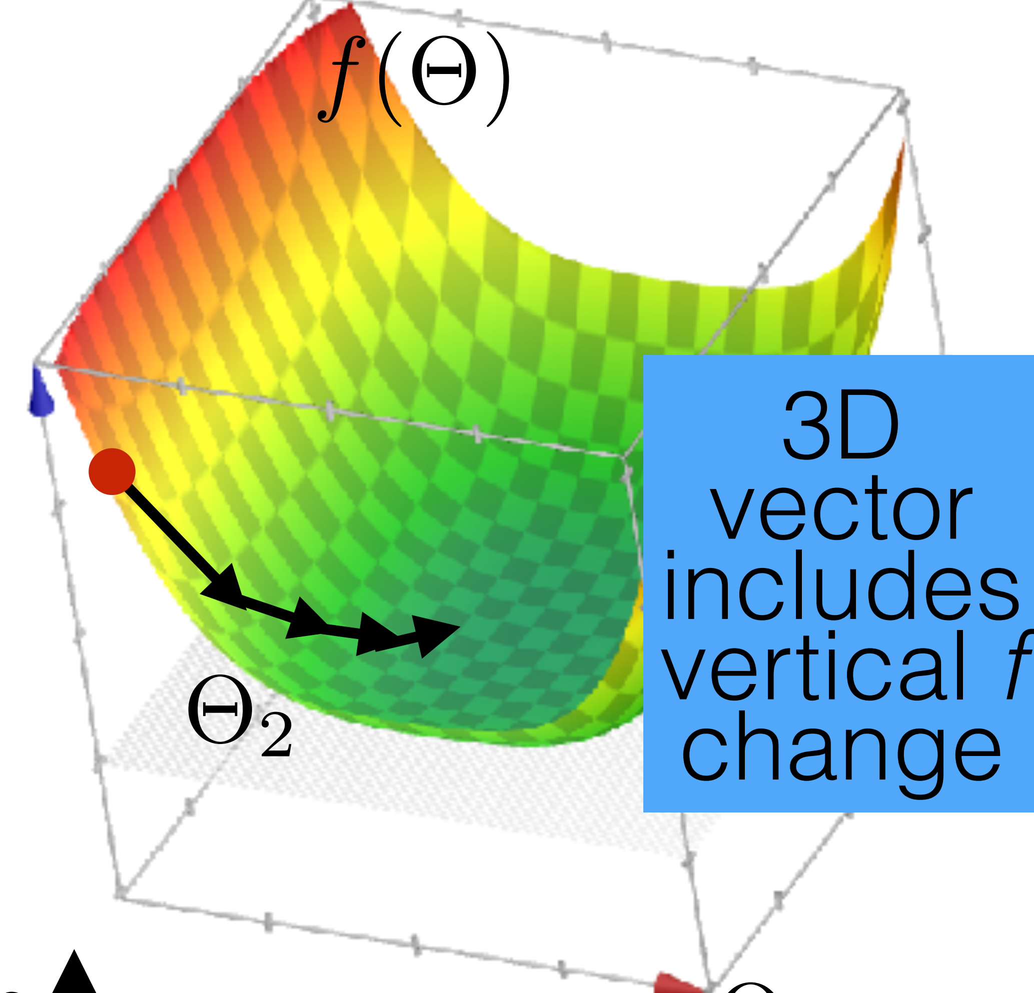

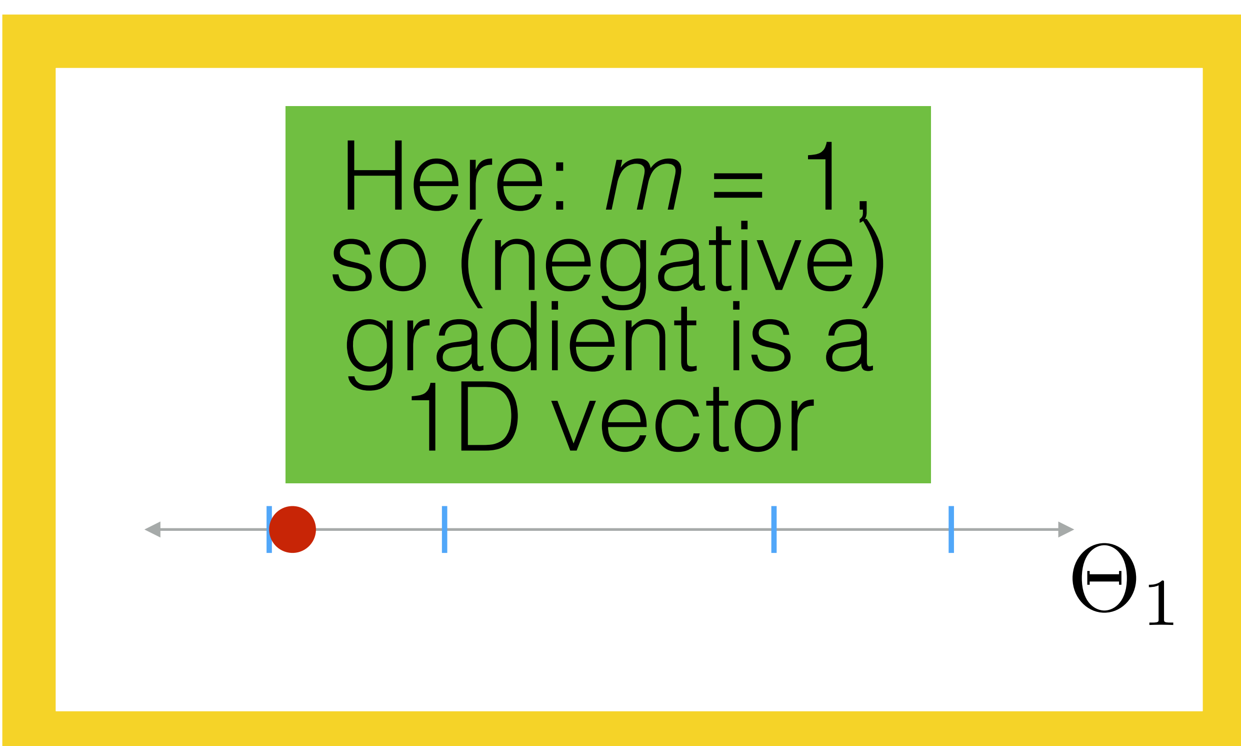

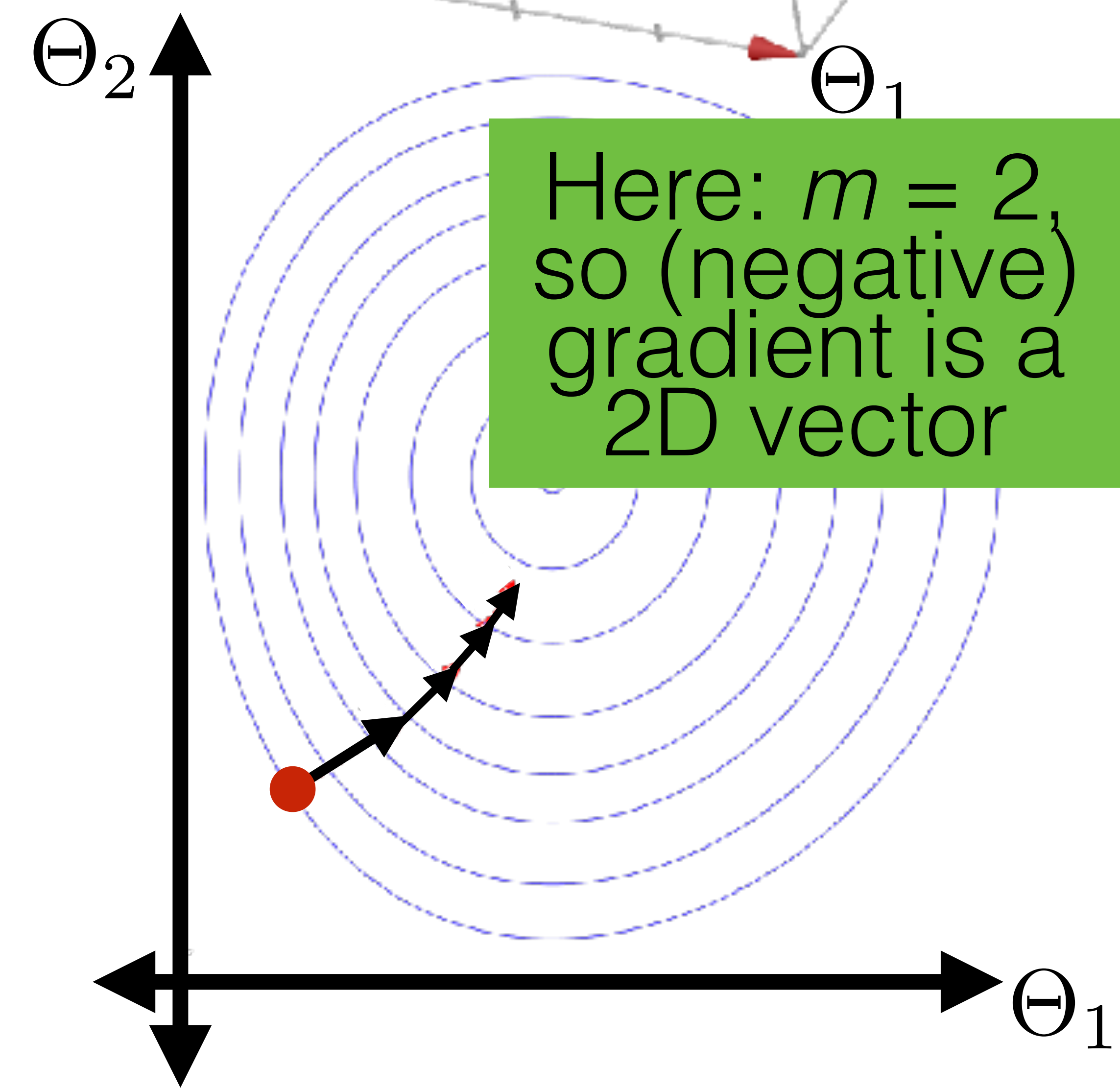

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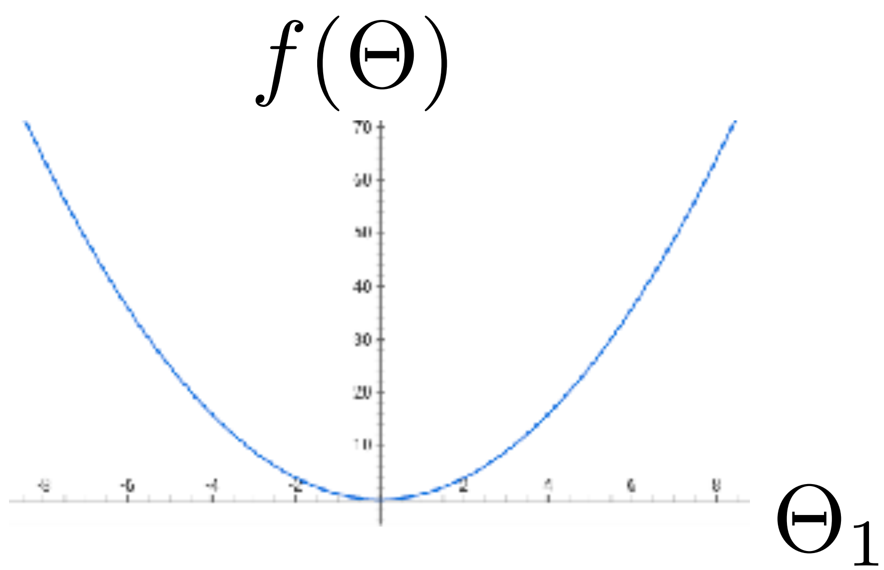

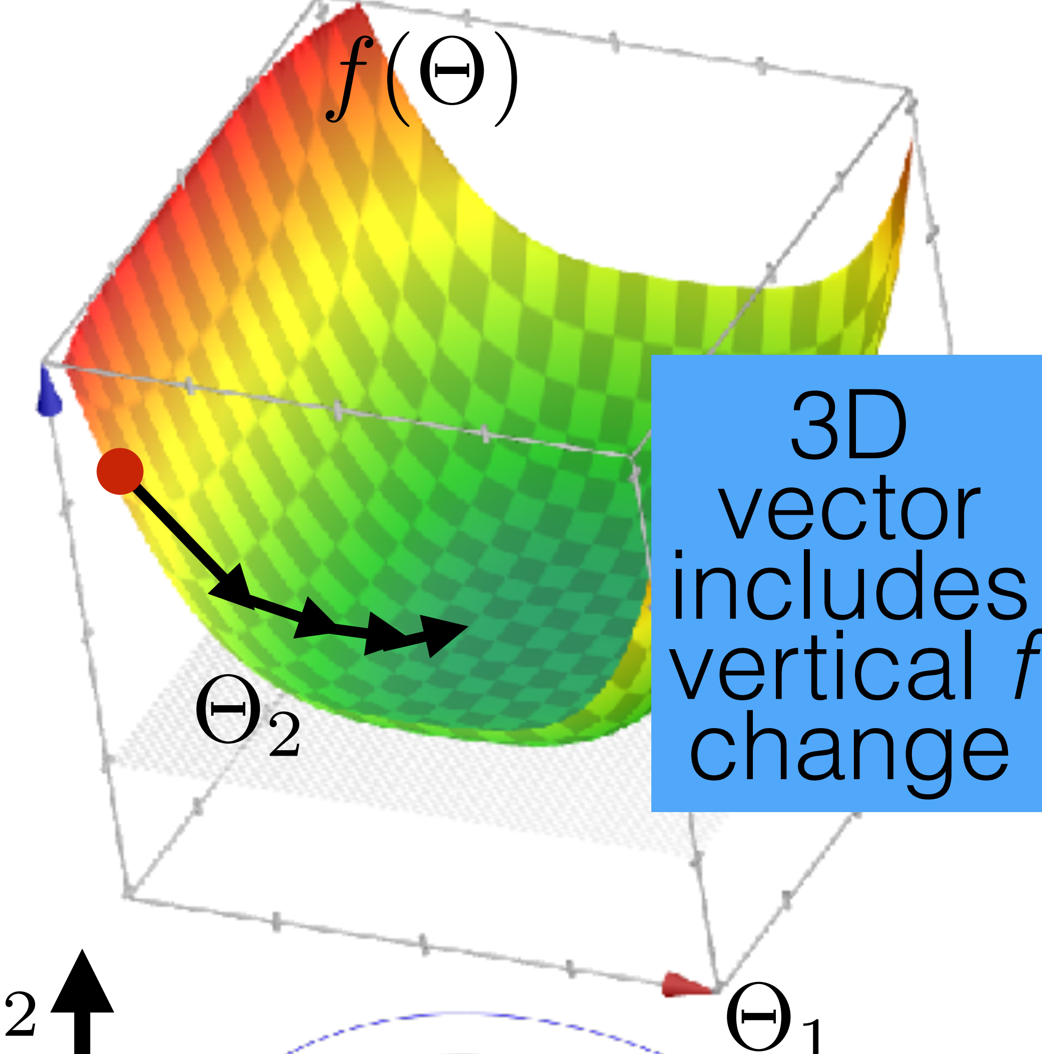

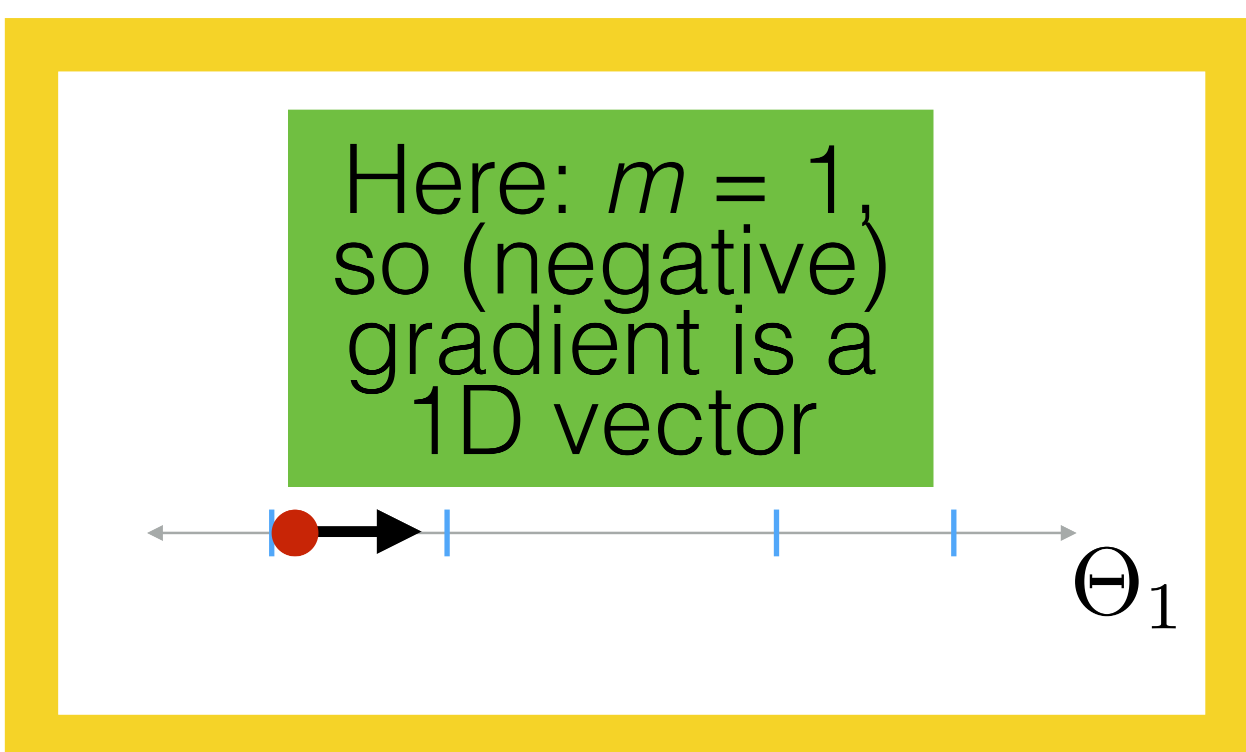

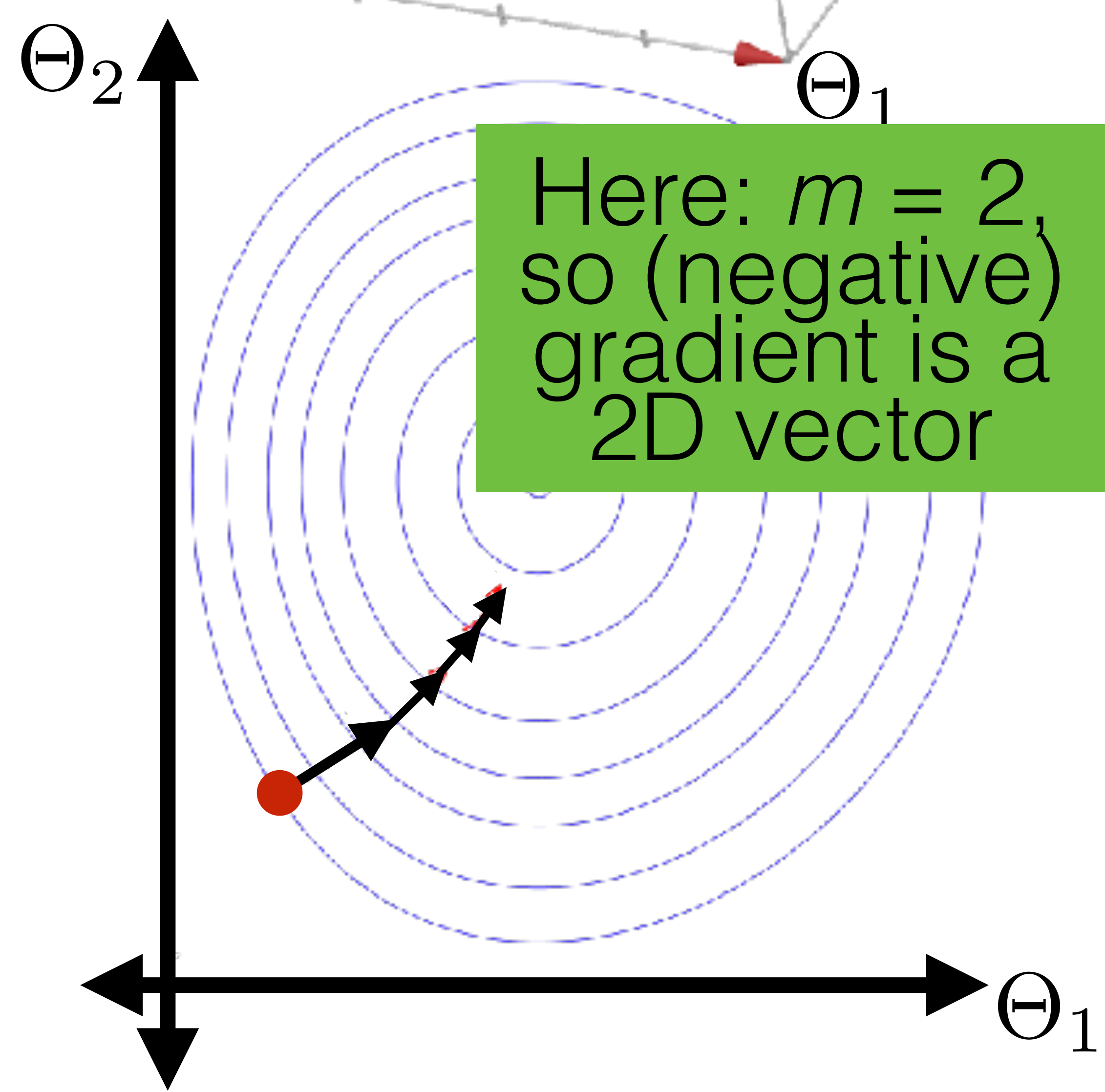

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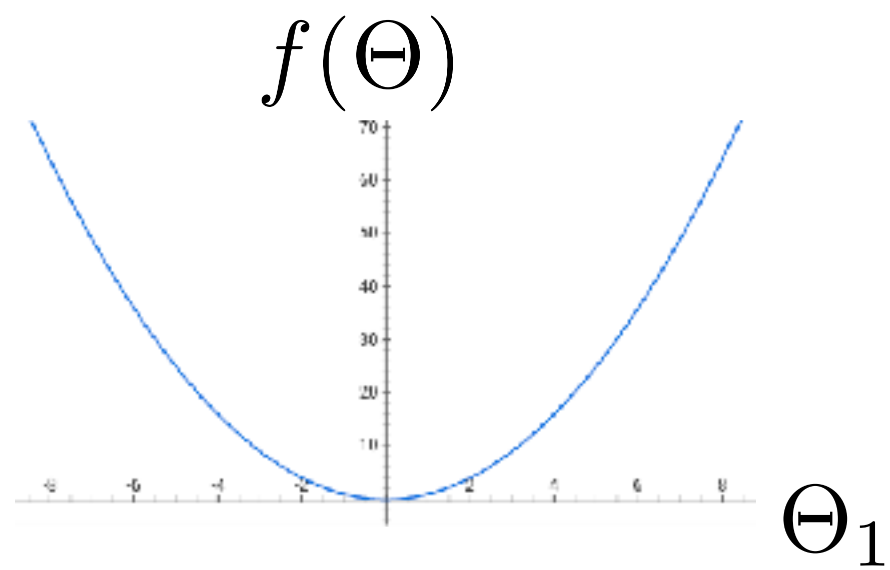

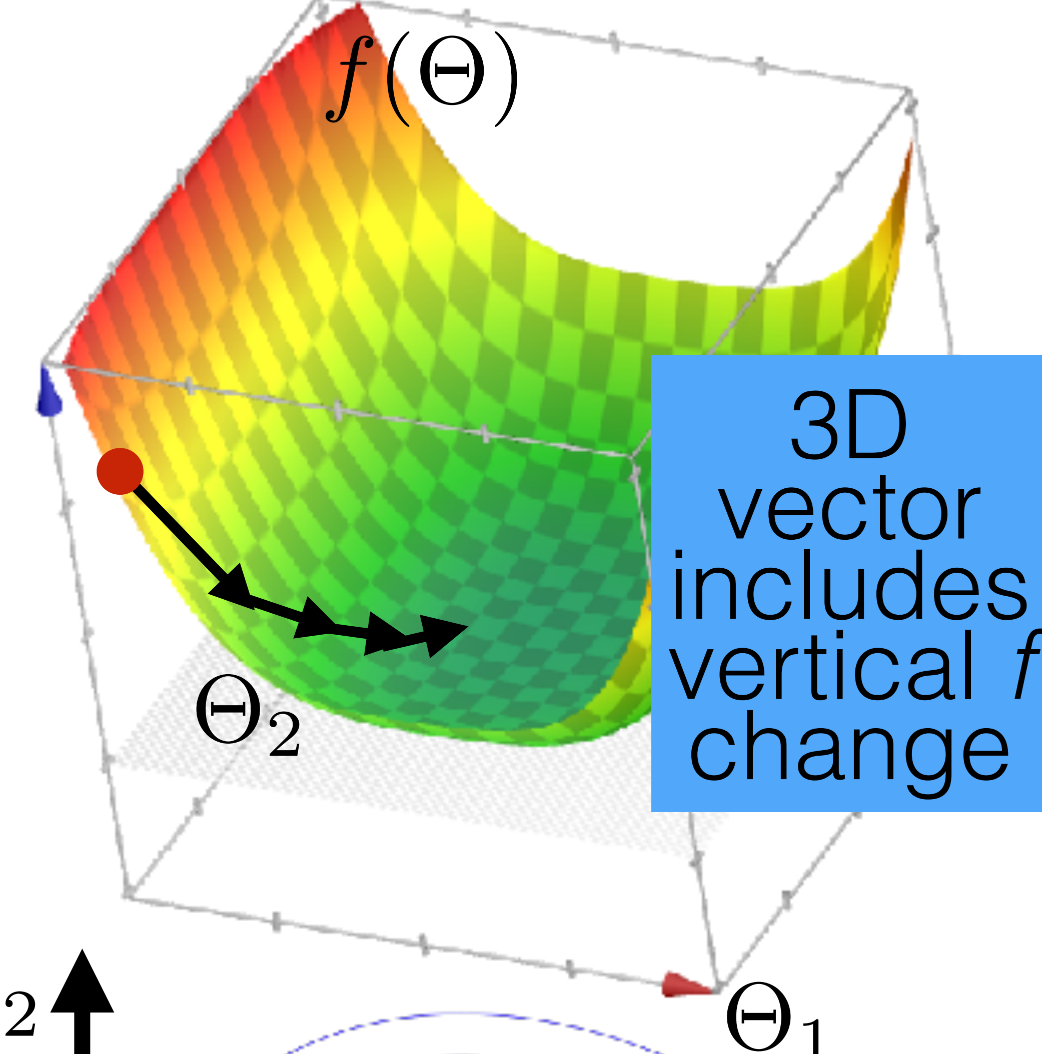

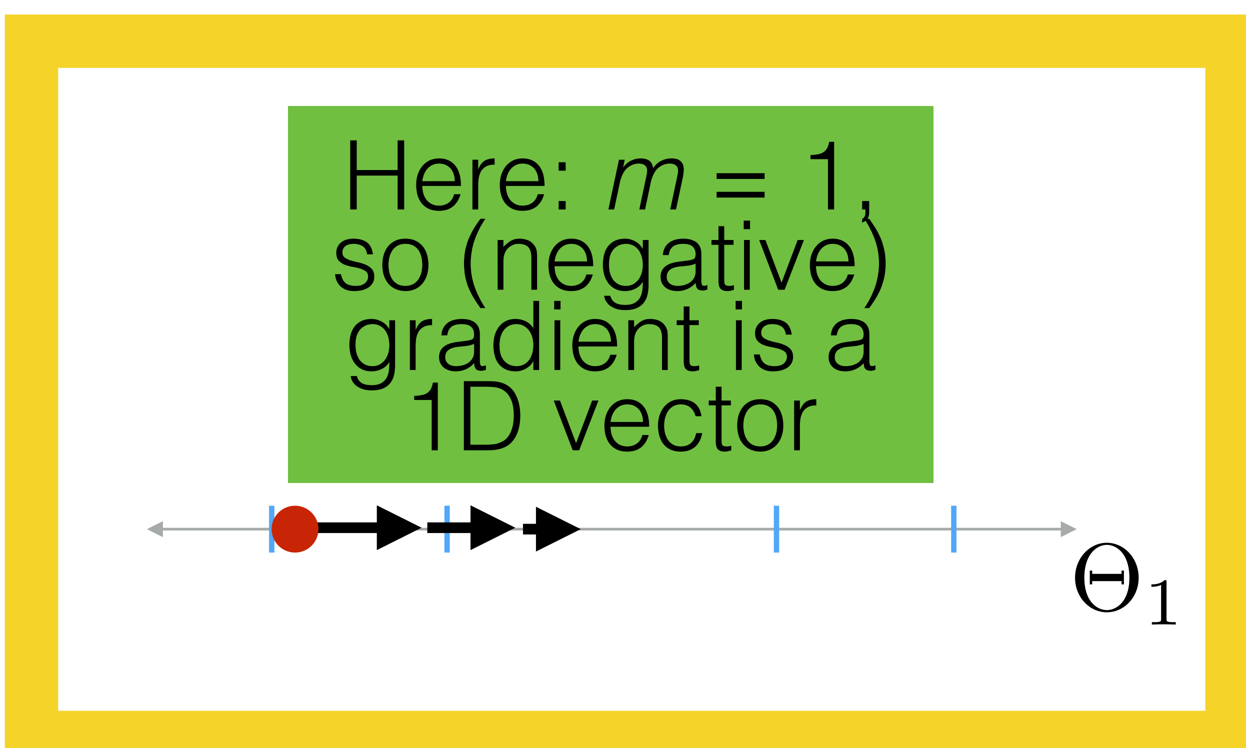

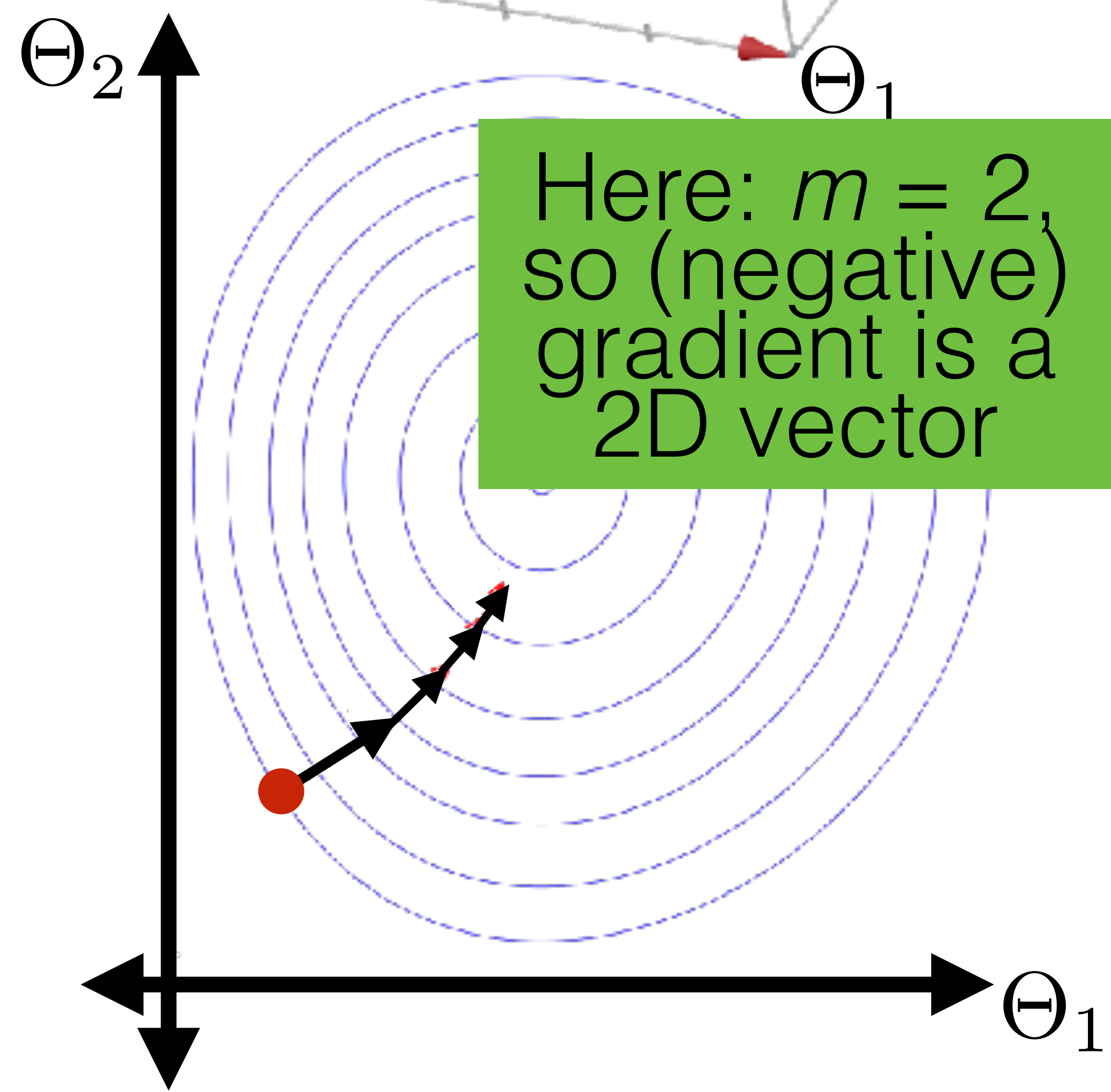

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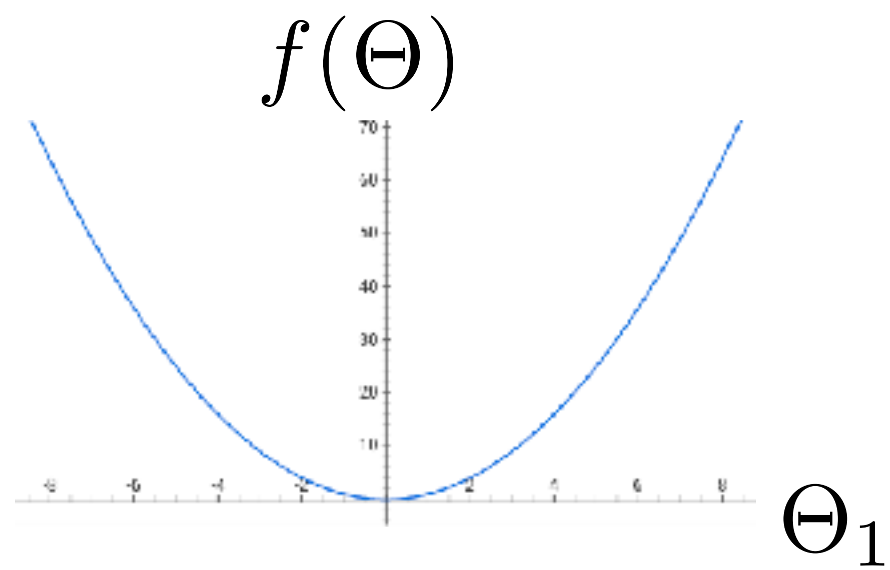
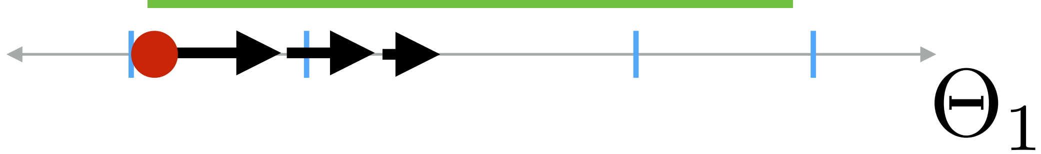
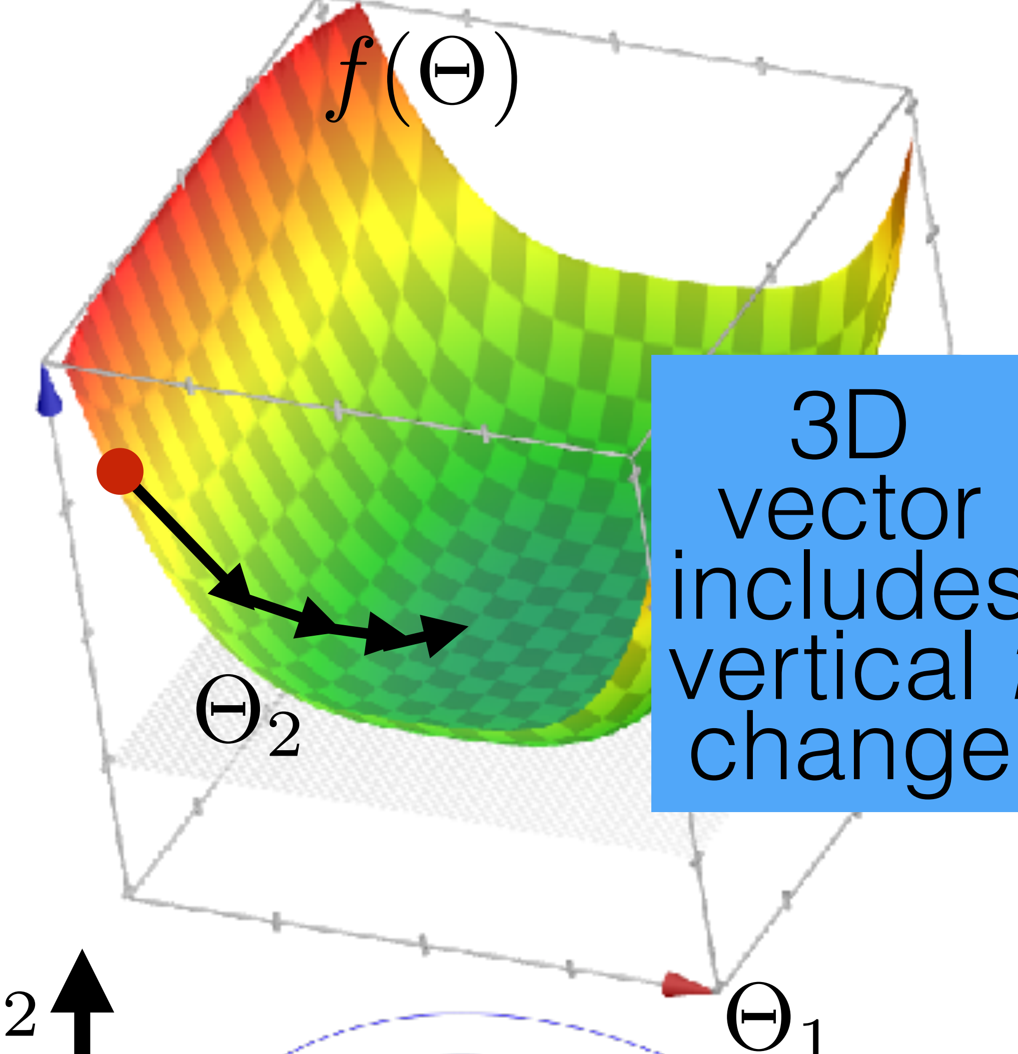
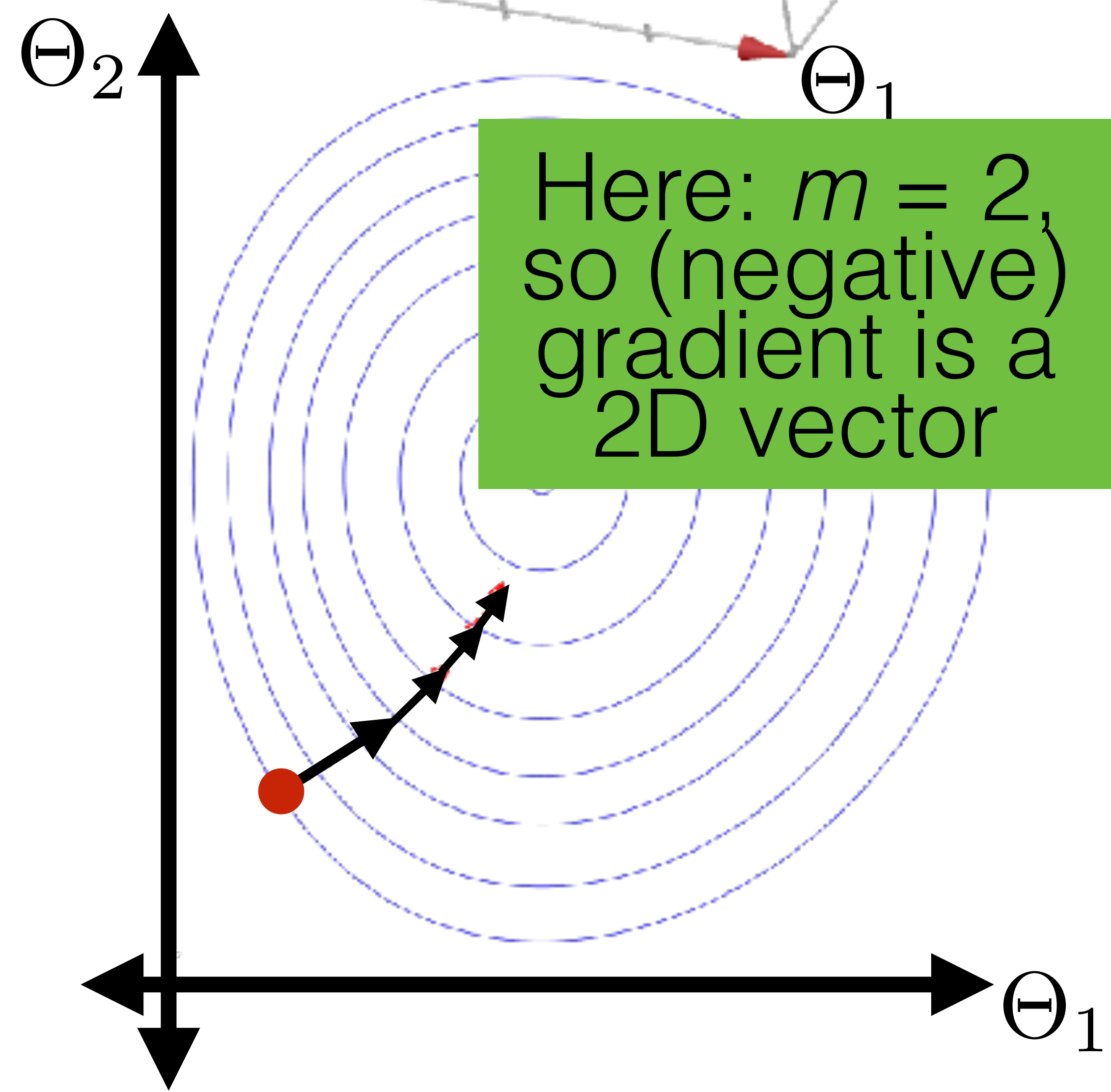


Here: m = 1, so (negative) gradient is a 1D vector

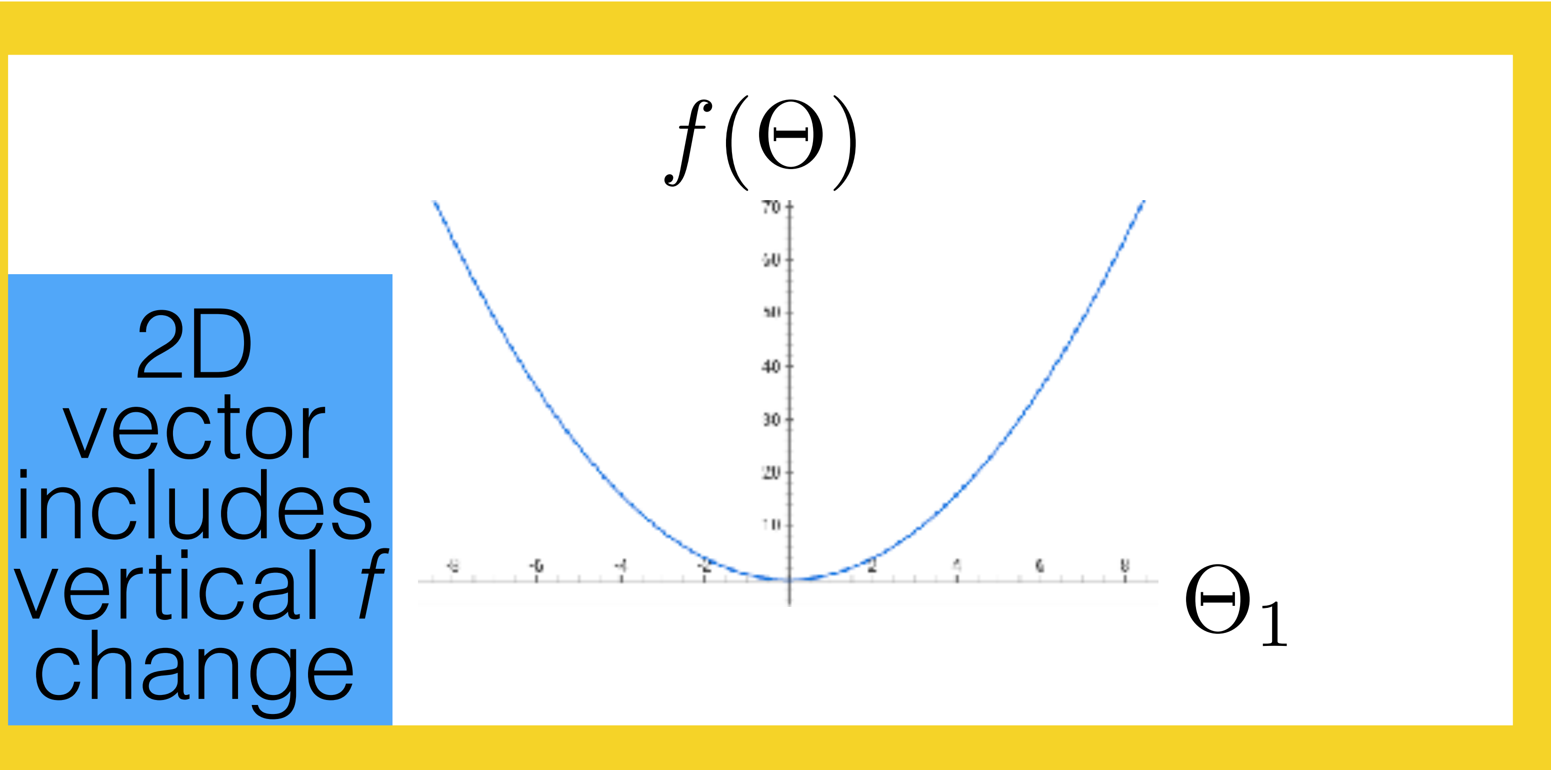


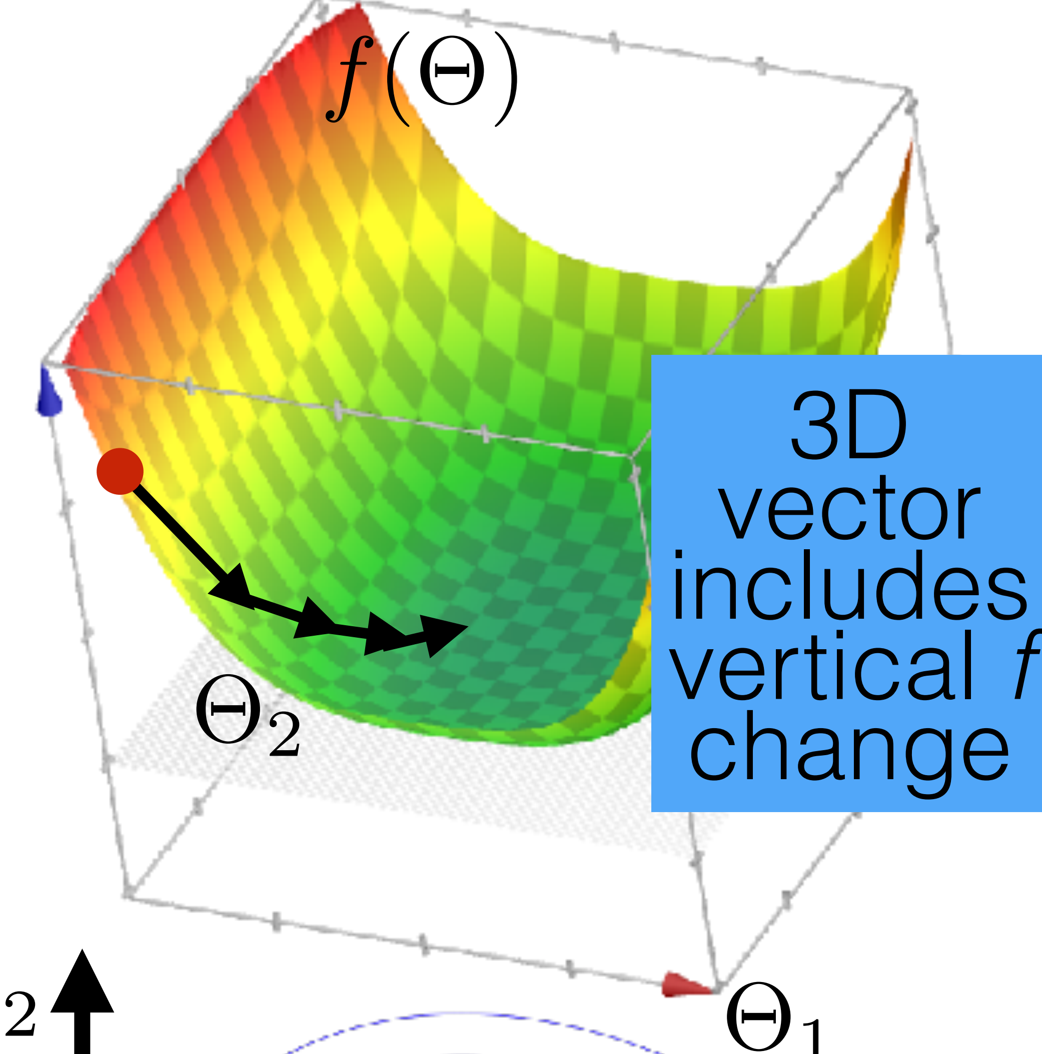




Here: m = 2, so (negative) gradient is a 2D vector

Gradient descent





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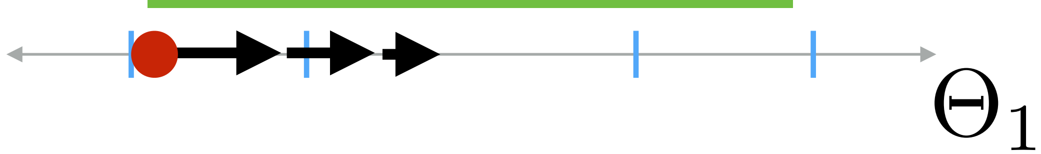

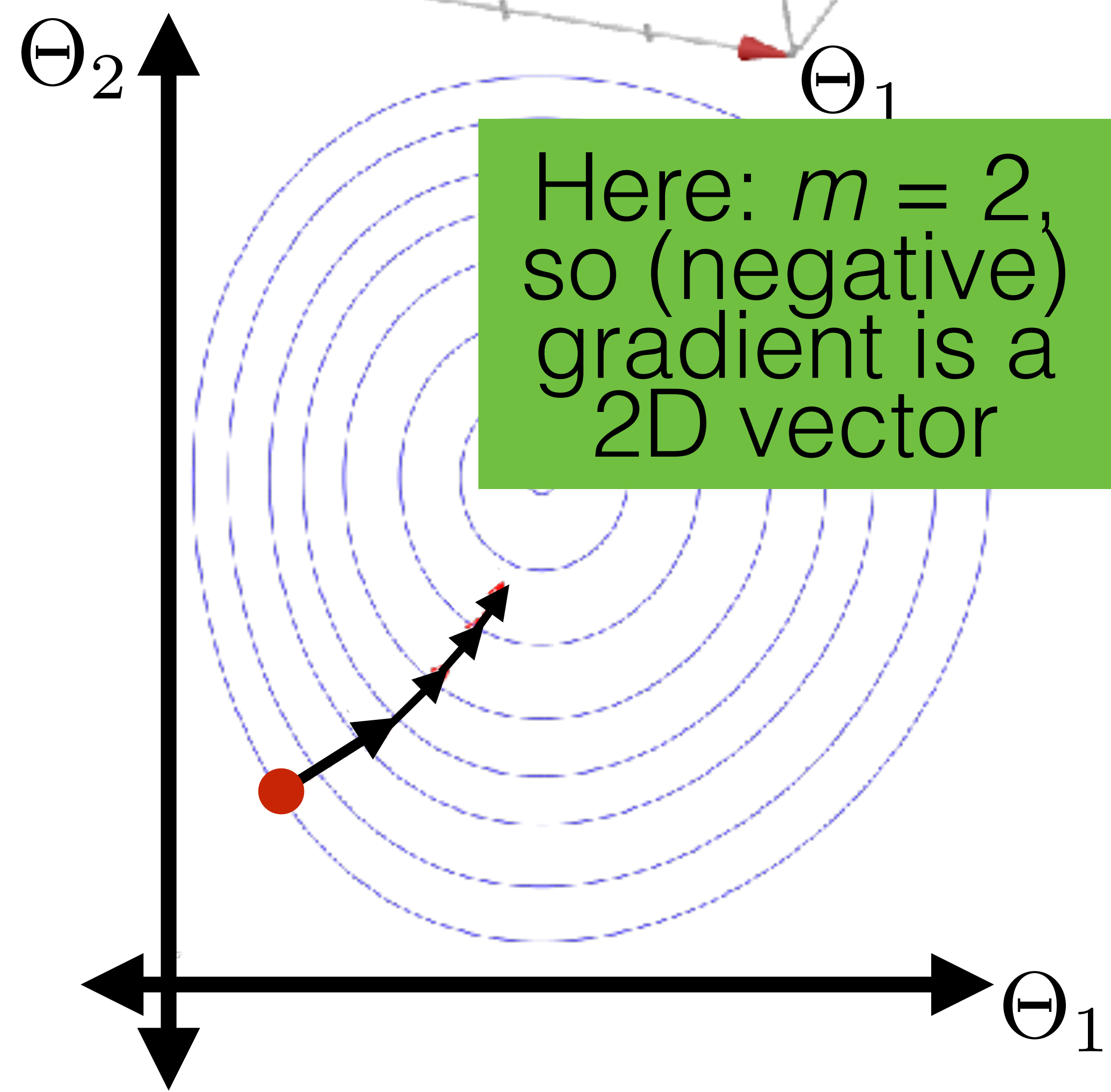

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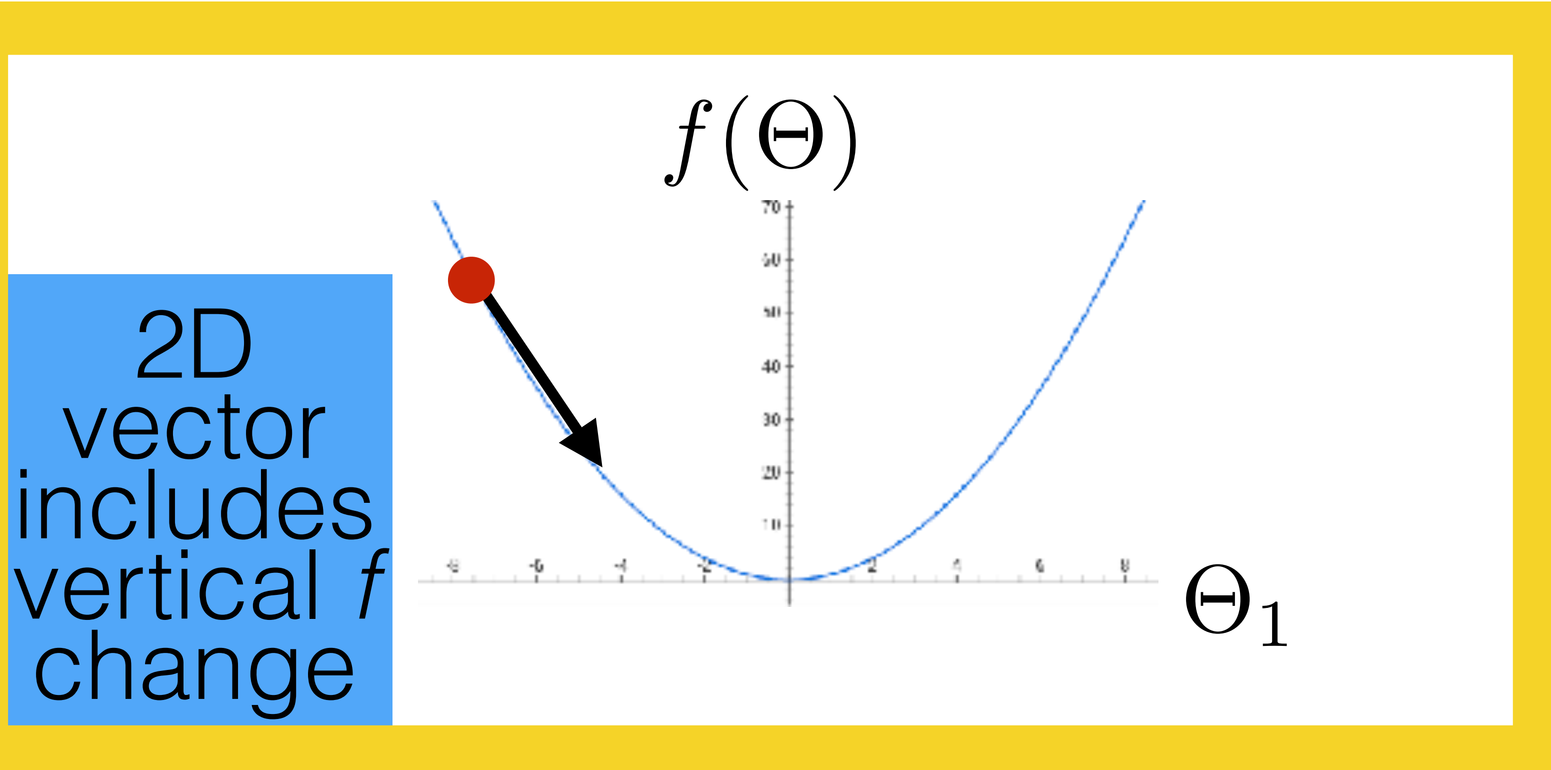

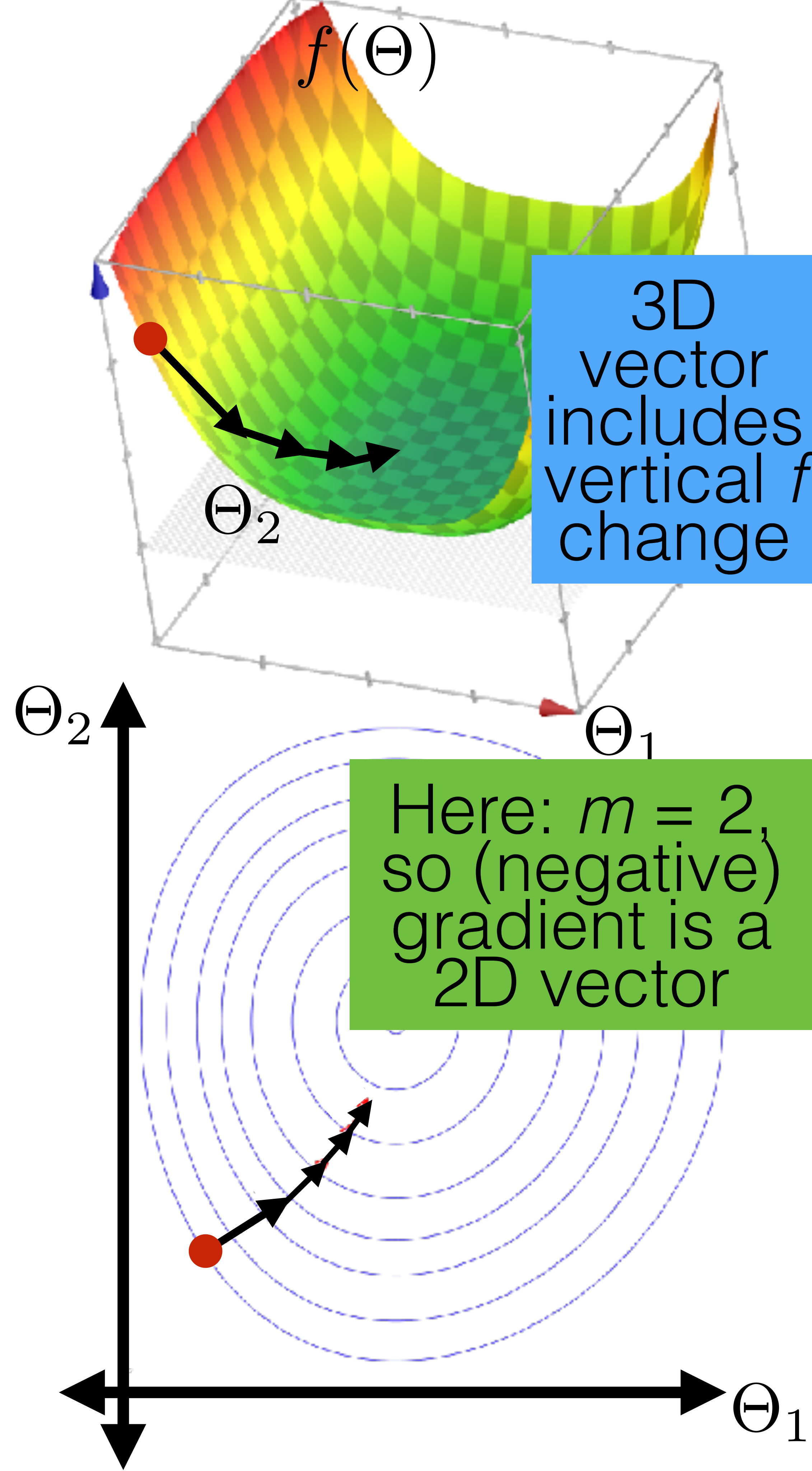

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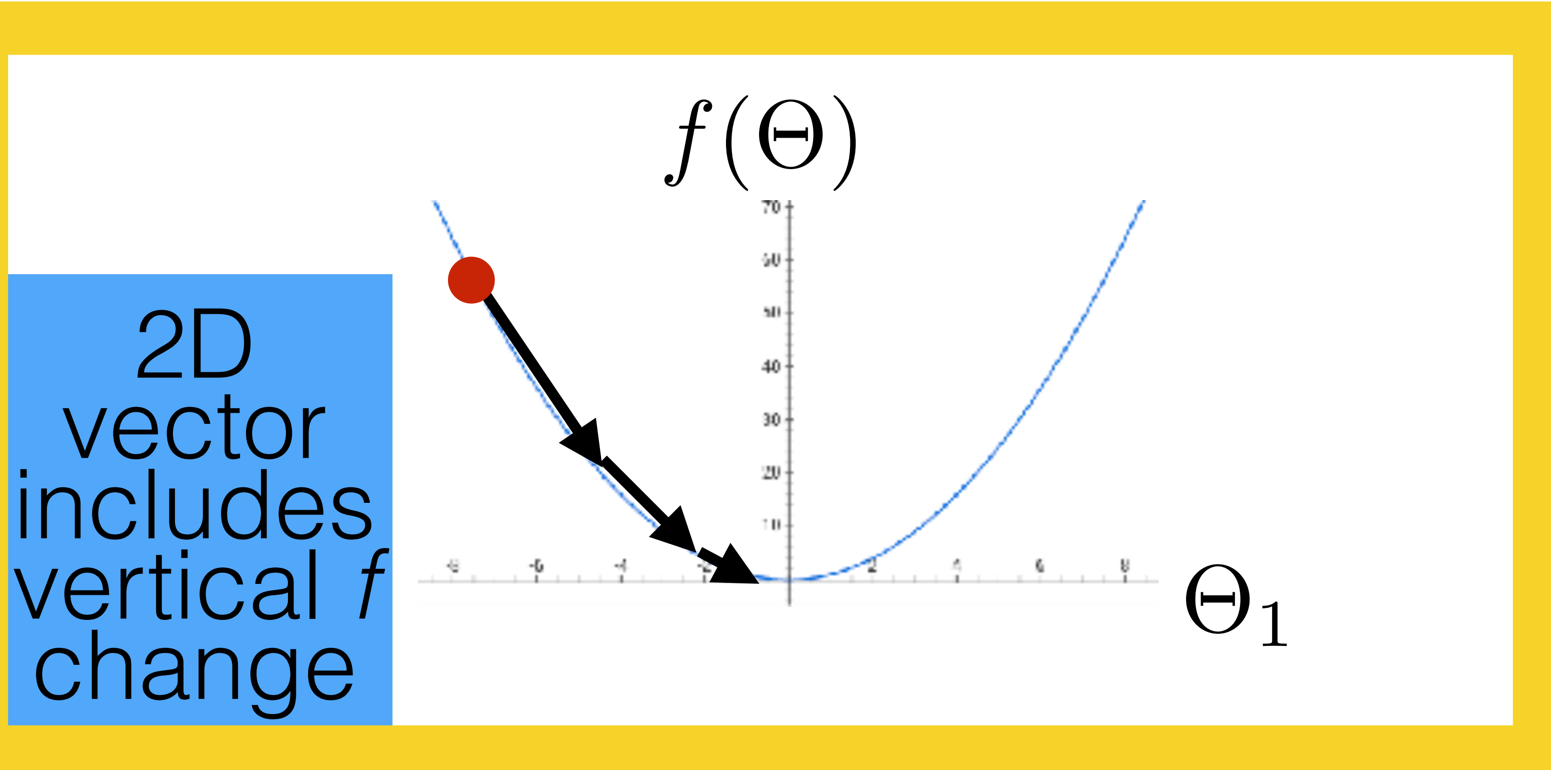
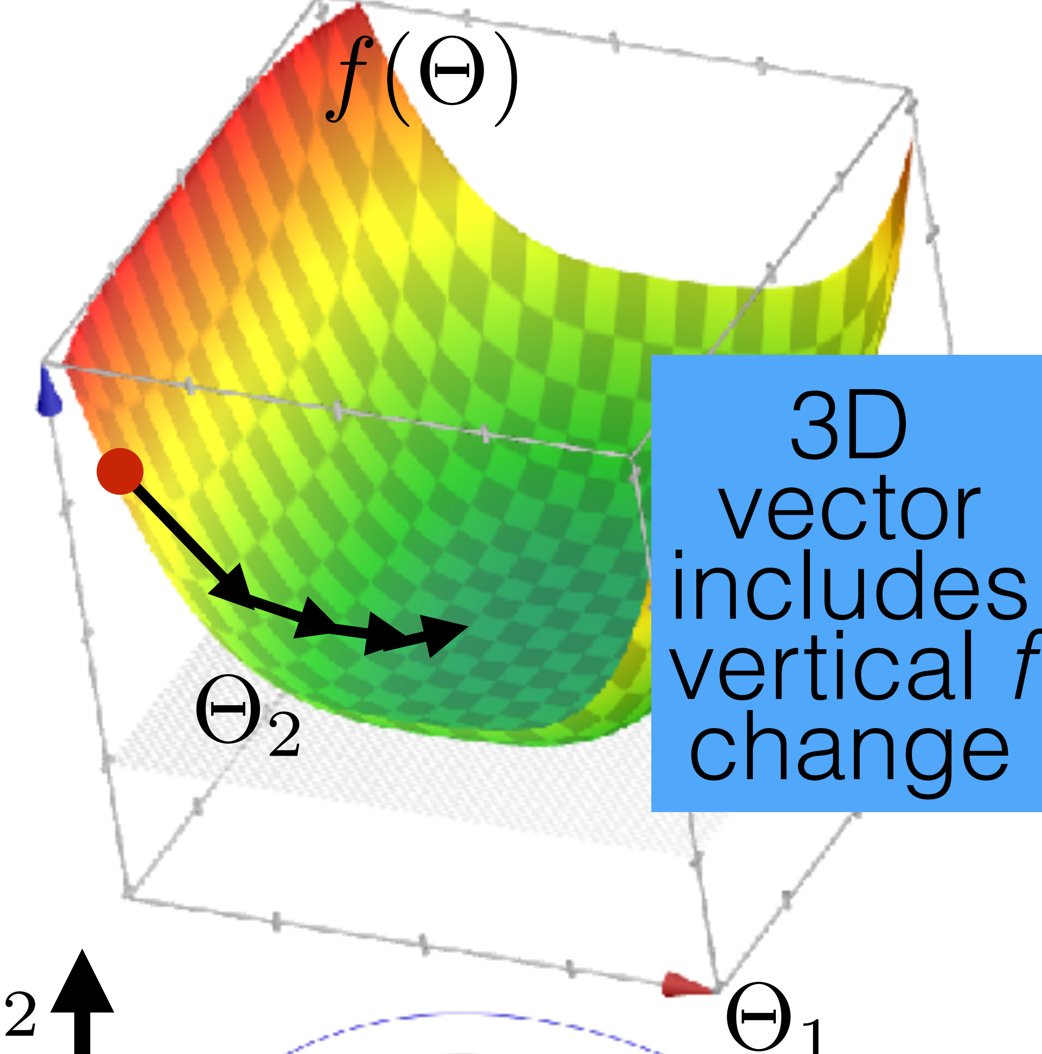
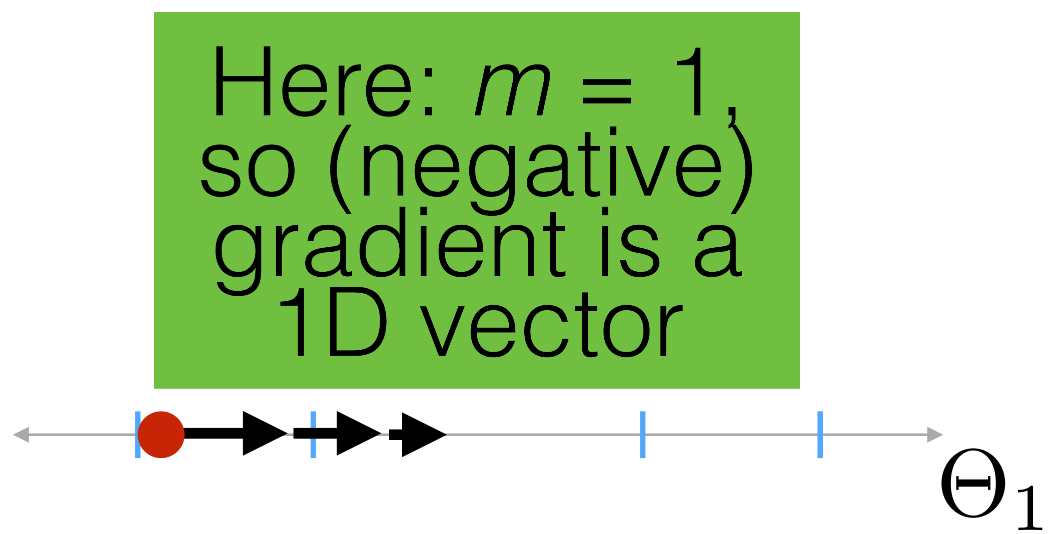
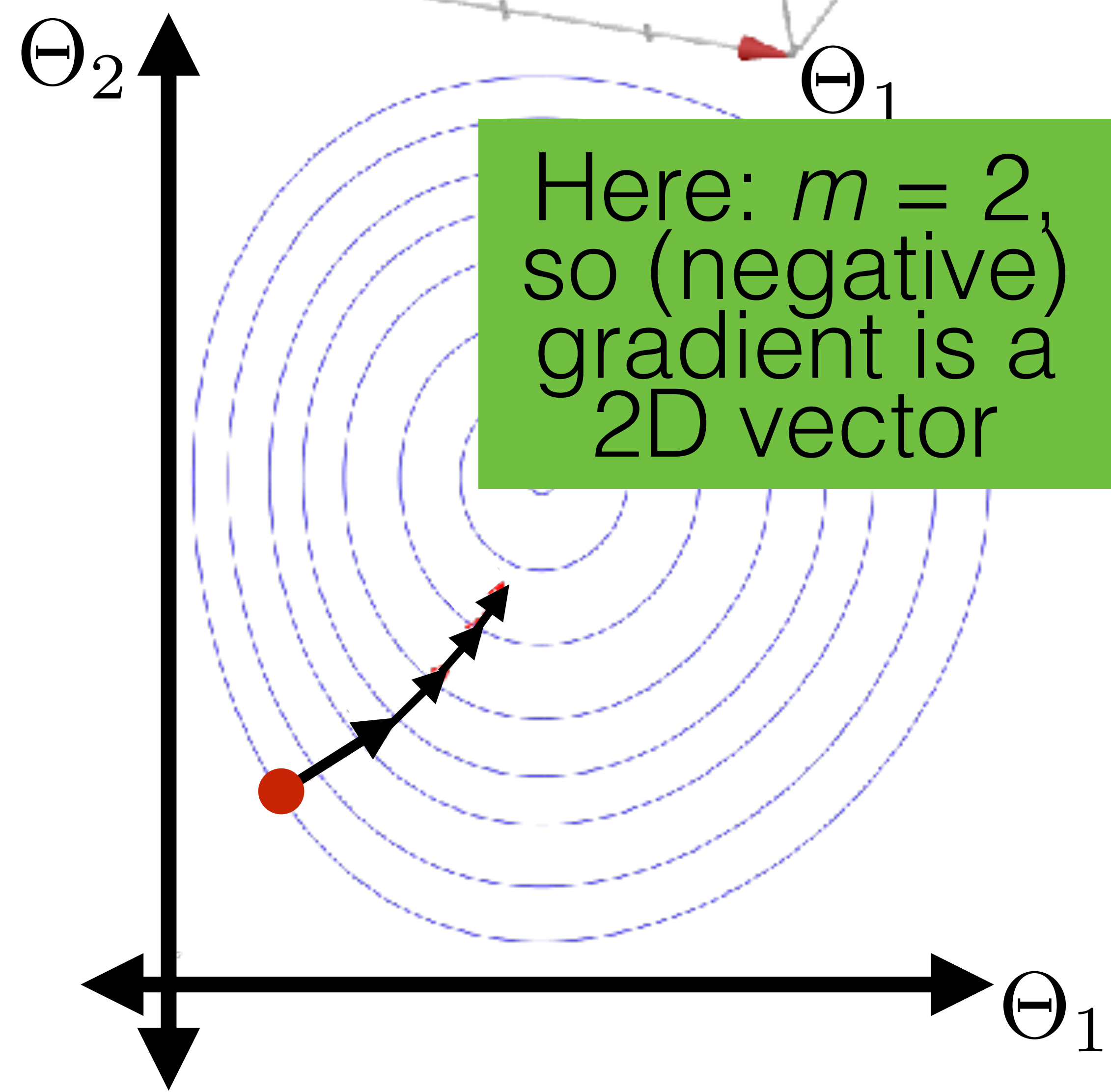
Gradient descent









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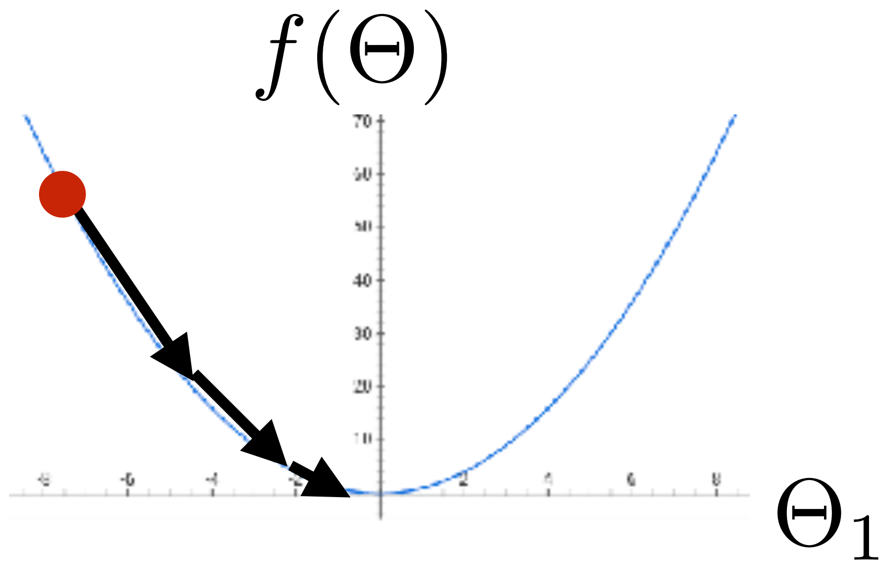
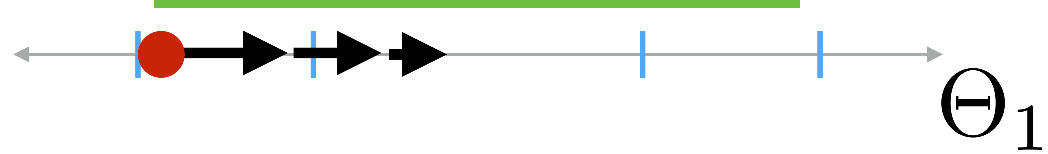
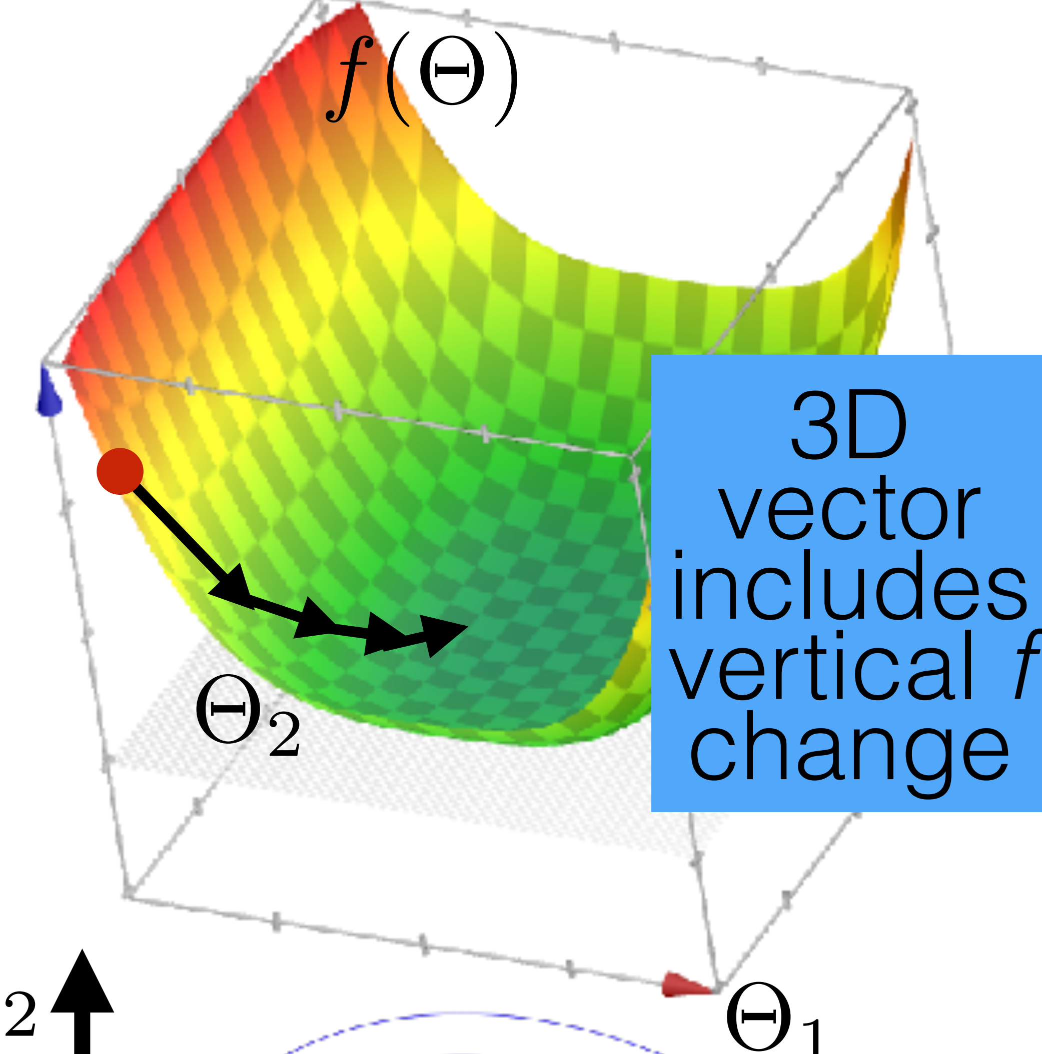


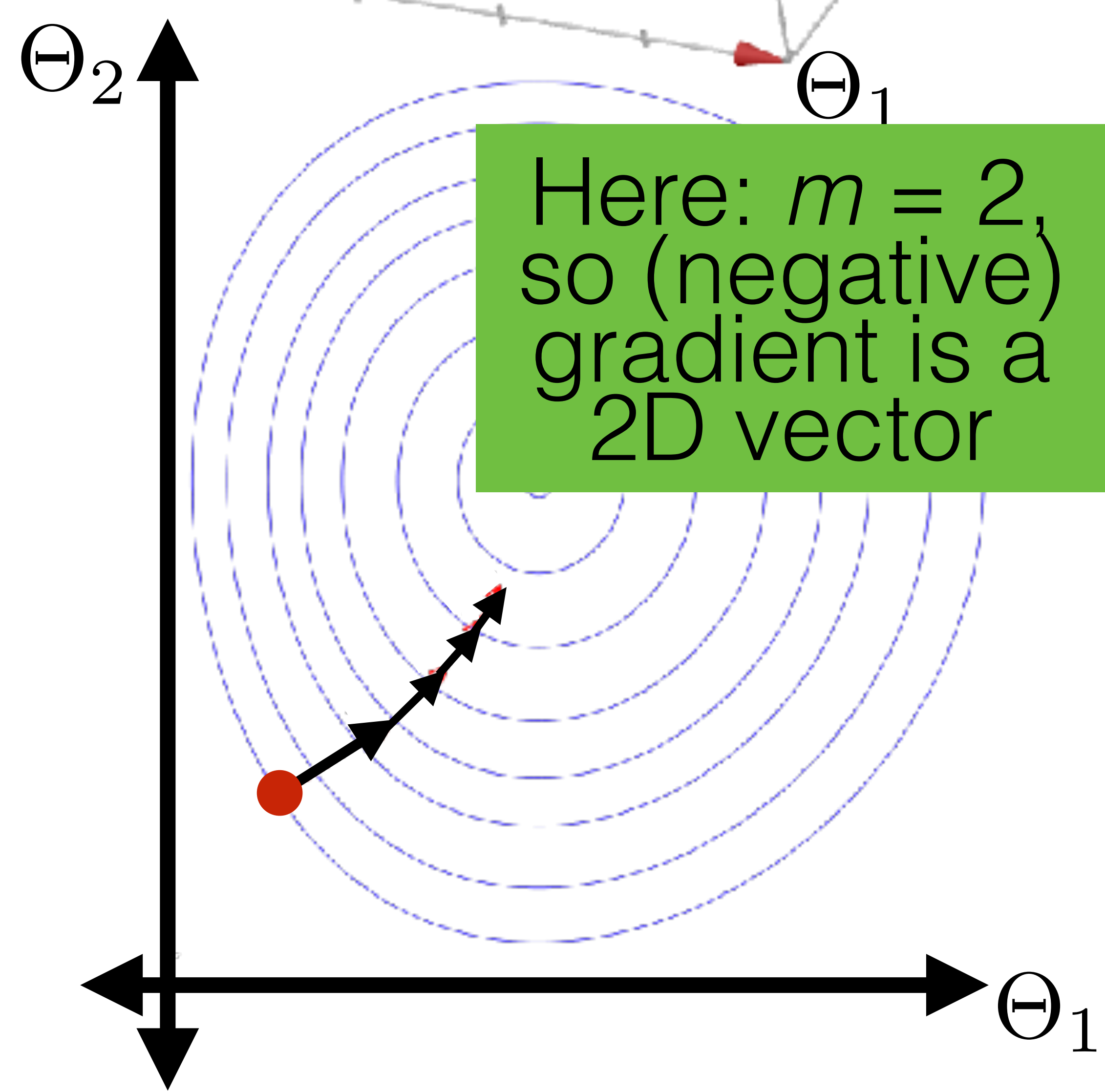
2D vector includes vertical *f* change

Here: *m* = 1, so (negative) gradient is a 1D vector





3D vector includes vertical *f* change



Here: *m* = 2, so (negative) gradient is a 2D vector

Gradient descent properties

Gradient descent properties

- A function *f* on $\mathbb{R}^m$ is convex if any line segment connecting two points of the graph of *f* lies above or on the graph

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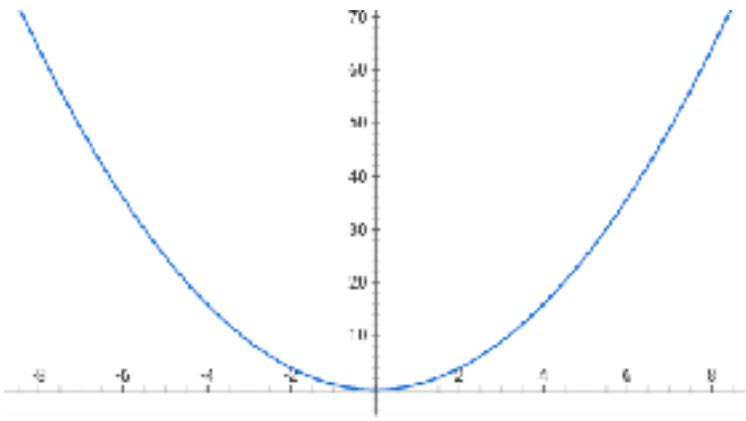

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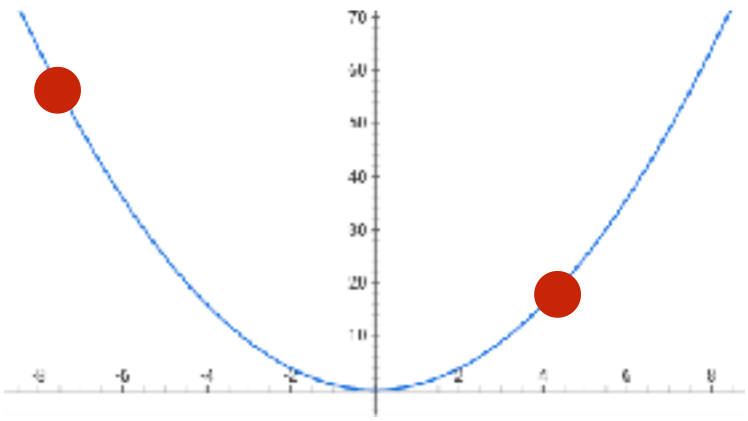

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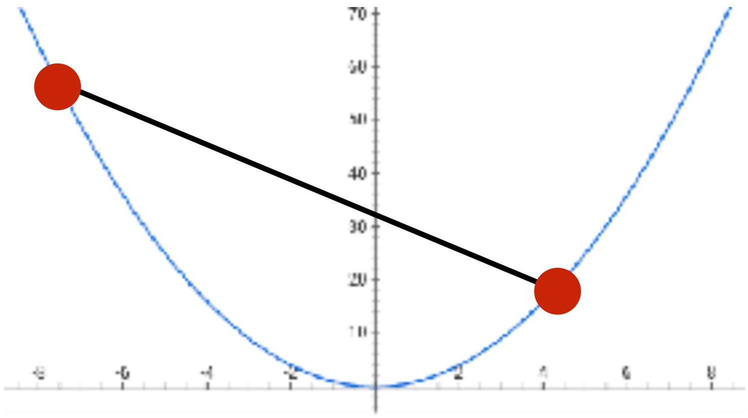

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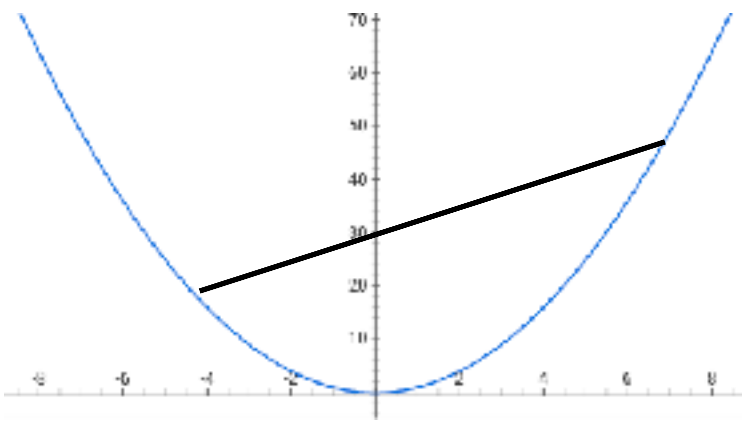

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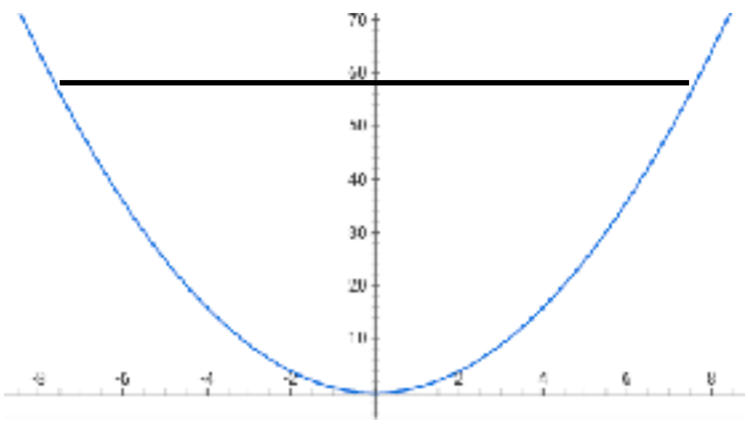

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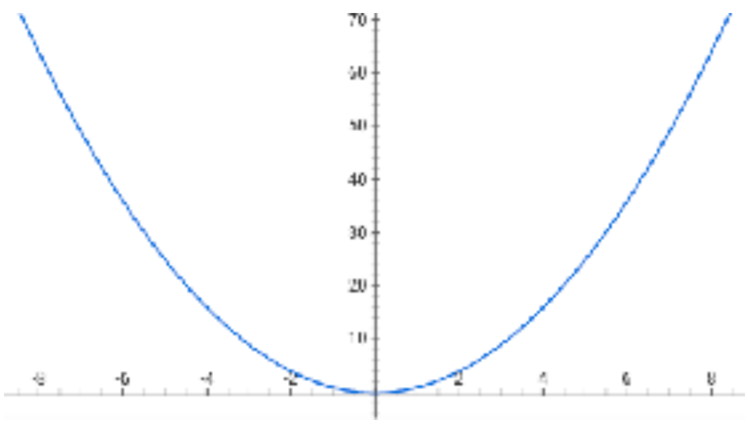

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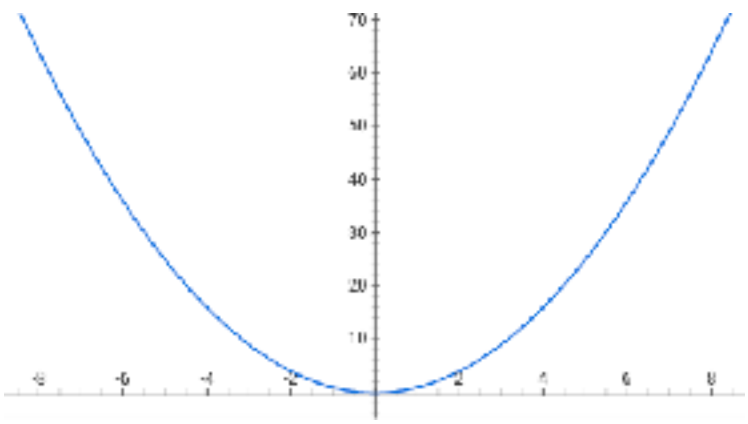

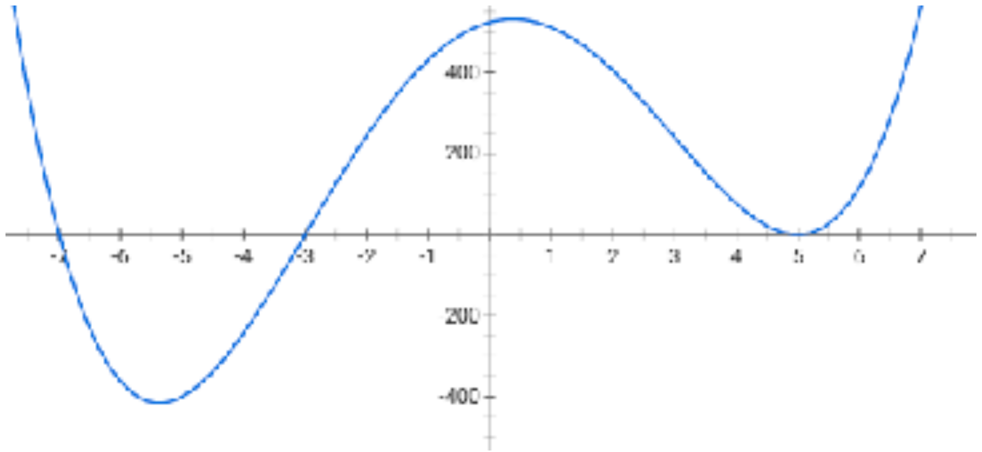

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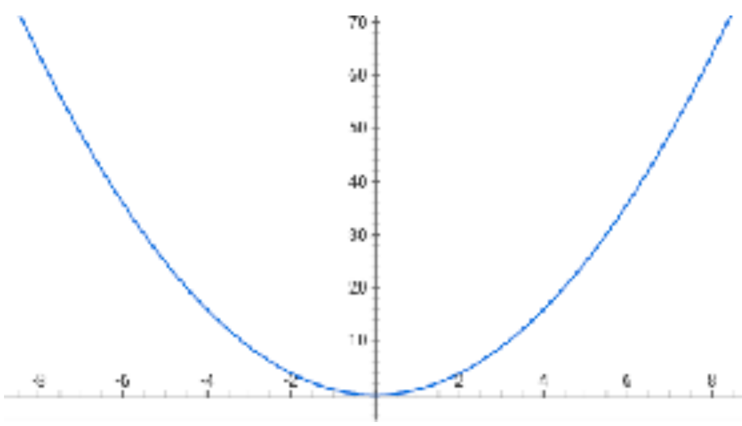
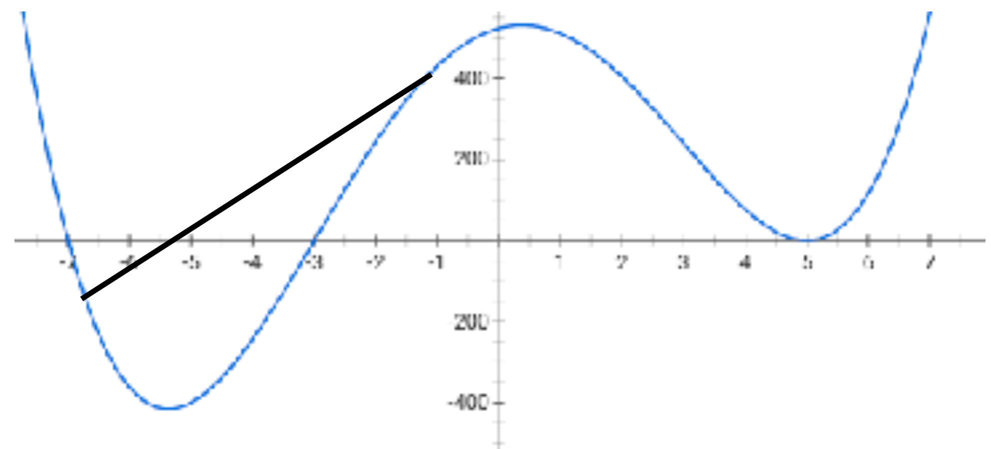
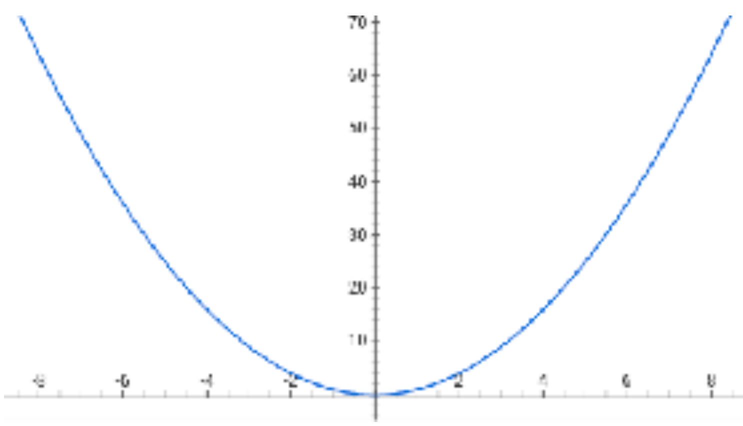
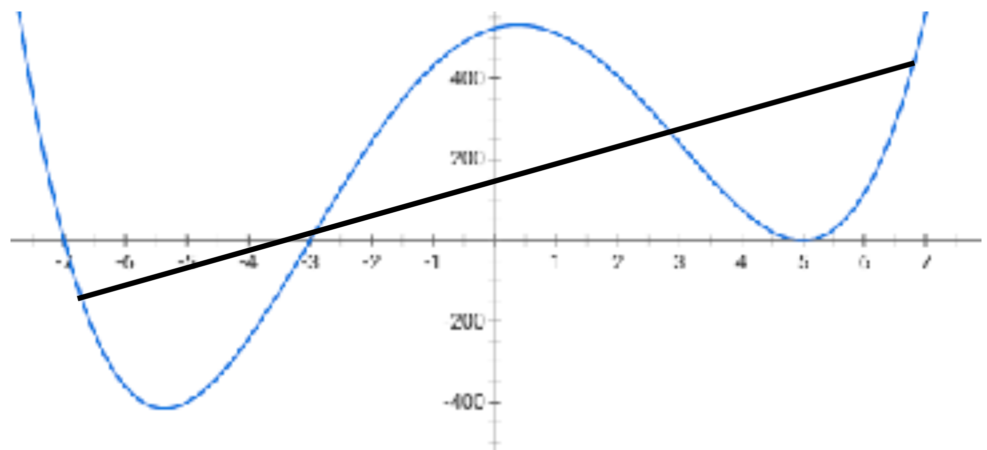




Gradient descent properties

- A function *f* on $\mathbb{R}^m$ is convex if any line segment connecting two points of the graph of *f* lies above or on the graph





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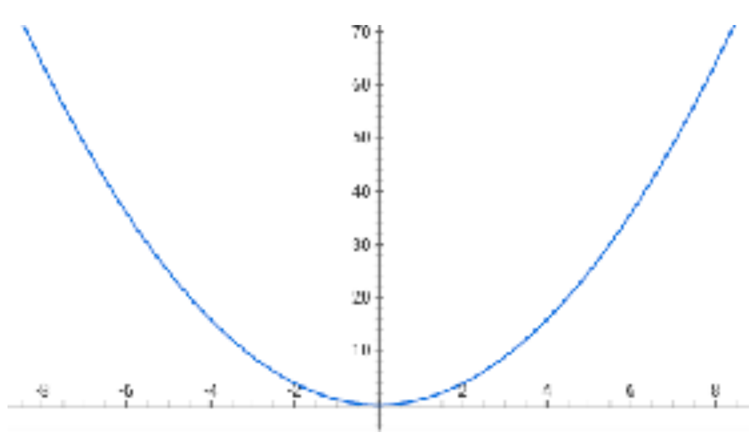

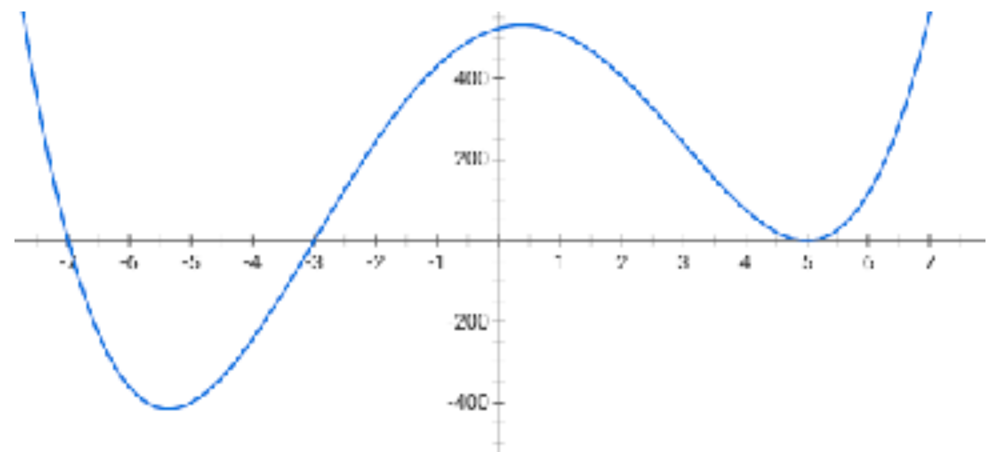

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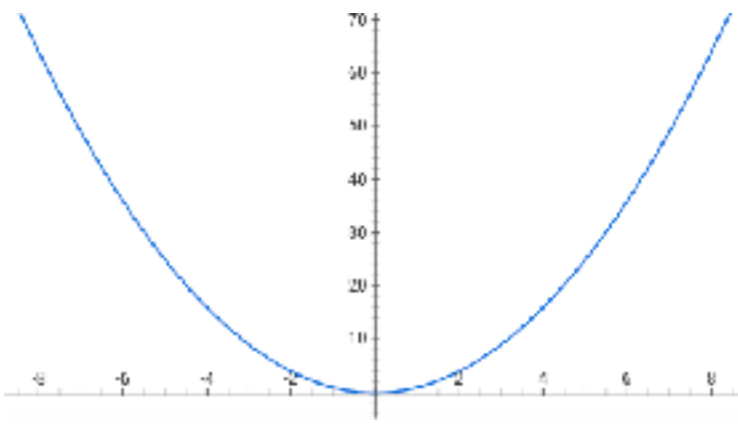

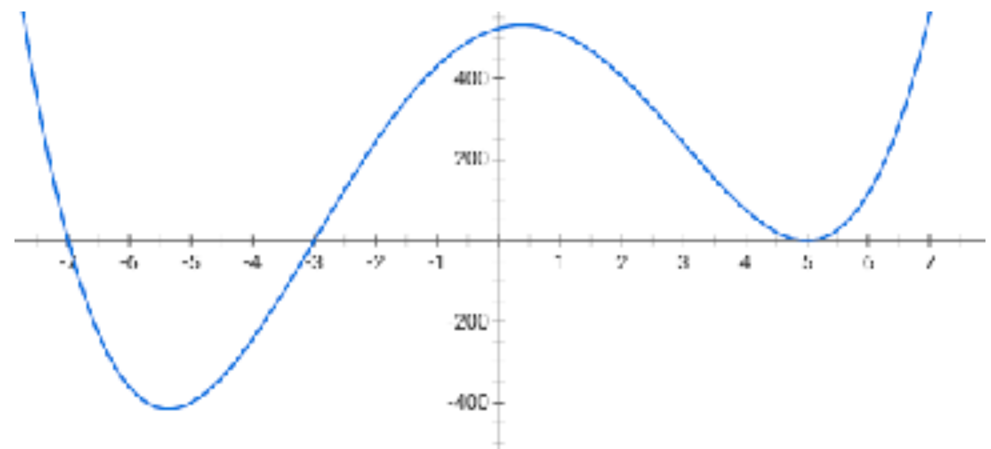

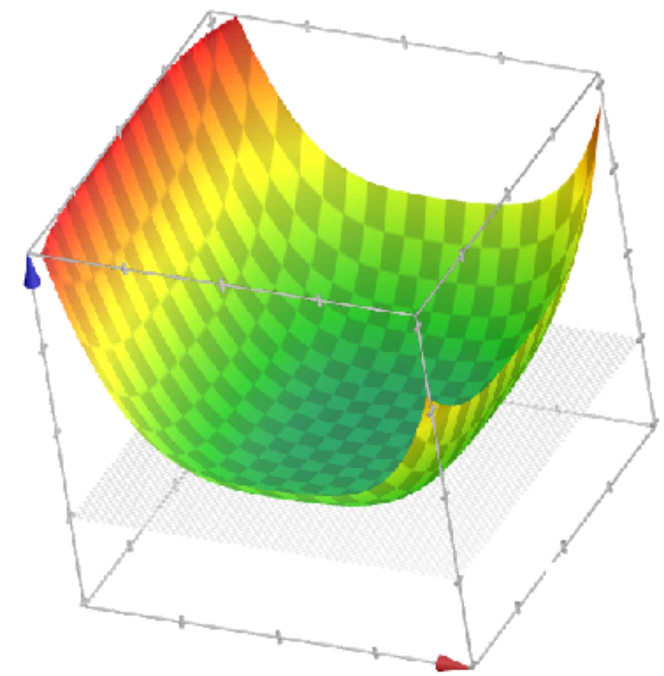

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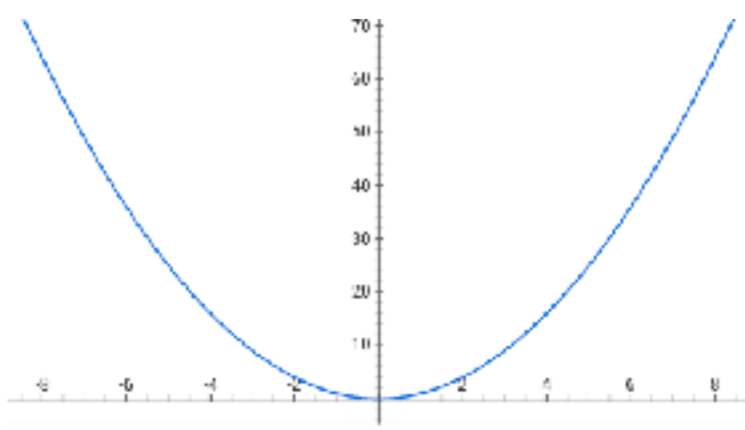

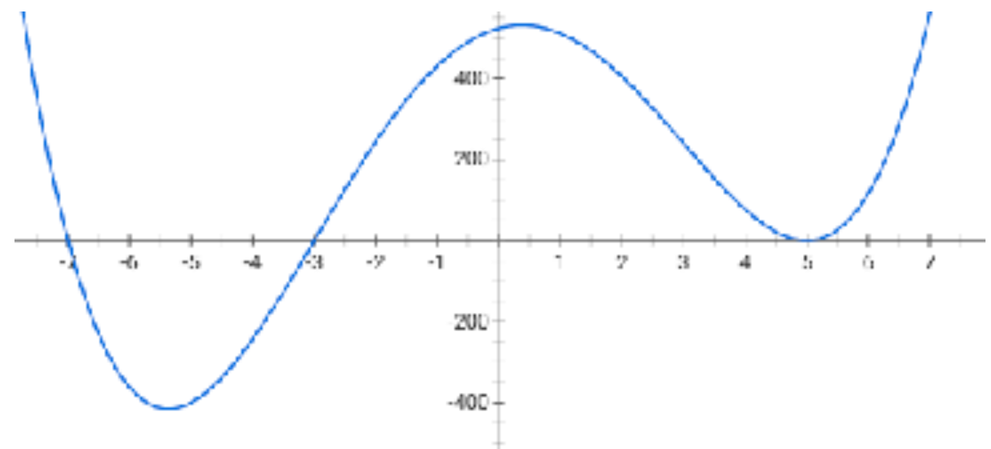

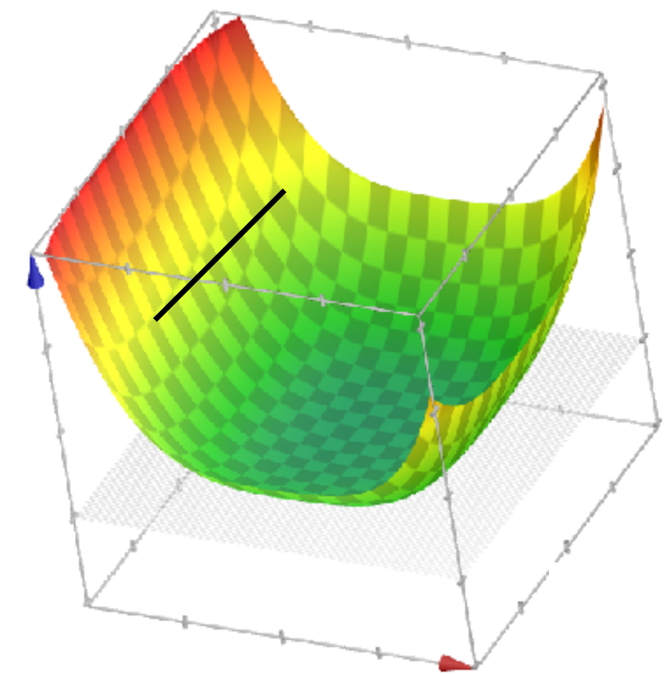

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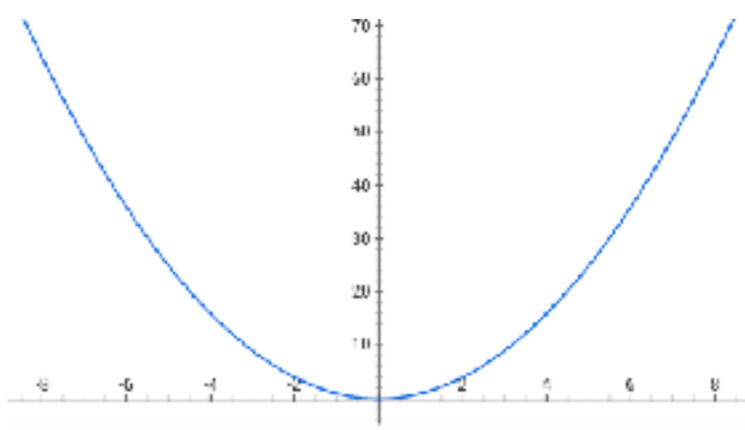

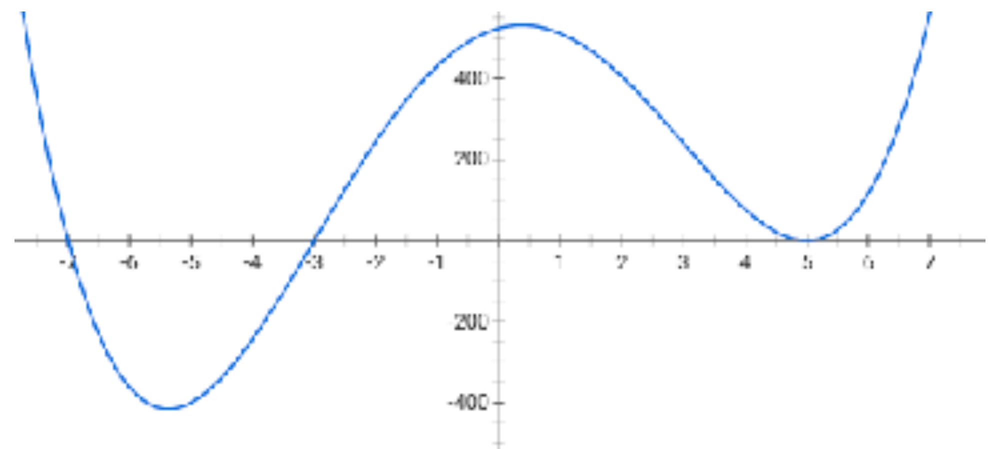

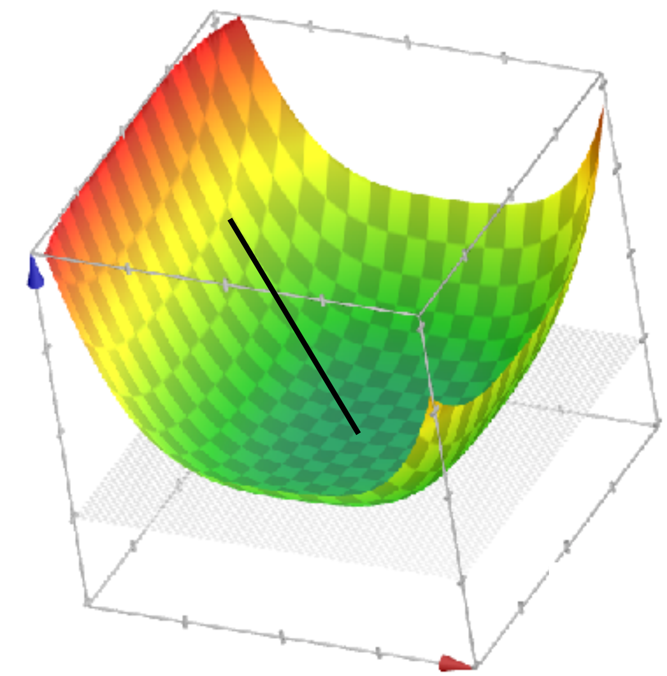

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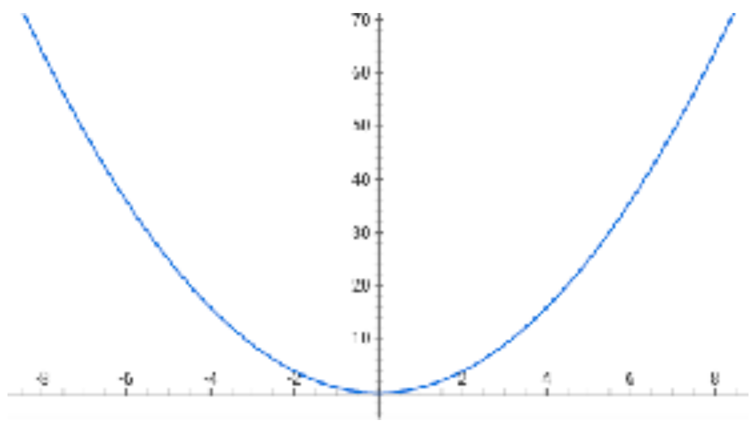

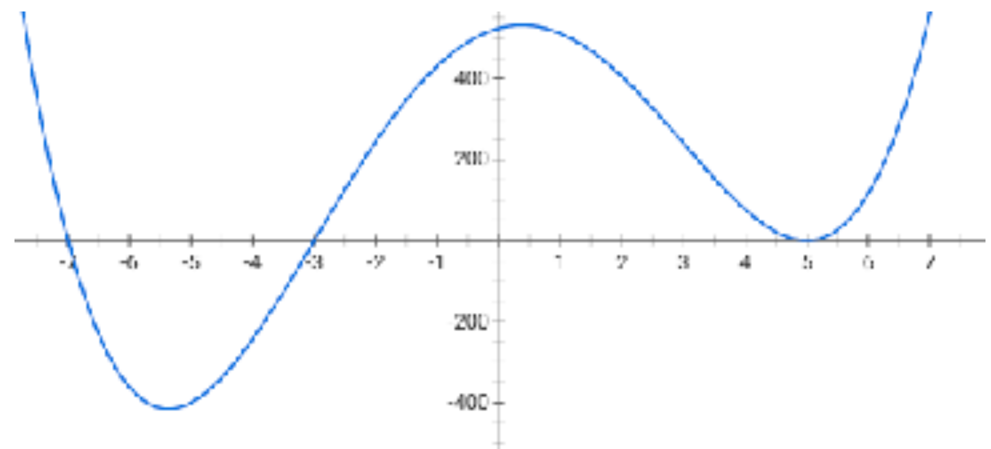

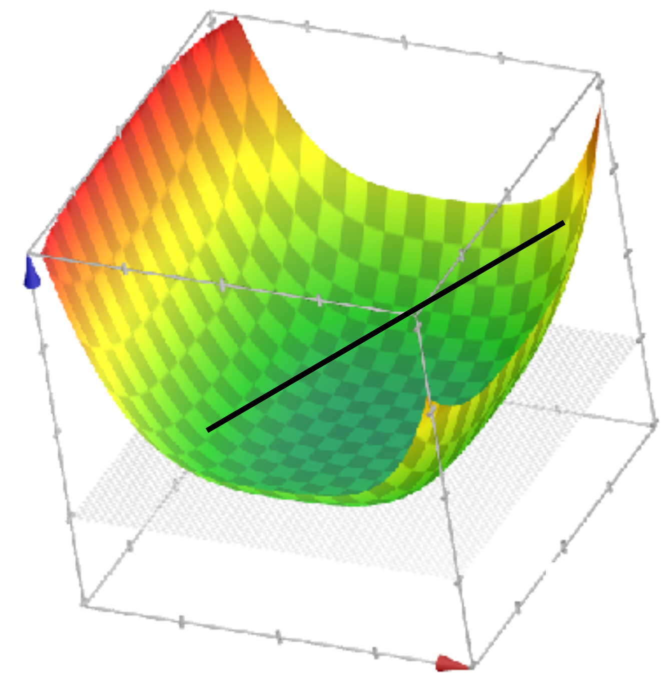

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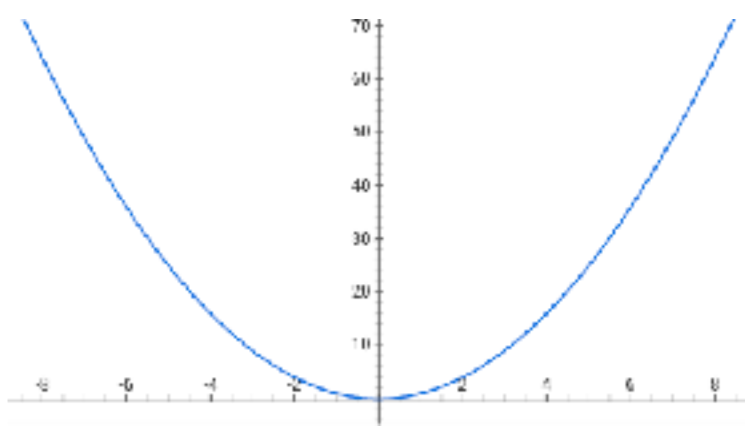

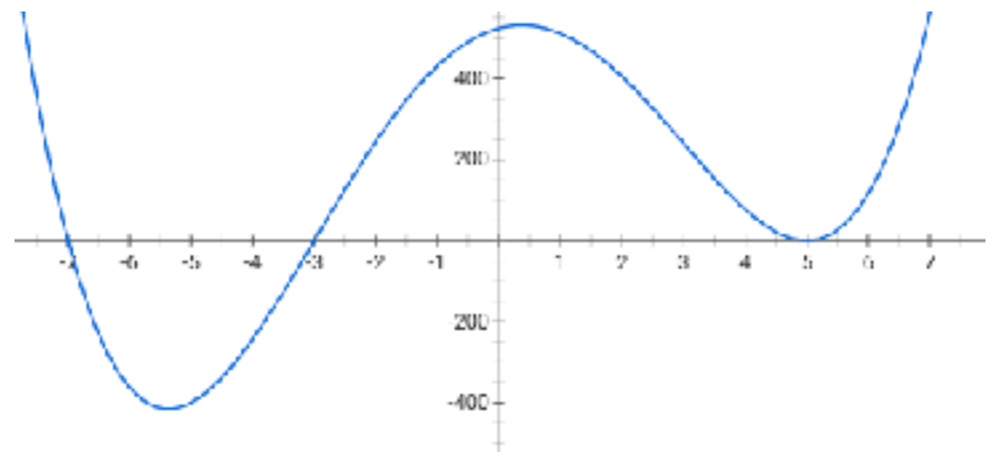

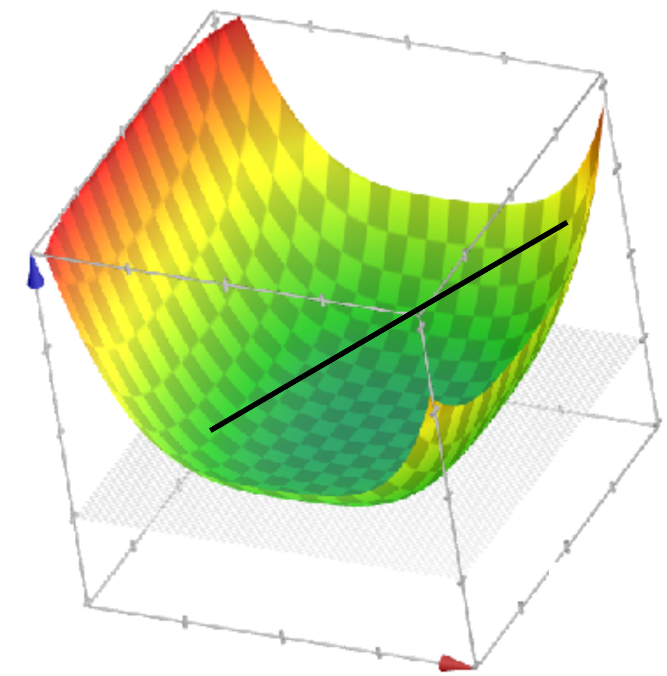

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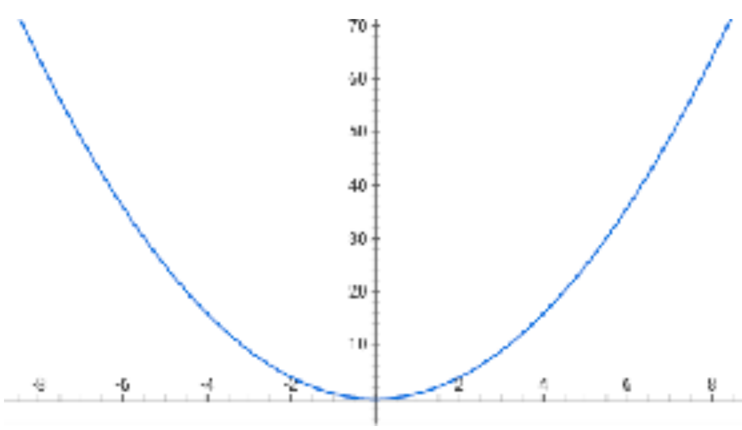

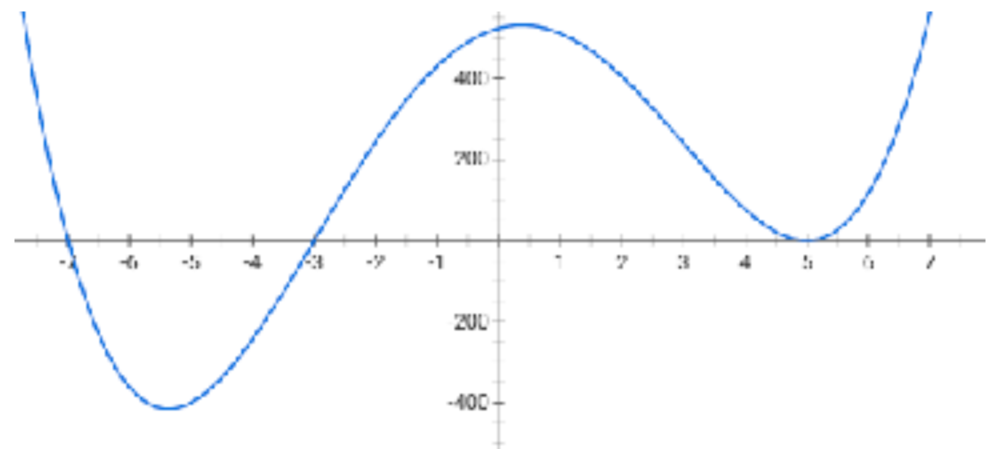

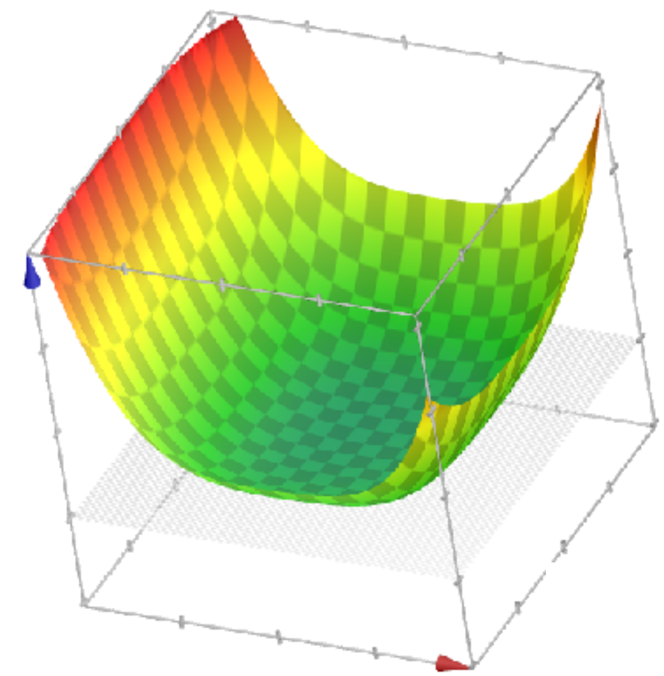

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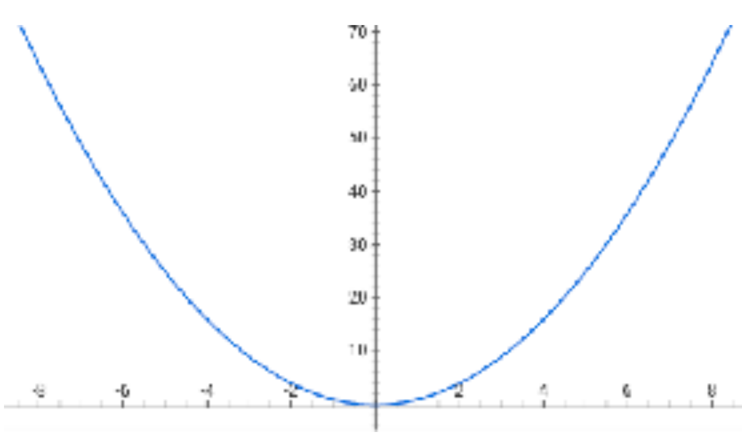

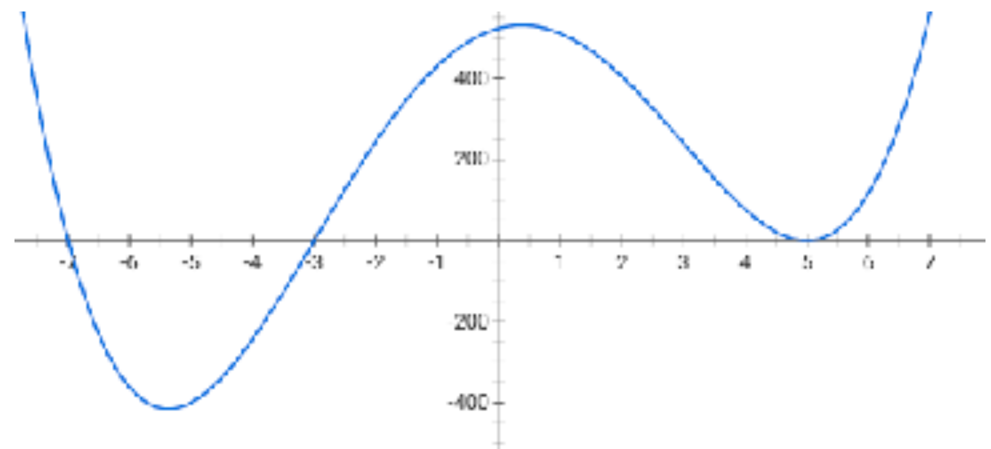

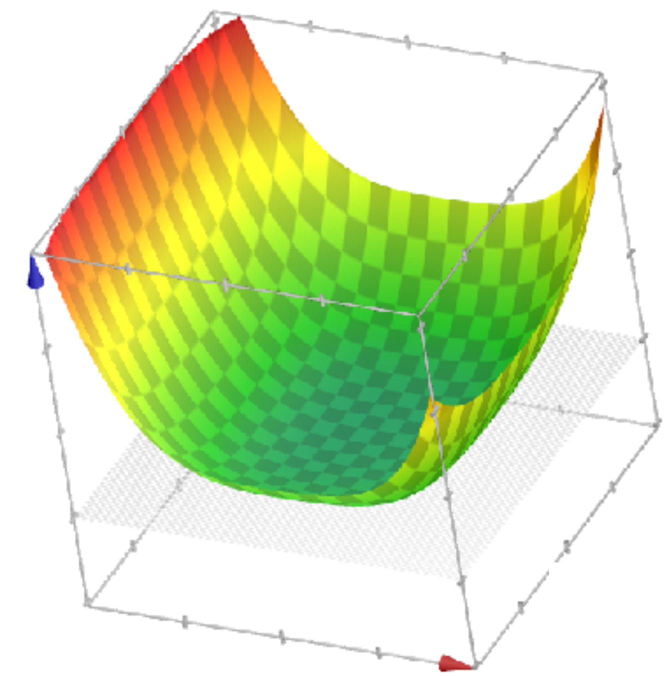

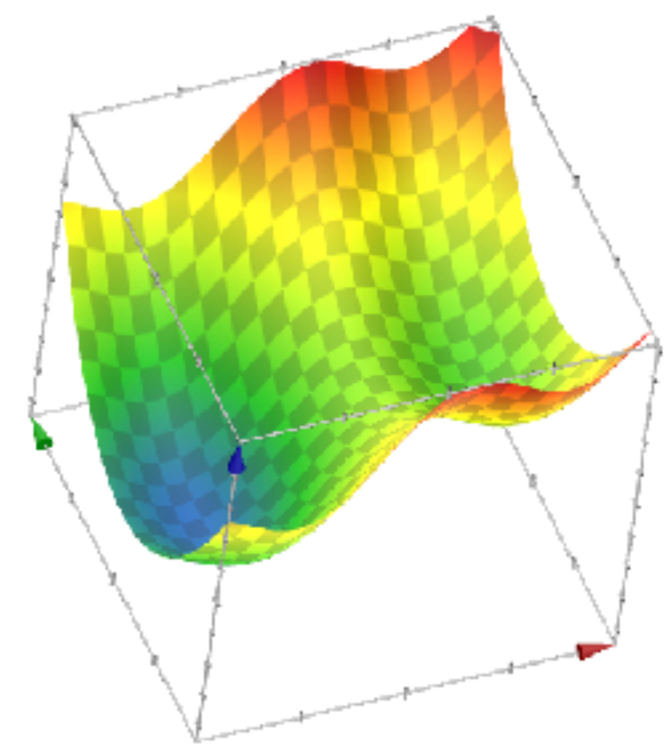

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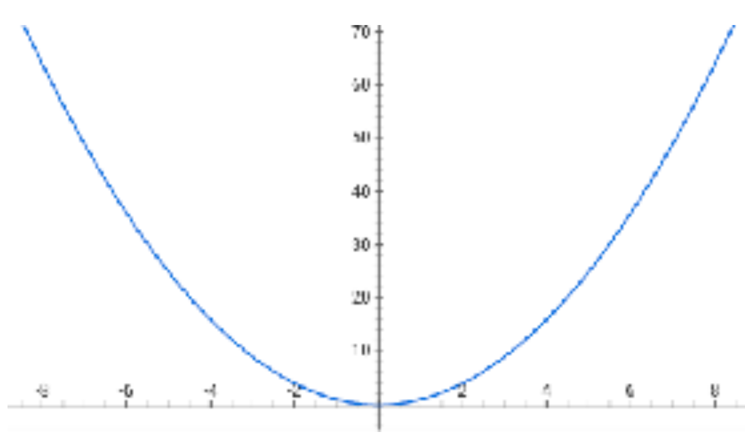
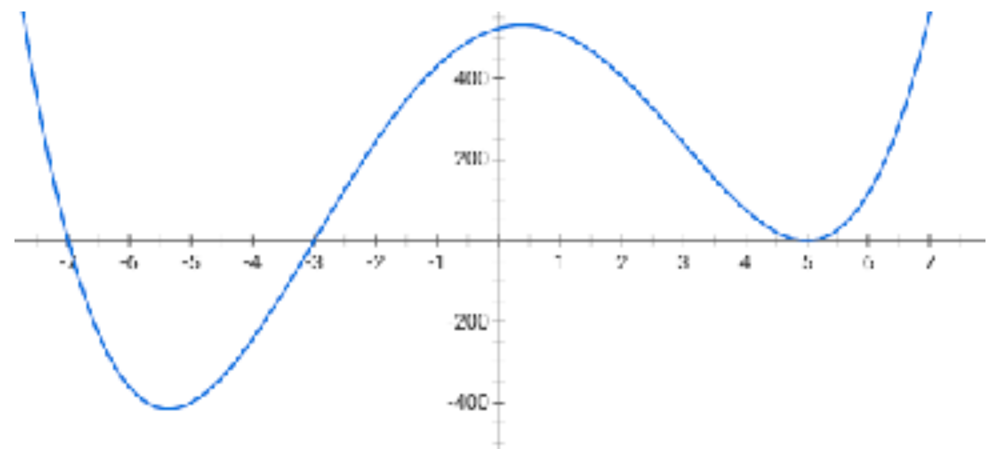
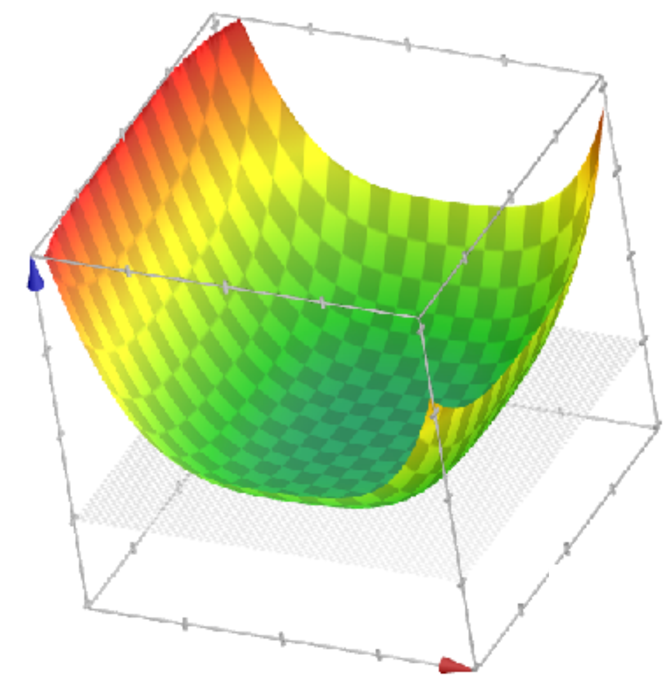
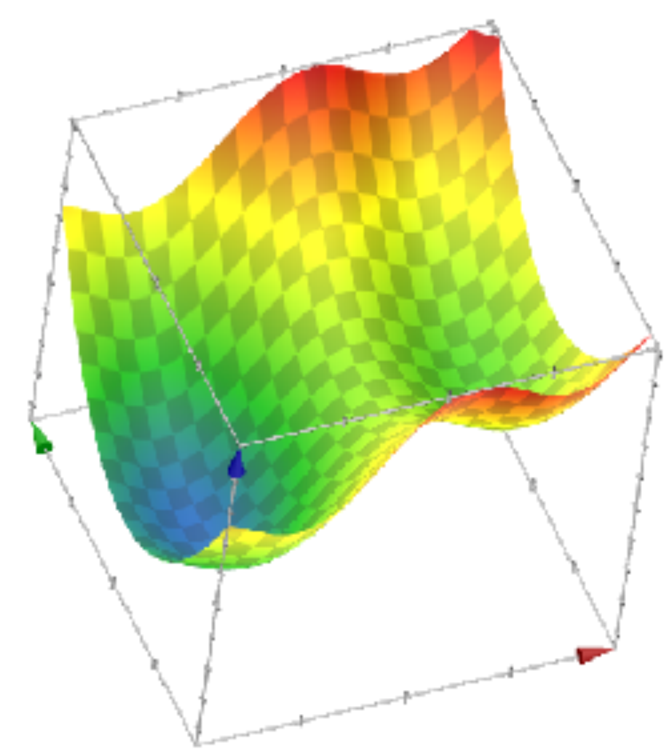






- **Theorem**: Gradient descent performance



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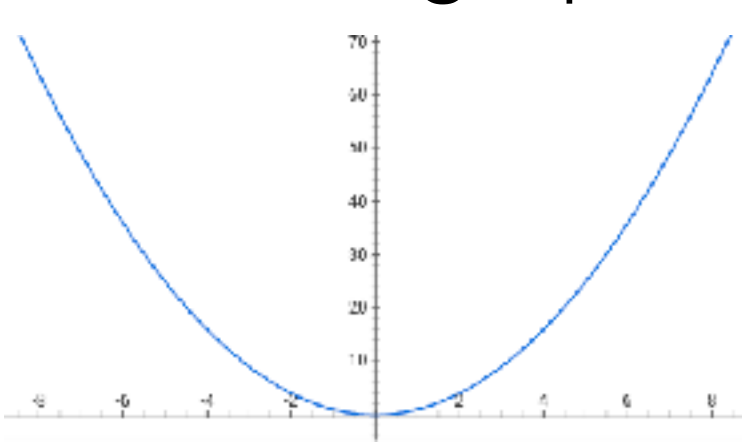
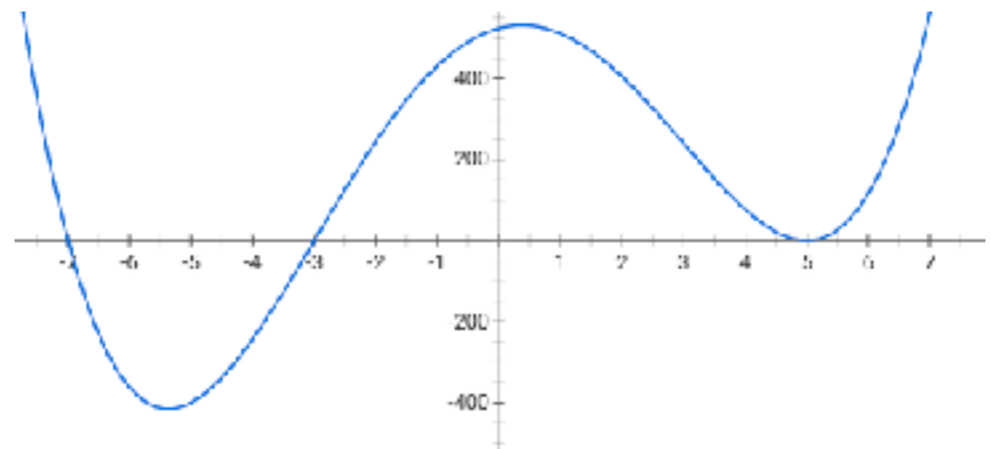
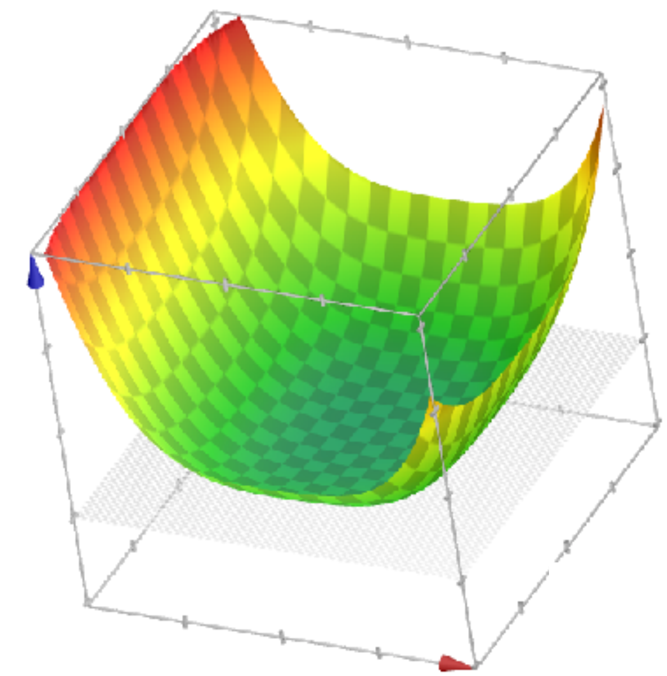
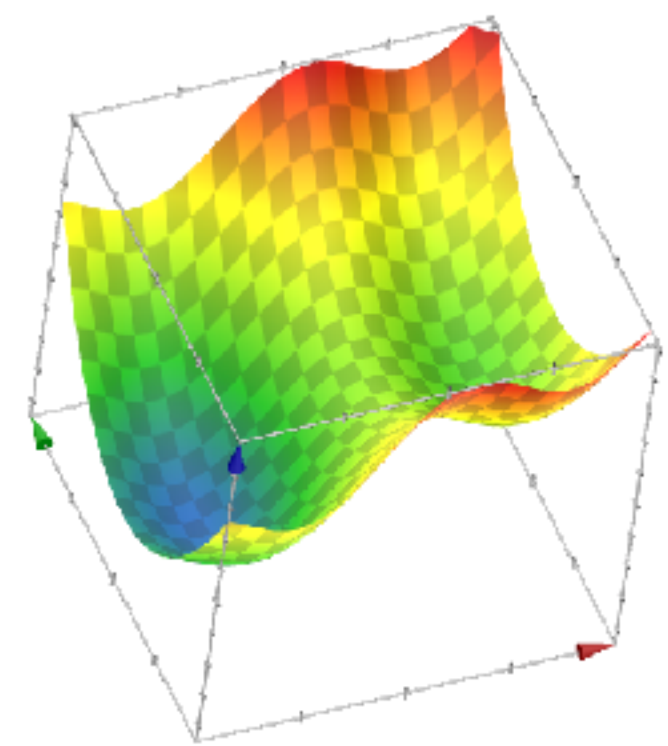
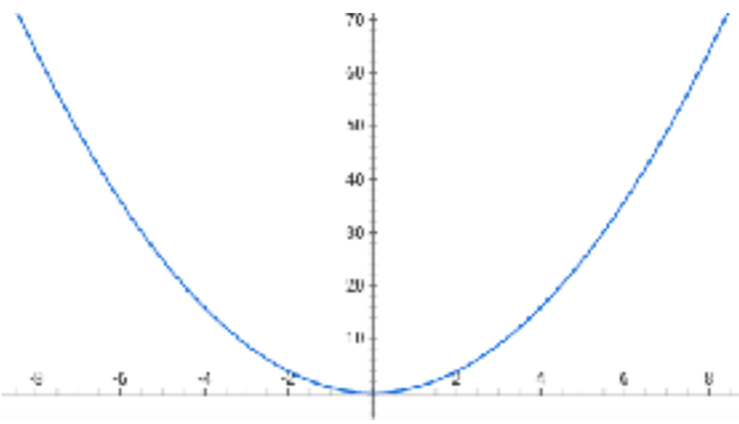




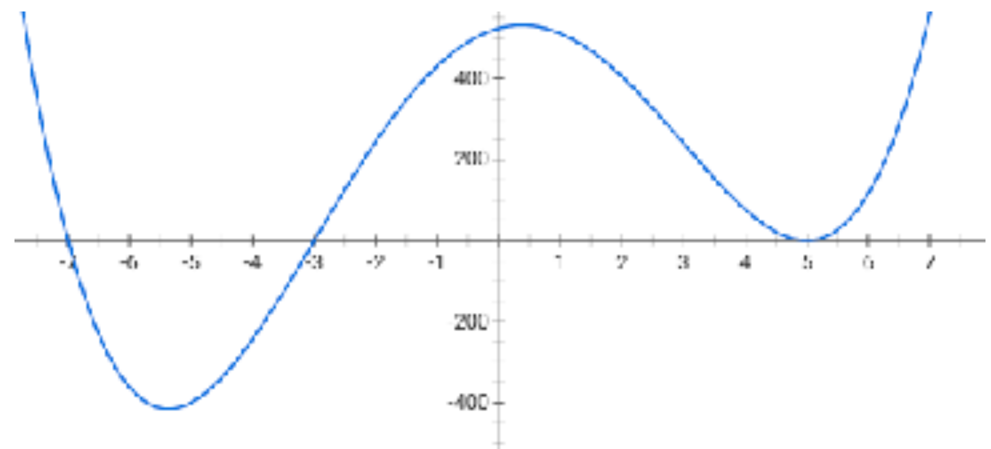
- **Theorem**: Gradient descent performance
    - **Assumptions**:

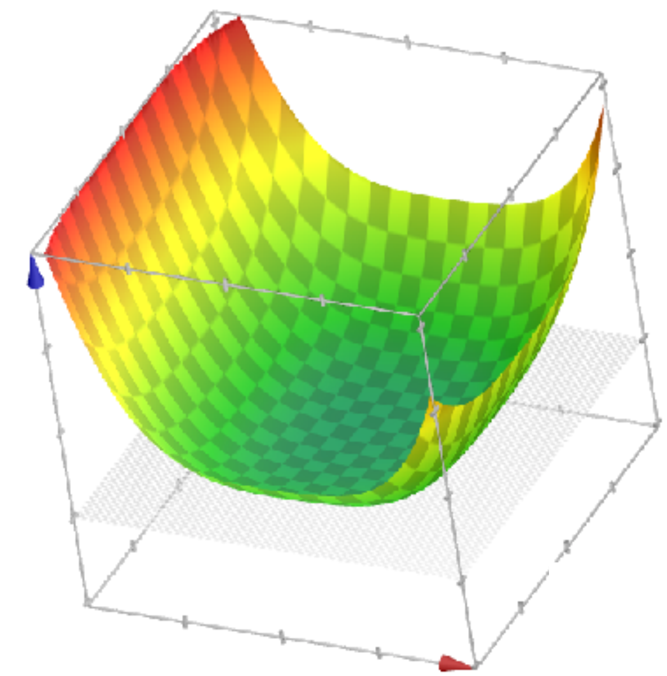


Gradient descent properties

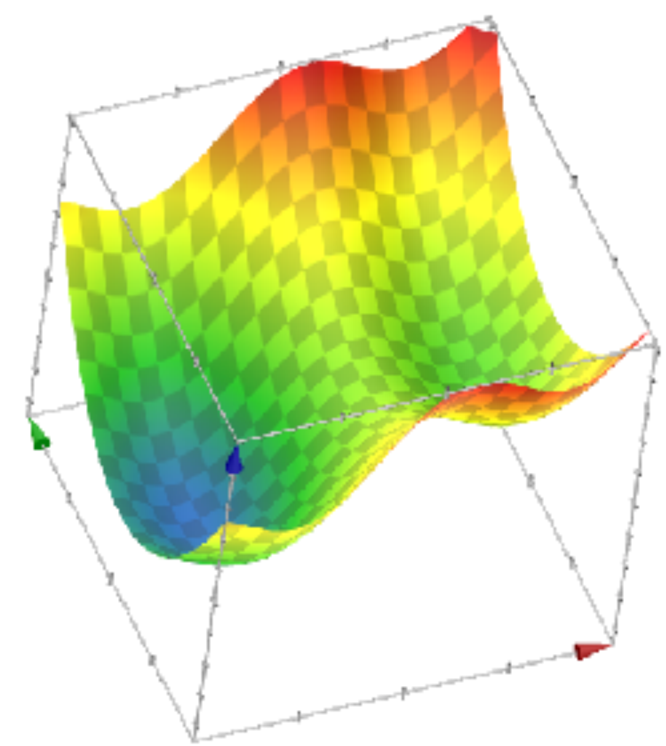
- A function *f* on $\mathbb{R}^m$ is convex if any line segment connecting two points of the graph of *f* lies above or on the graph







- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)



Gradient descent properties

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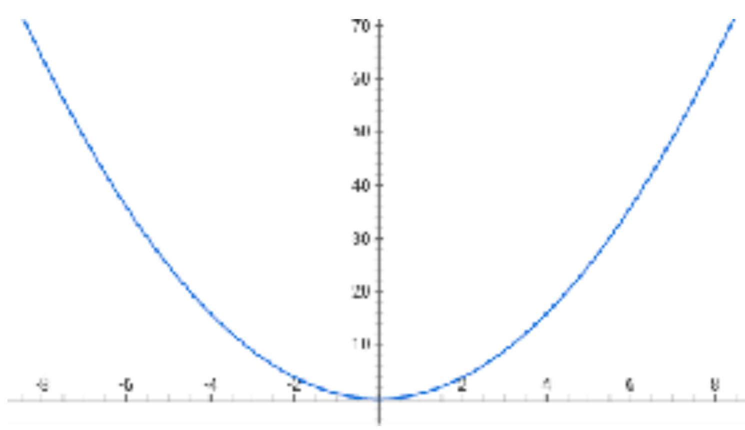
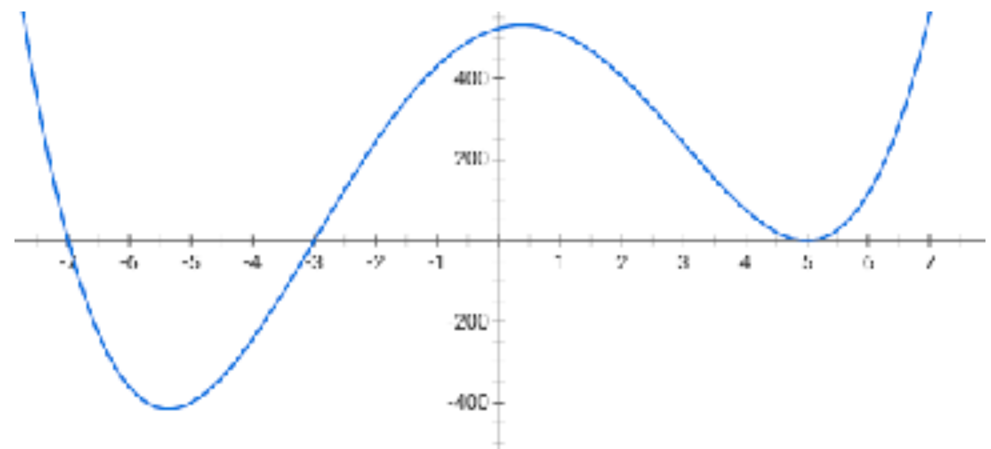
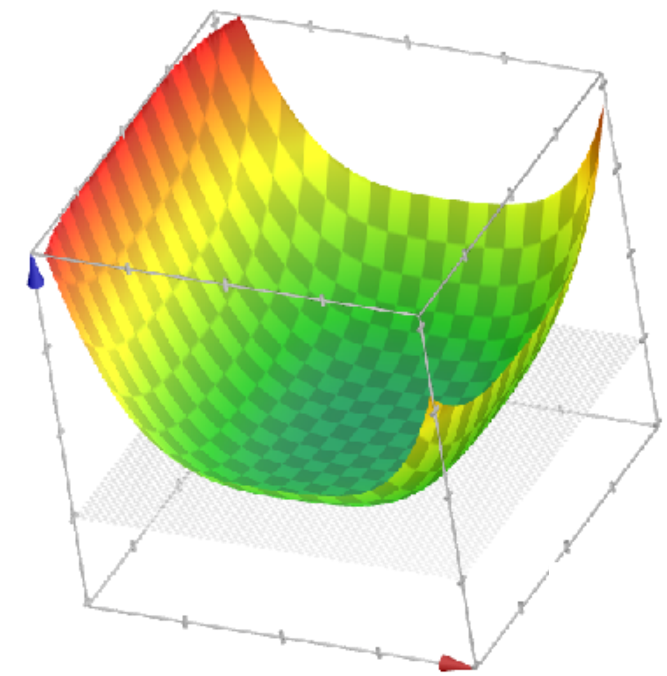
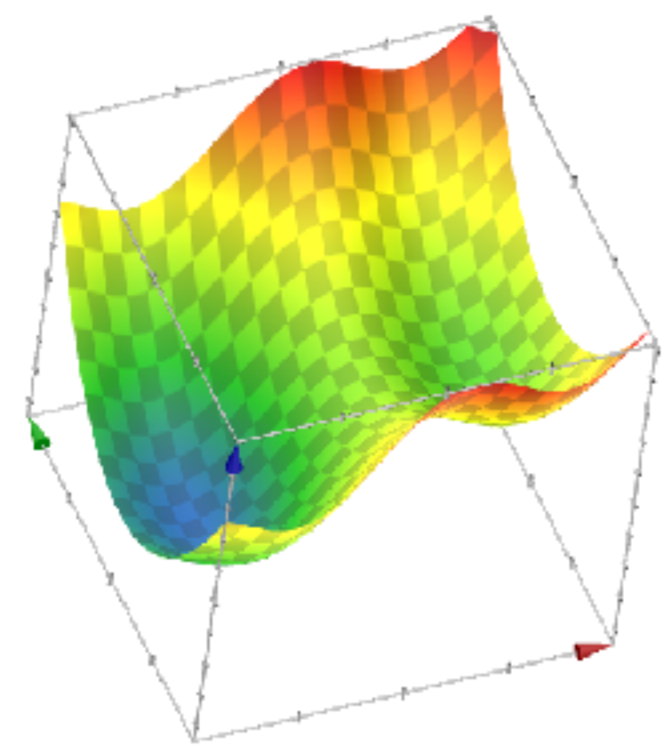
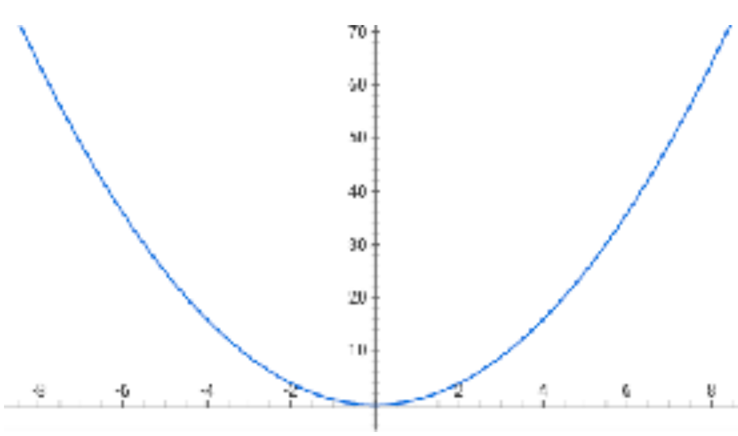
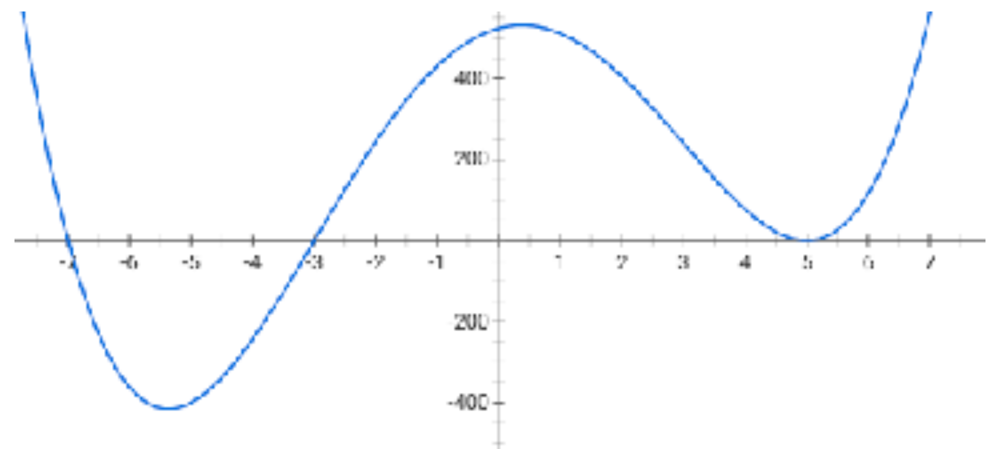
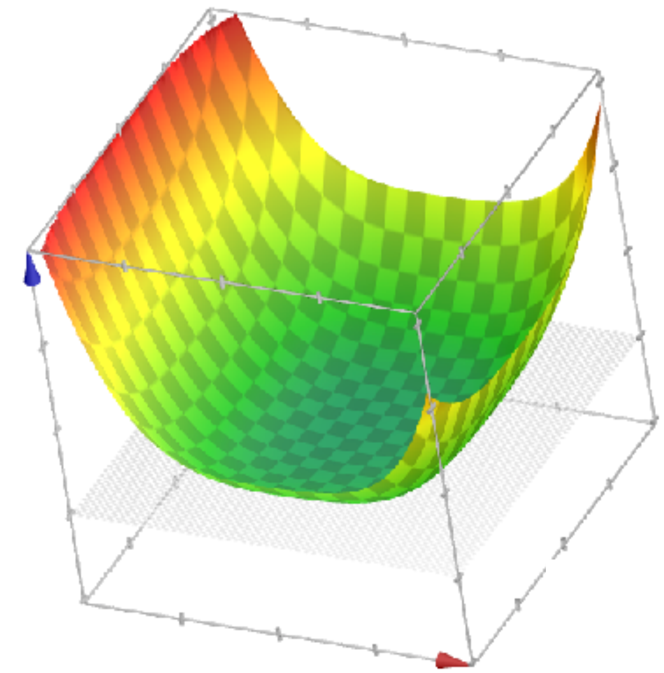




- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is sufficiently "smooth" and convex

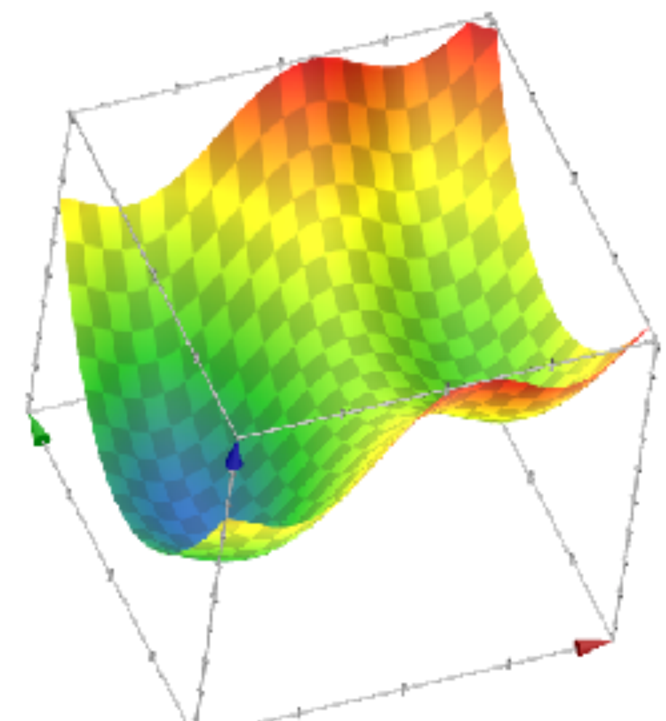


Gradient descent properties

- A function f on $\mathbb{R}^m$ is convex if any line segment connecting two points of the graph of f lies above or on the graph

- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - f is sufficiently "smooth" and convex
    - f has at least one global optimum



Gradient descent properties

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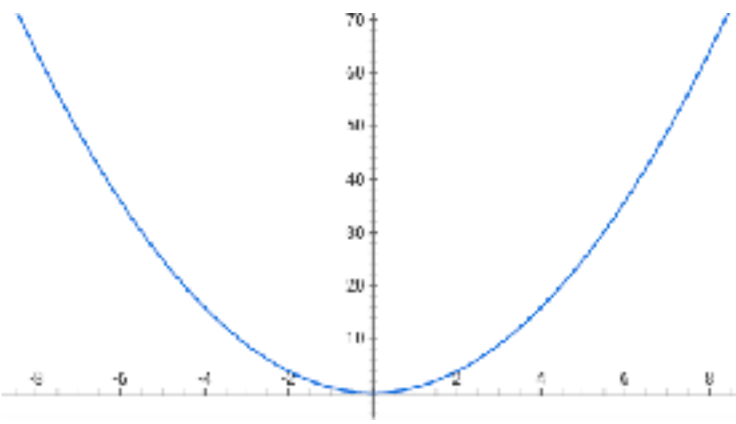

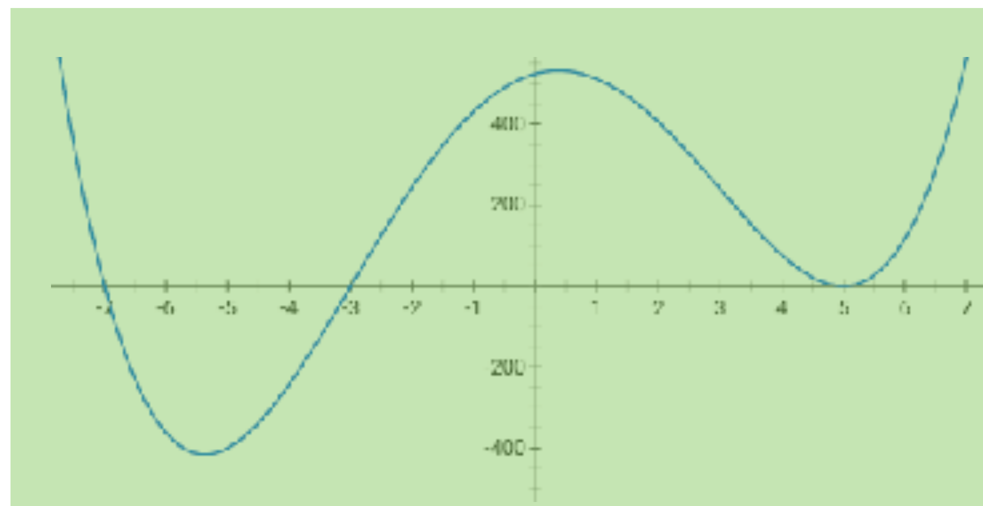

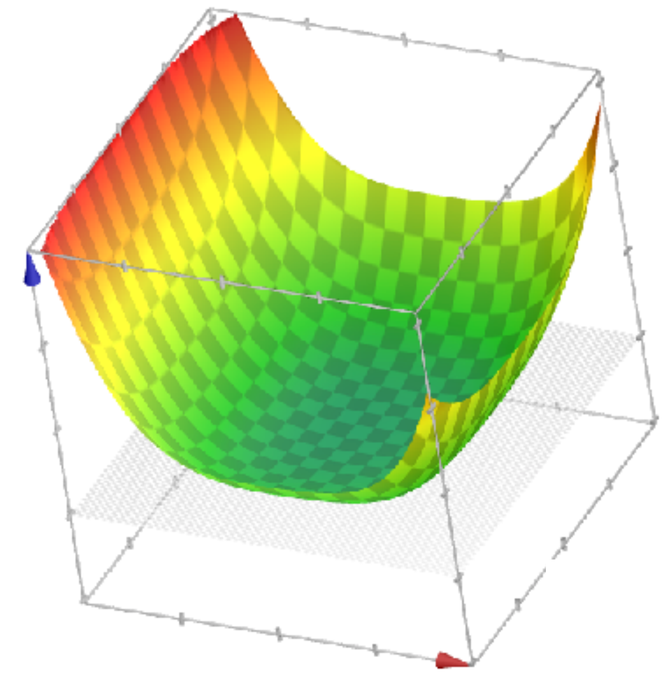

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  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
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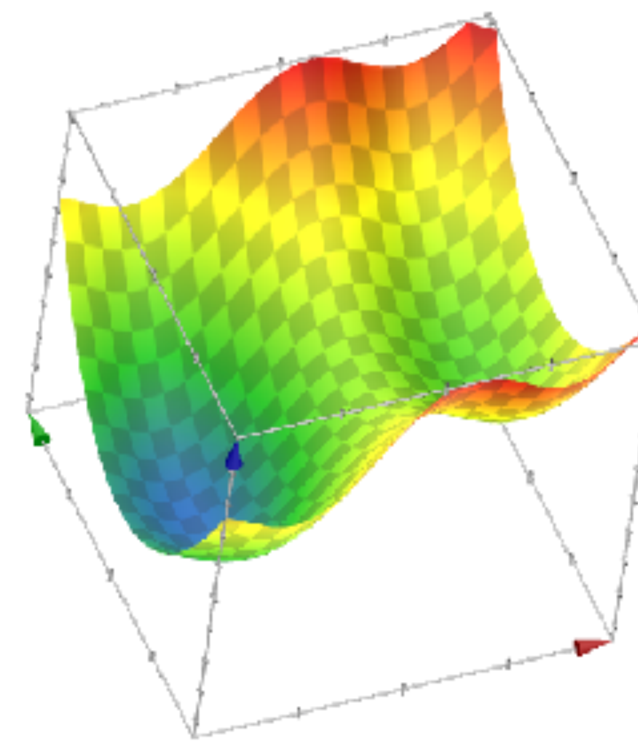
    - *f* has at least one global optimum



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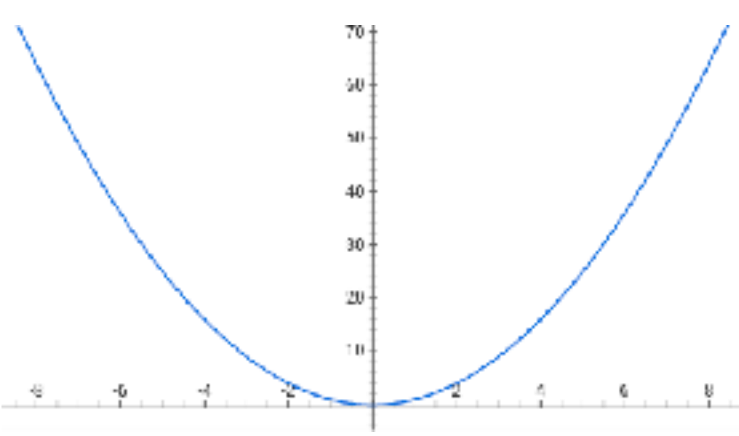

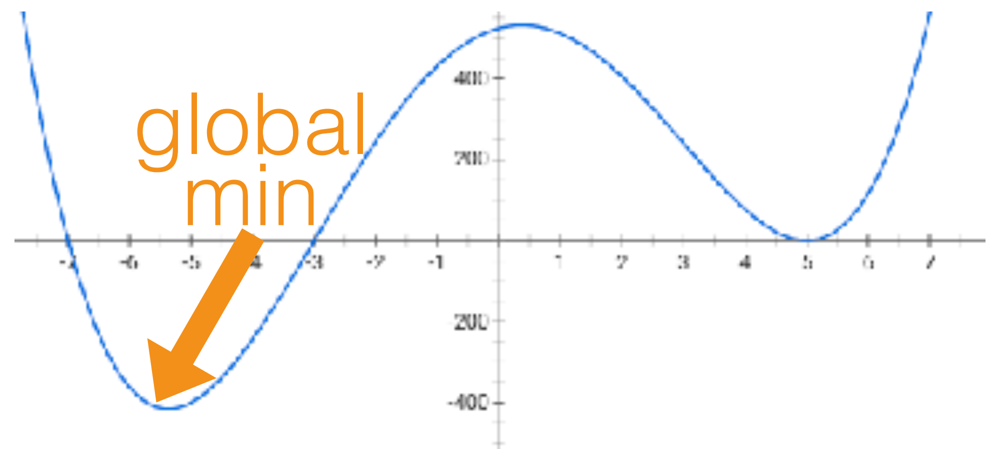

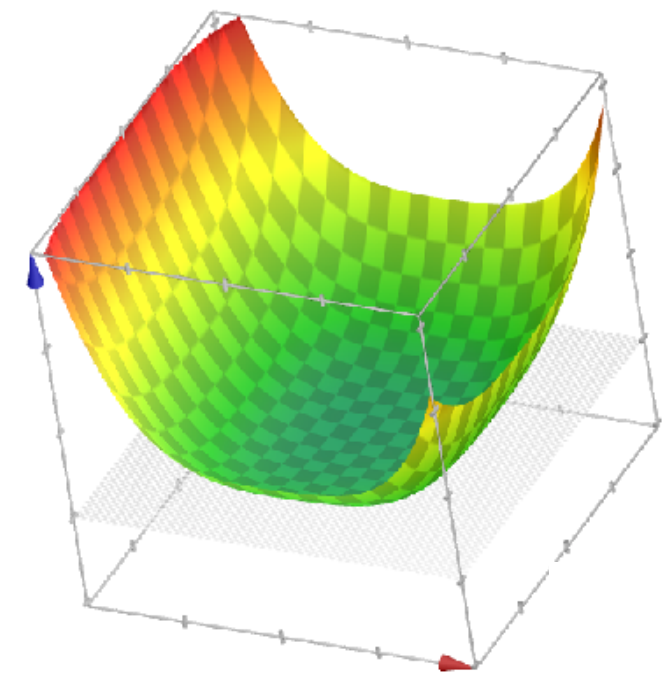

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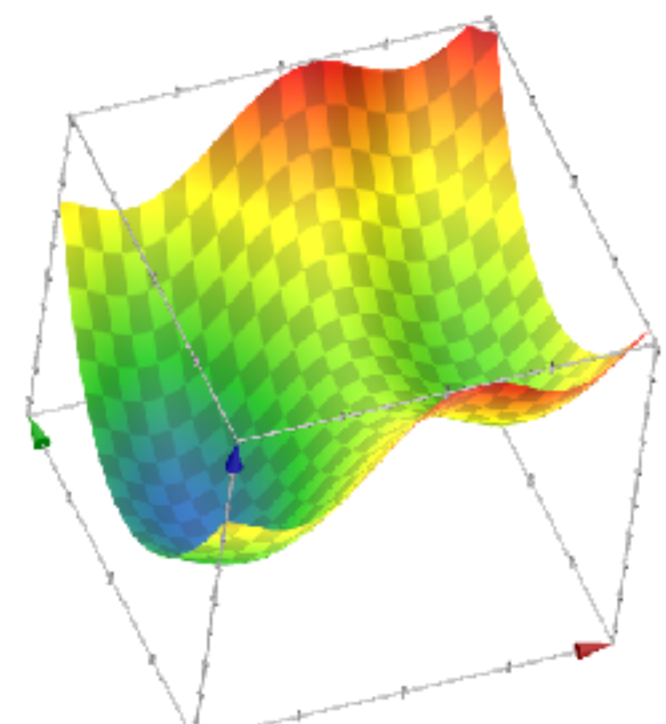

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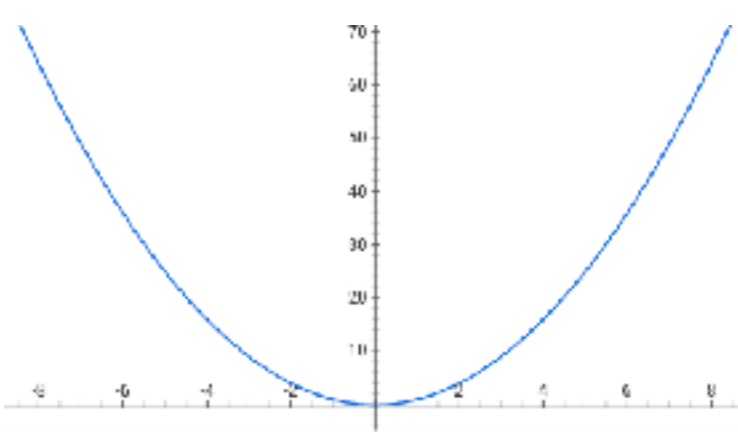

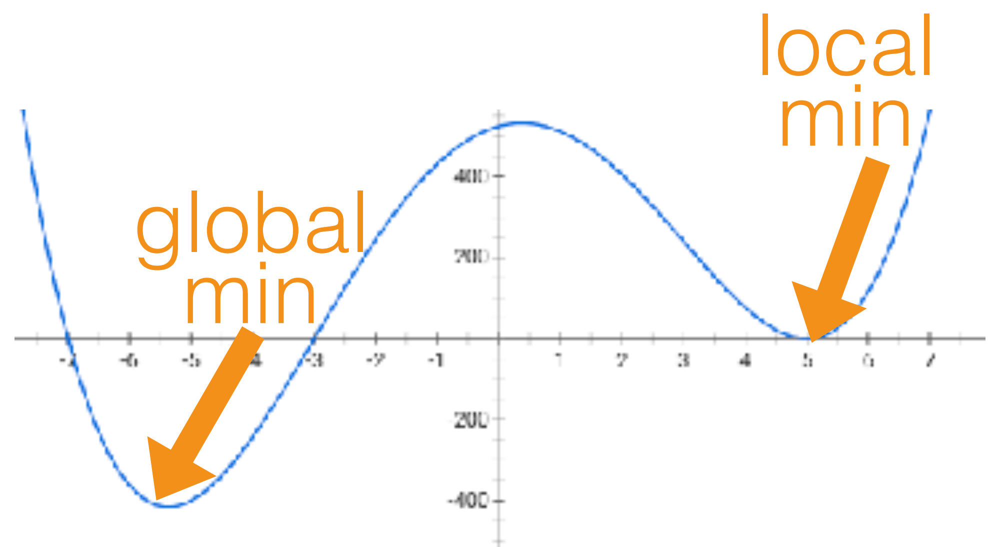

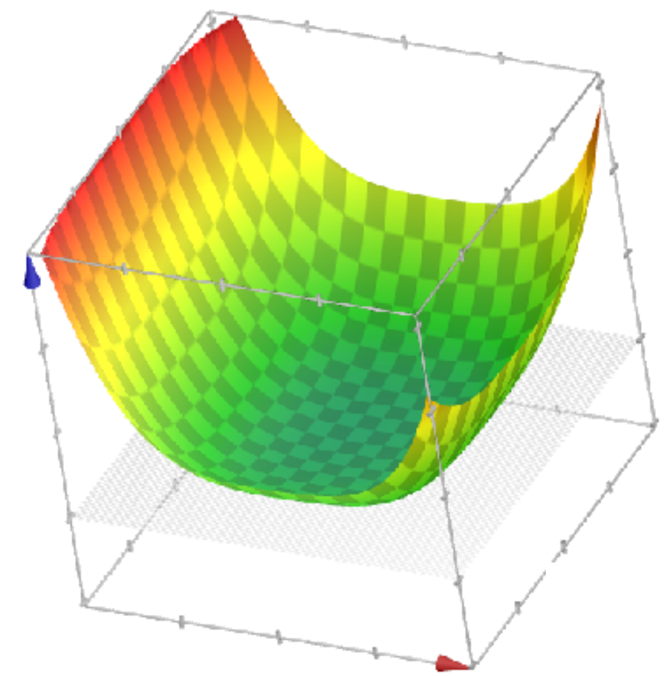

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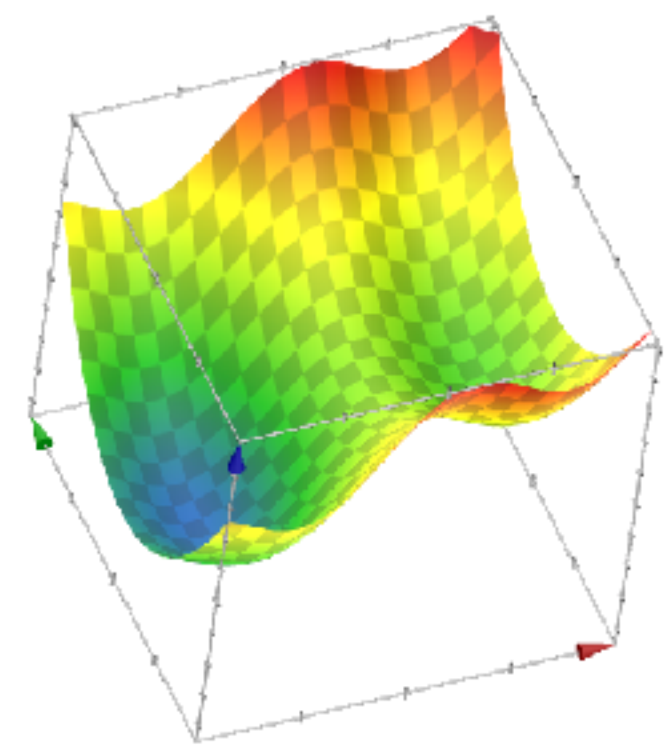

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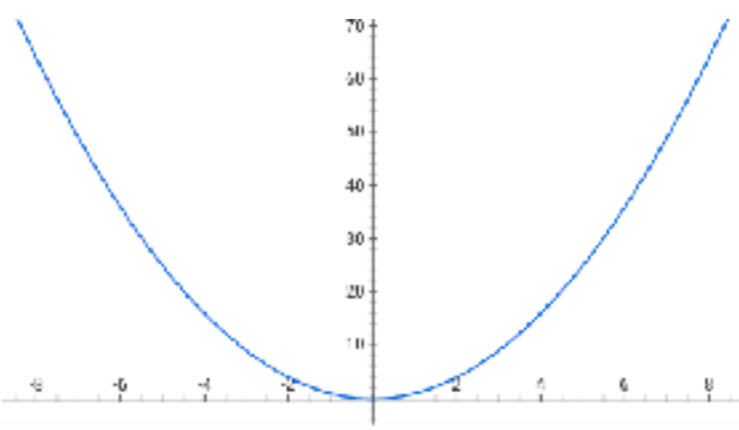
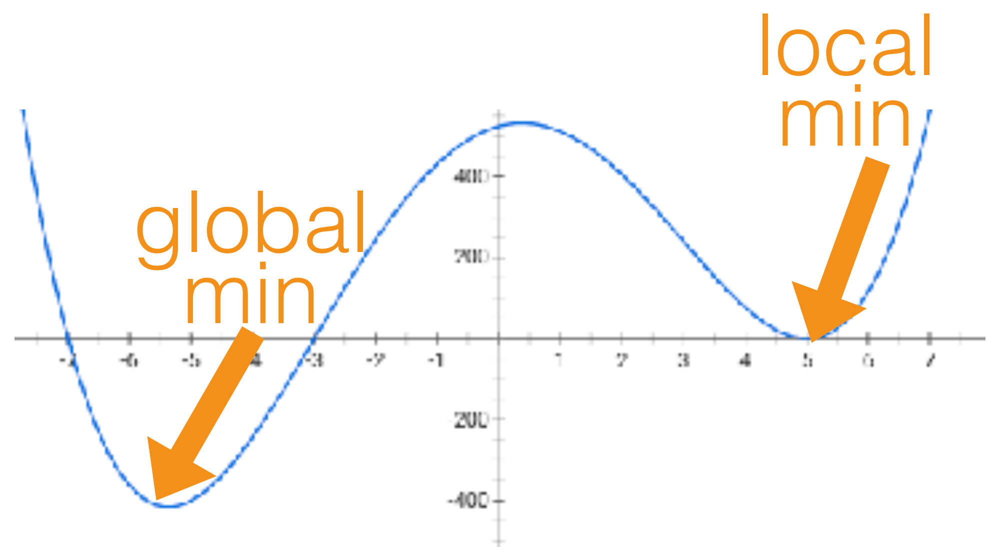
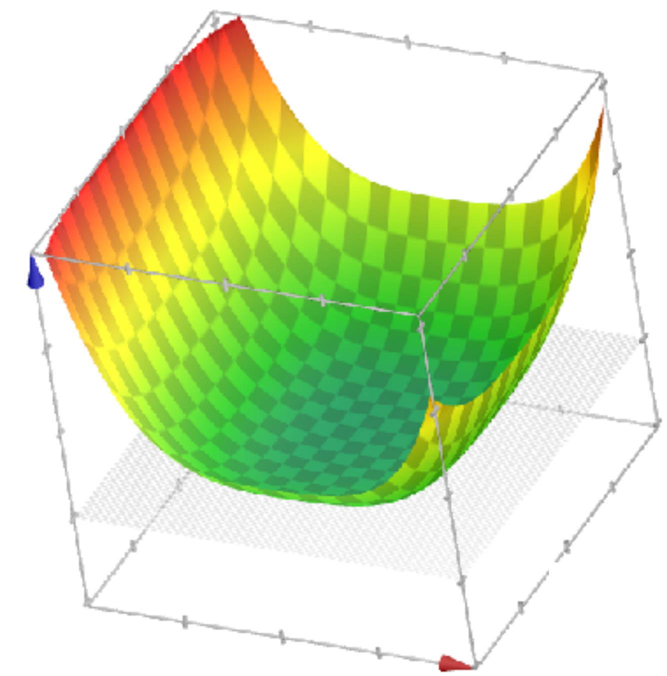
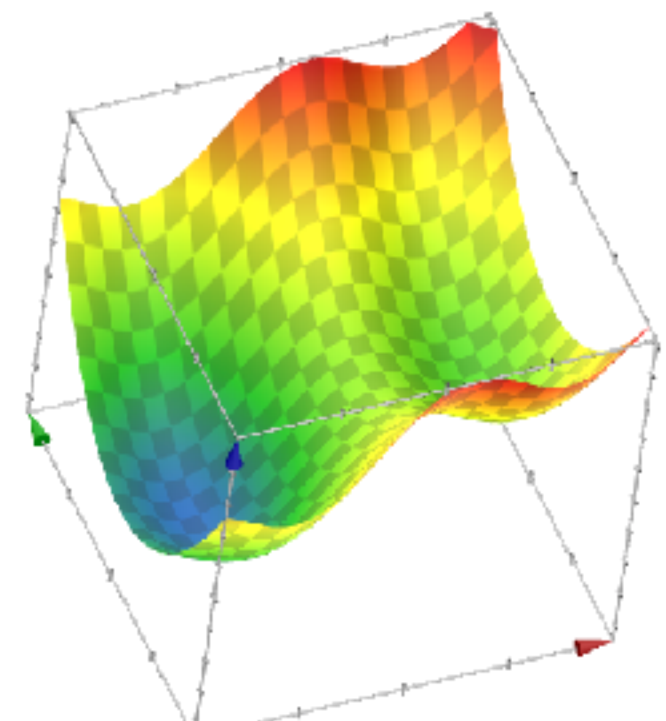






- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is sufficiently "smooth" and convex
    - *f* has at least one global optimum
    - η is sufficiently small



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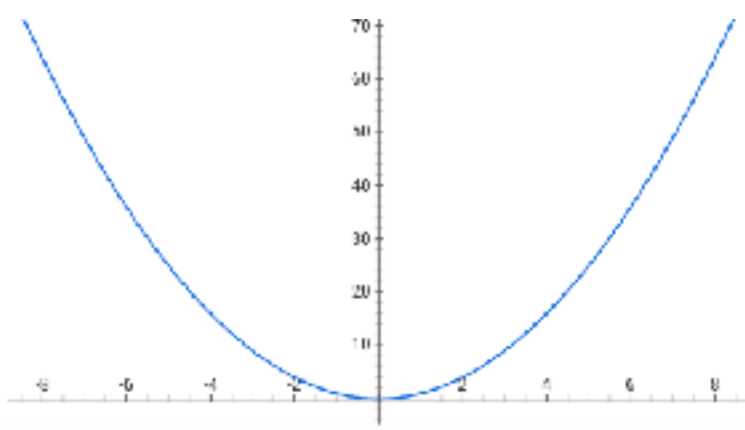

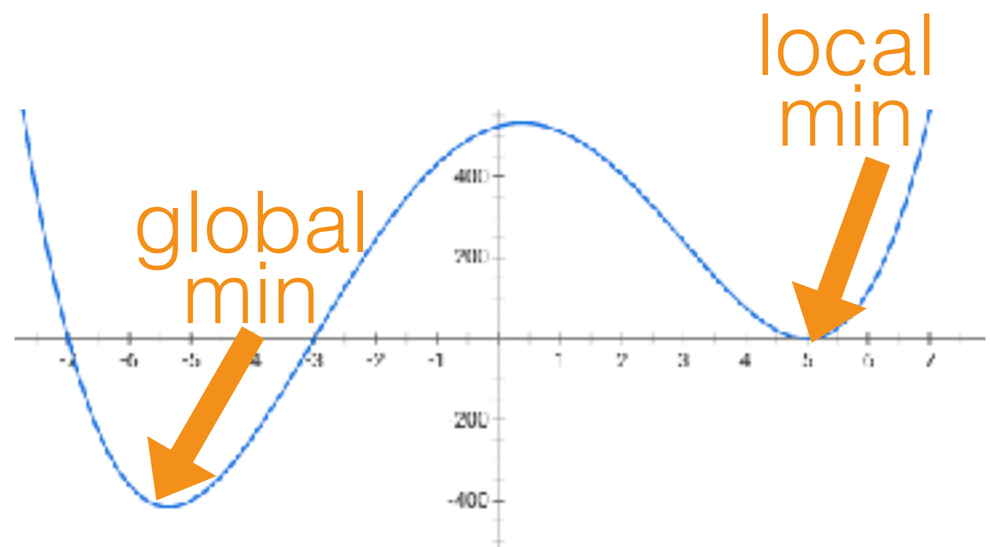

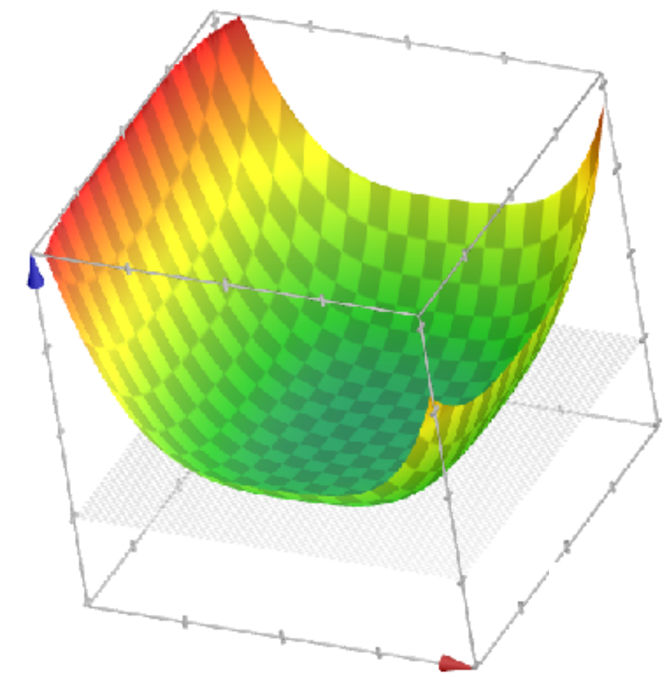

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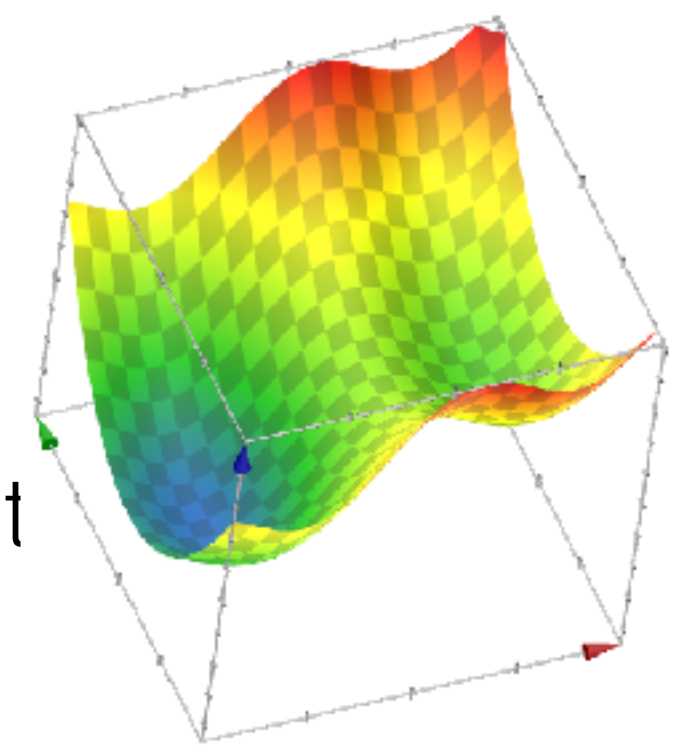

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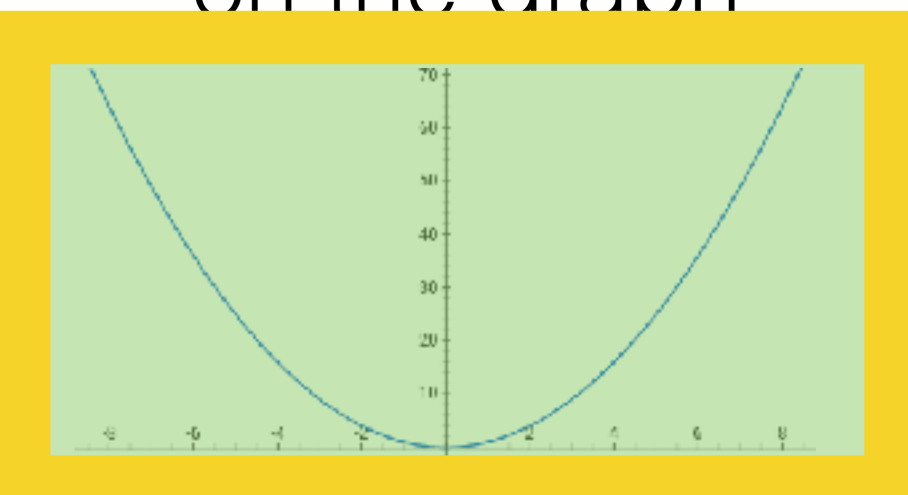

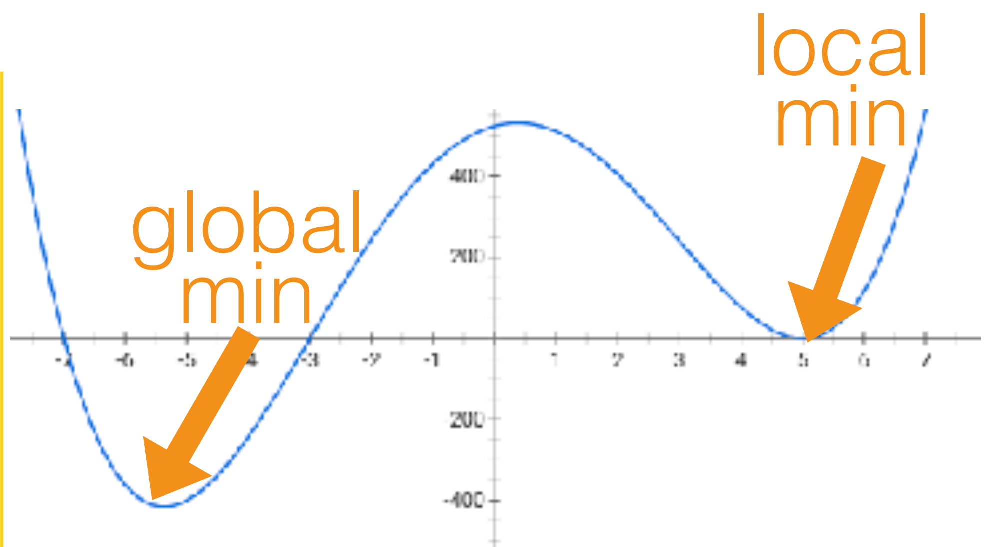

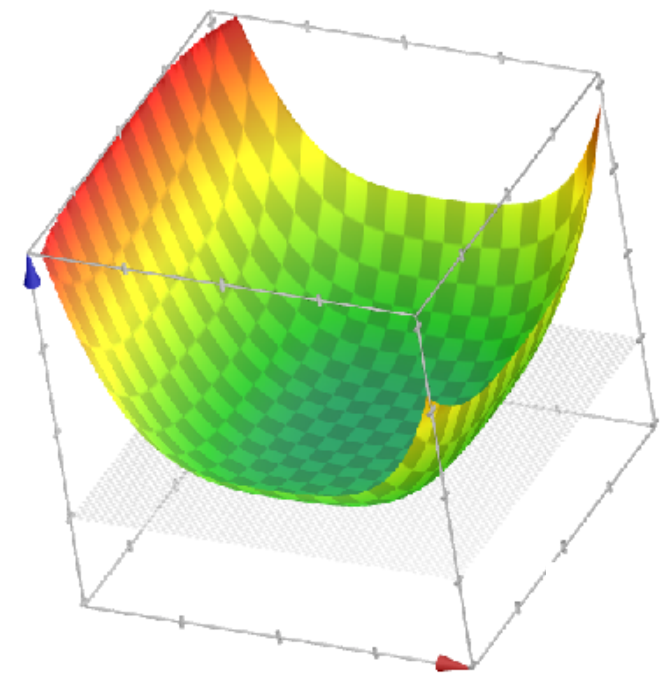

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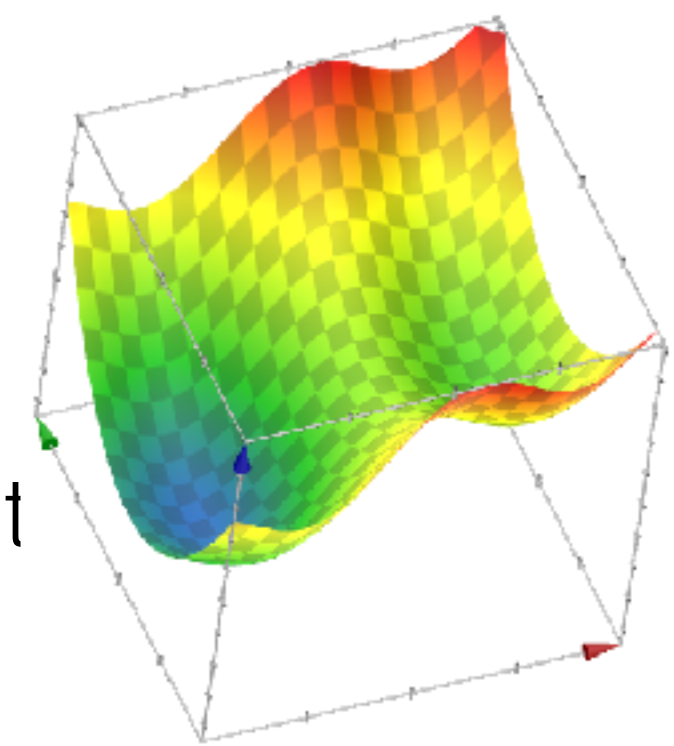

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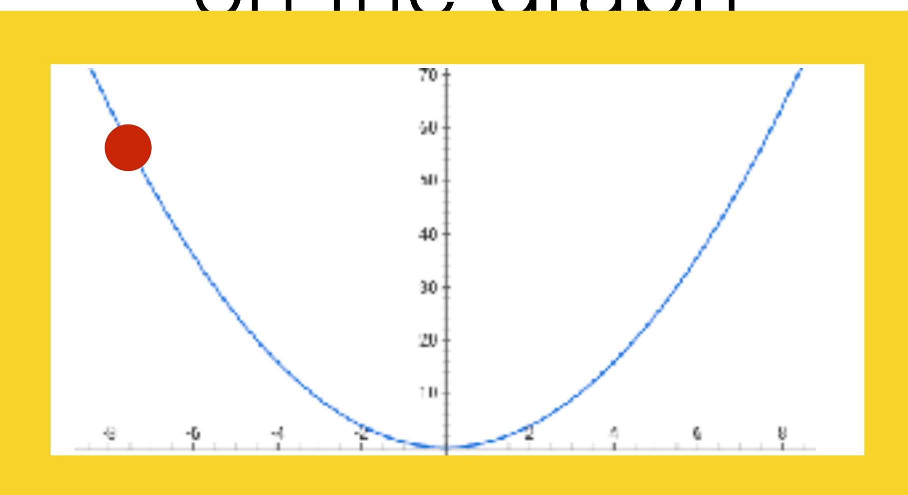

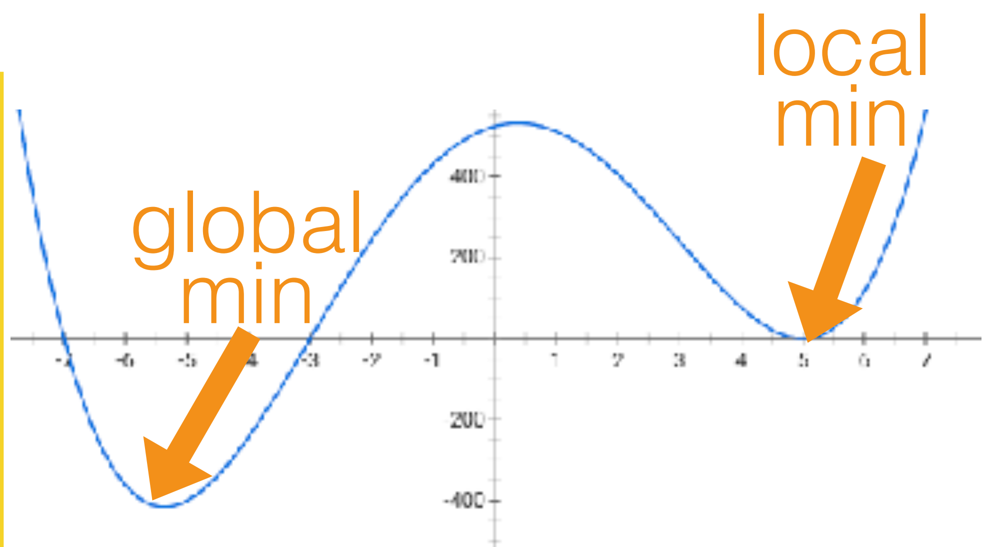

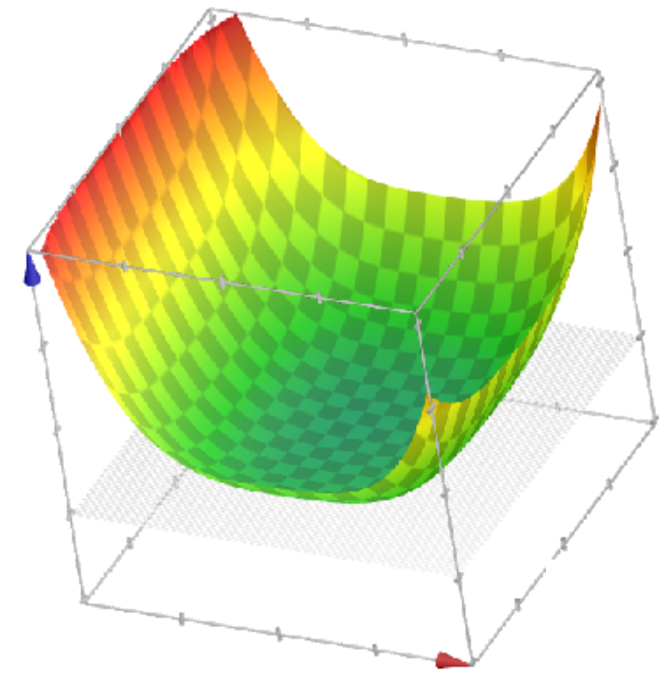

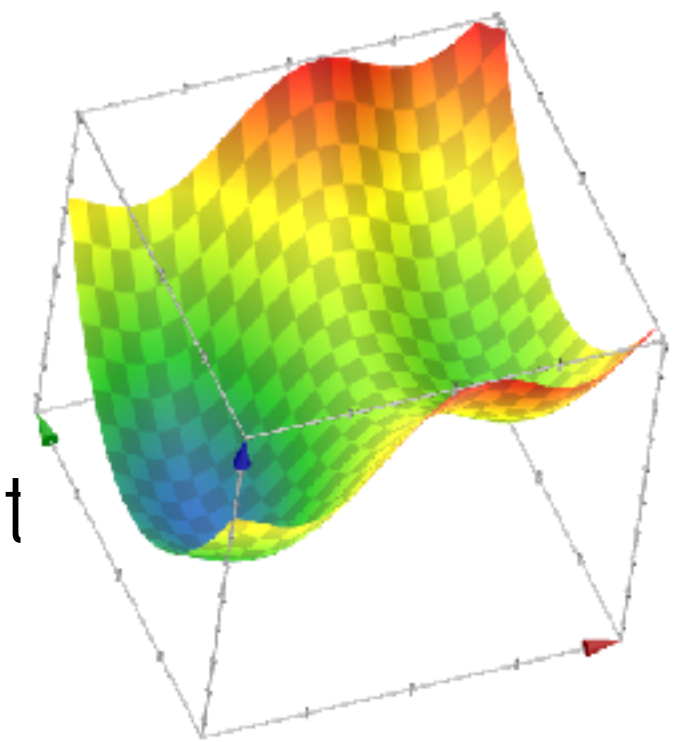

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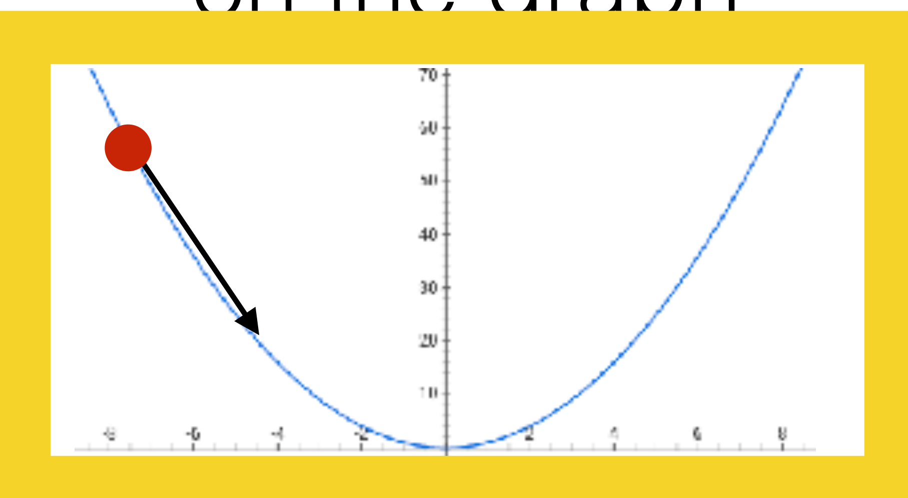

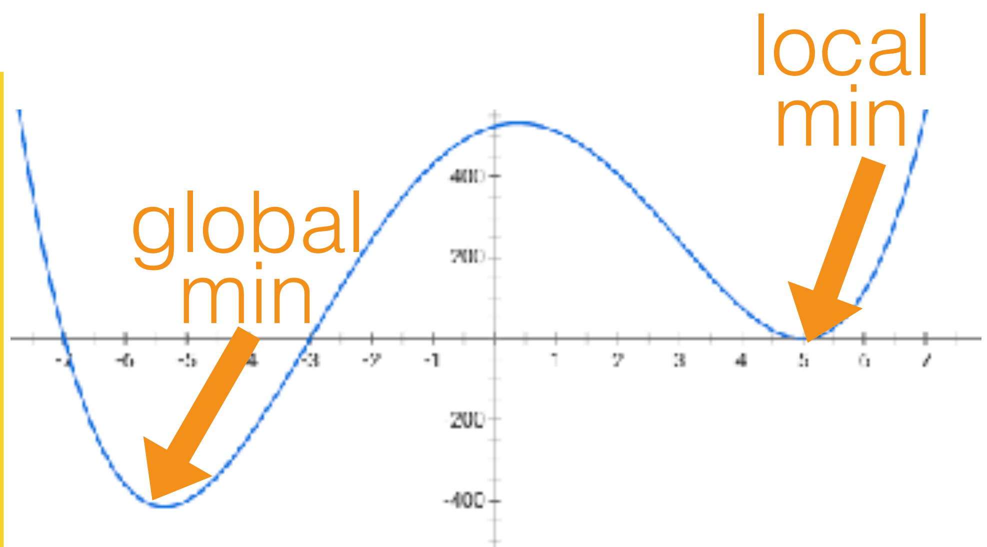

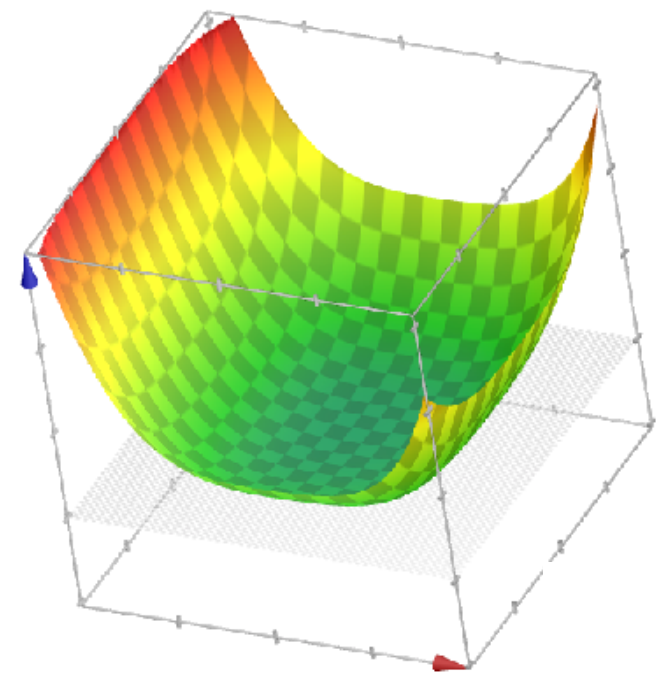

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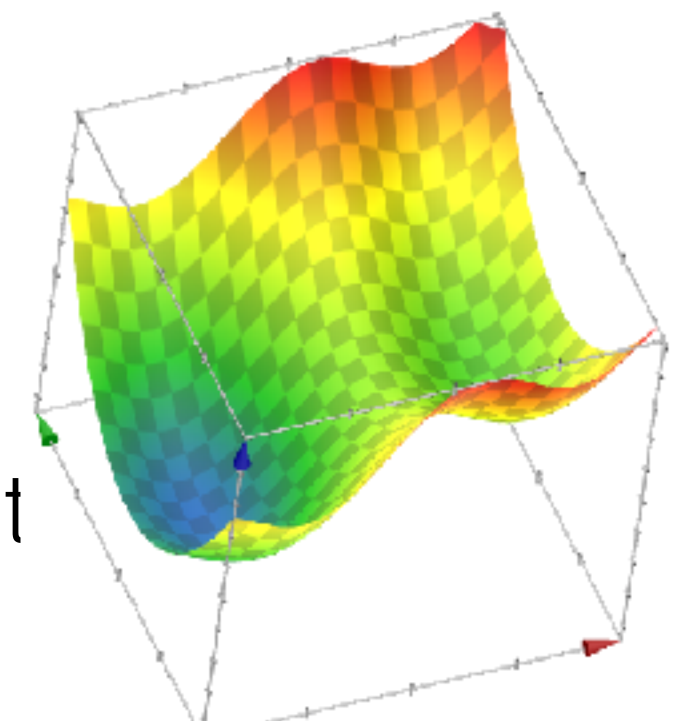

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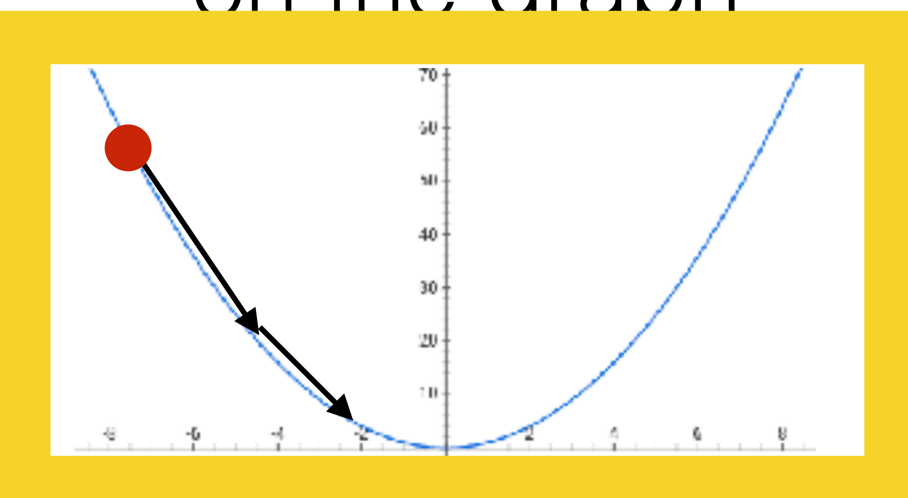

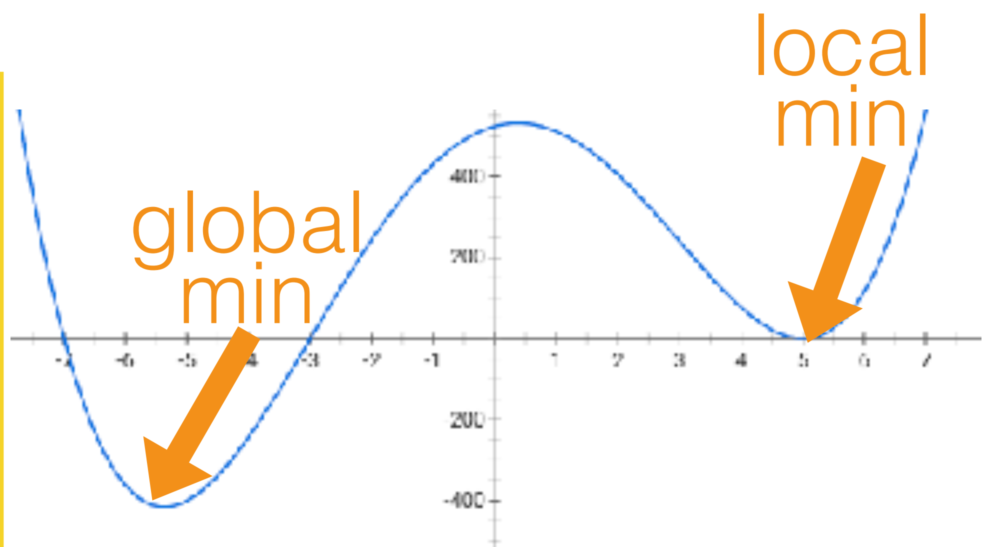

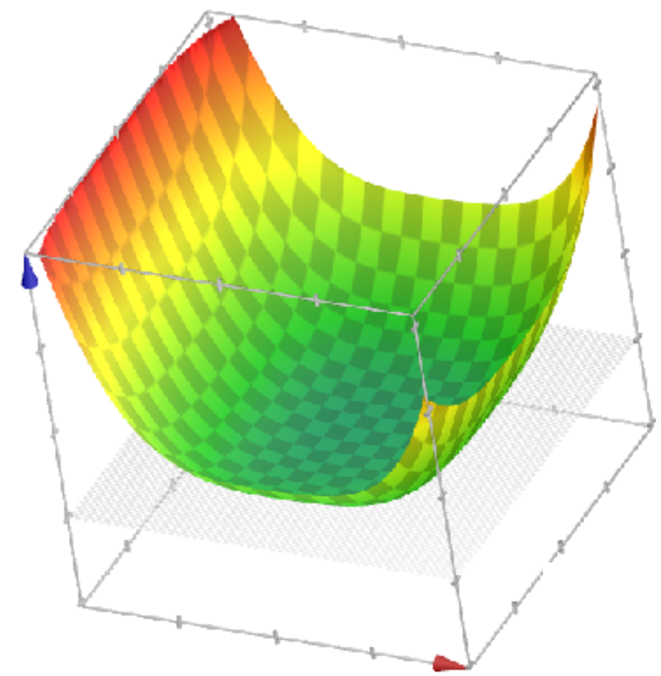

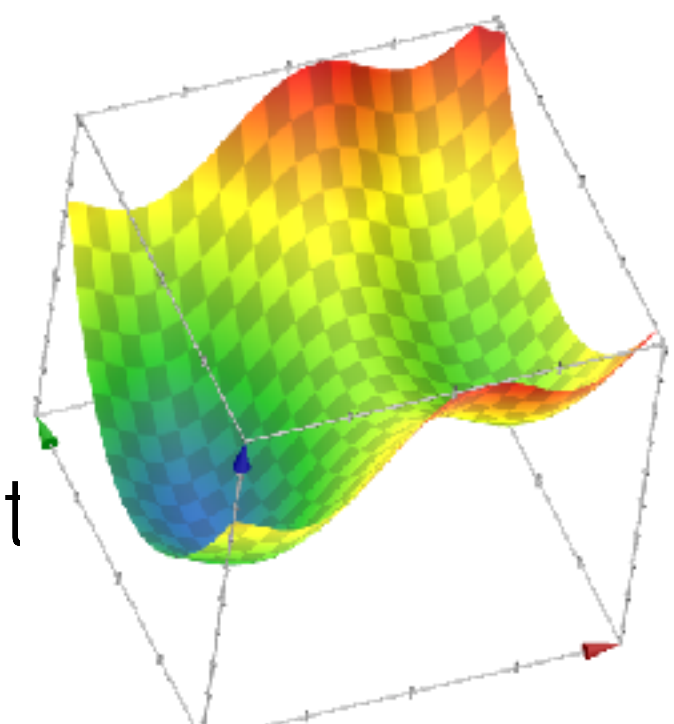

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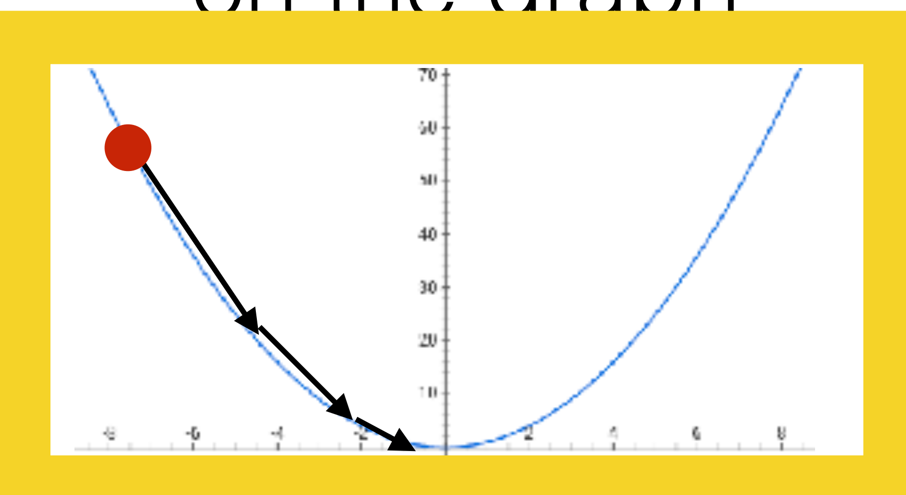

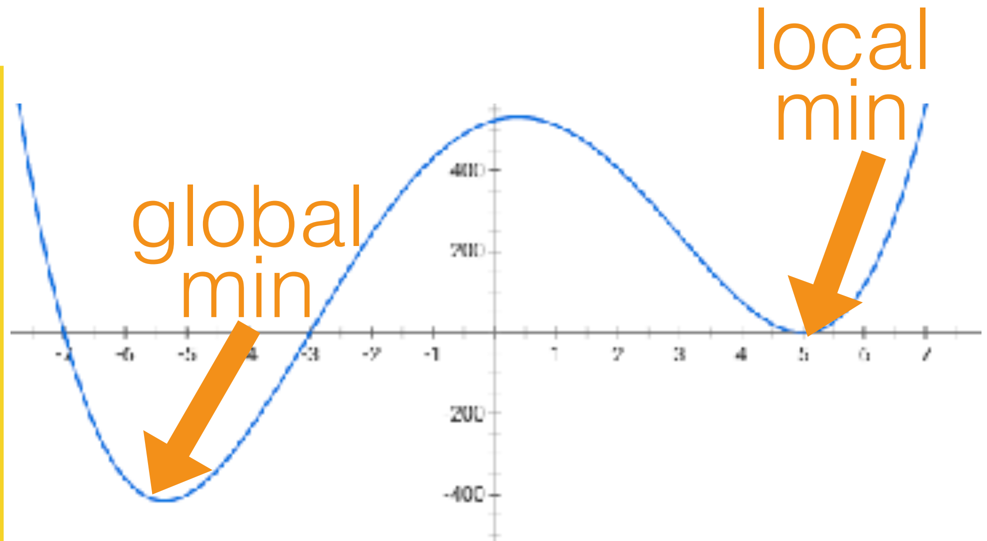

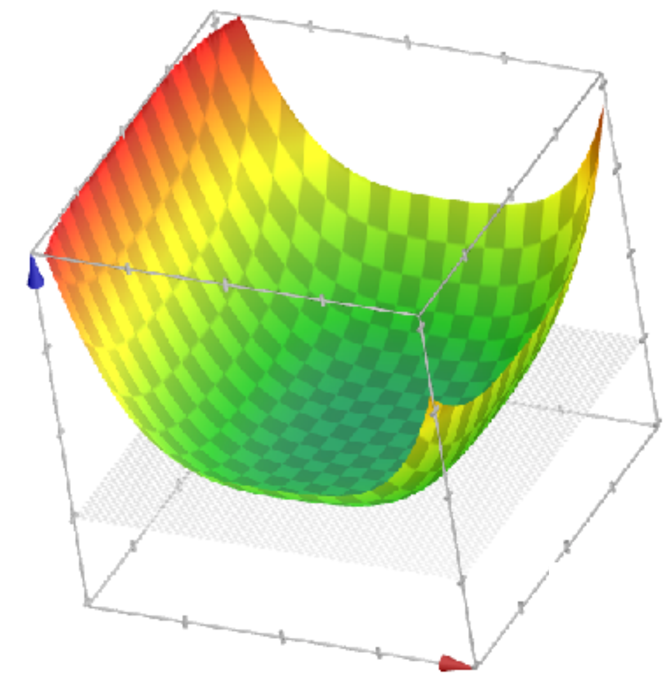

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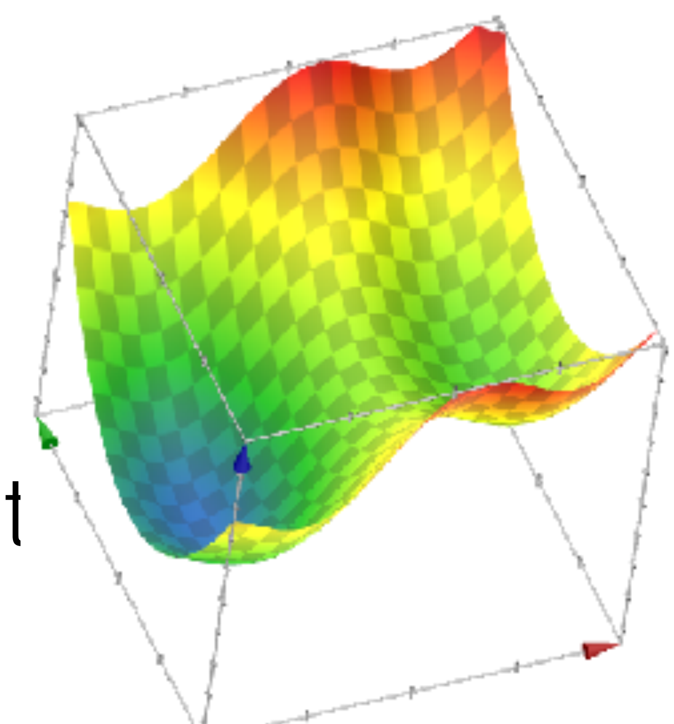

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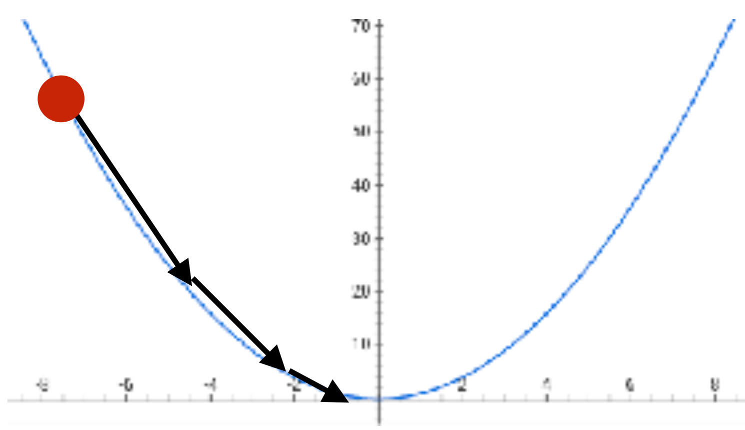

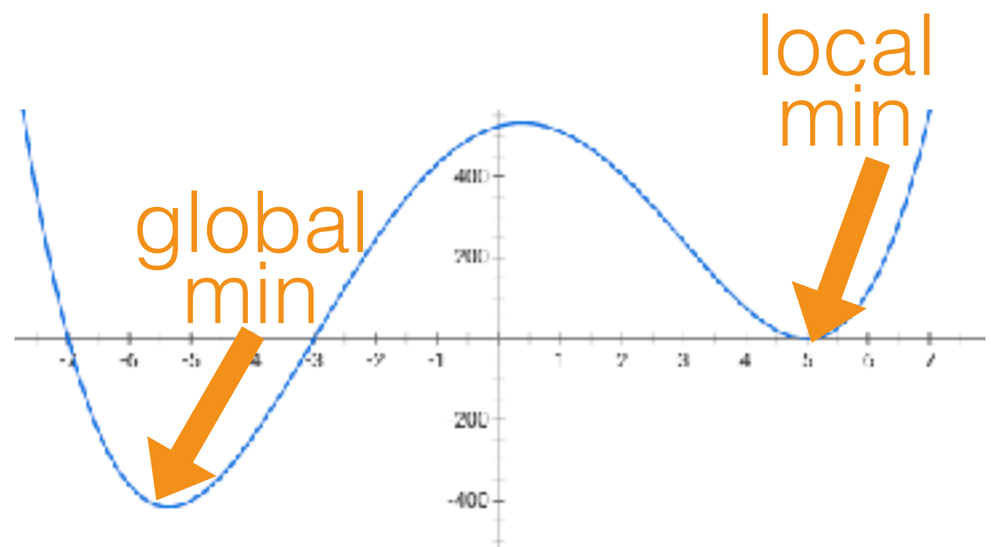

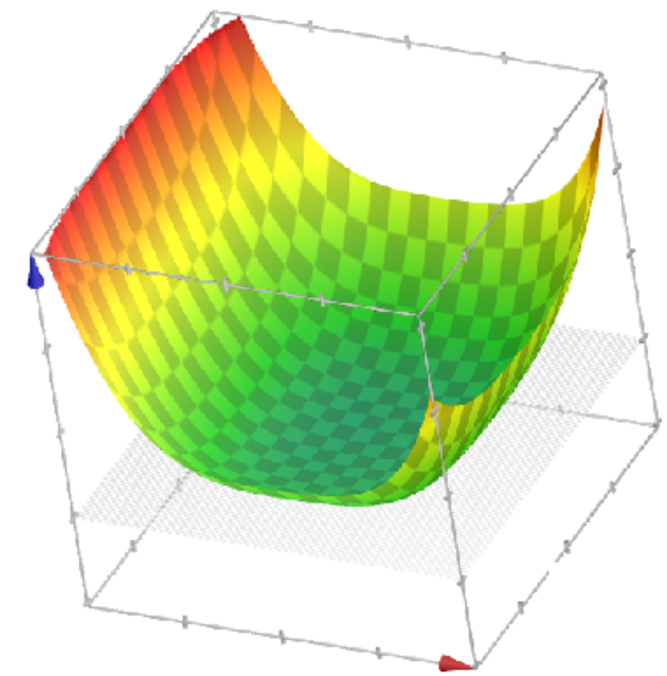

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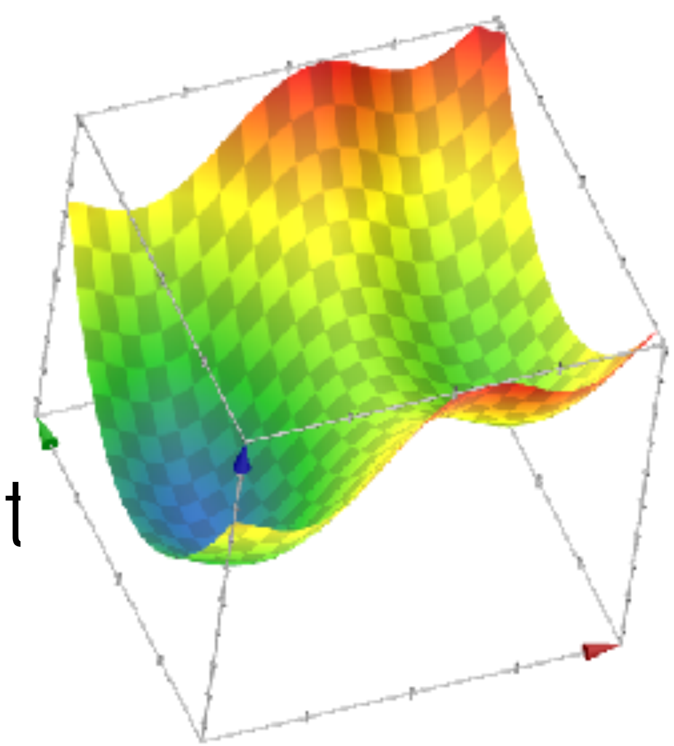

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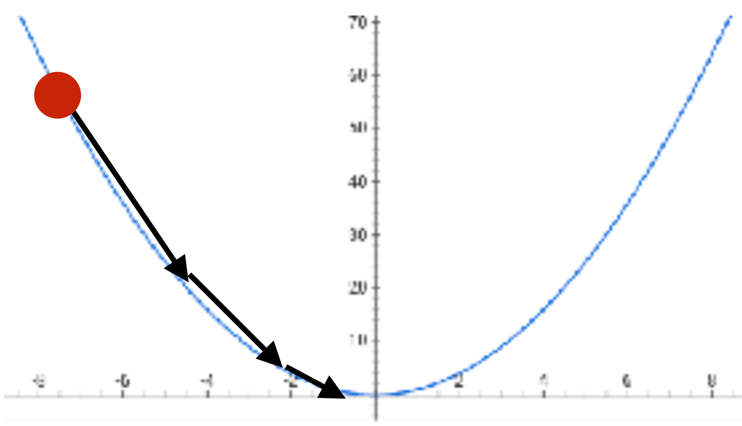

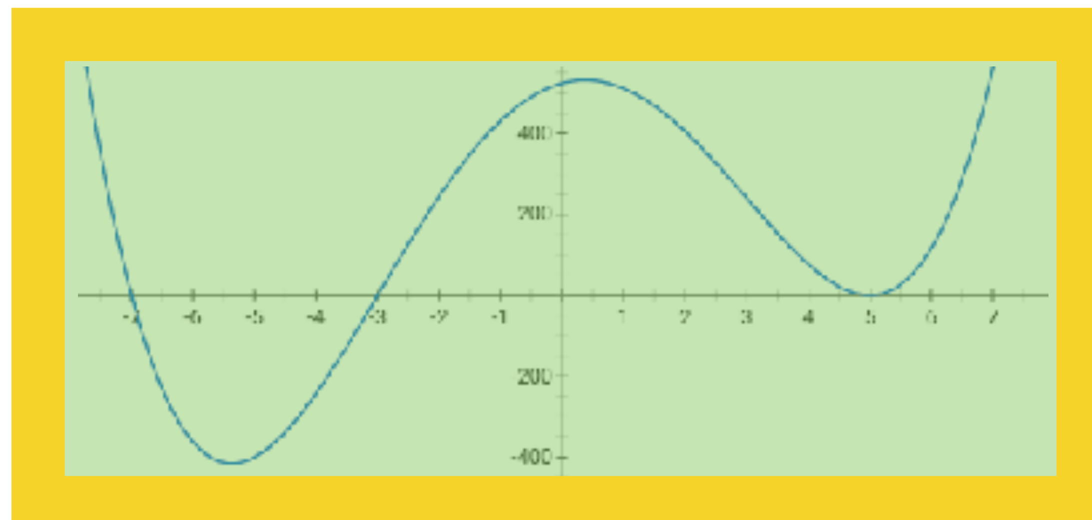

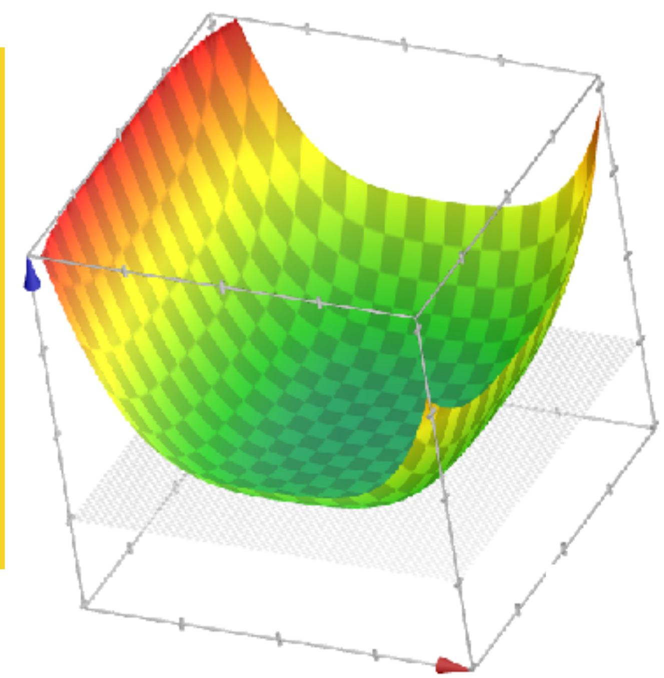

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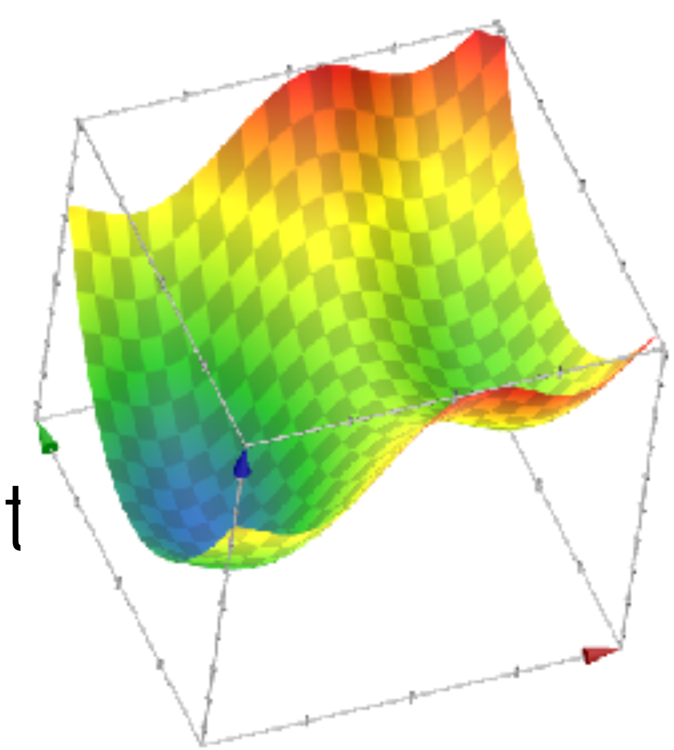

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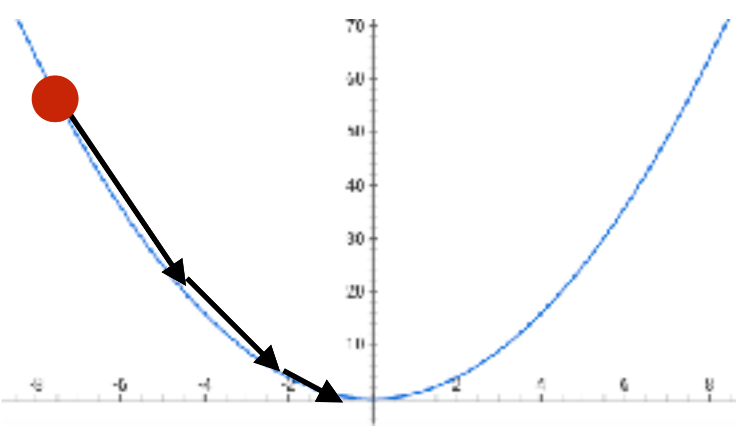

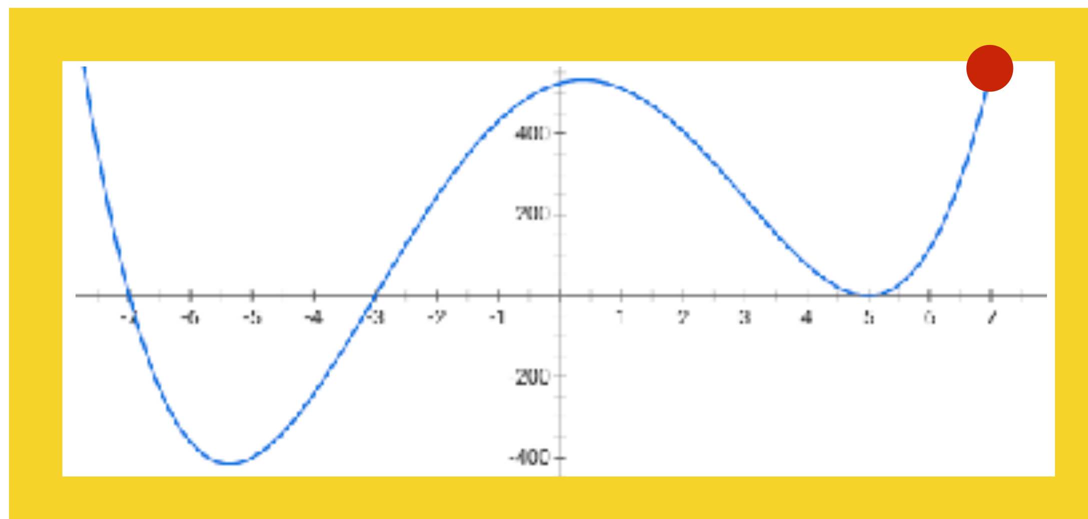

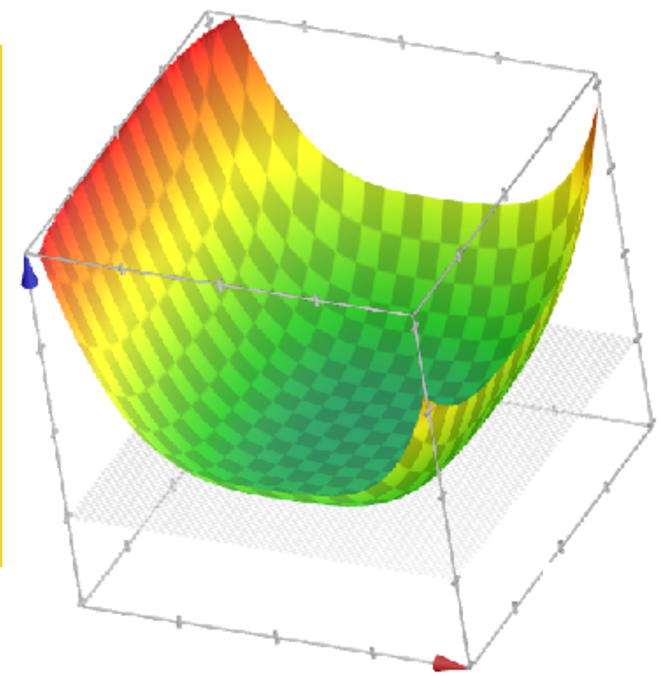

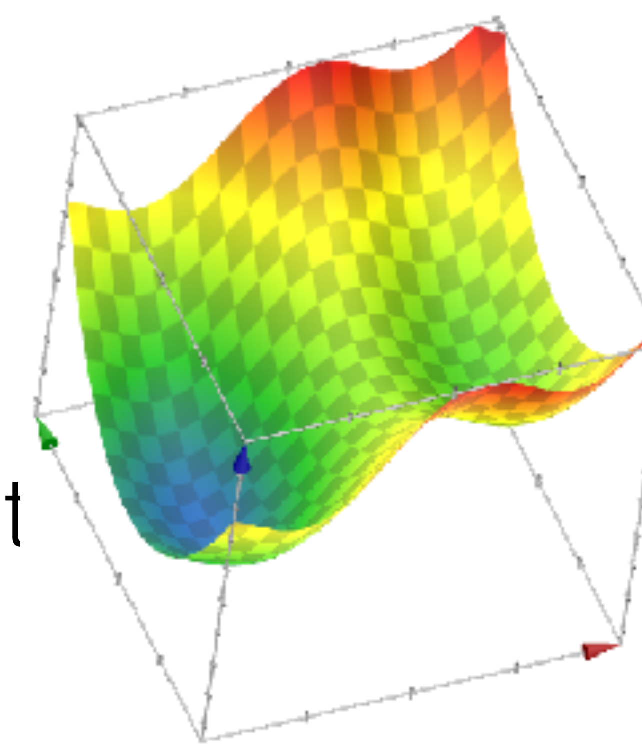

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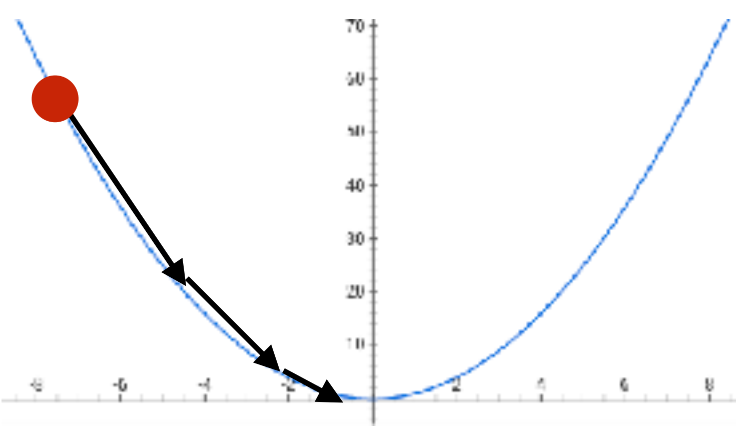

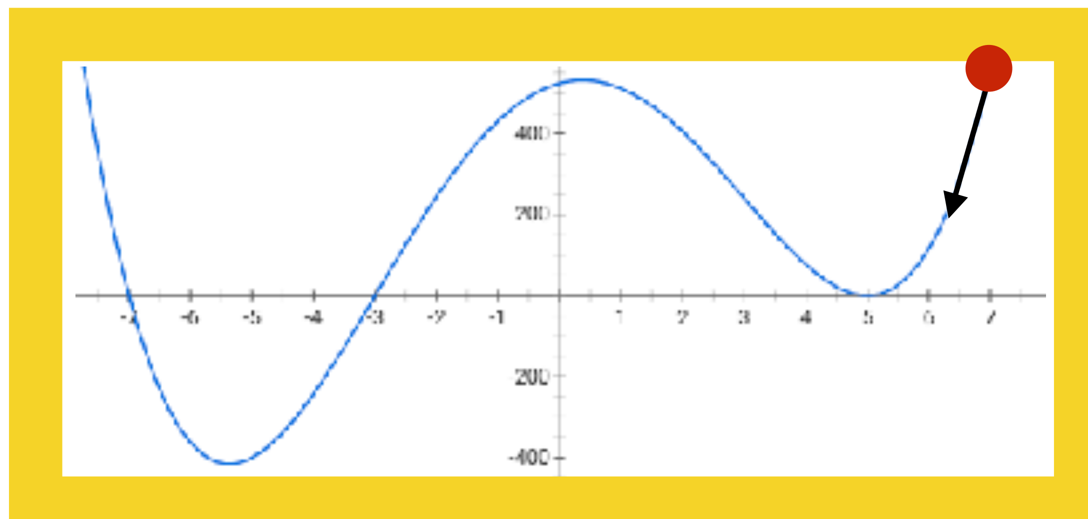

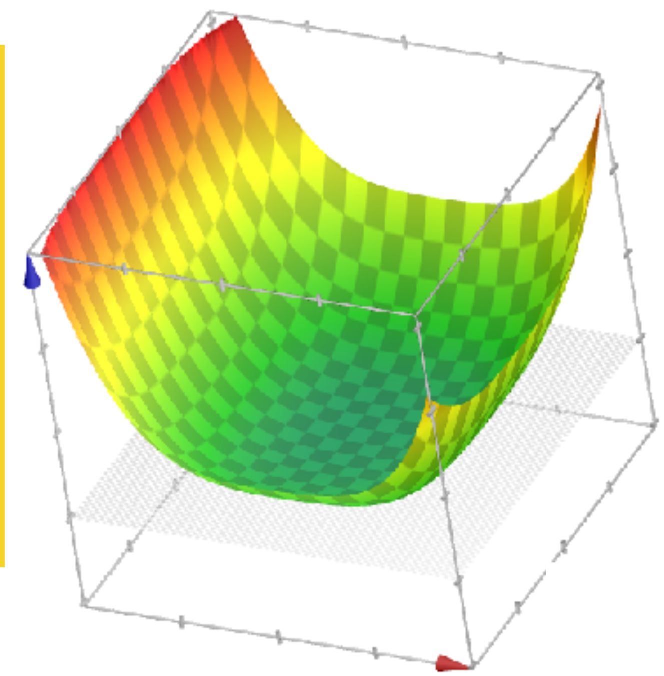

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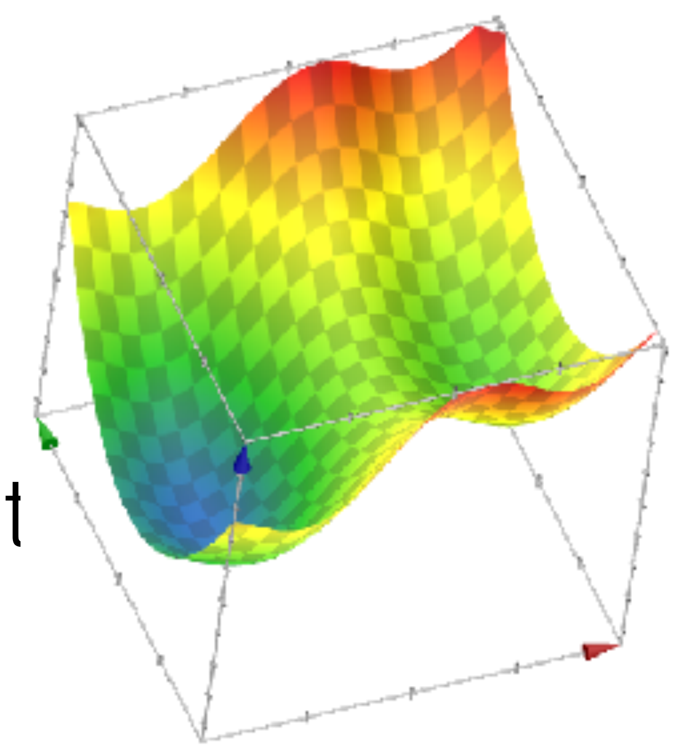

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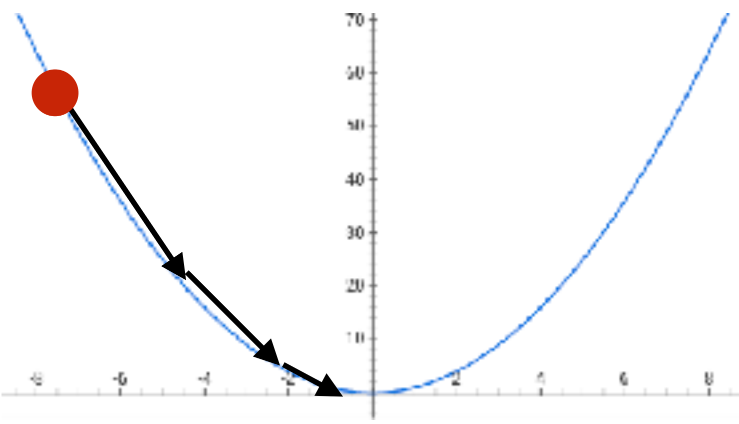

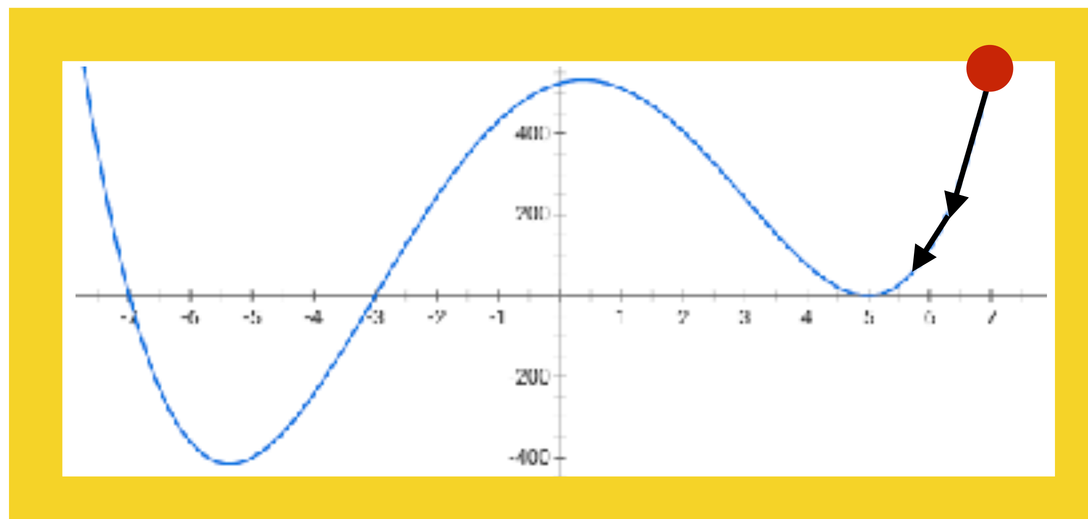

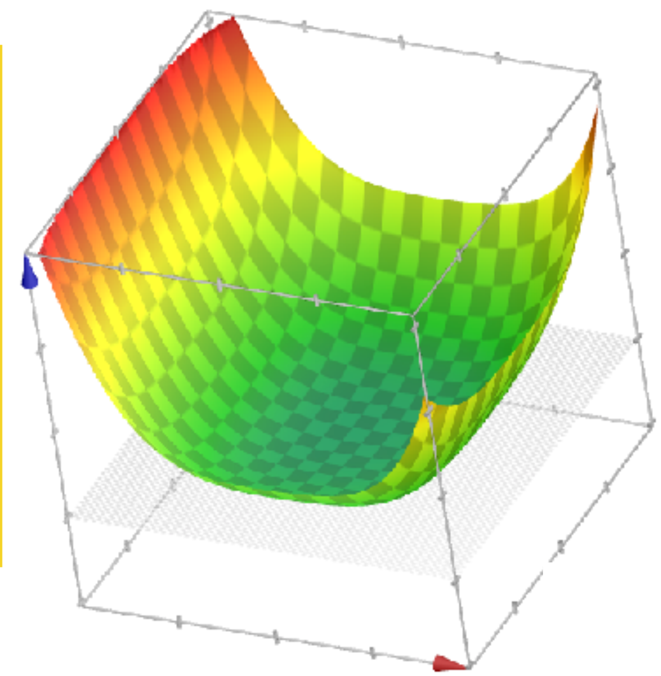

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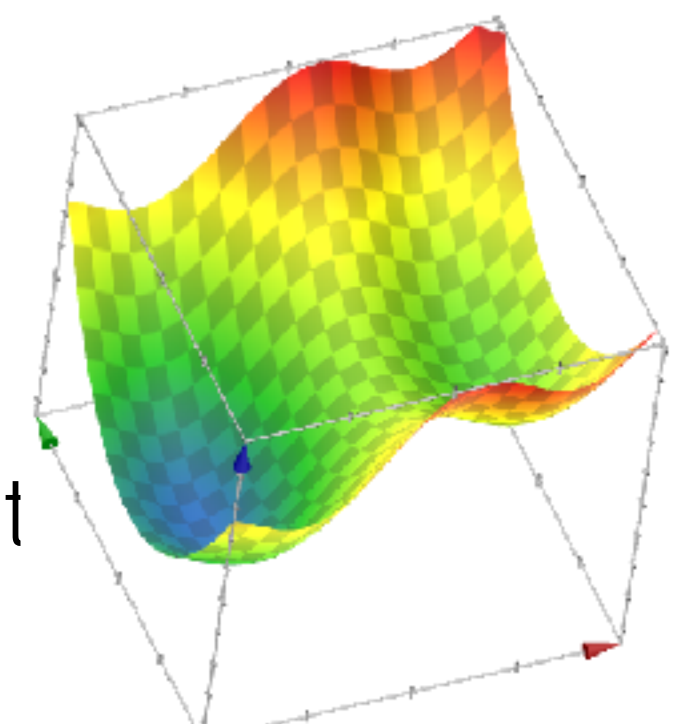

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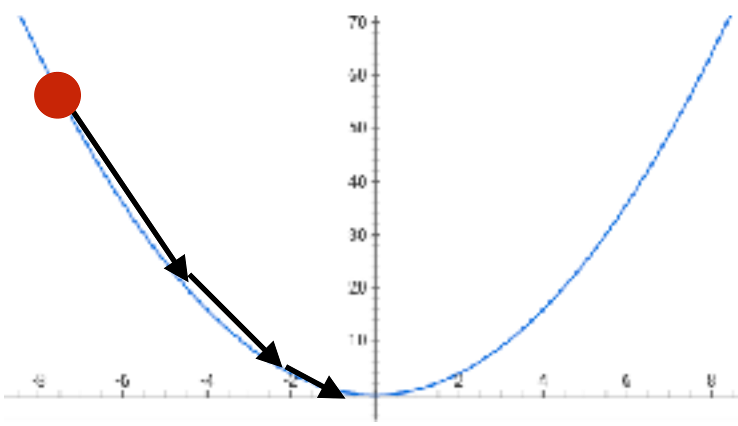

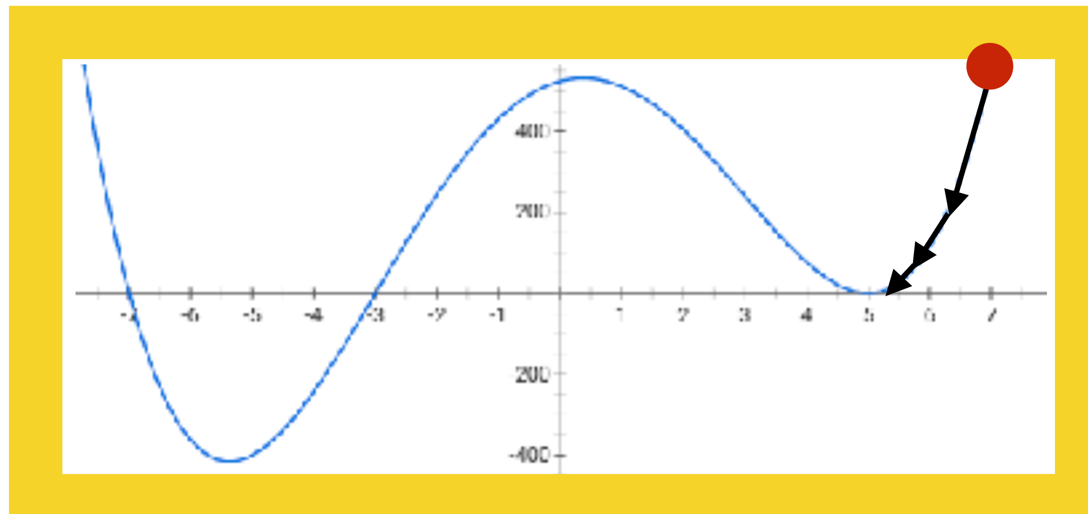

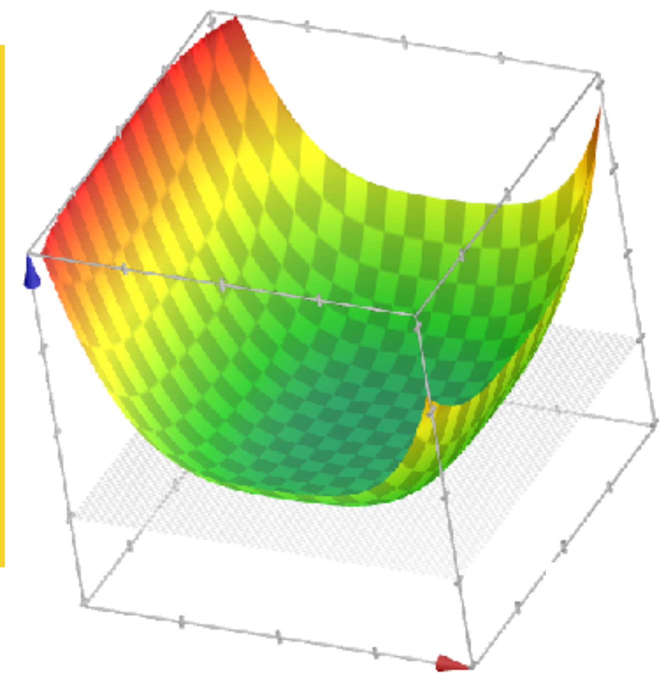

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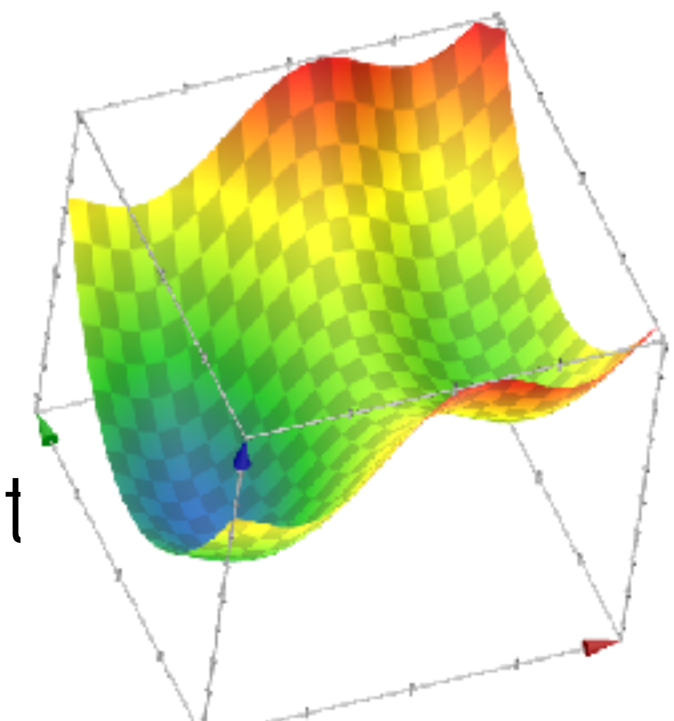

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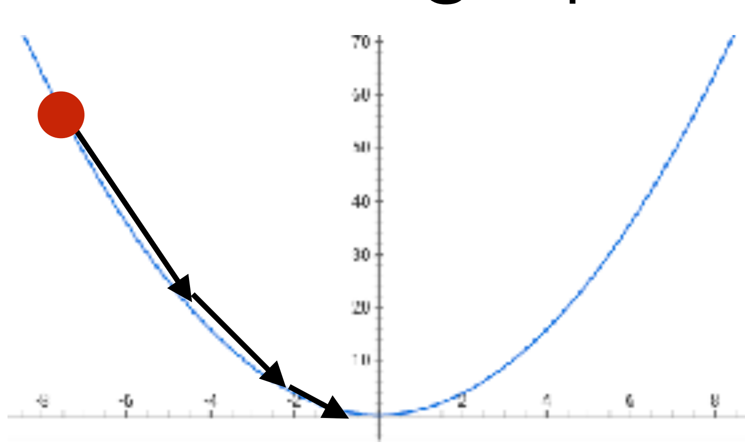

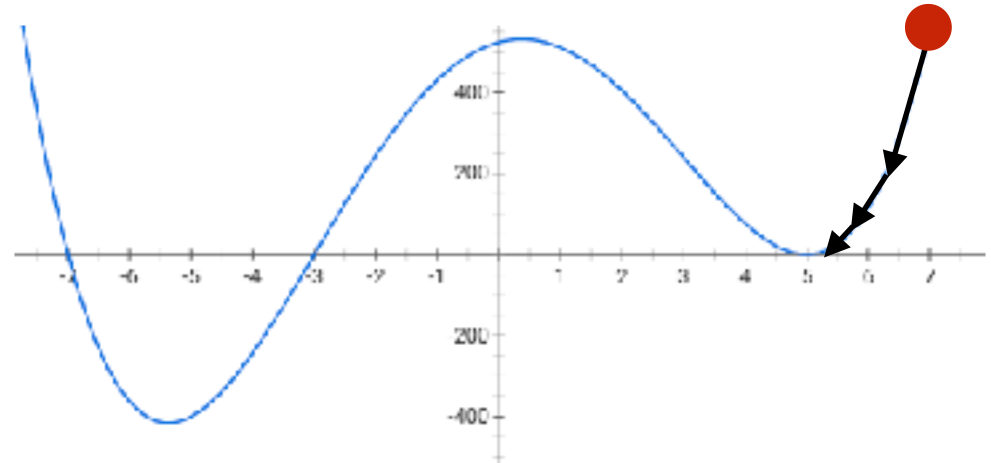

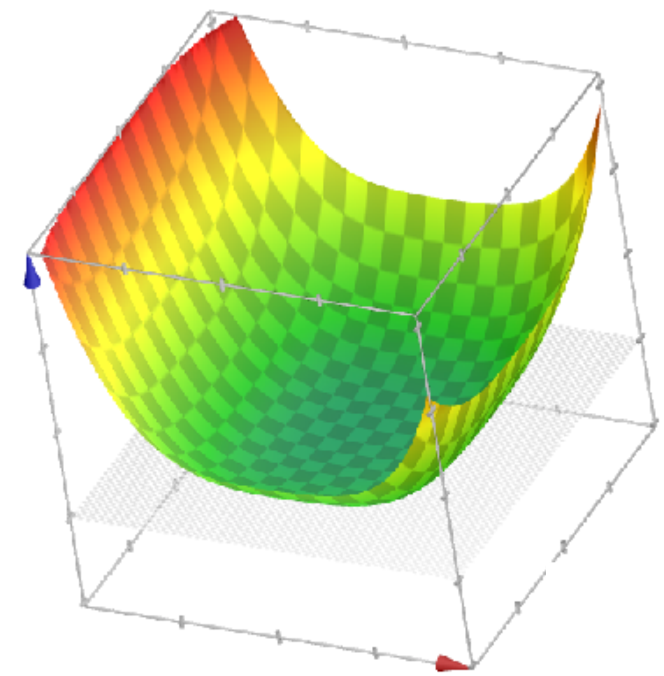

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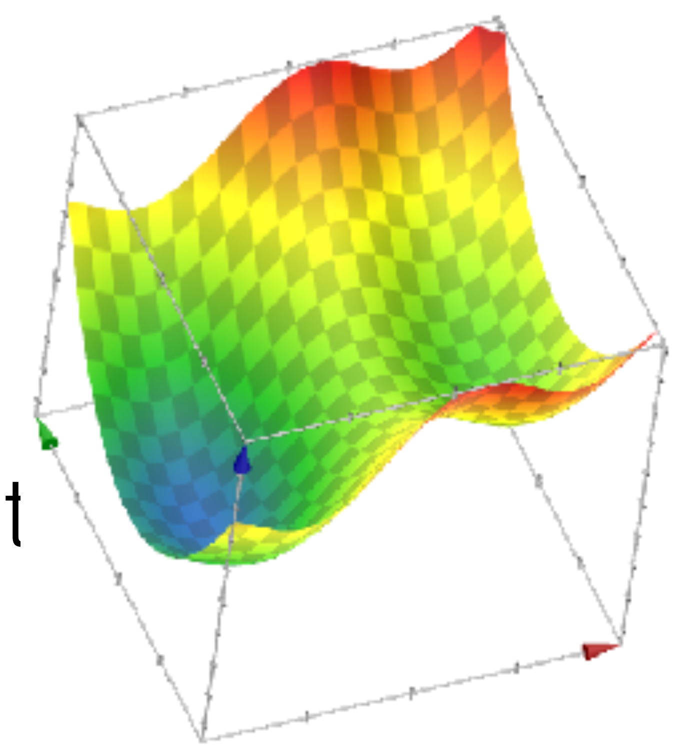

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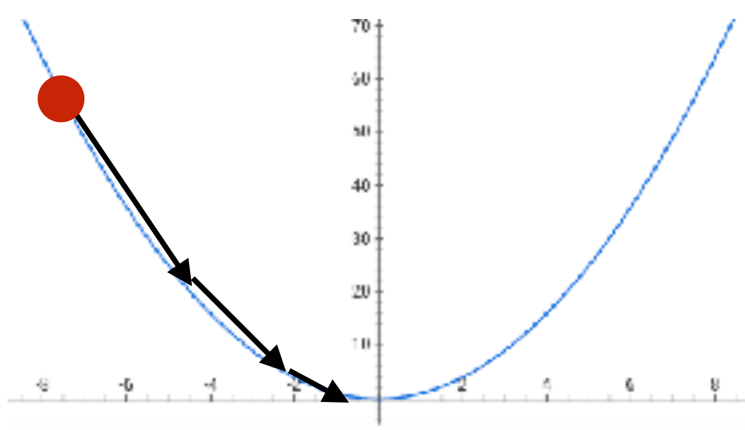

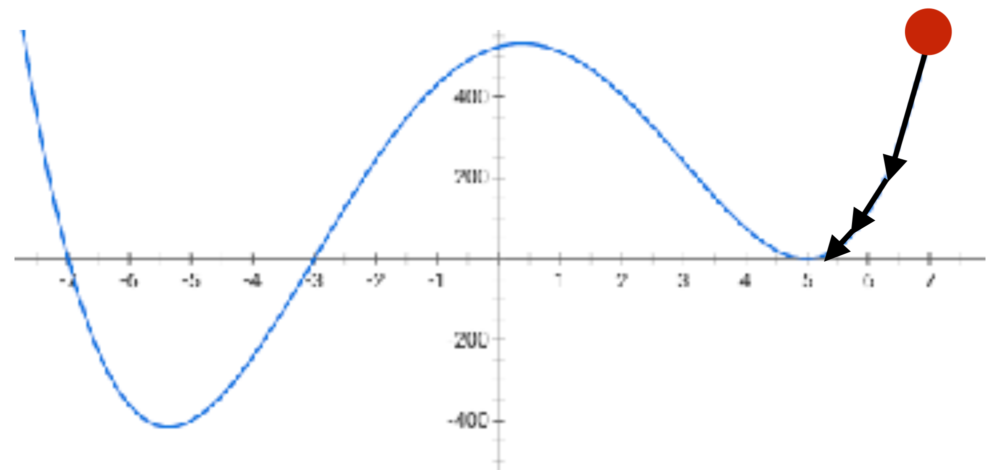

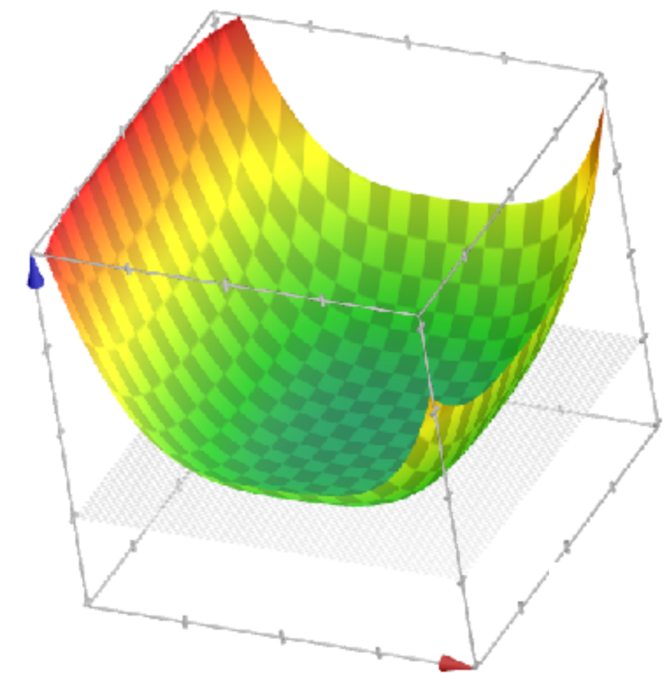

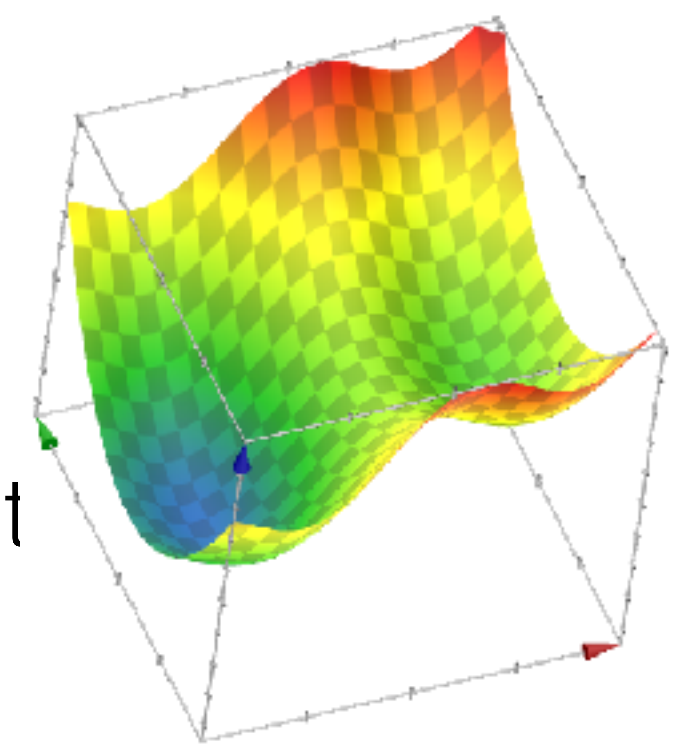

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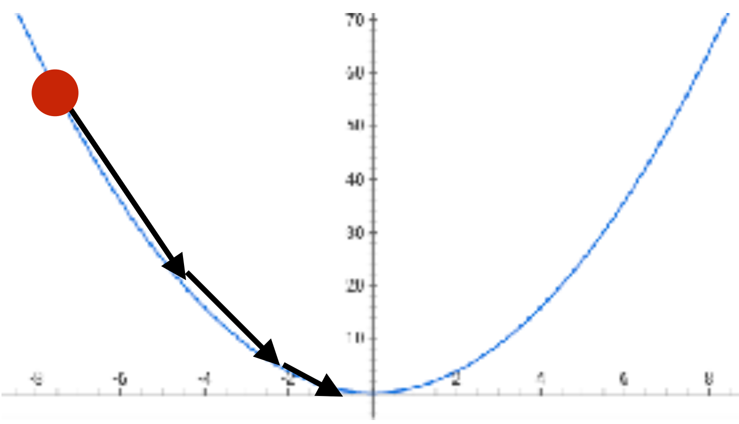

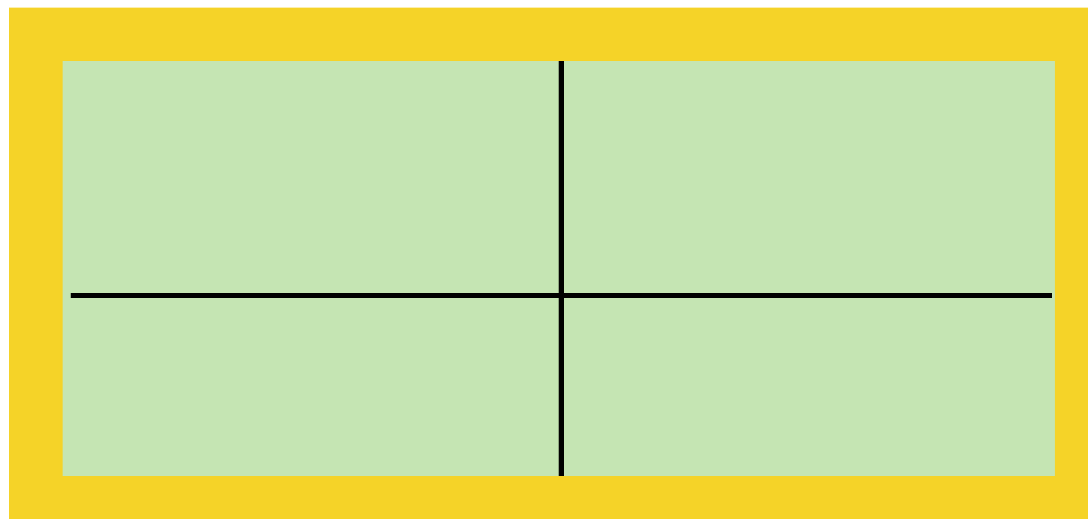

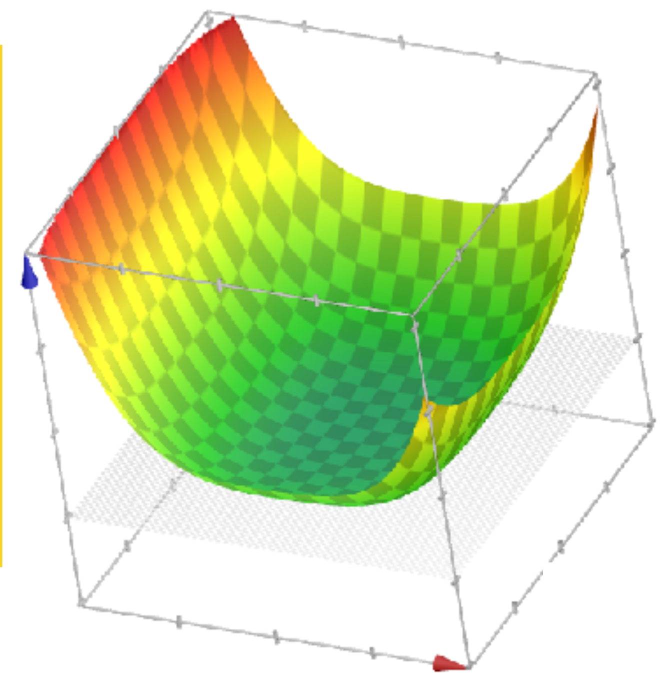

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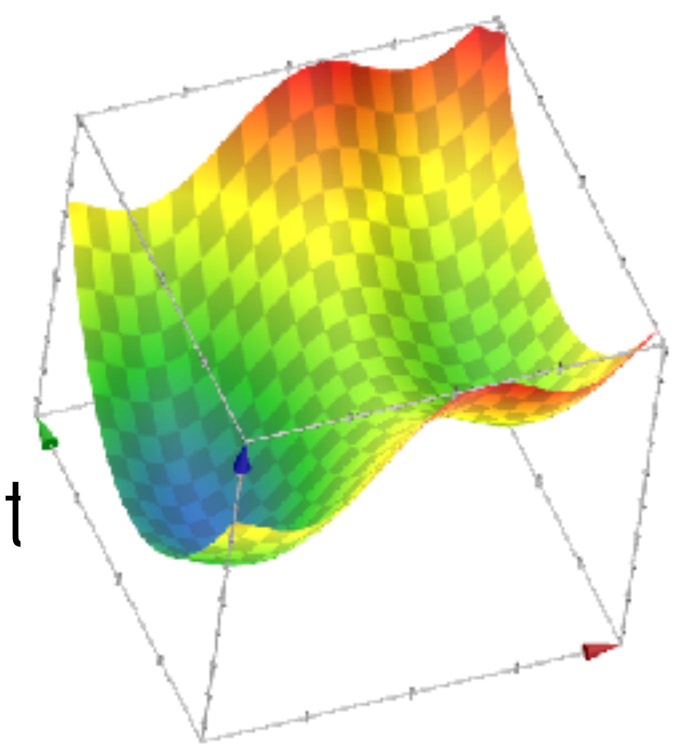

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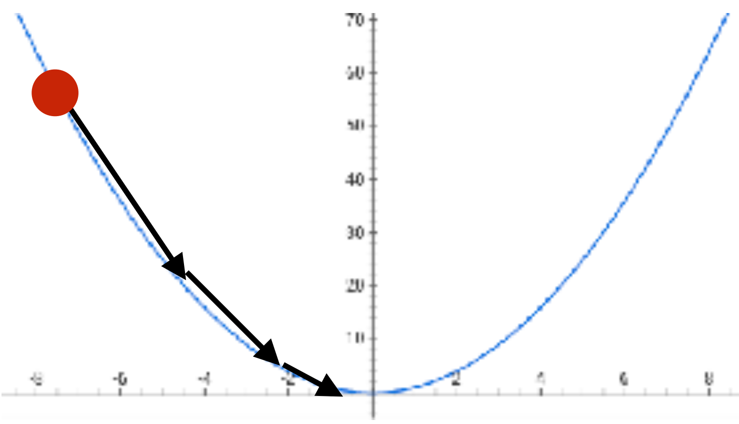

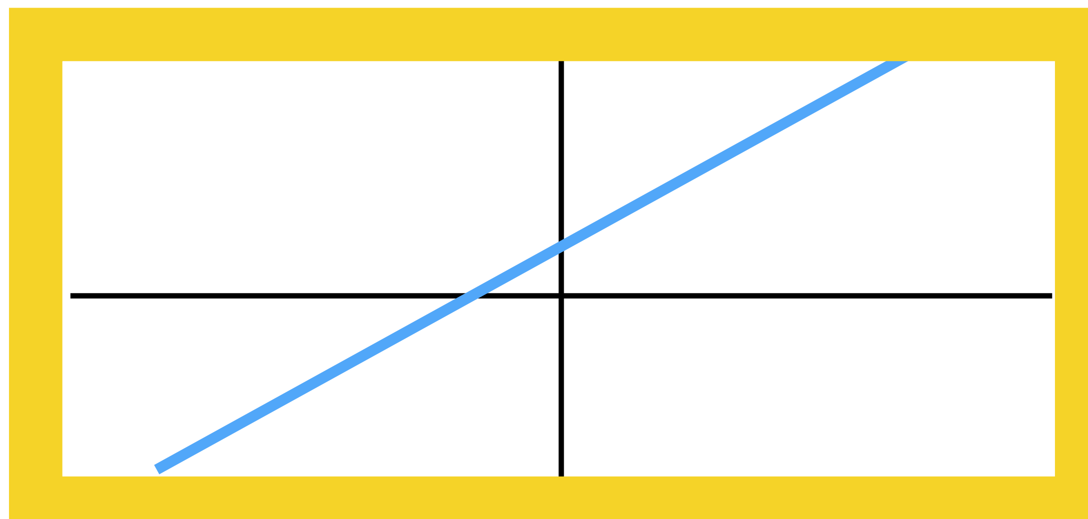

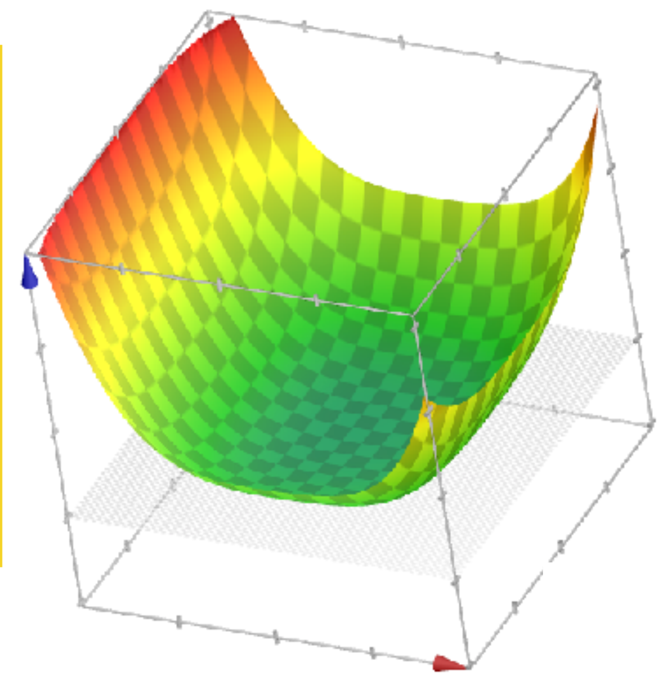

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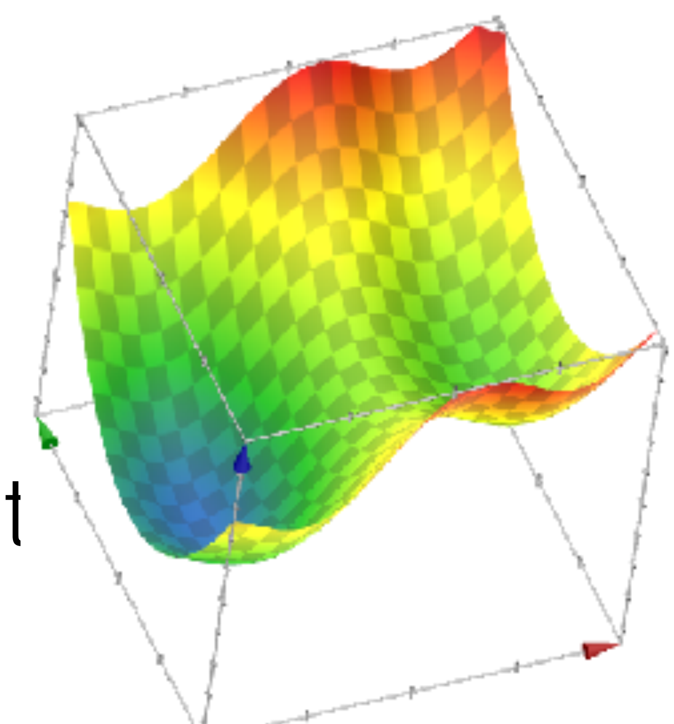

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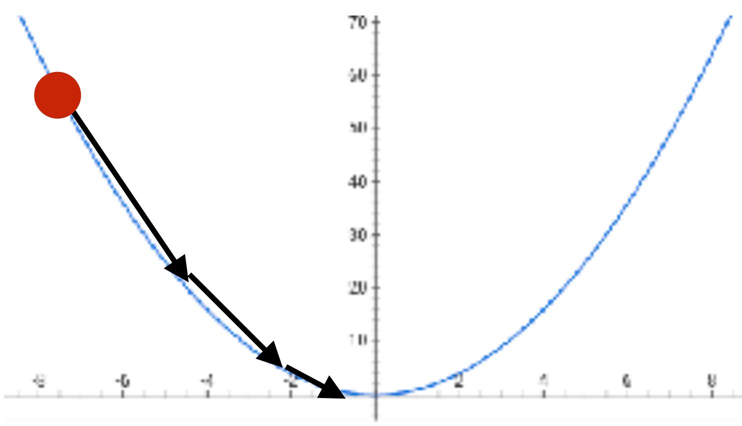

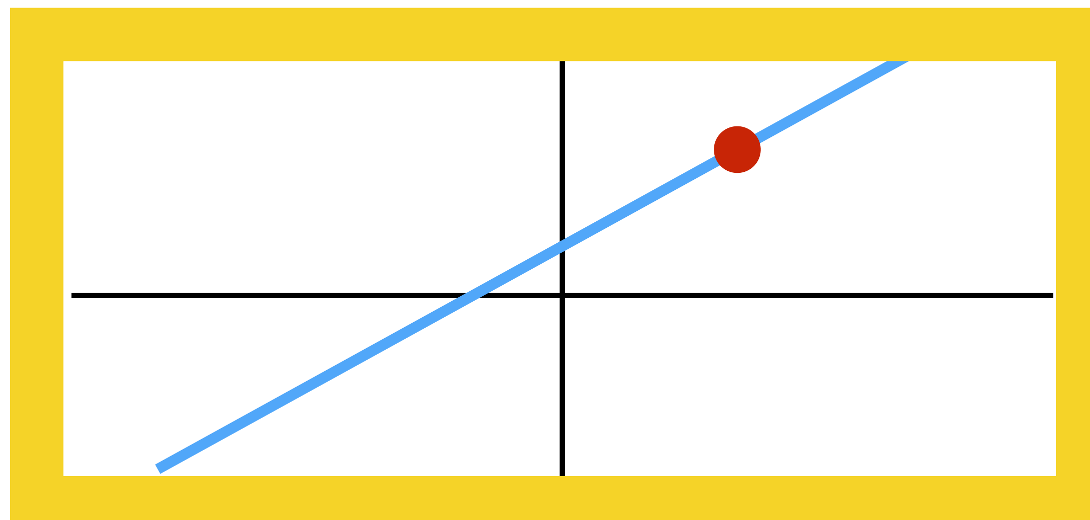

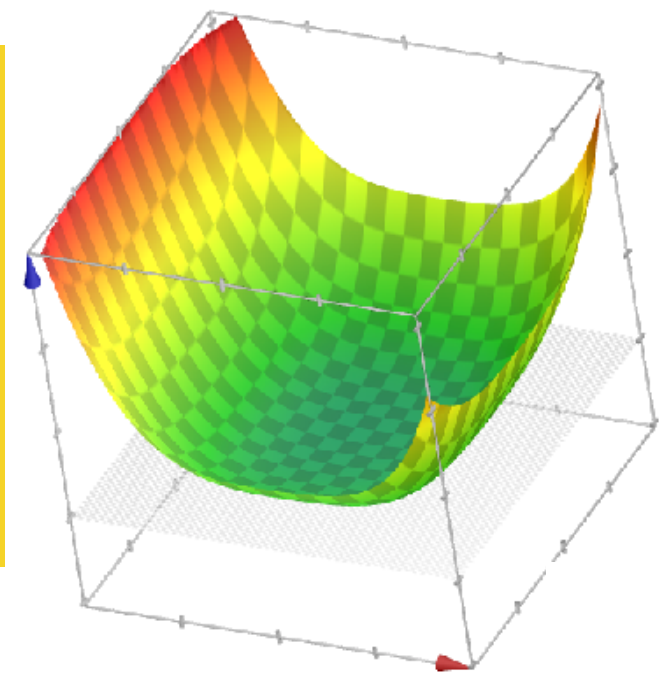

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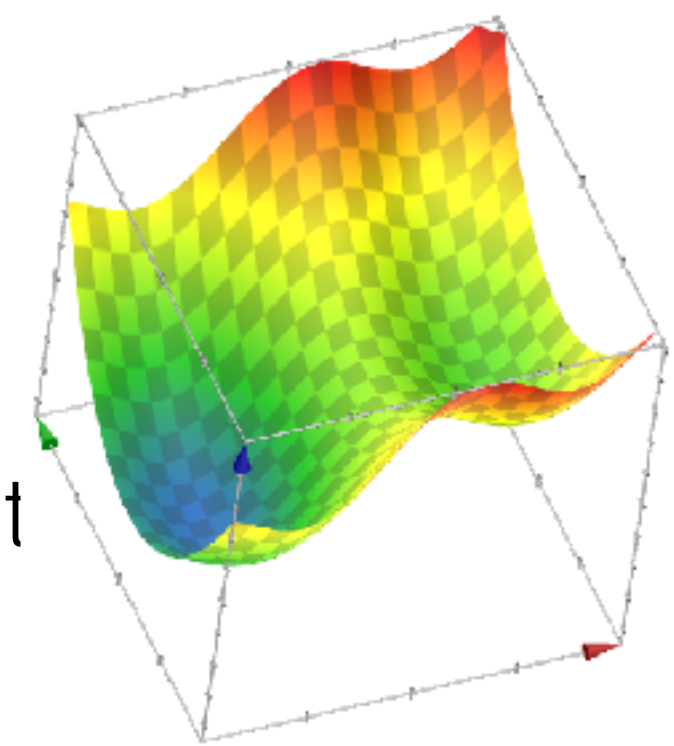

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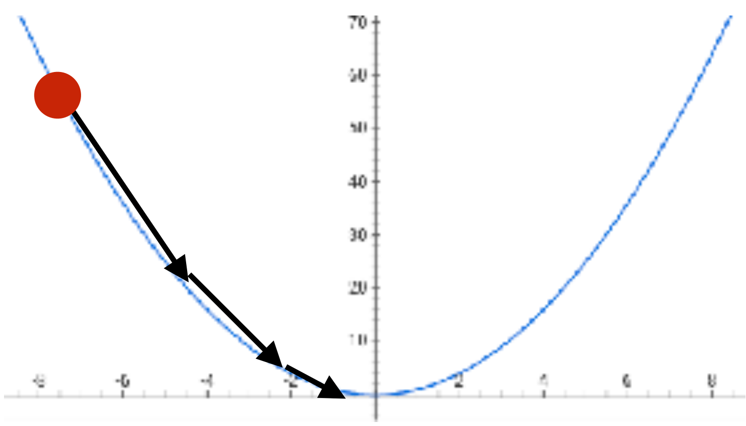

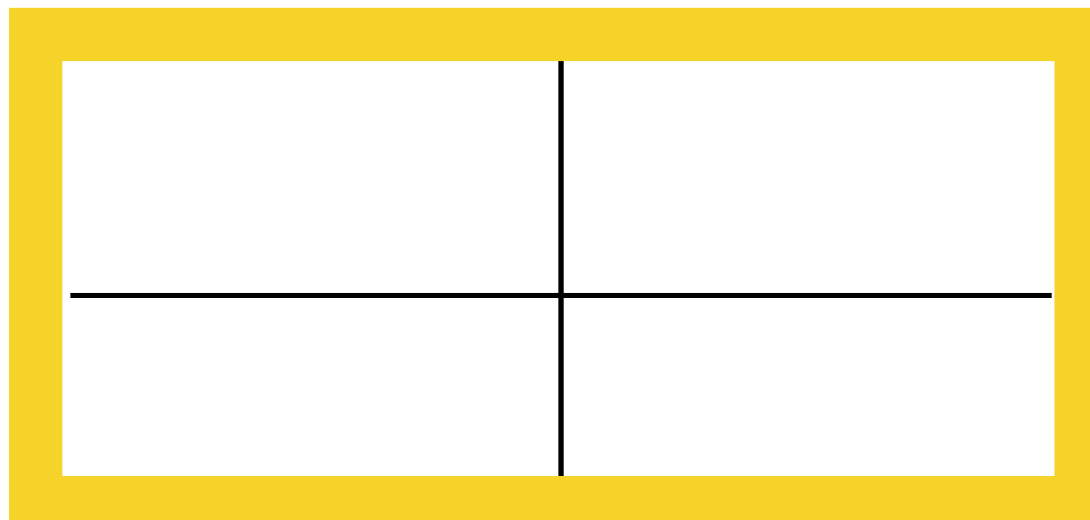

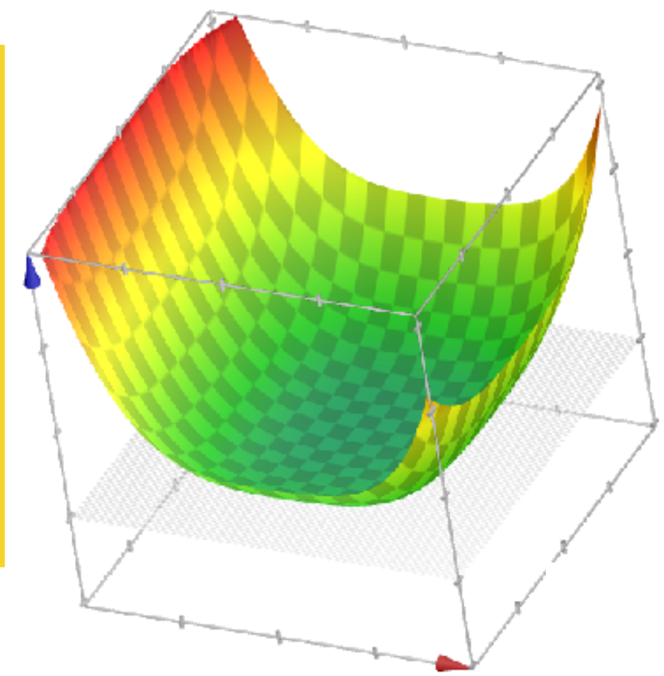

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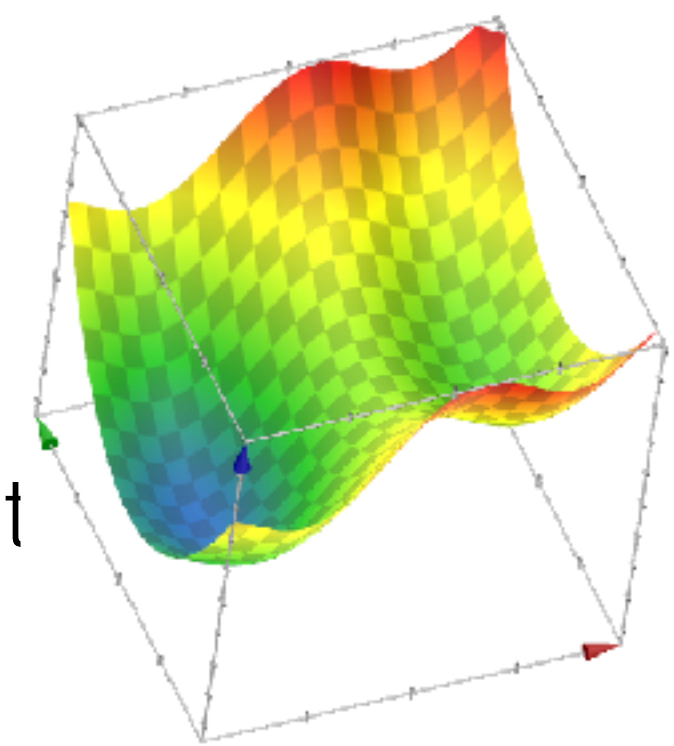

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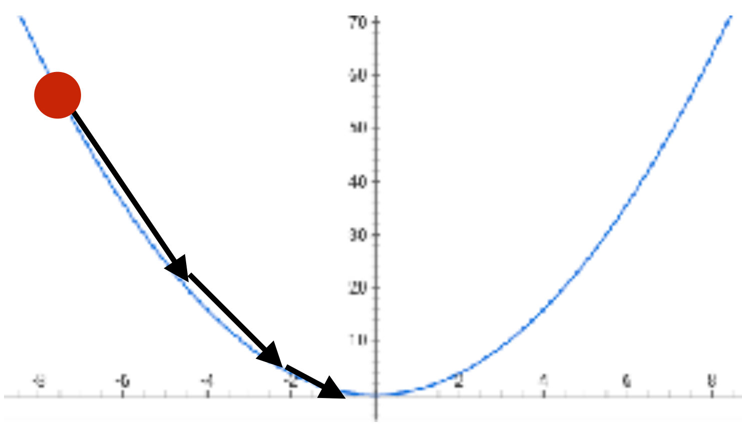

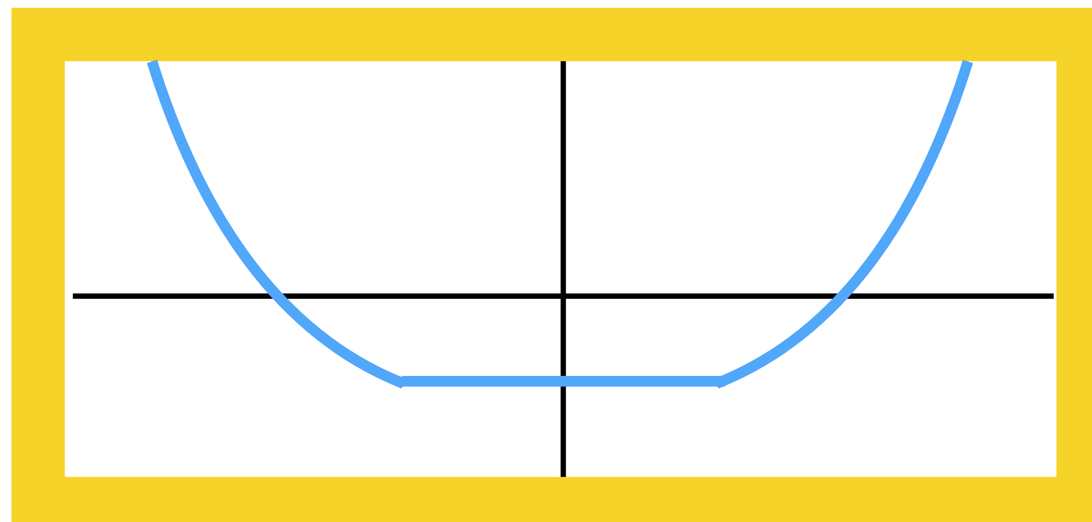

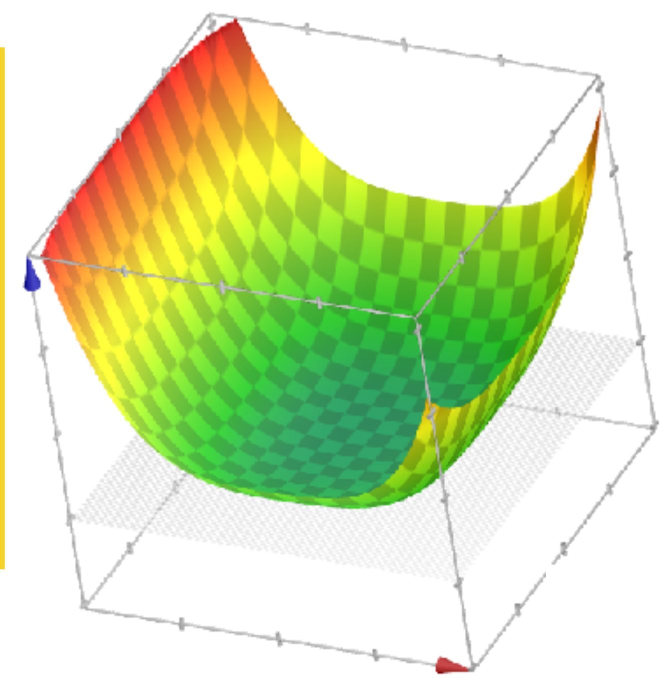

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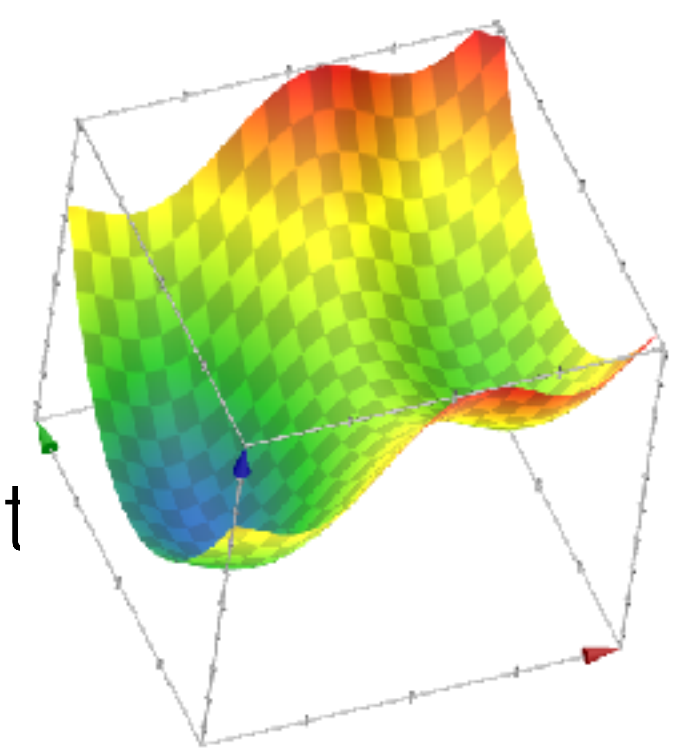

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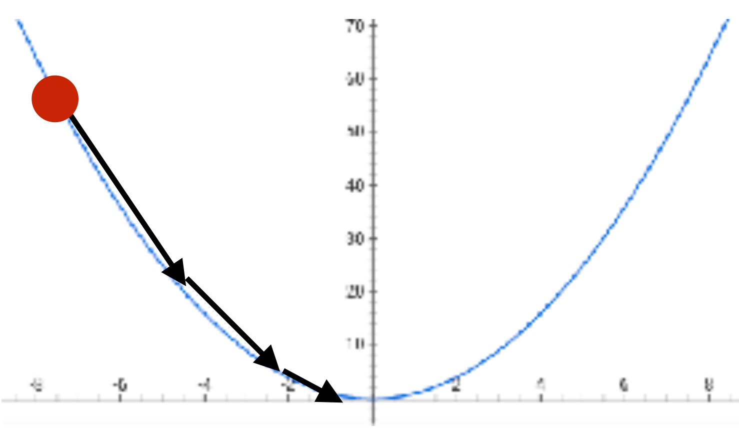

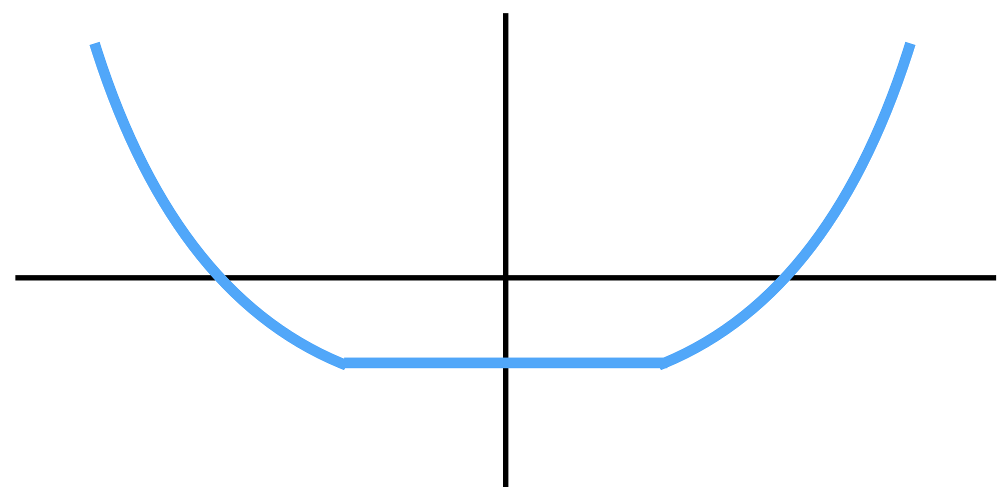

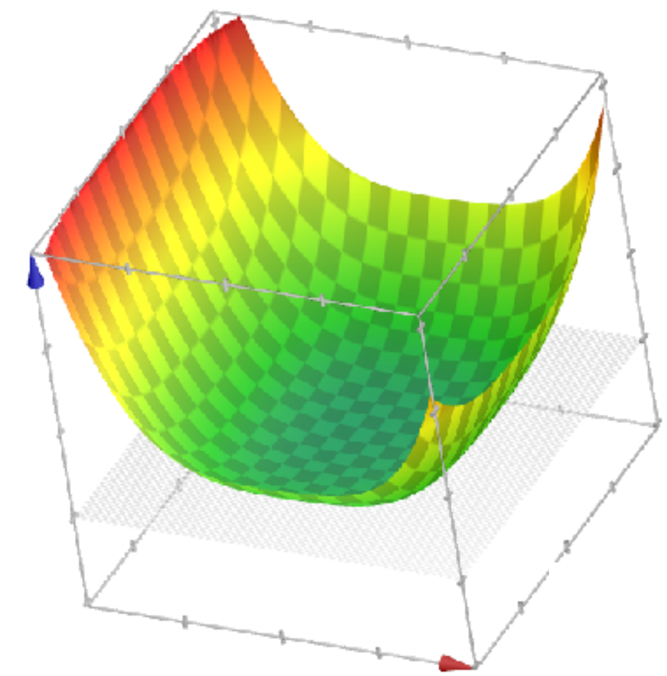

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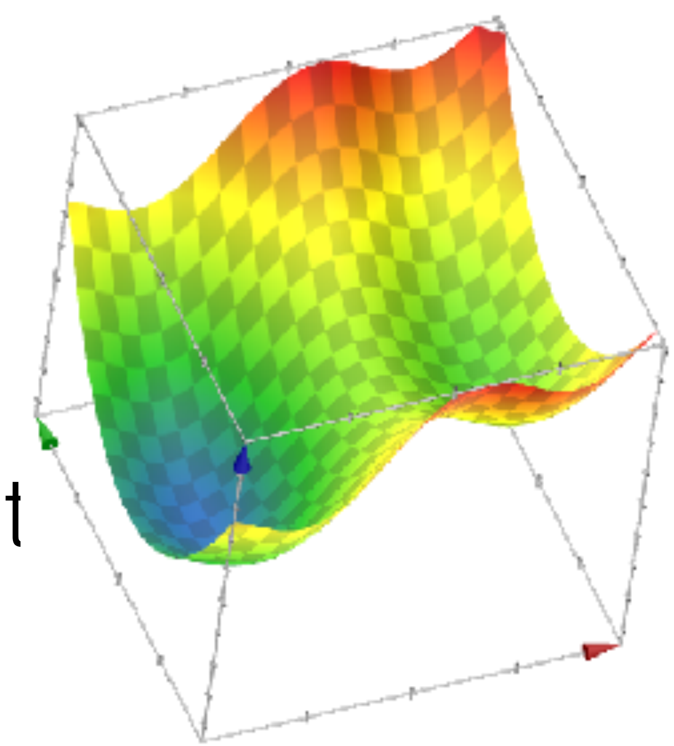

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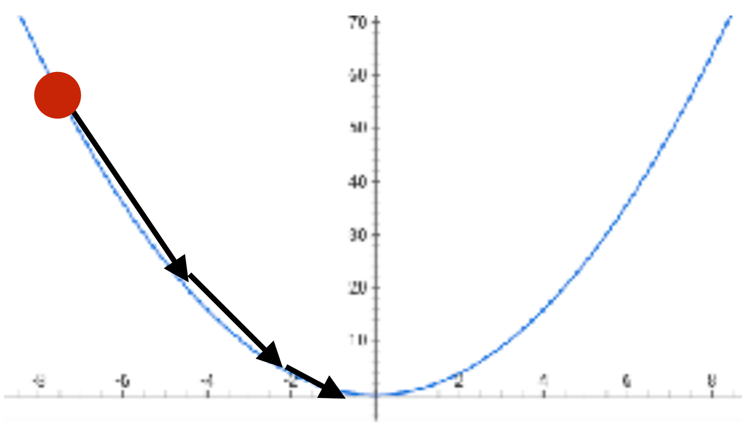

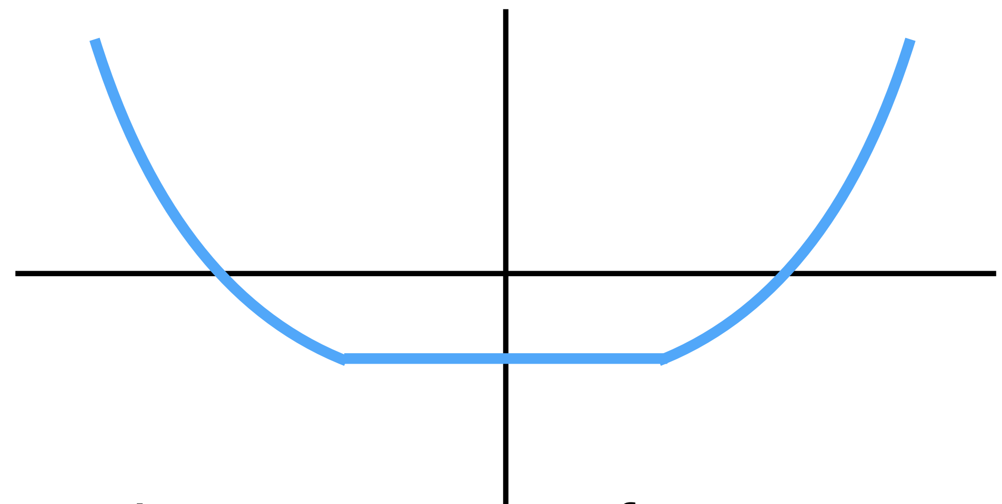

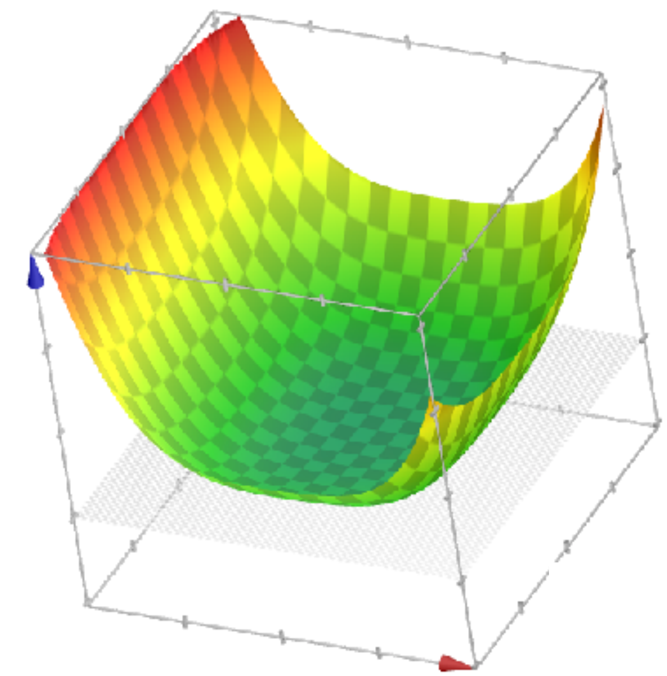

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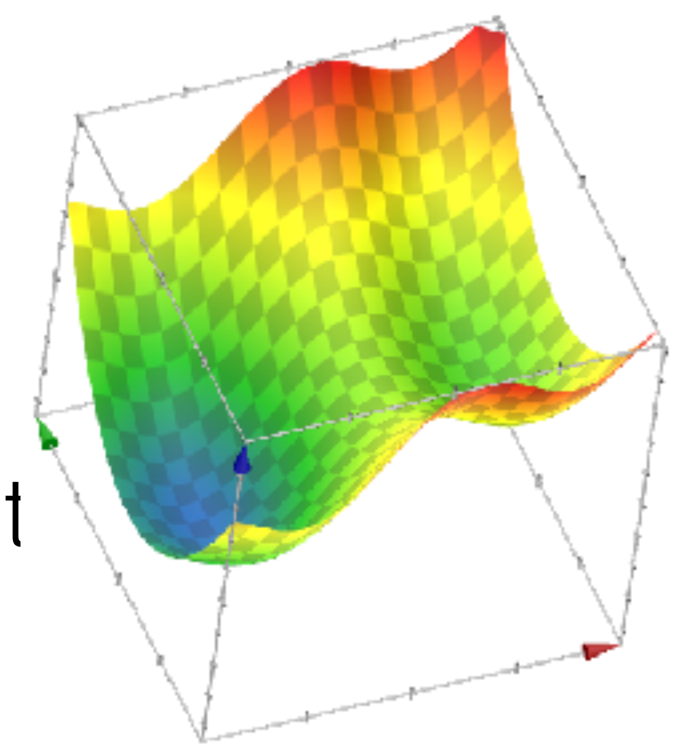

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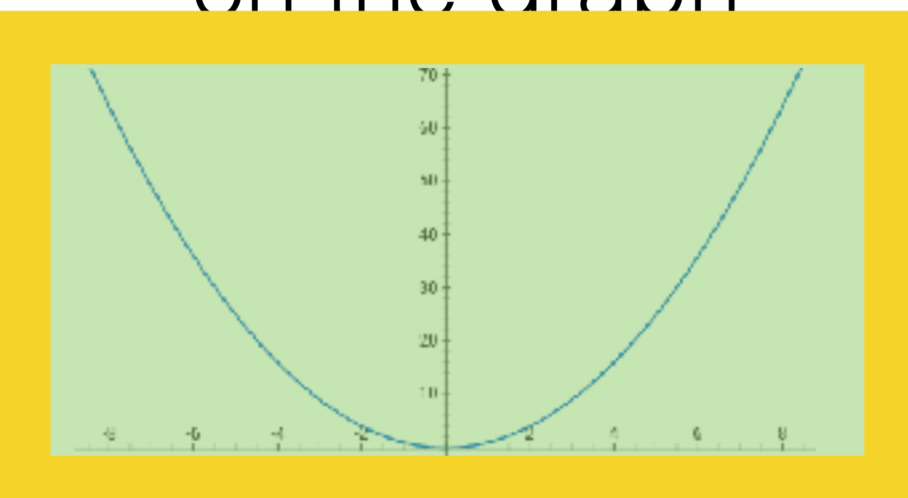

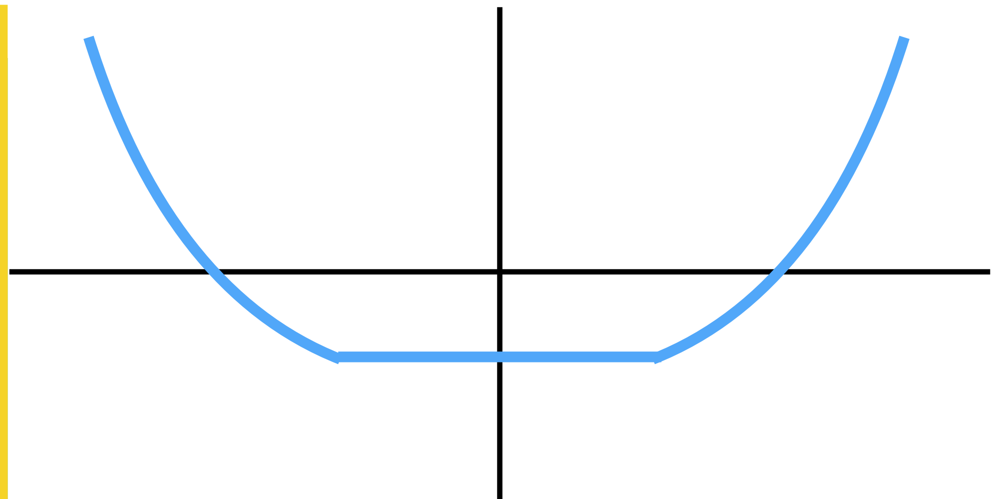

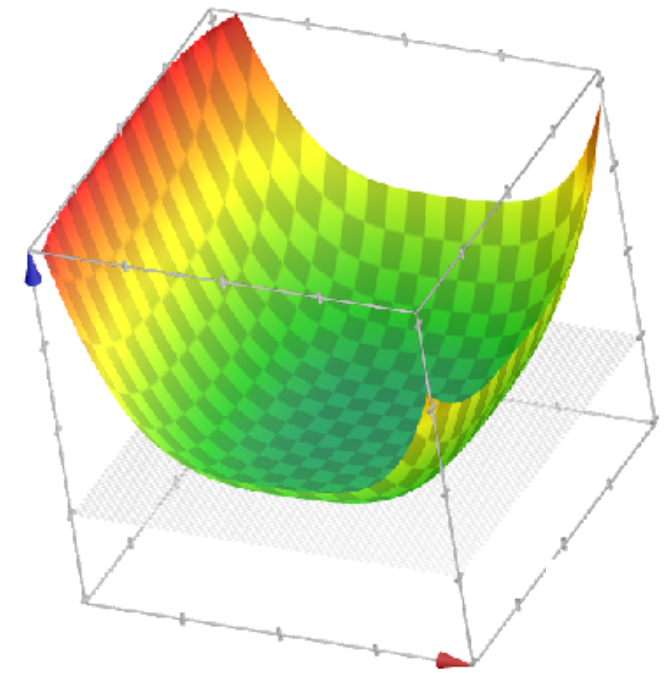

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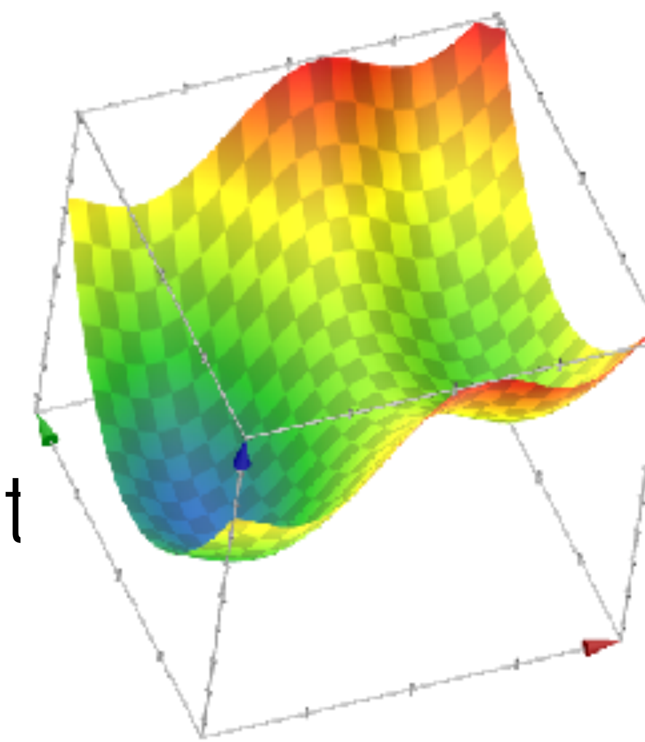

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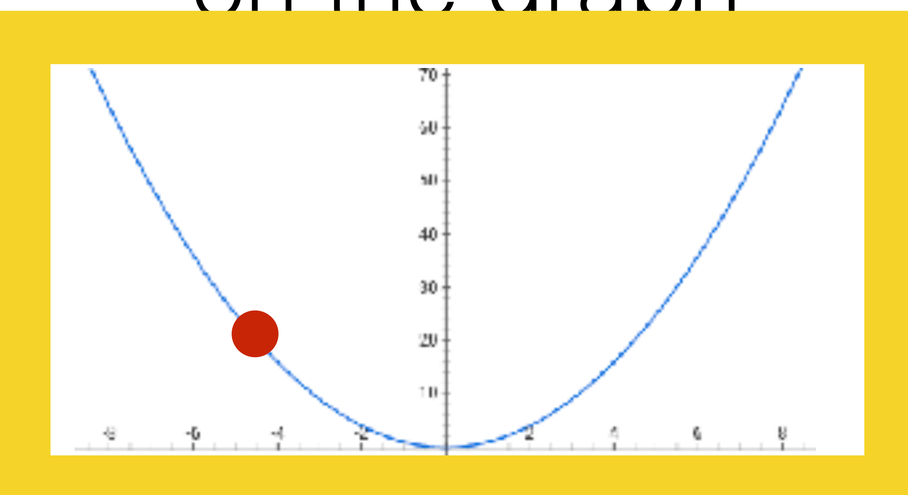

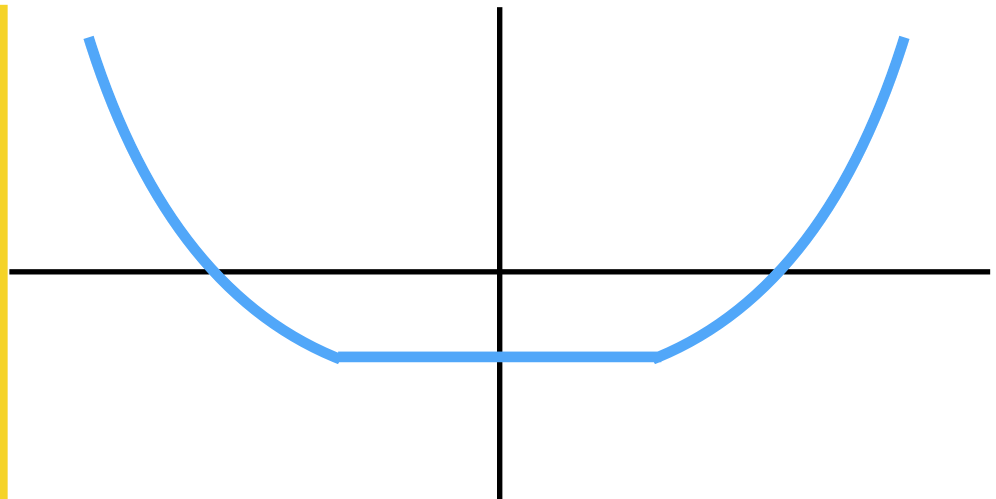

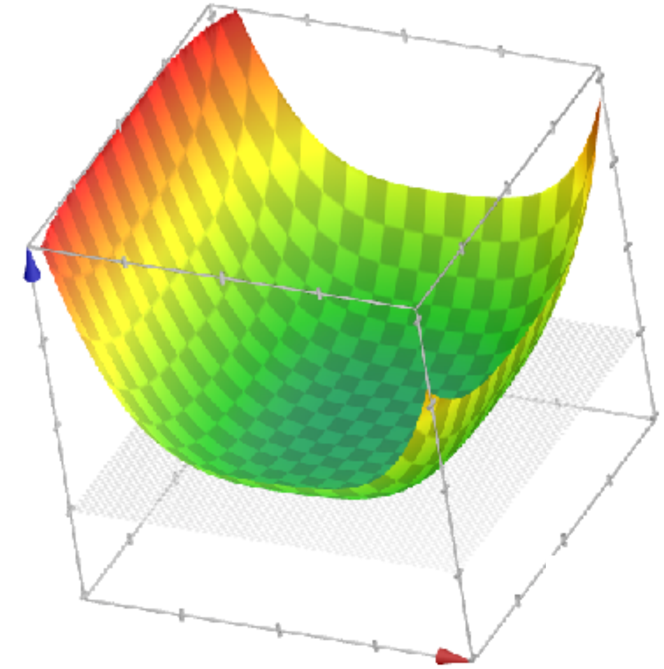

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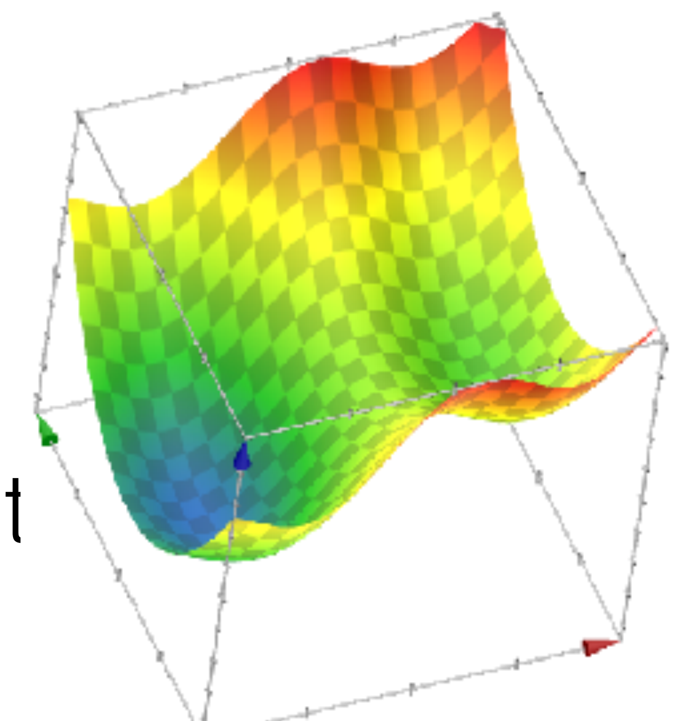

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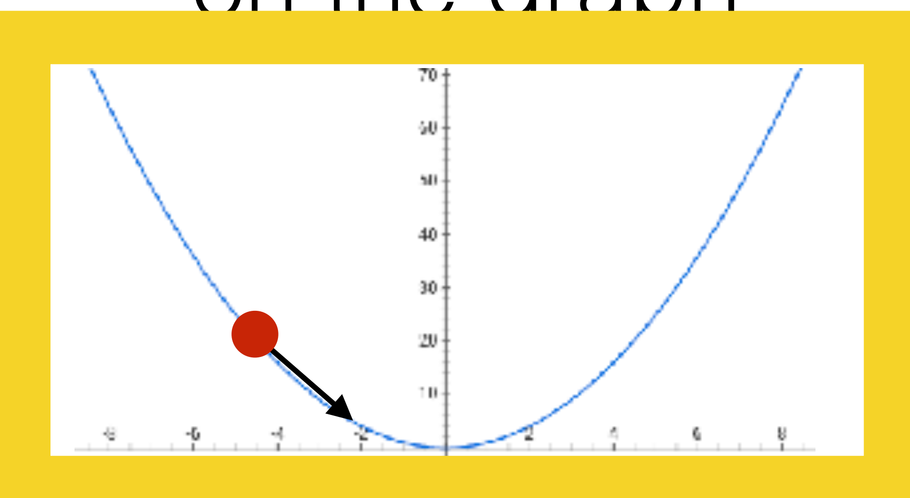

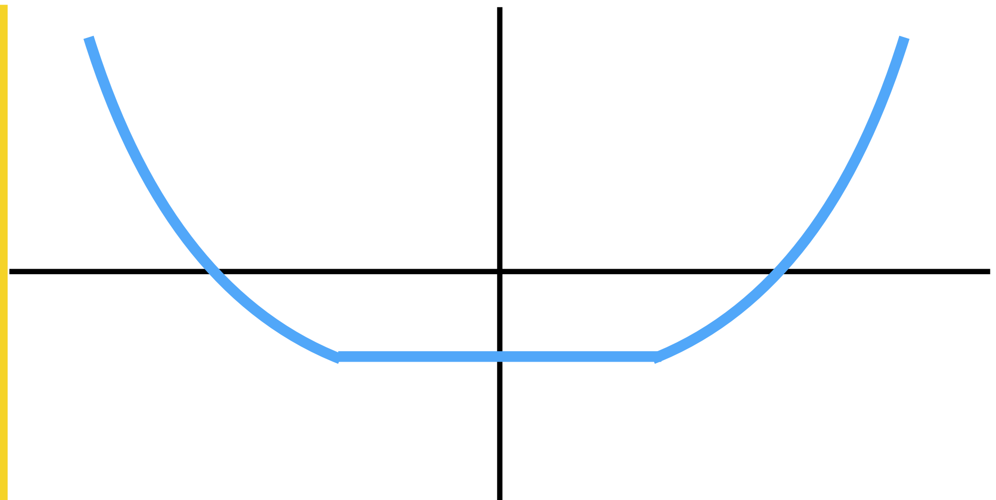

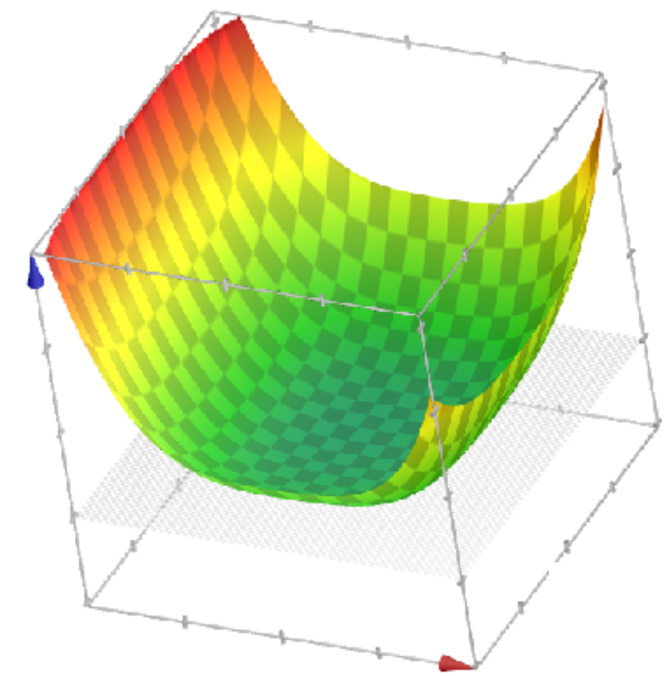

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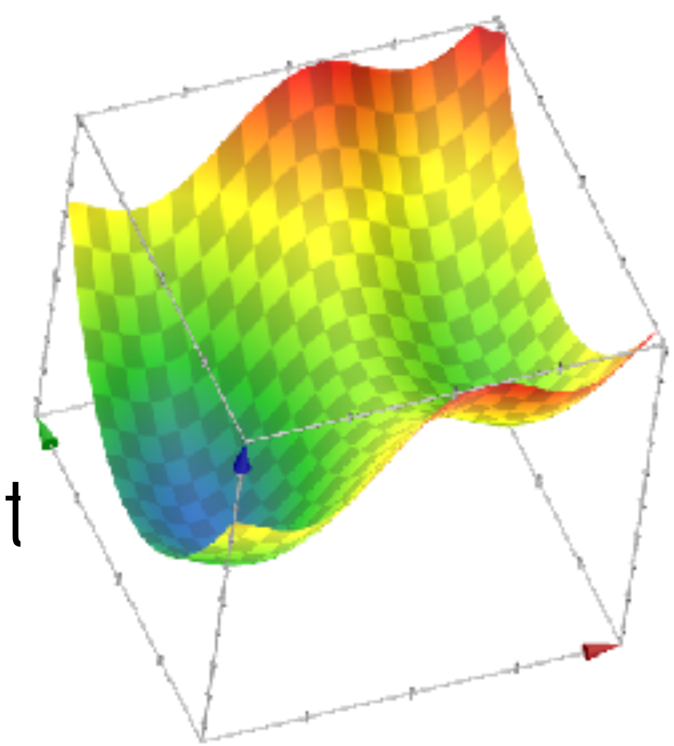

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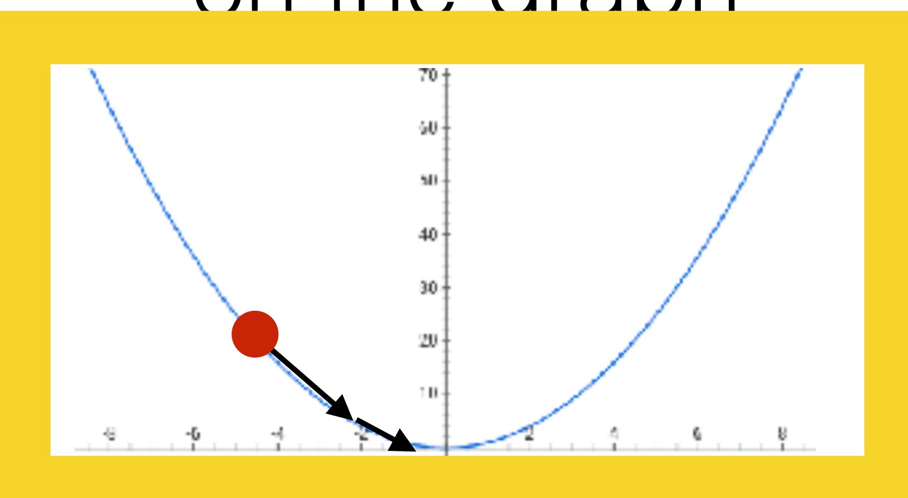

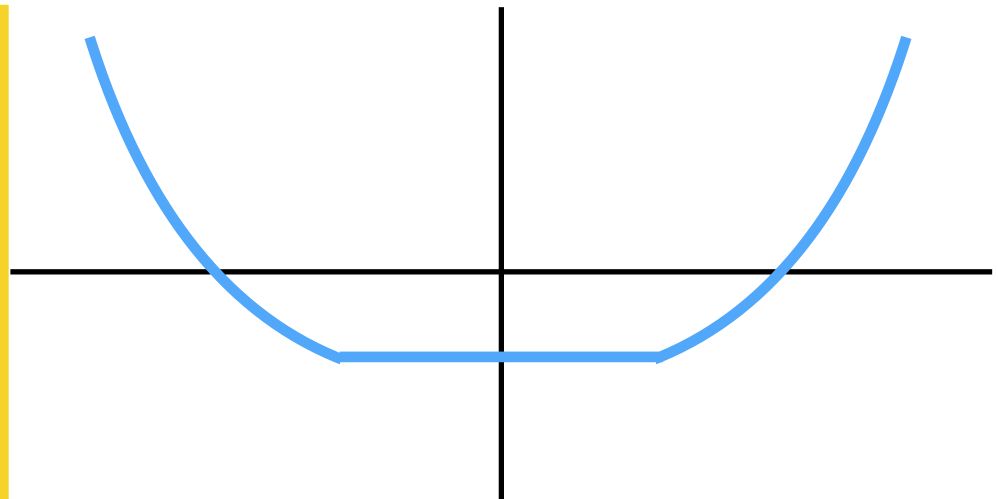

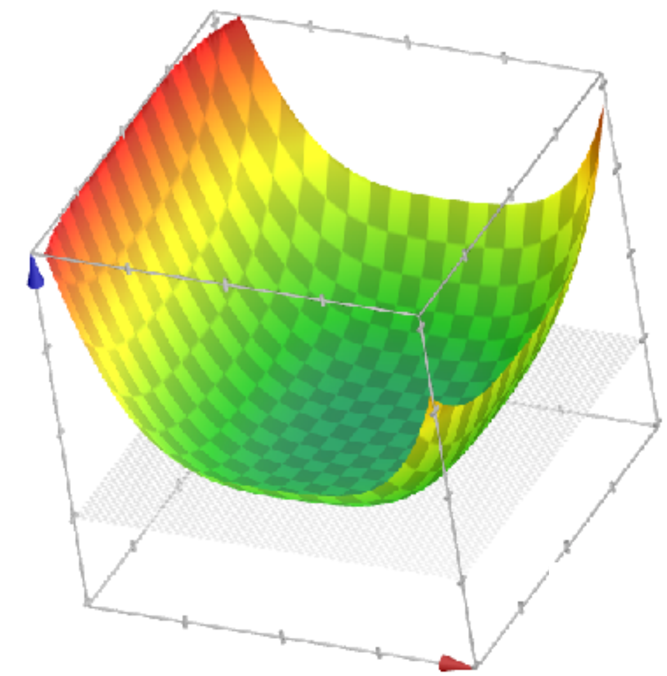

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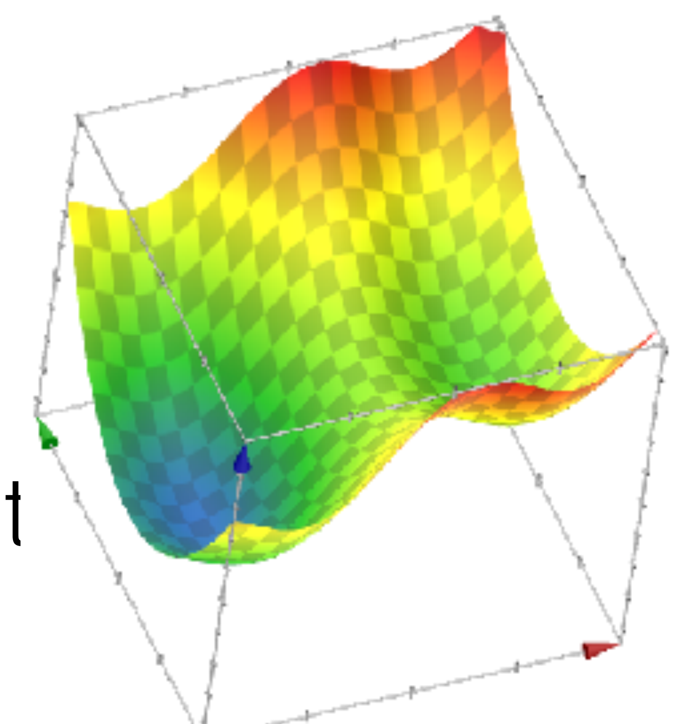

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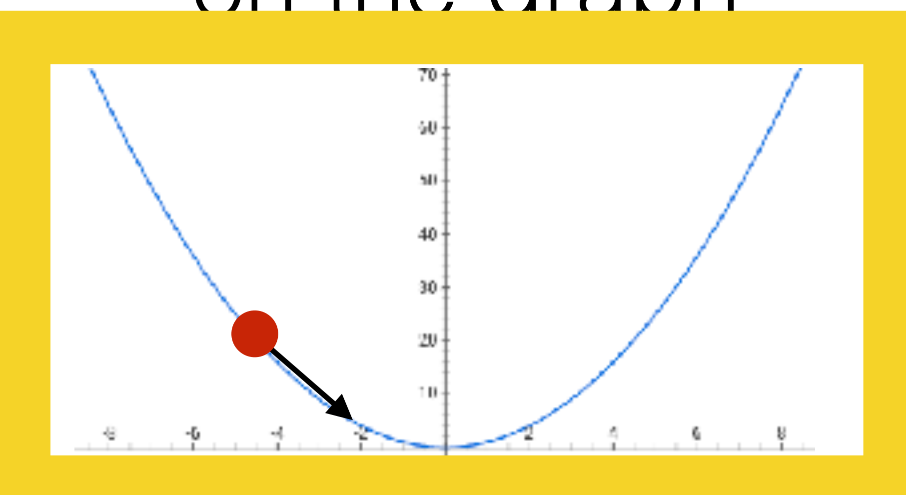

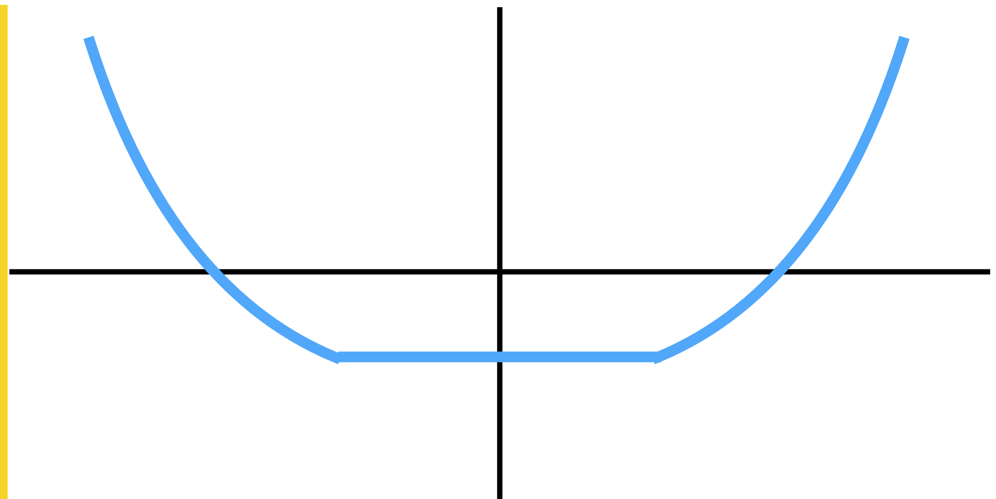

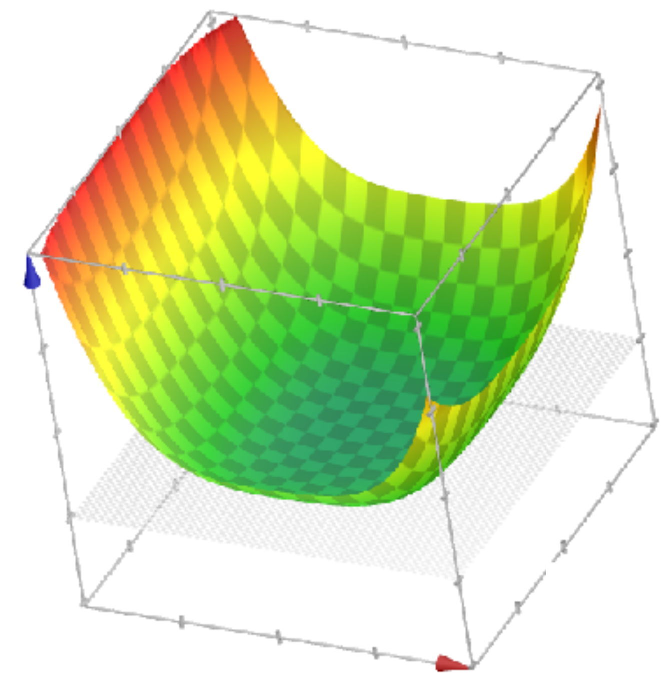

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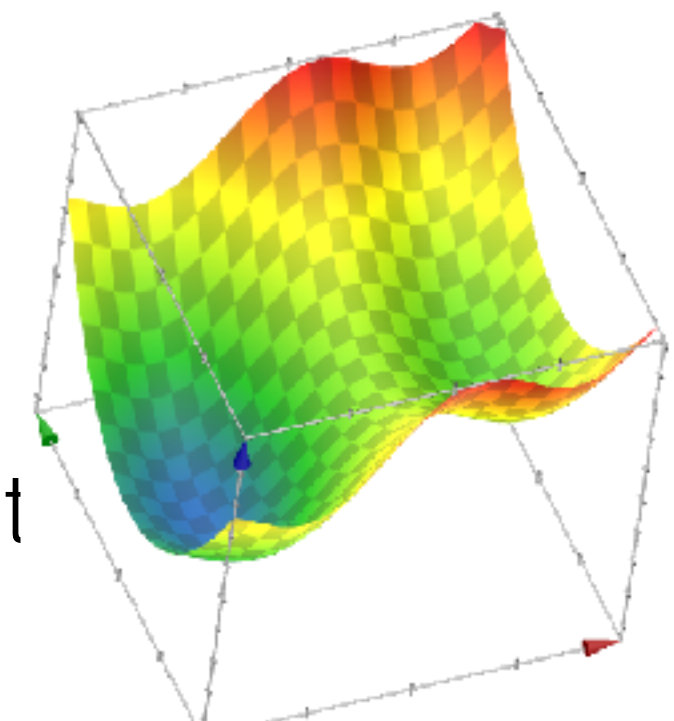

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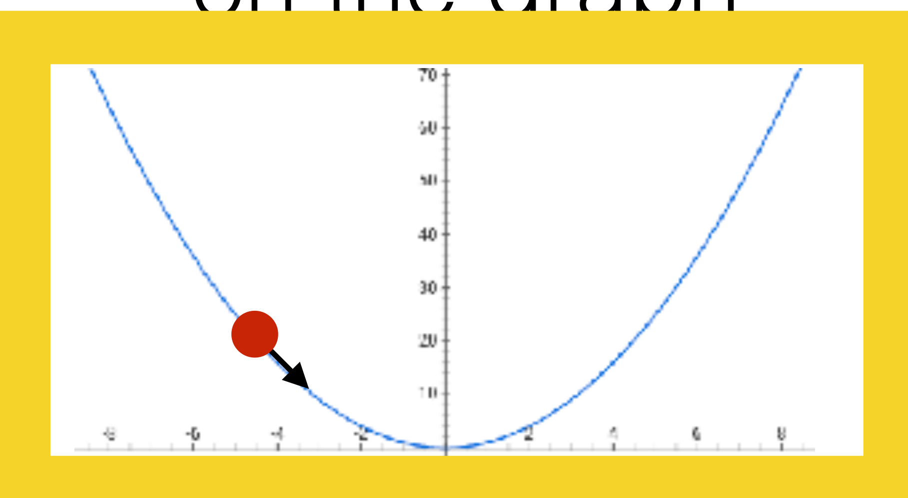

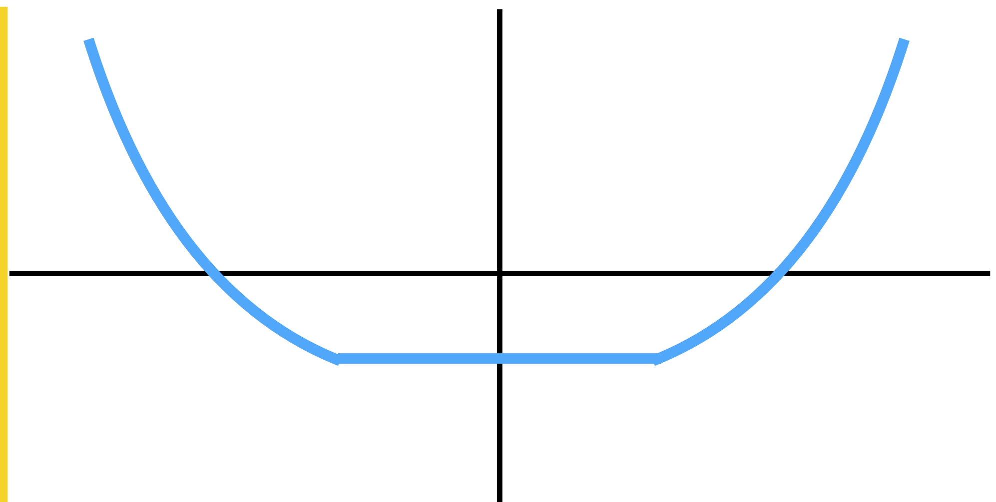

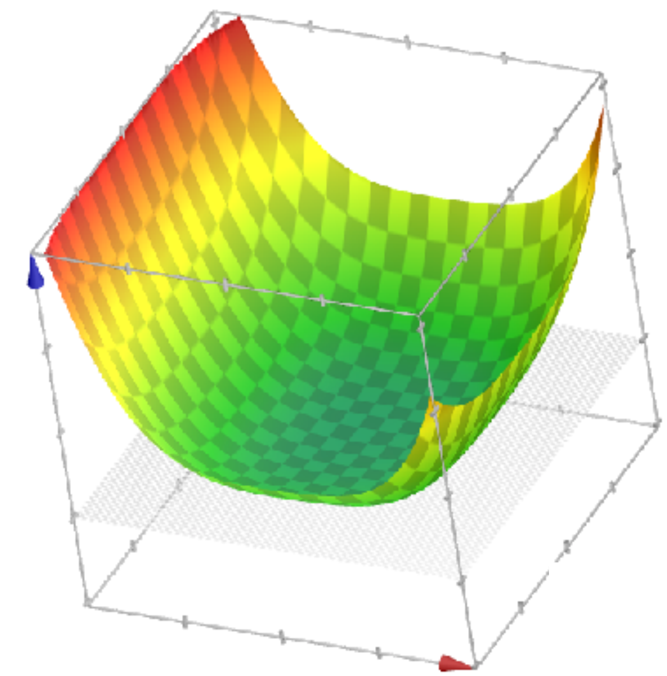

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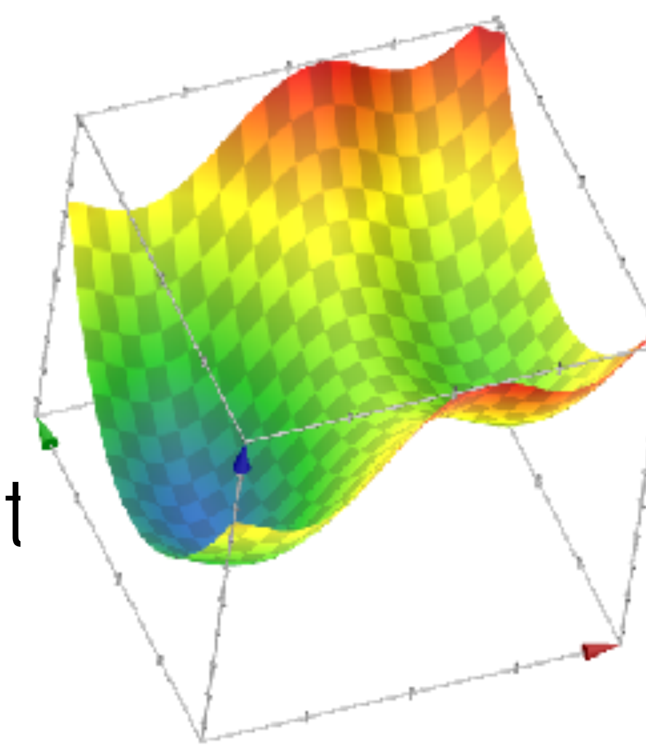

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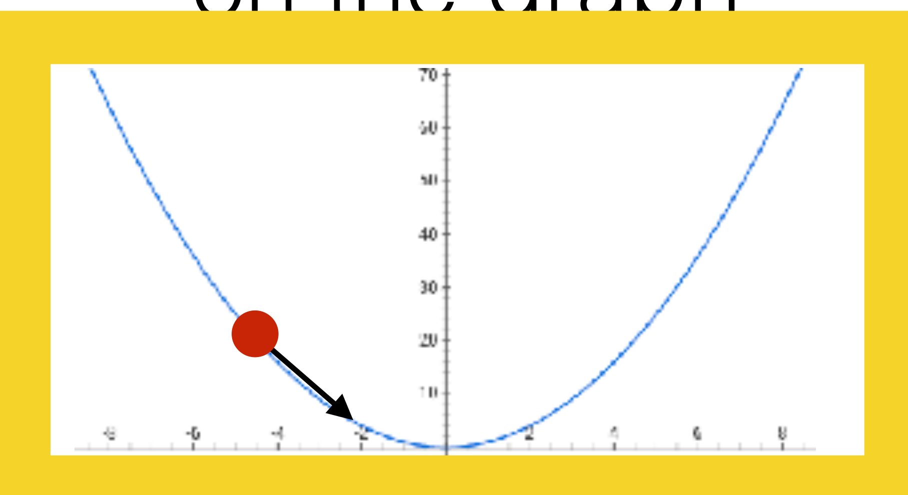

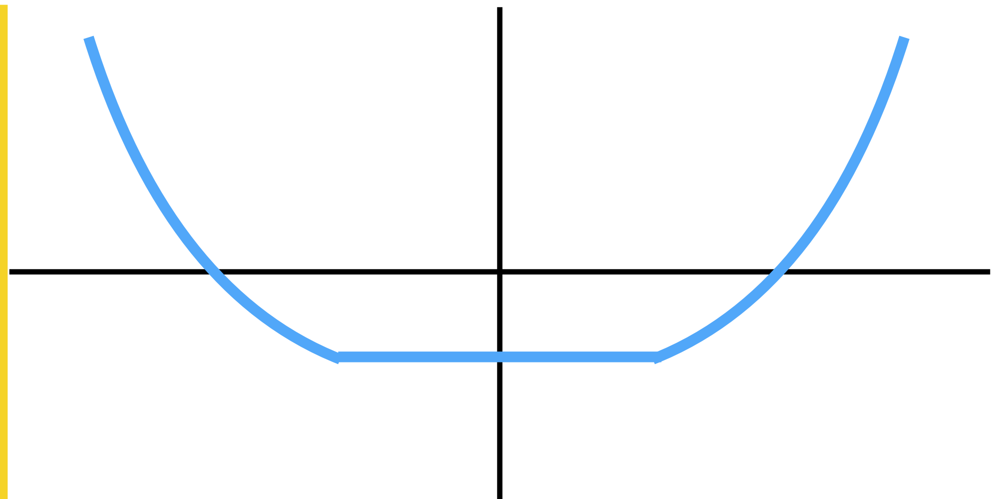

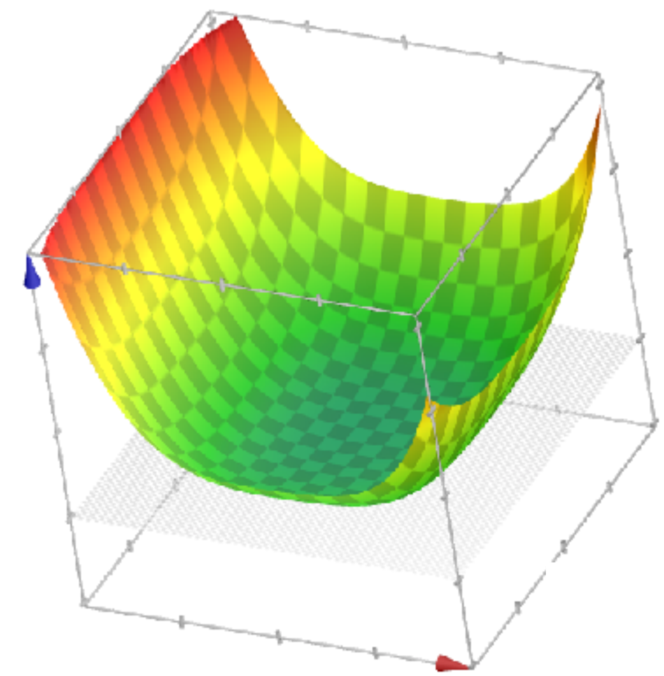

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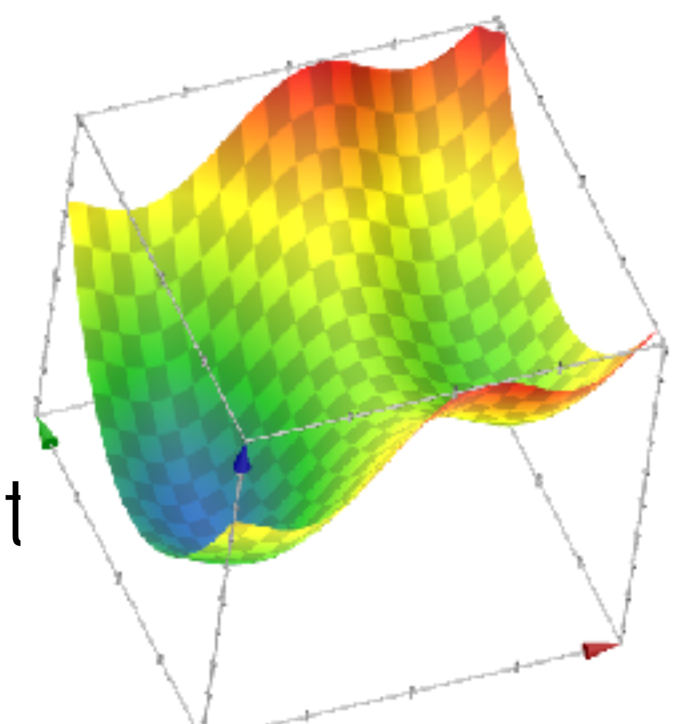

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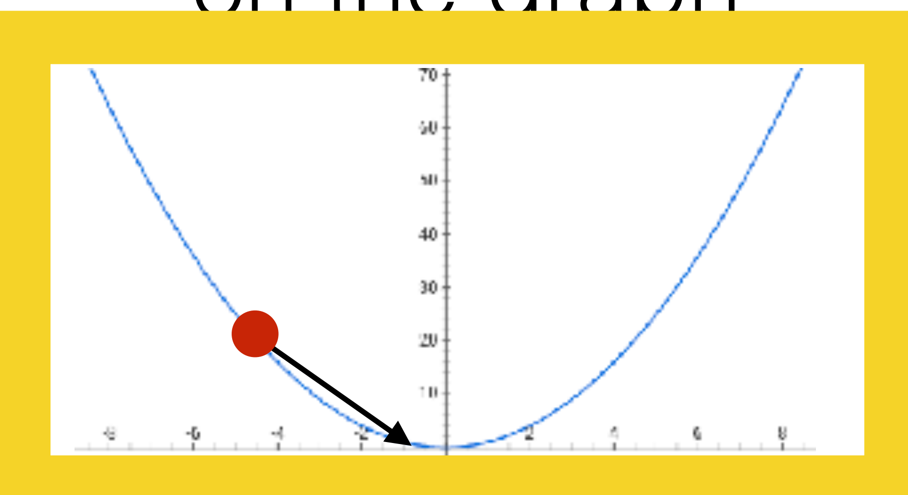

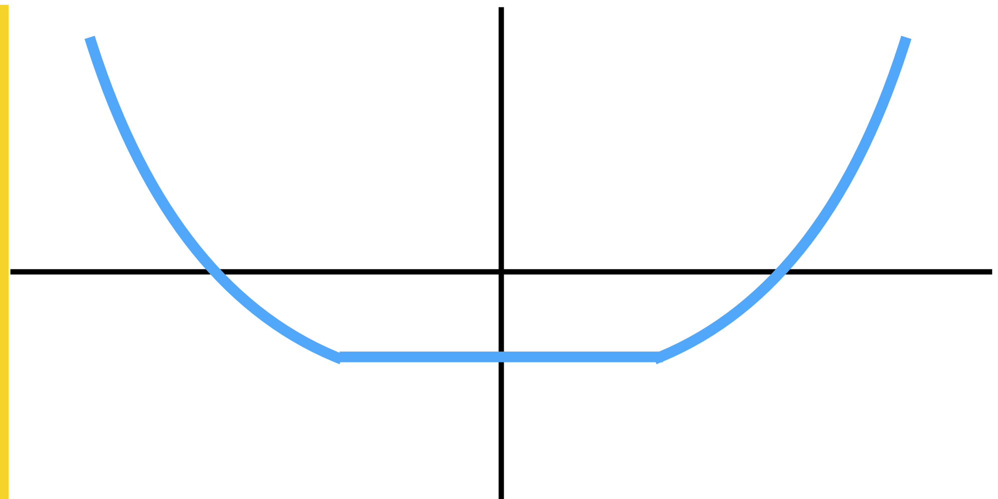

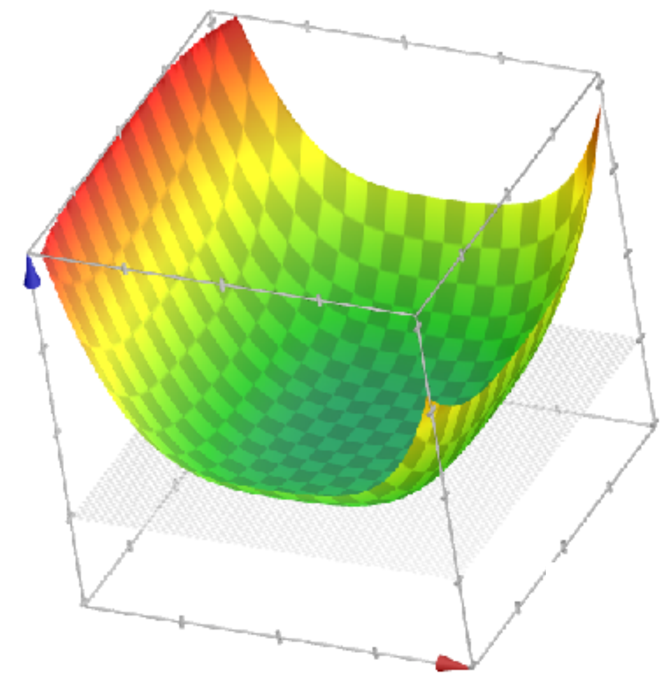

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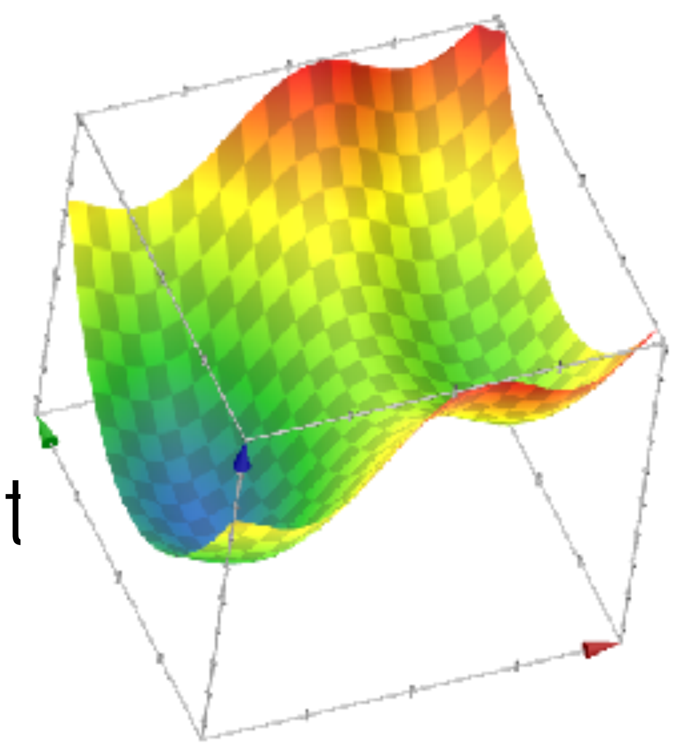

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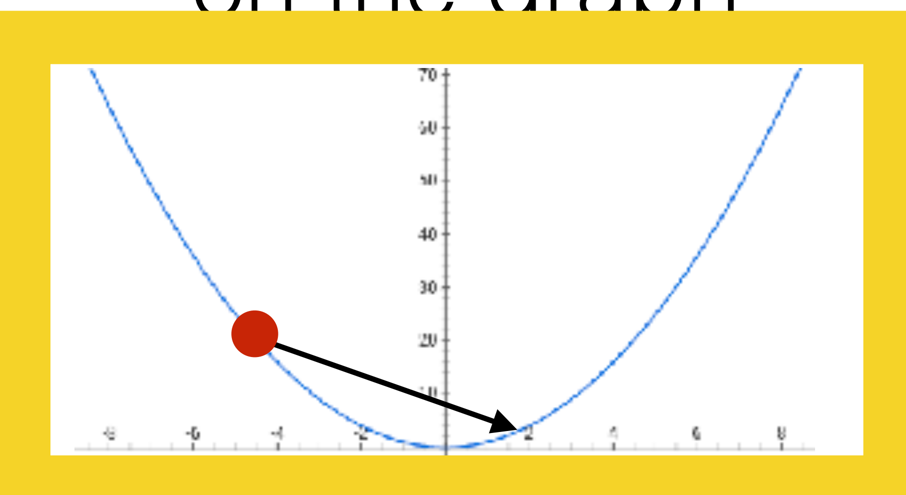

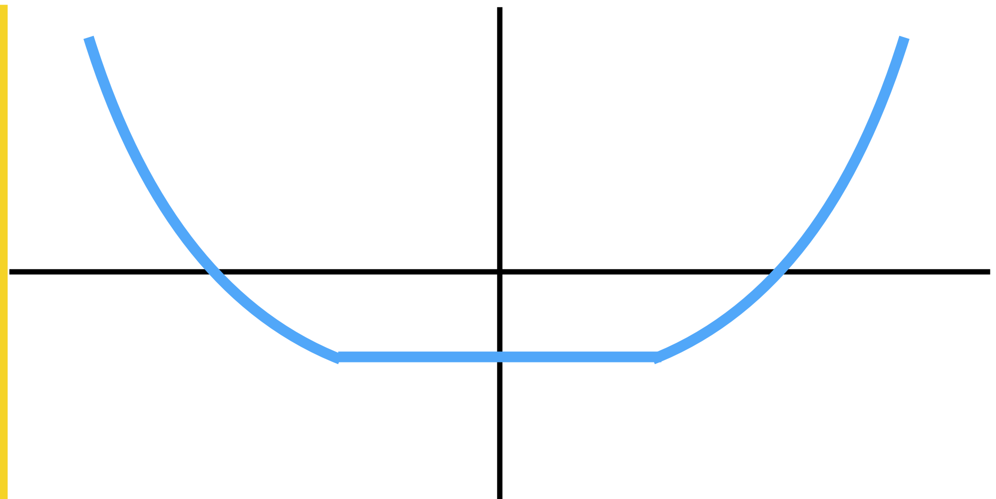

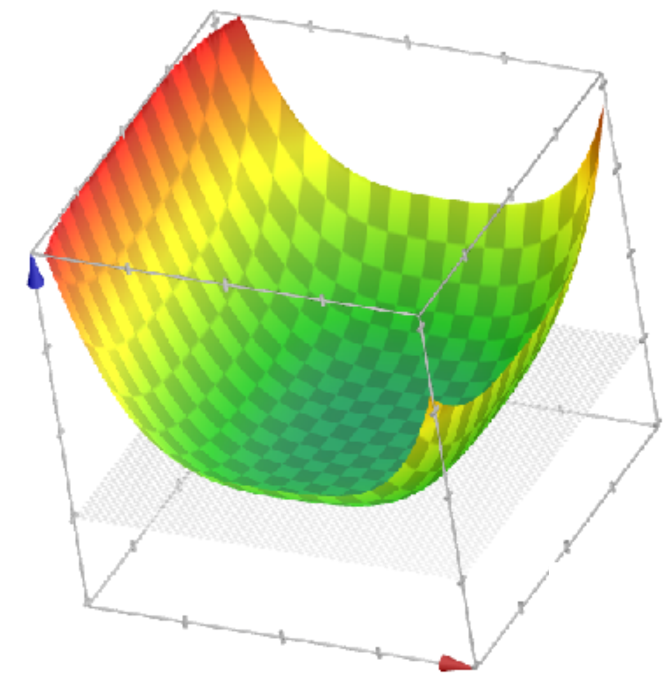

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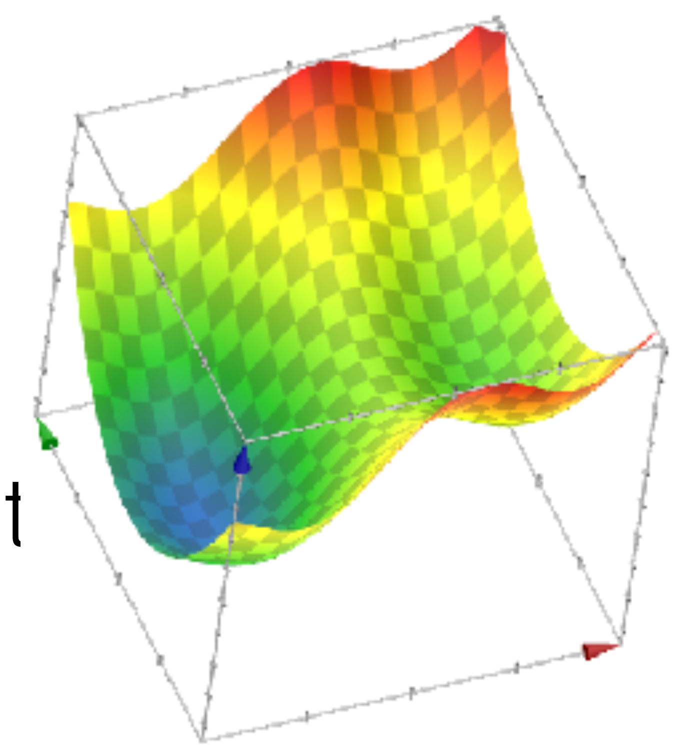

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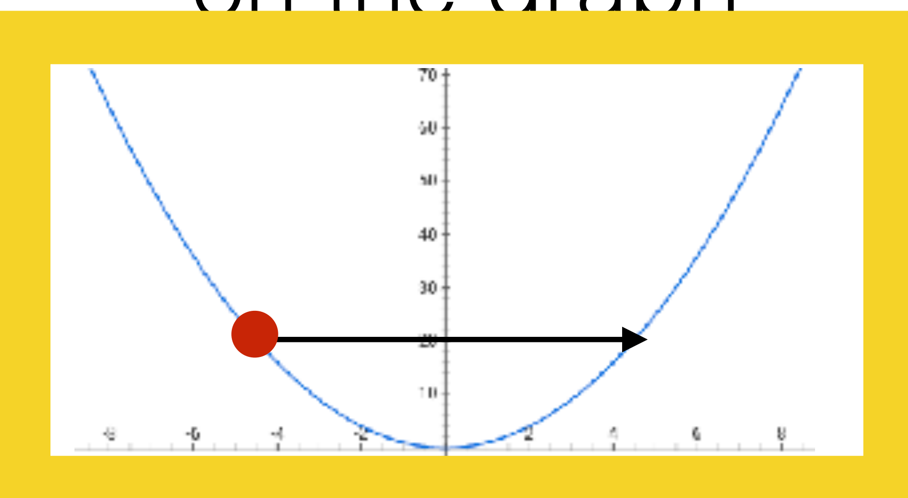

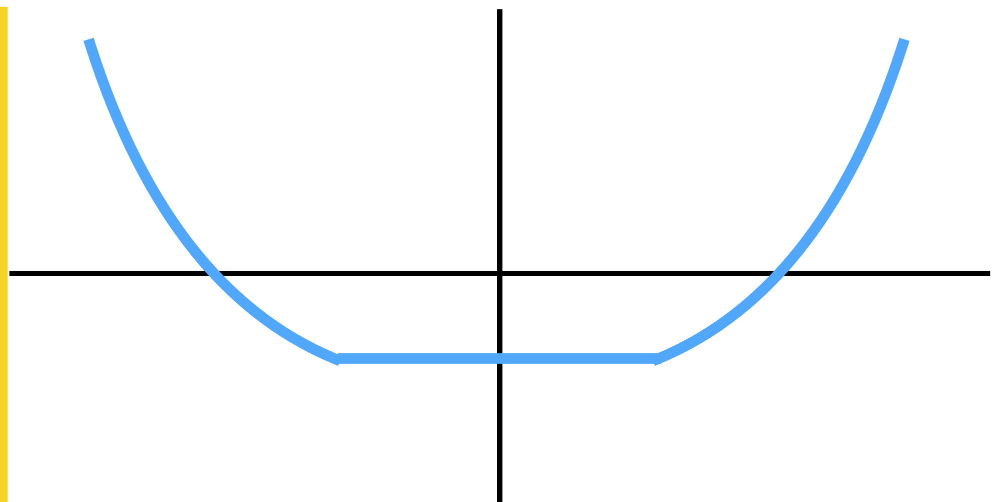

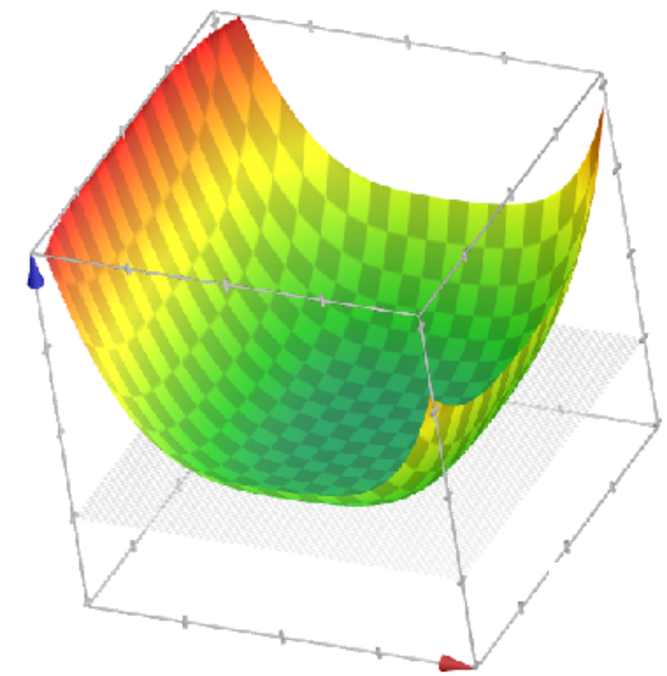

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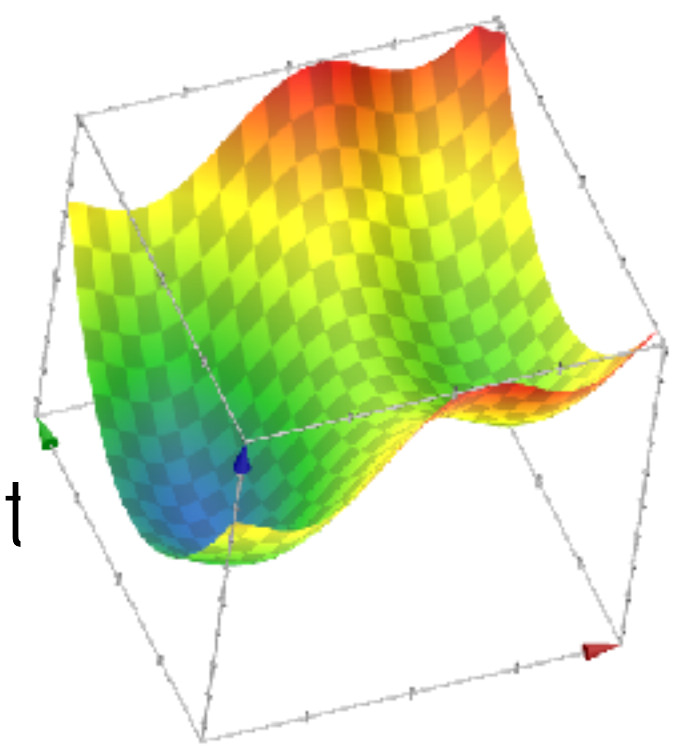

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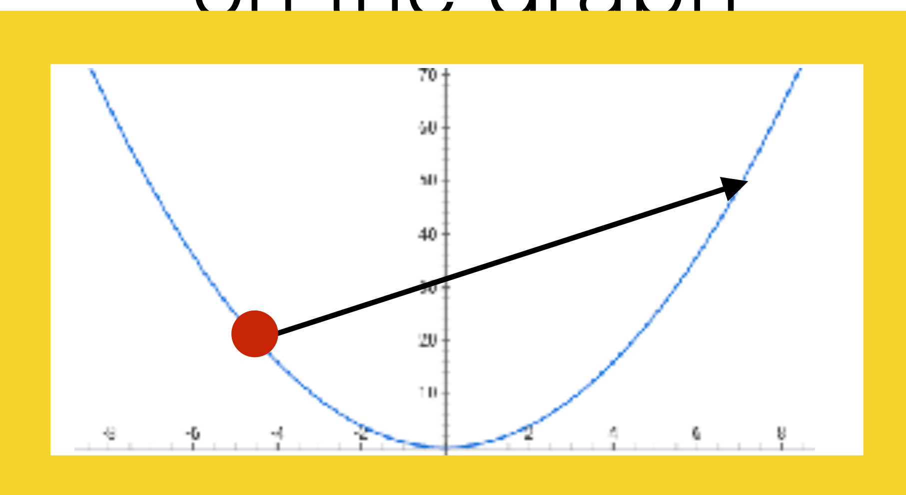

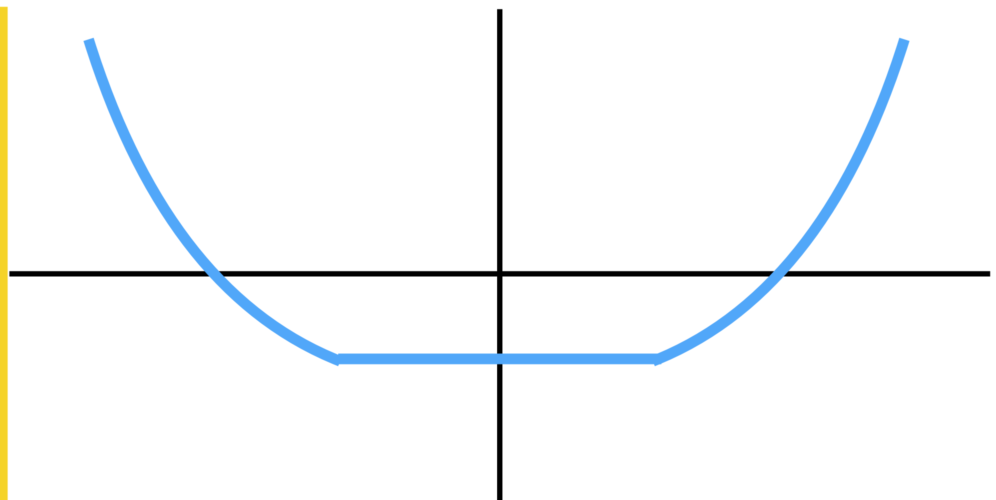

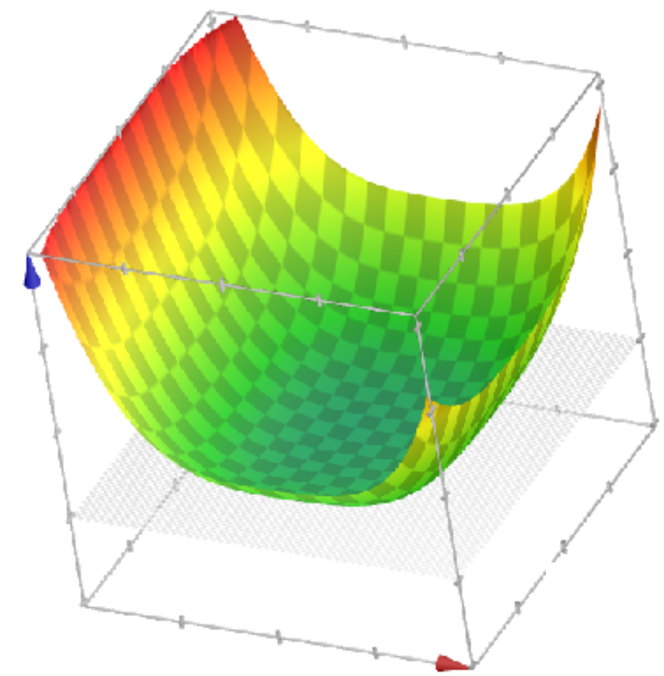

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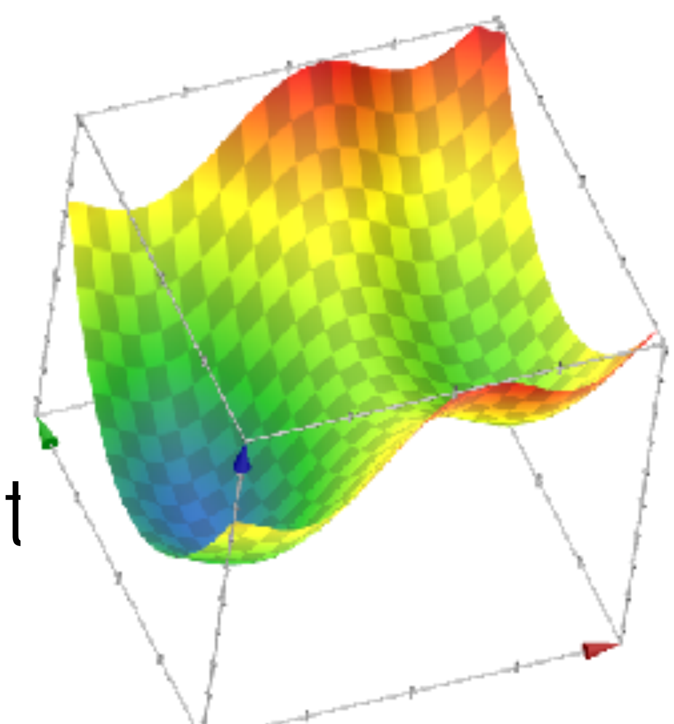

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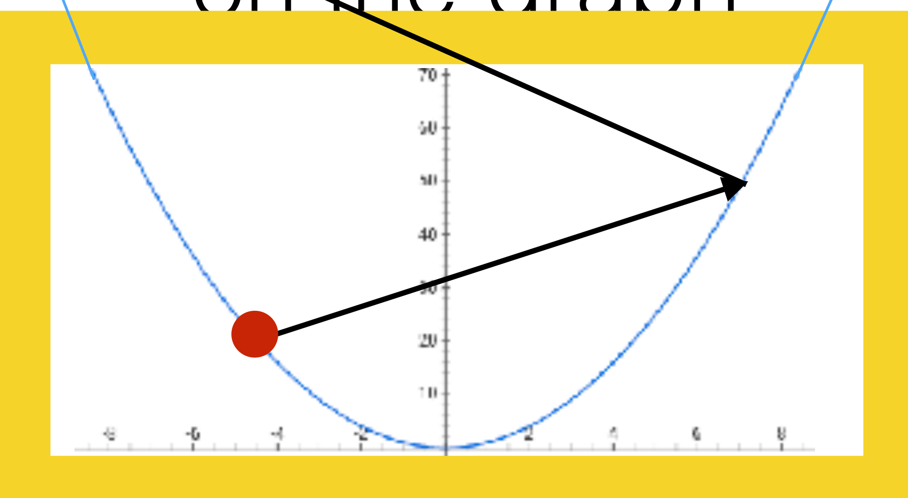

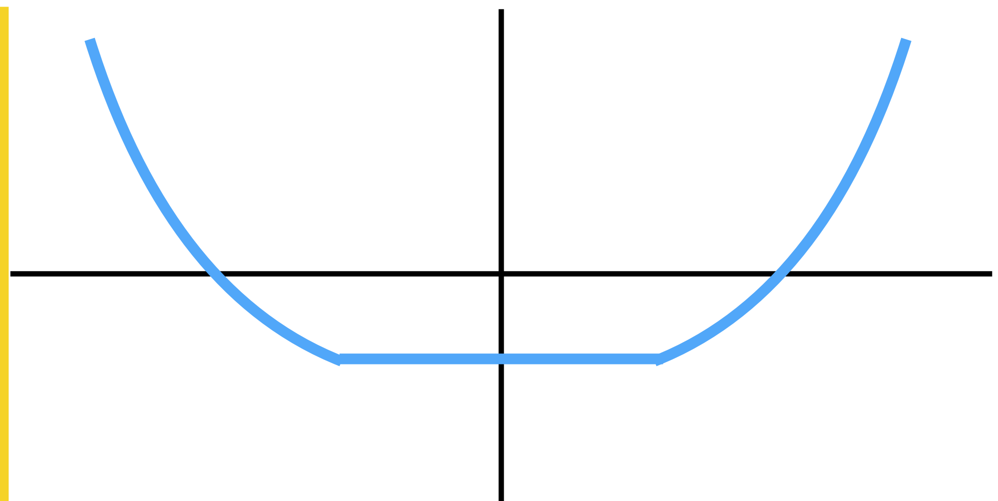

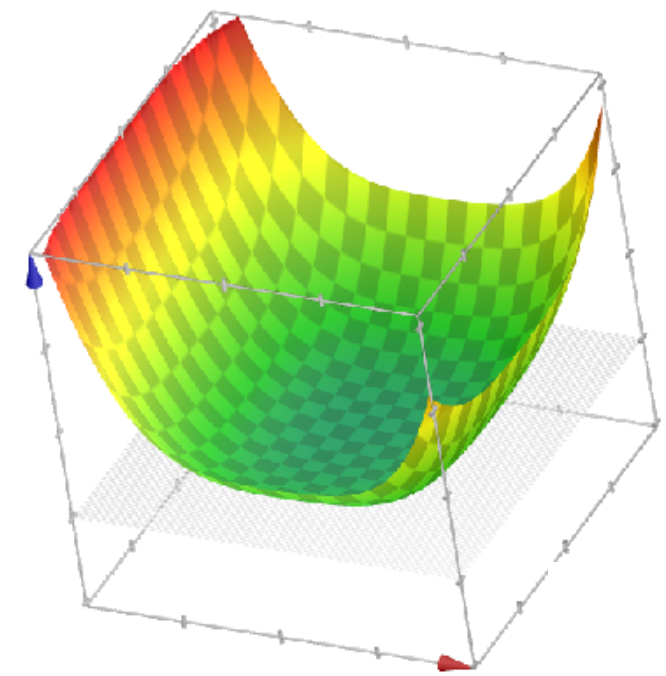

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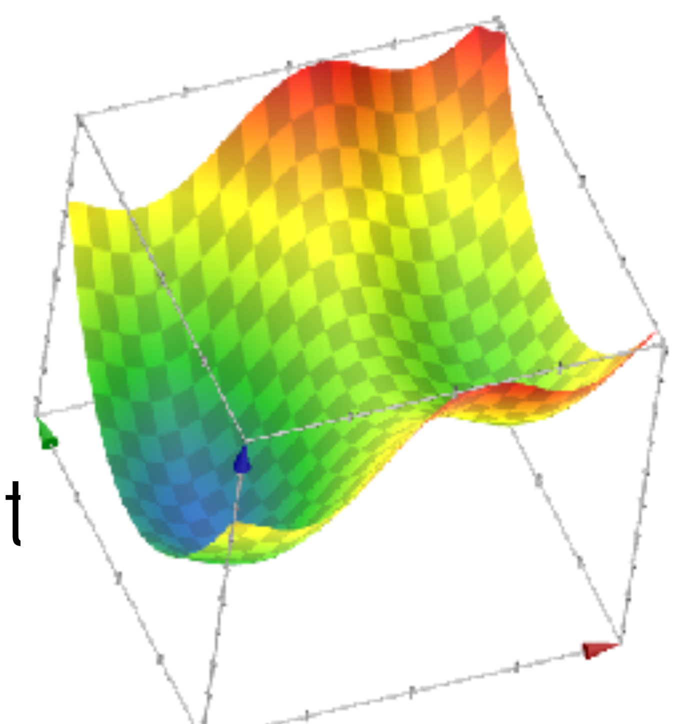

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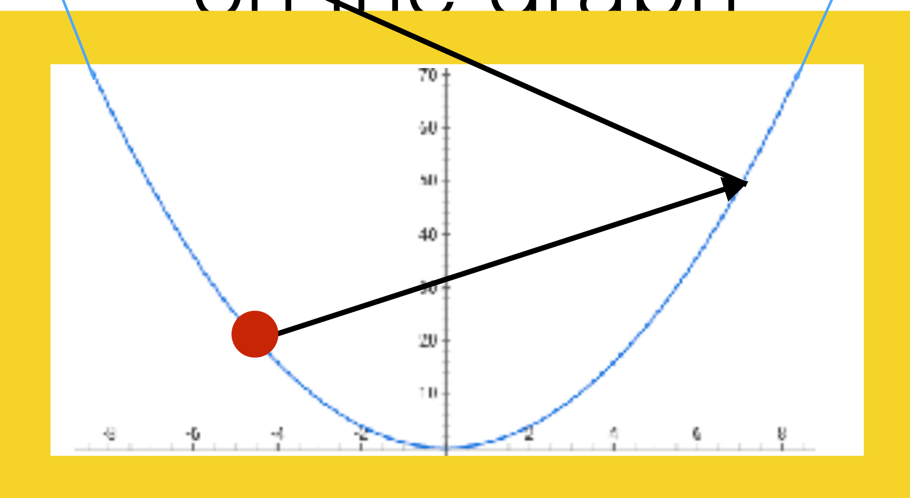

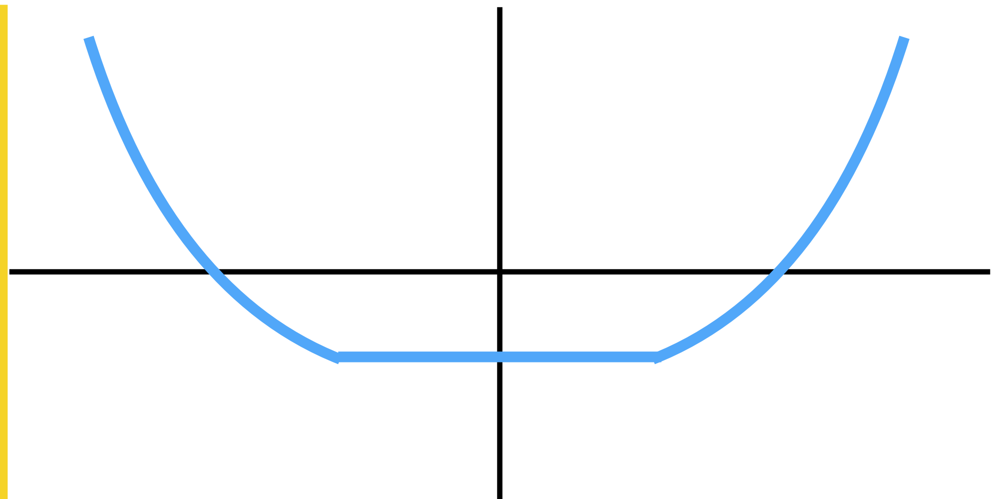

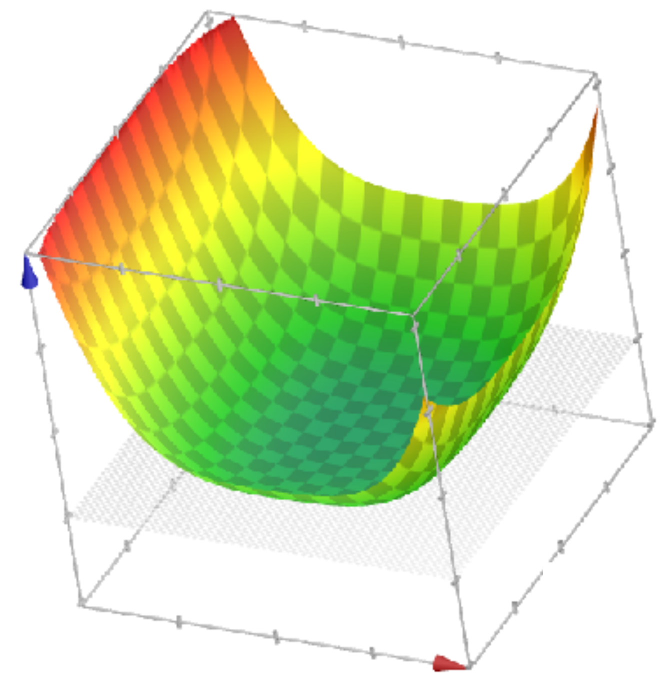

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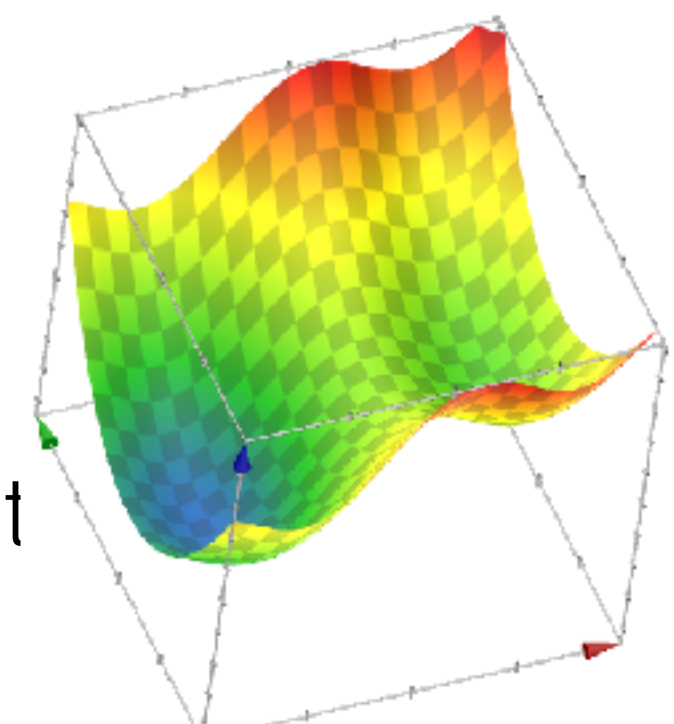

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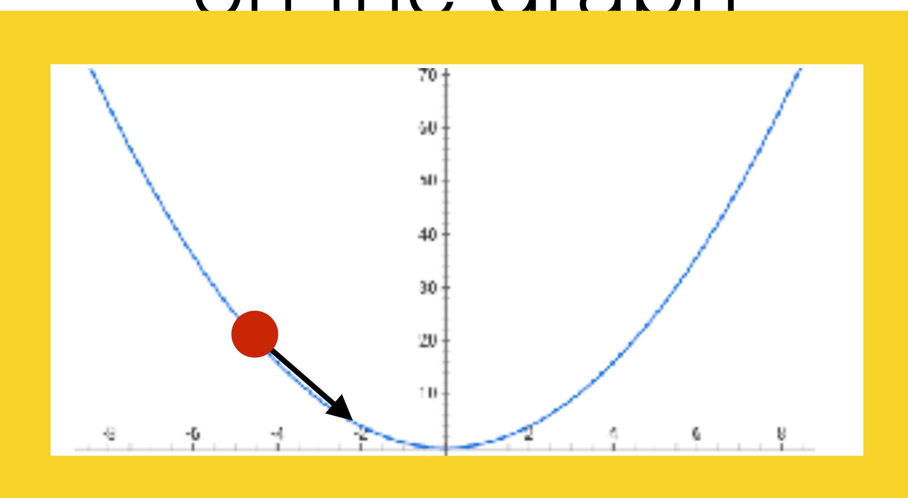

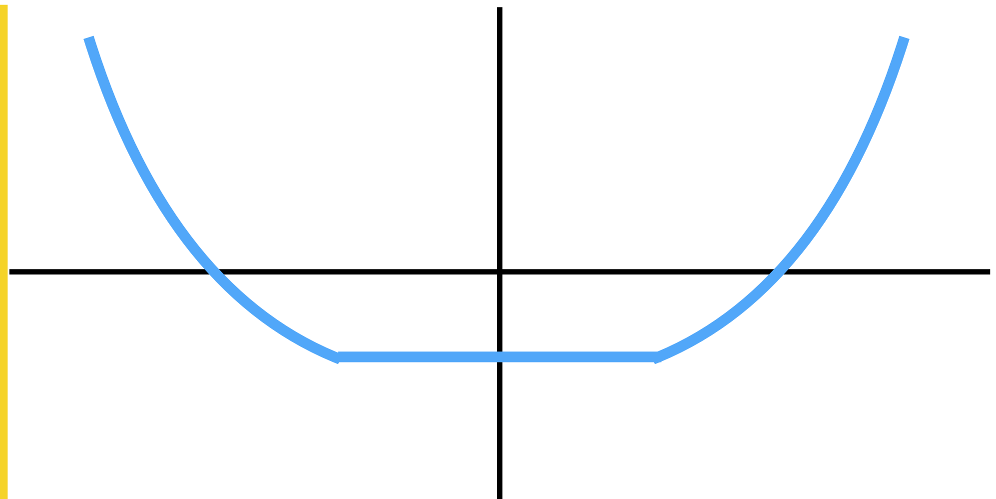

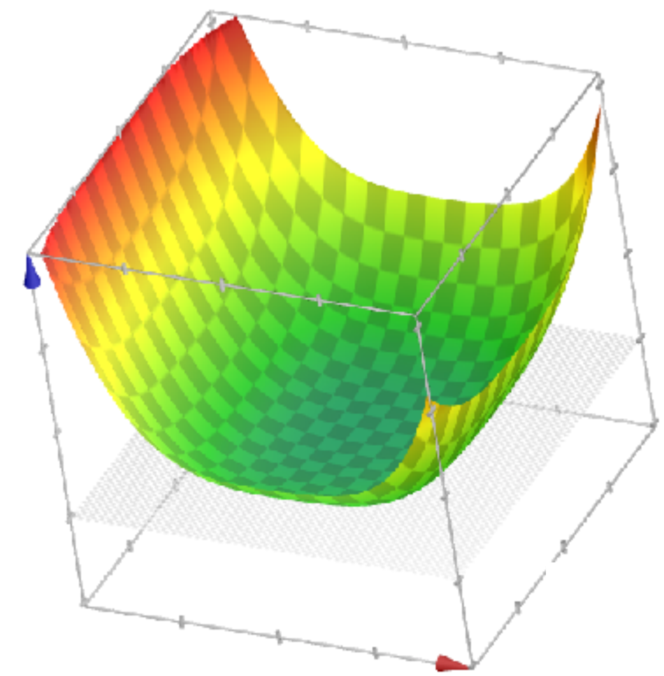

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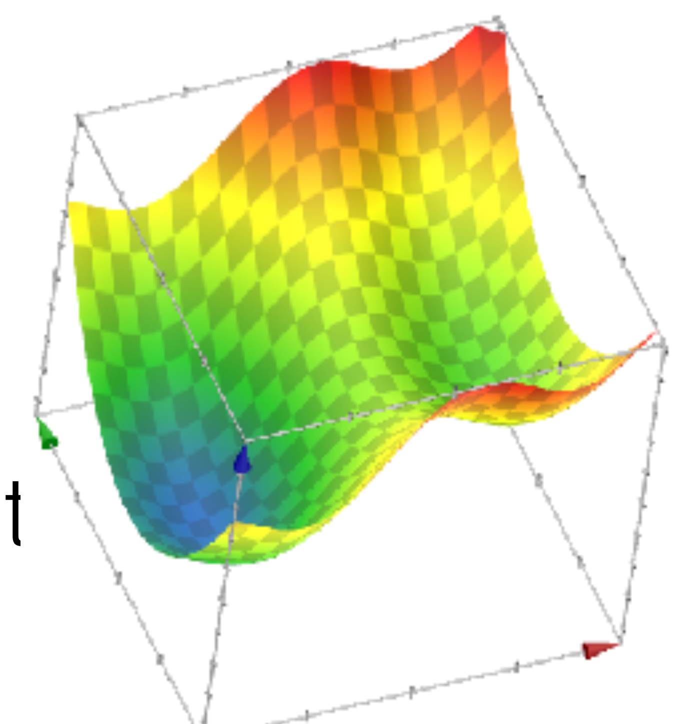

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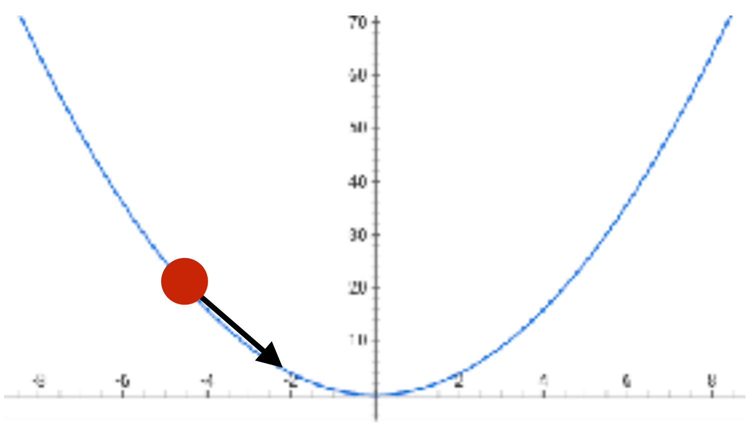

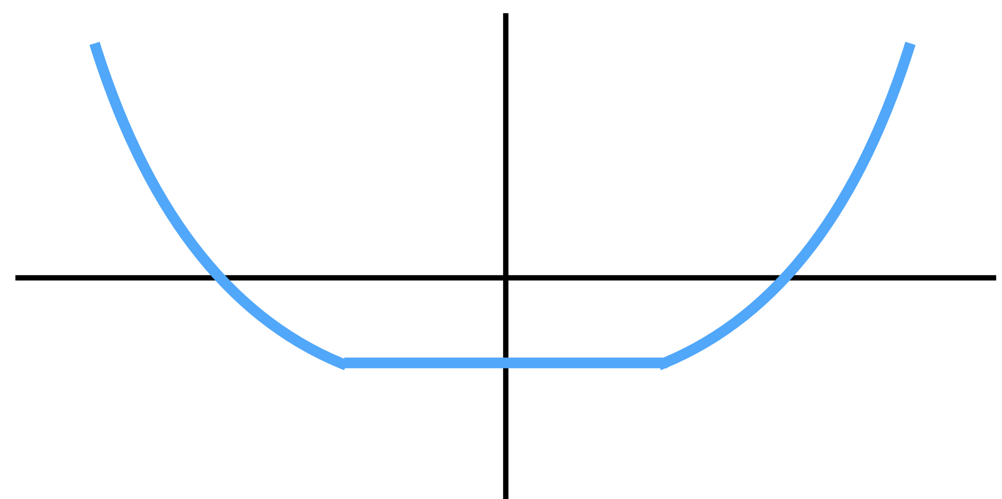

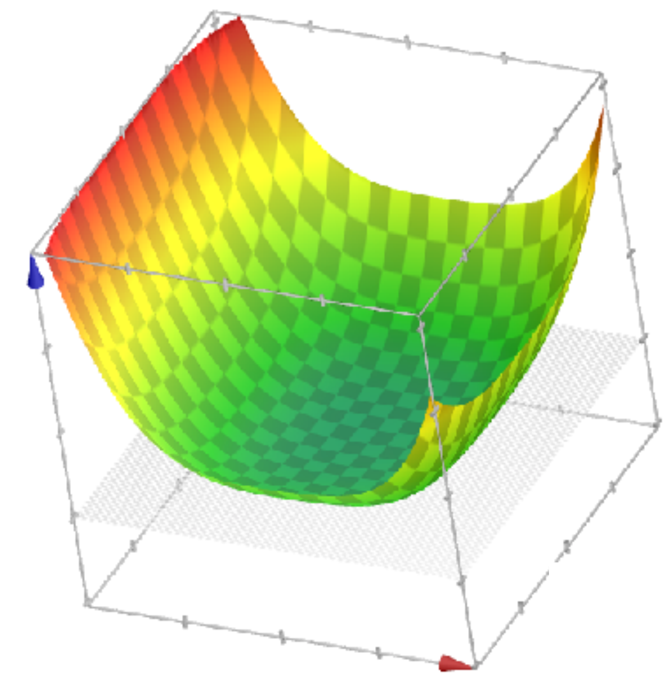

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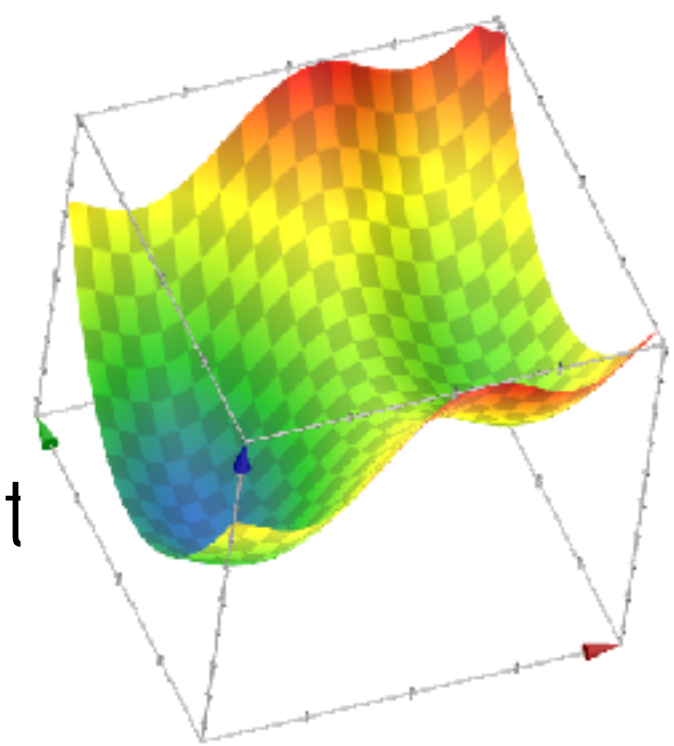

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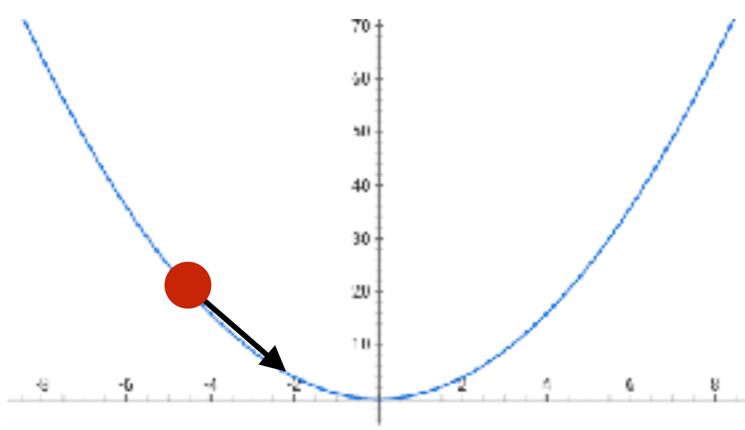

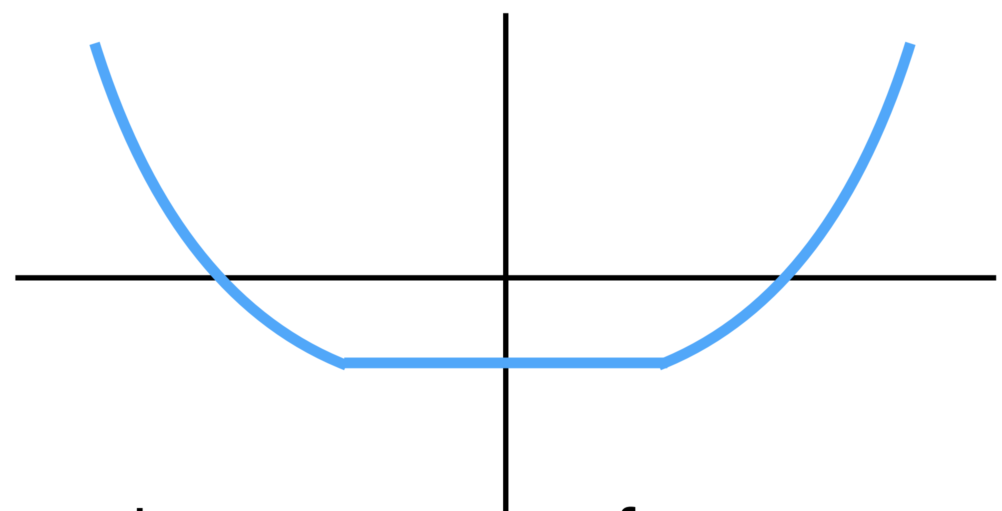

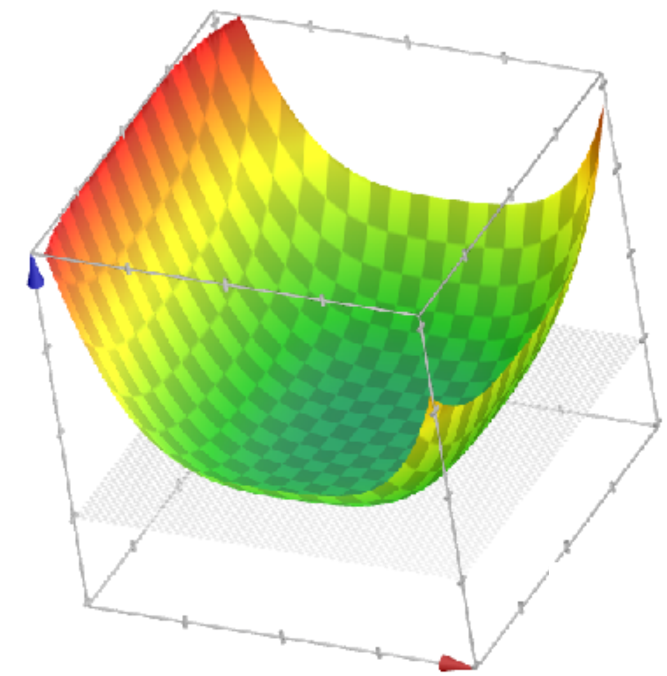

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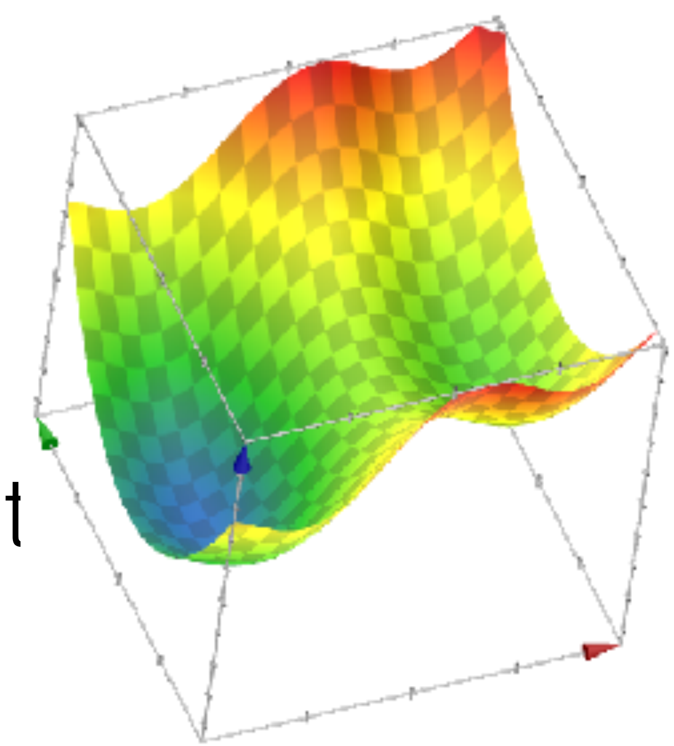

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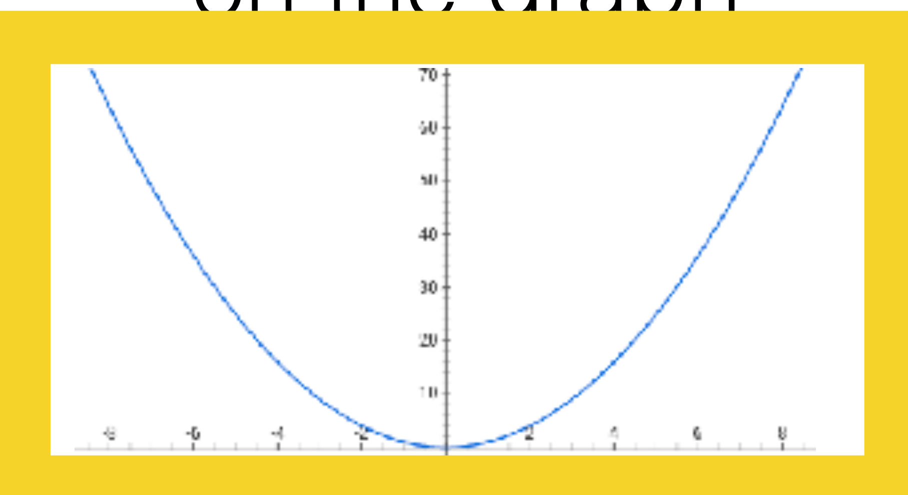

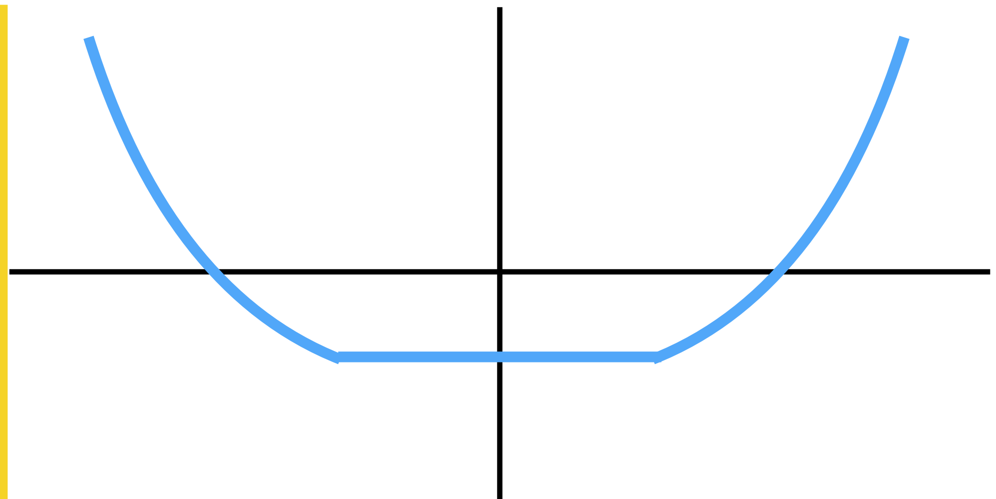

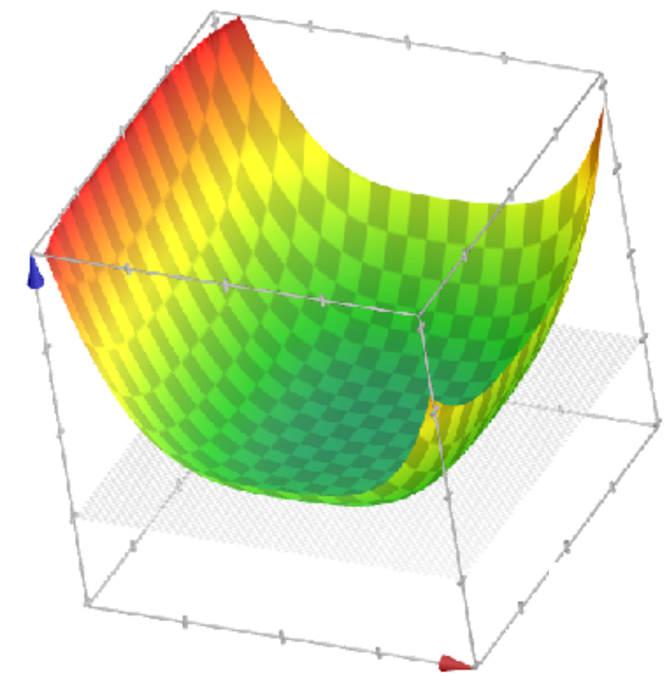

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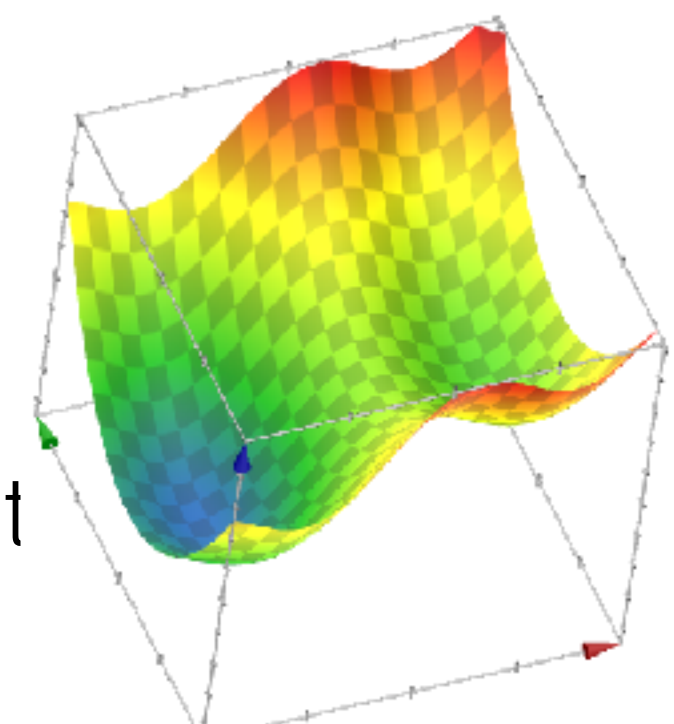

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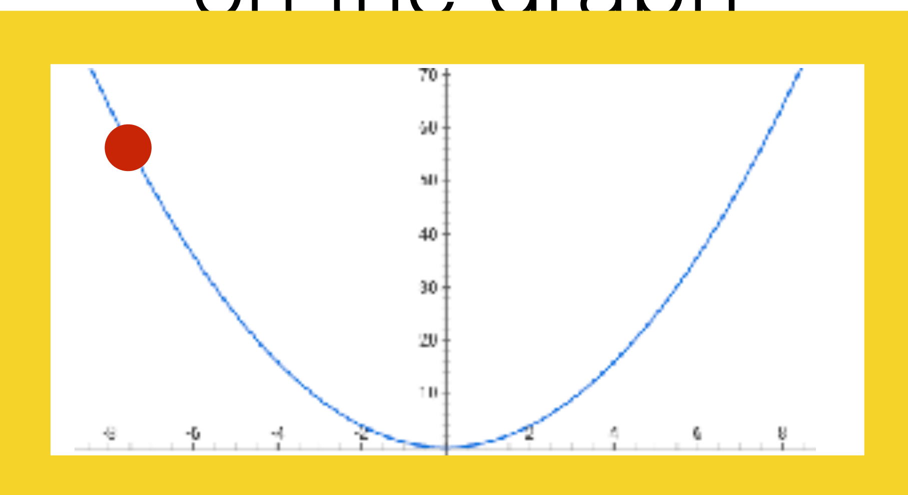

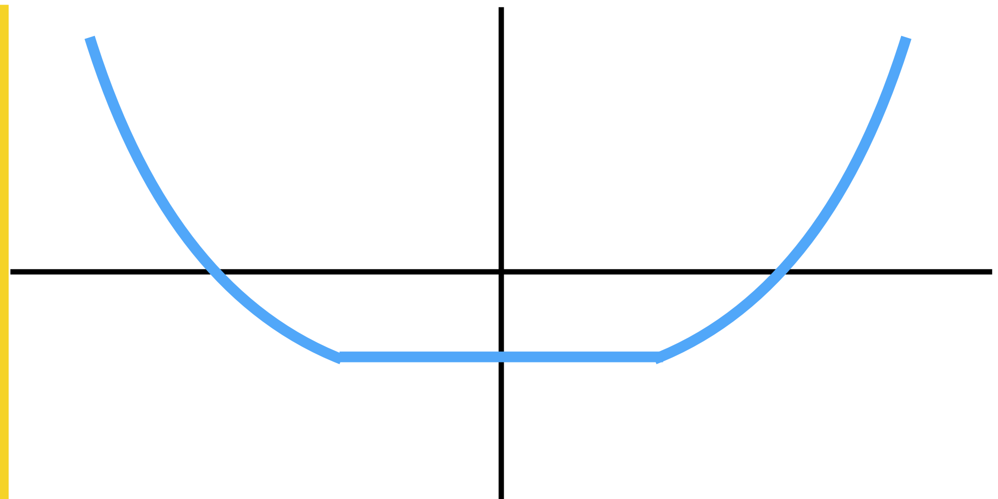

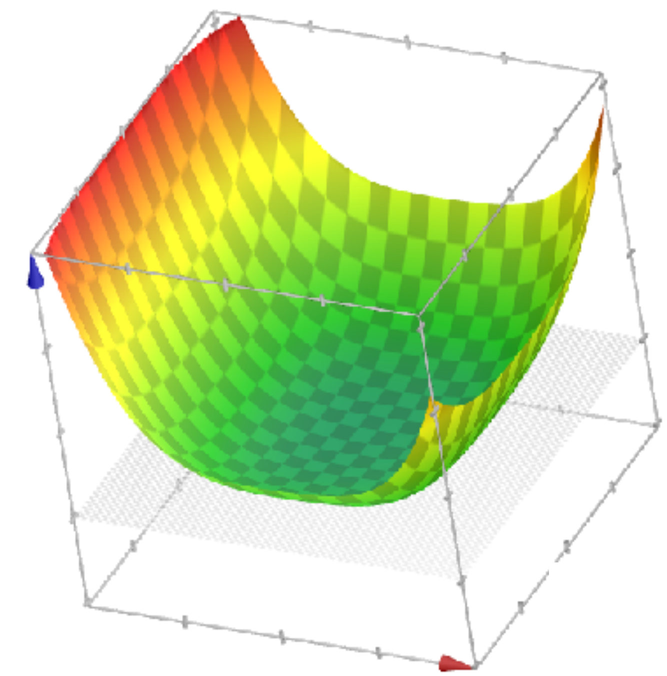

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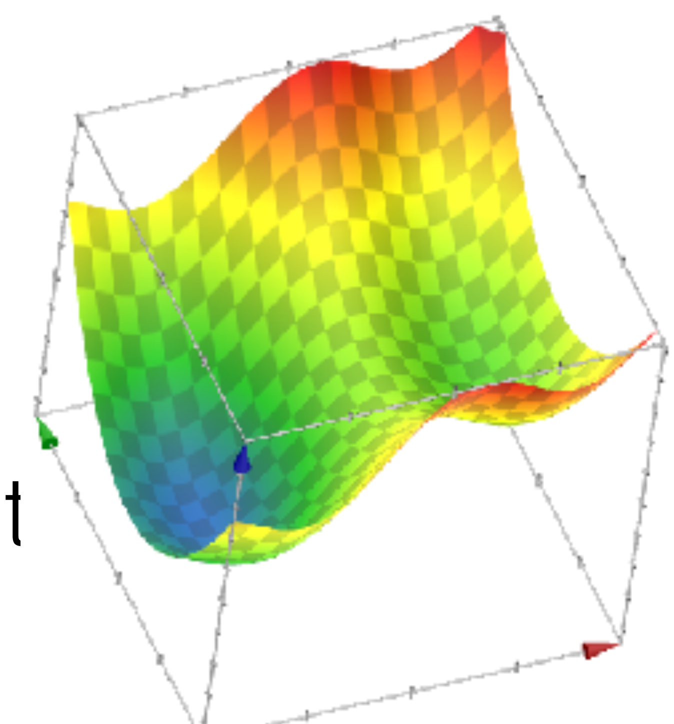

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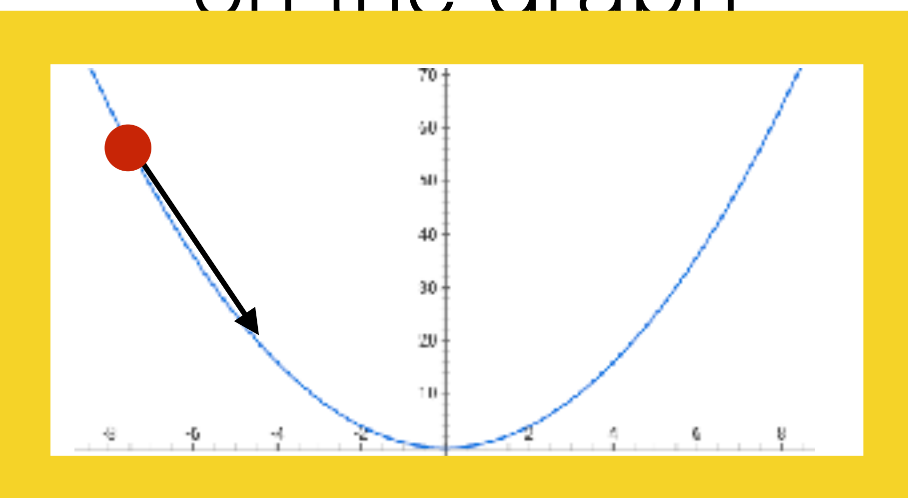

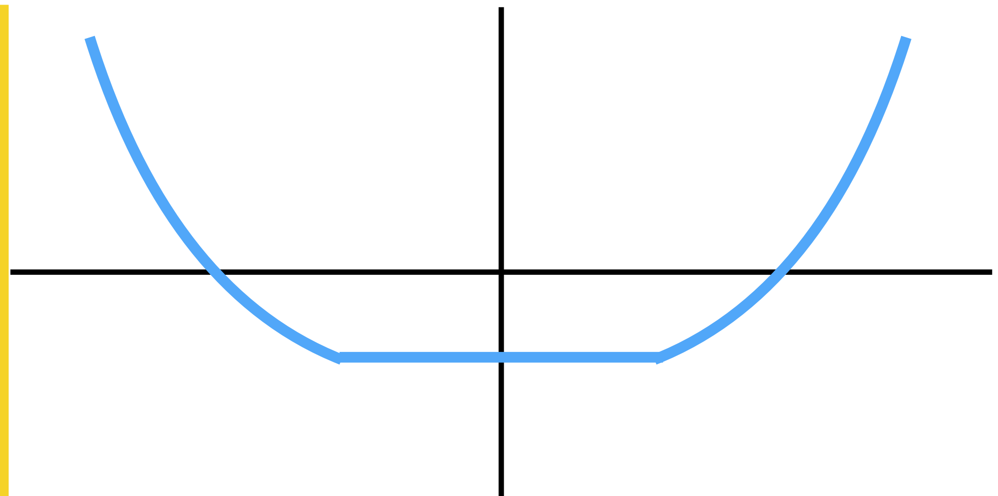

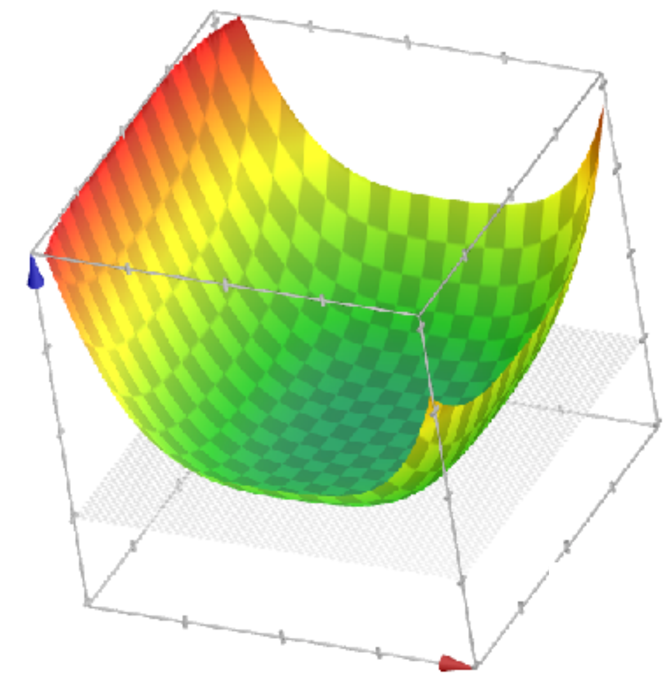

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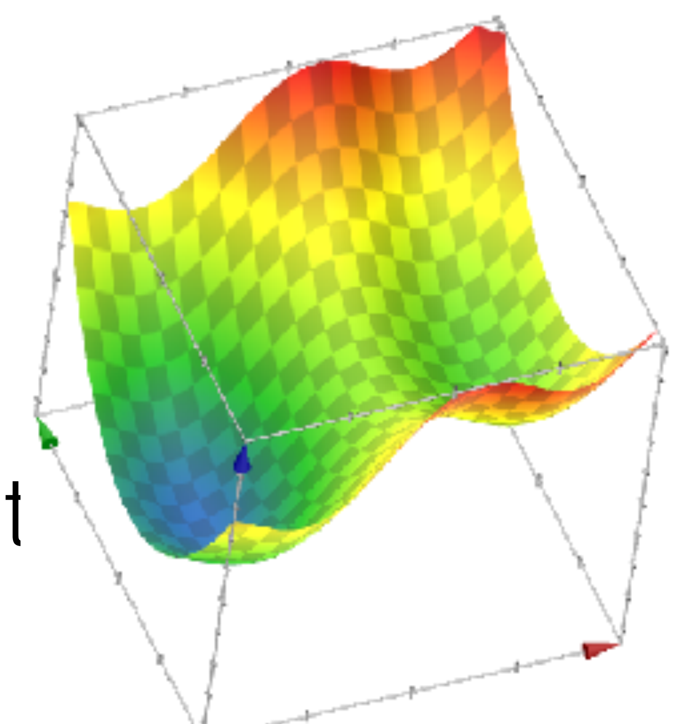

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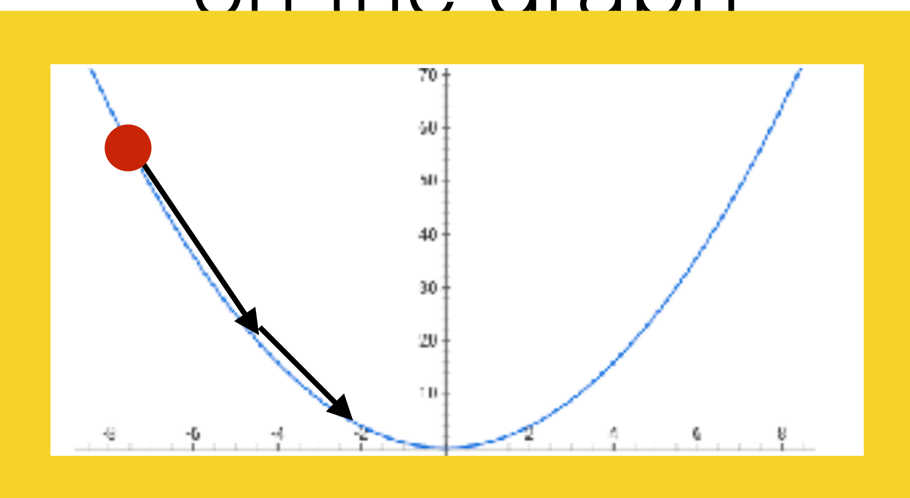
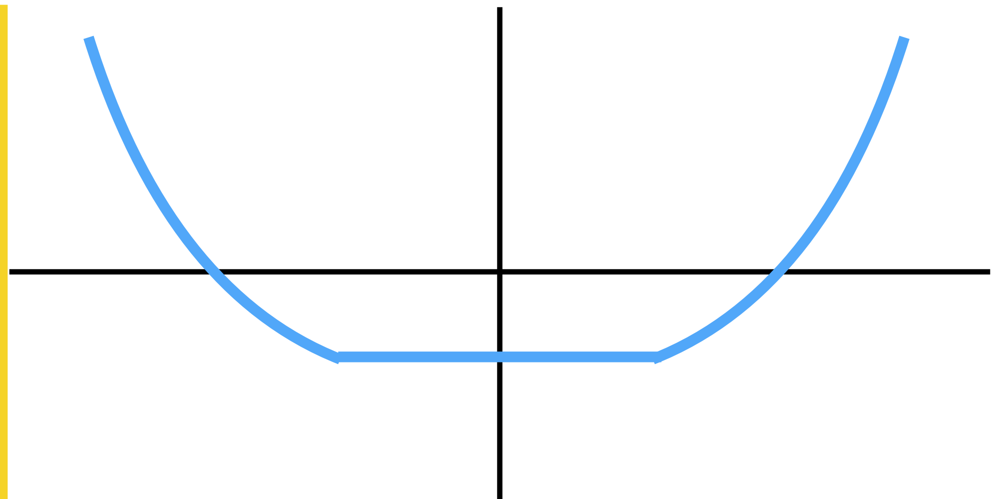
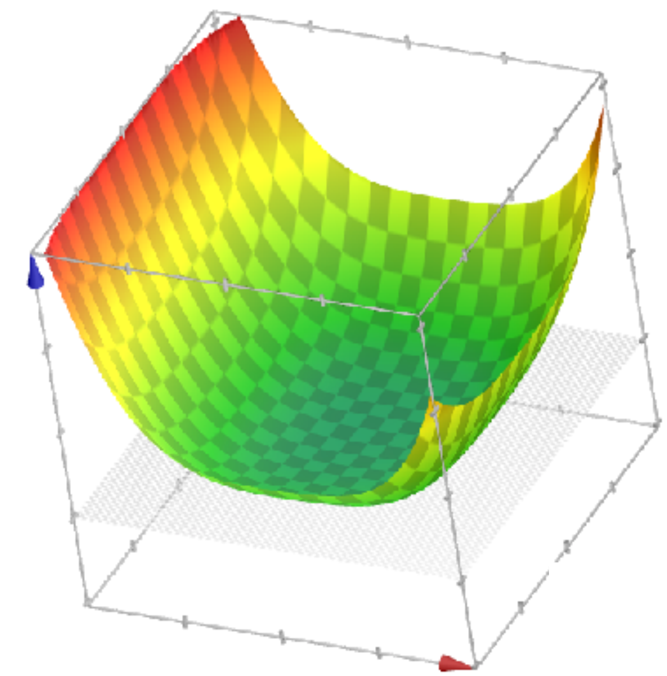

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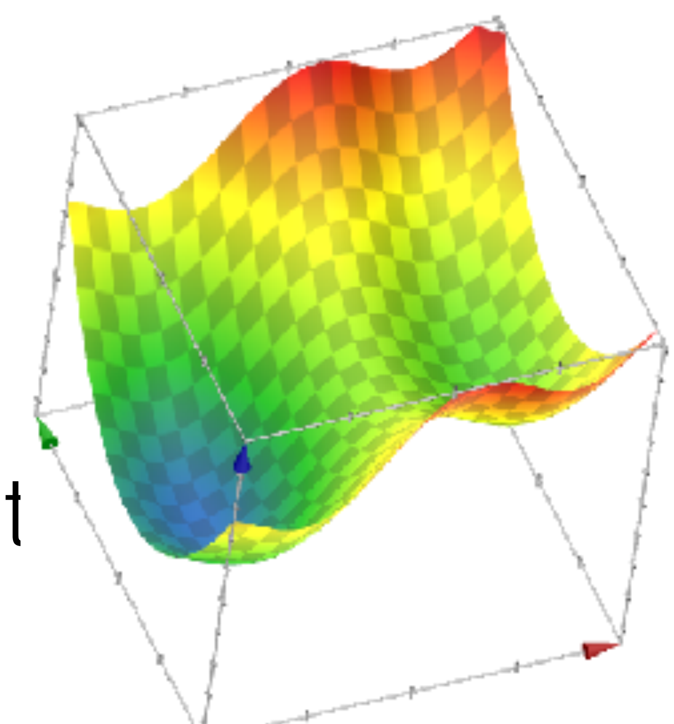

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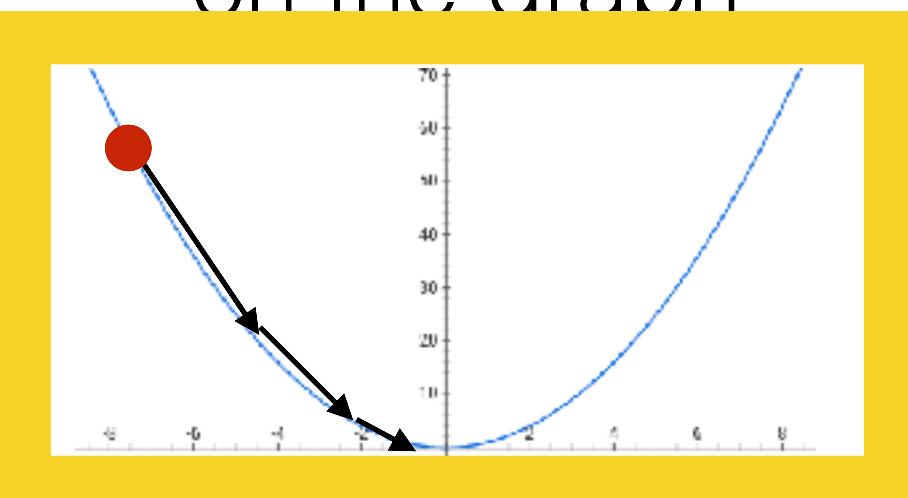

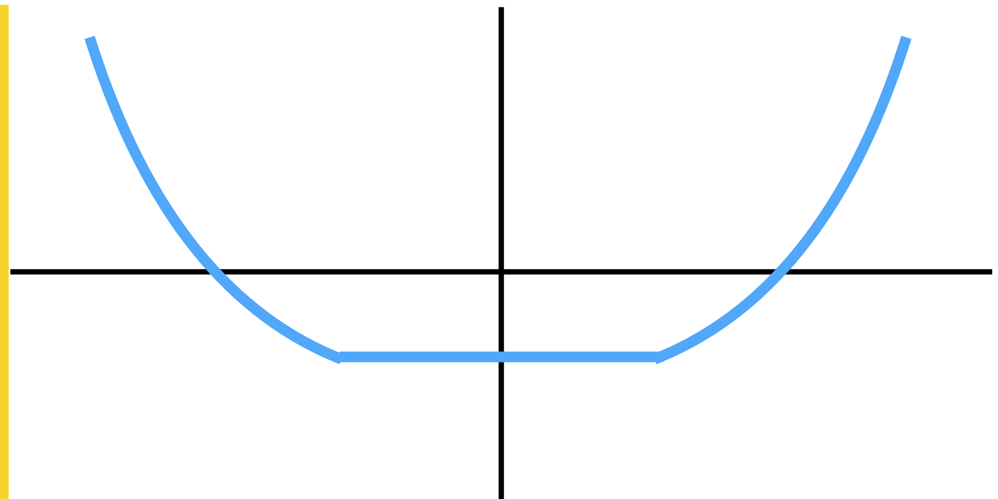

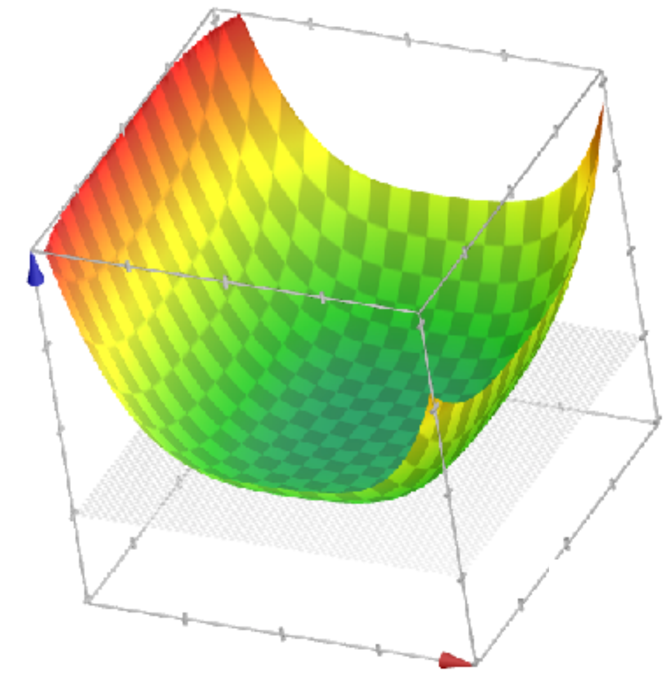

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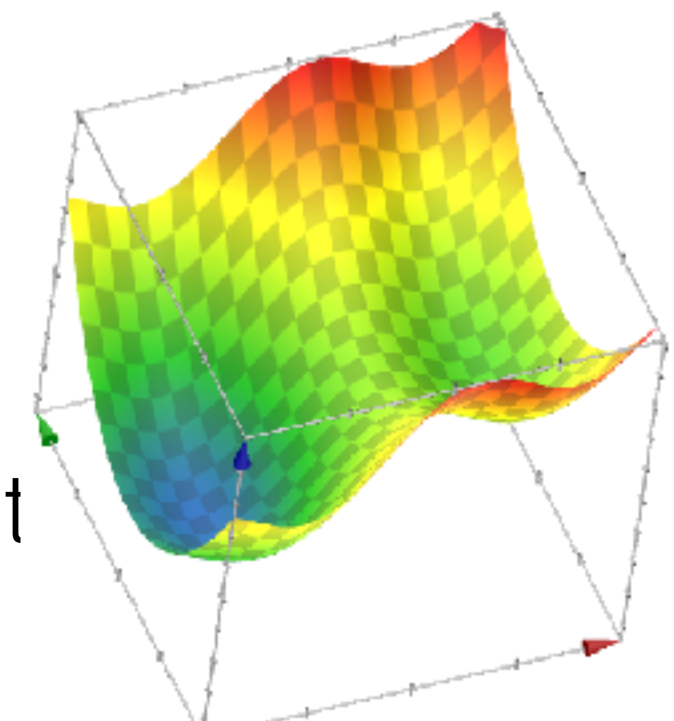

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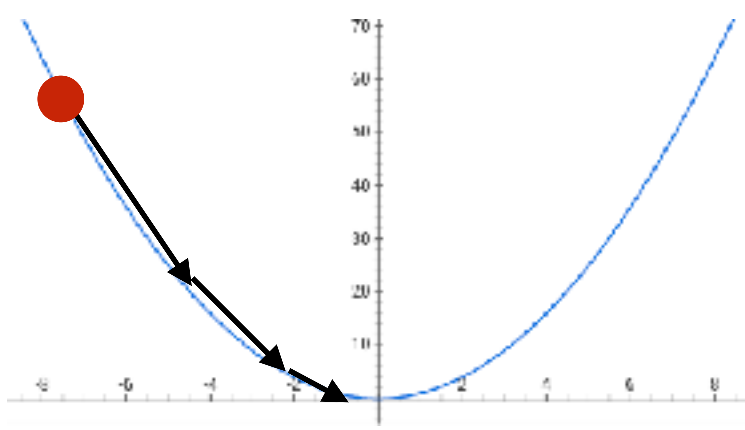

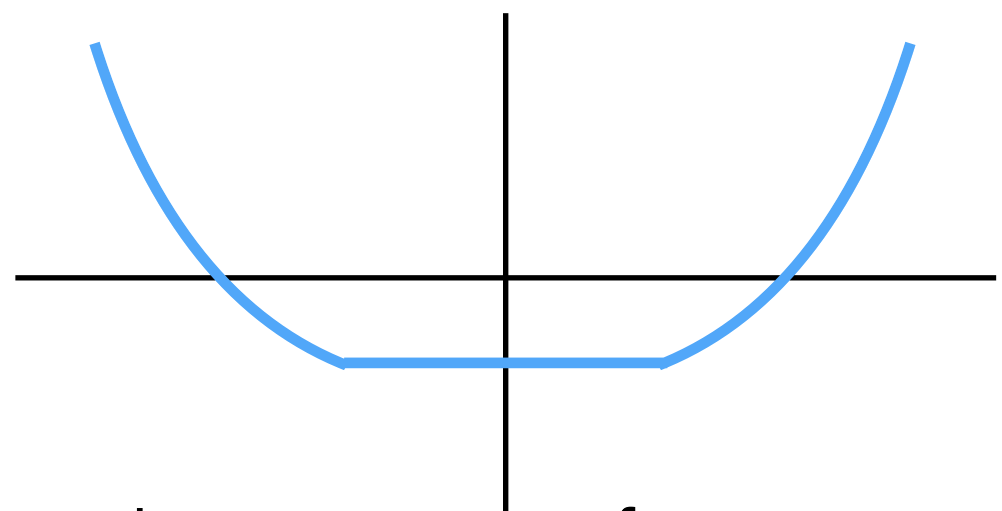

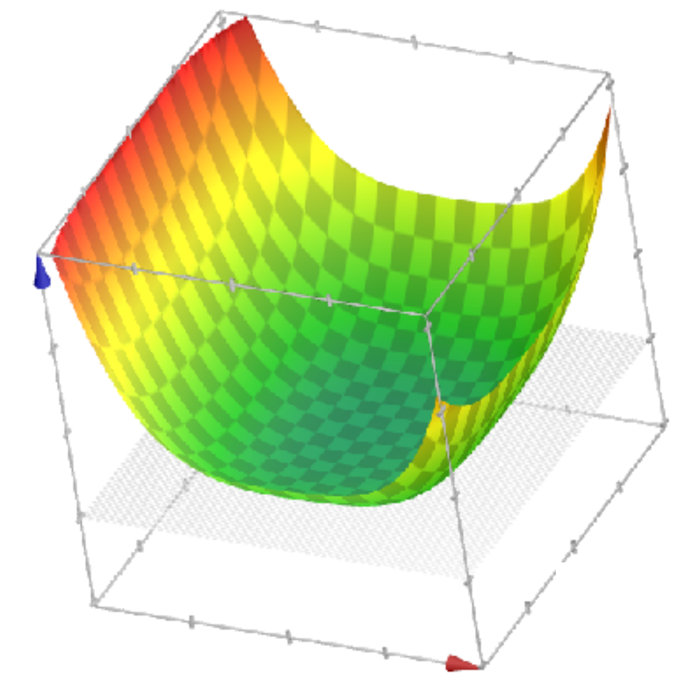

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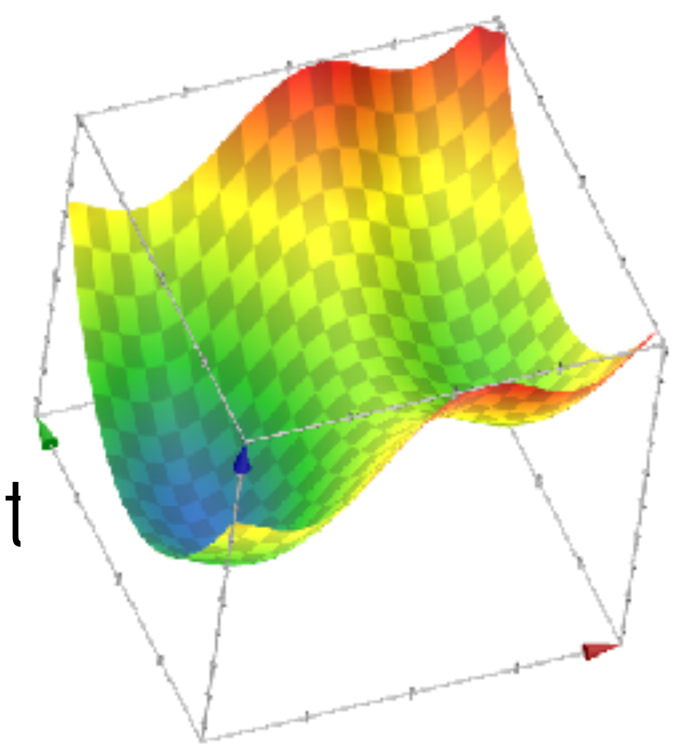

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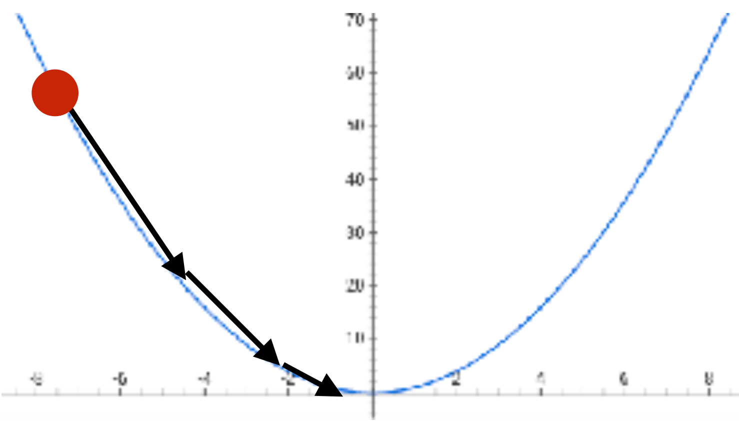

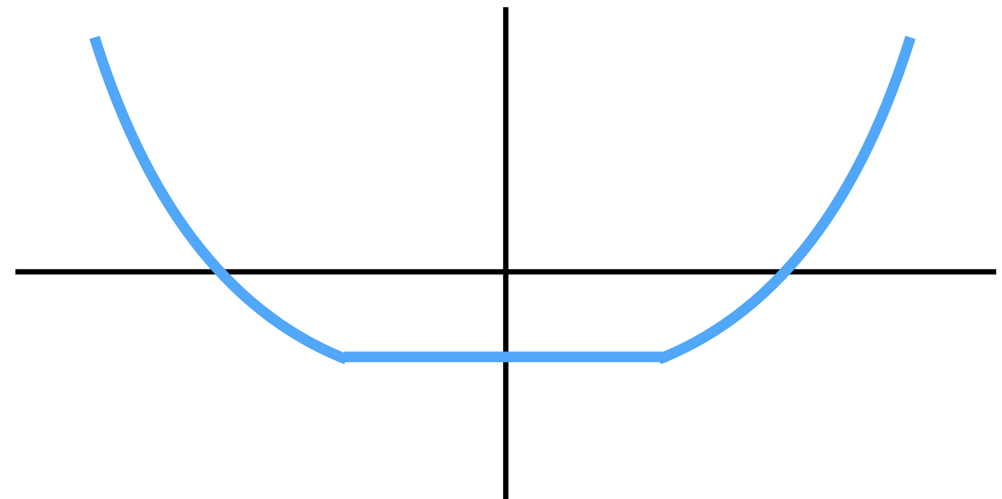

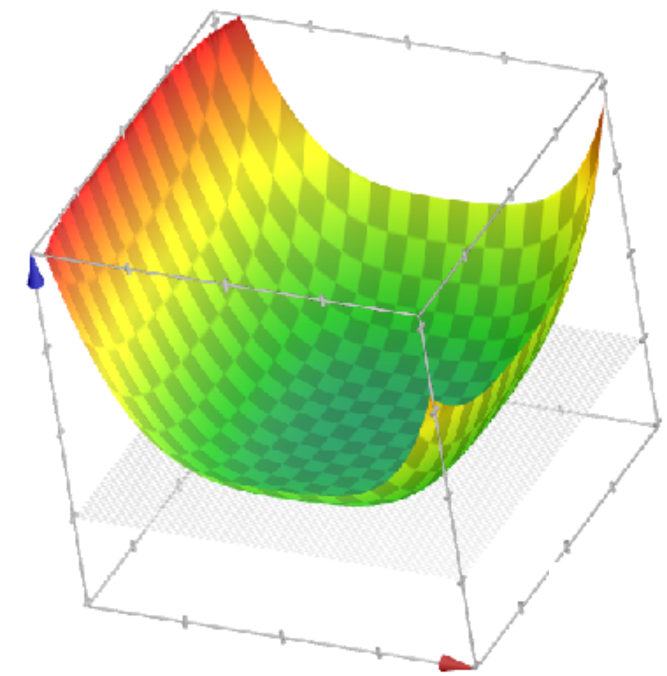

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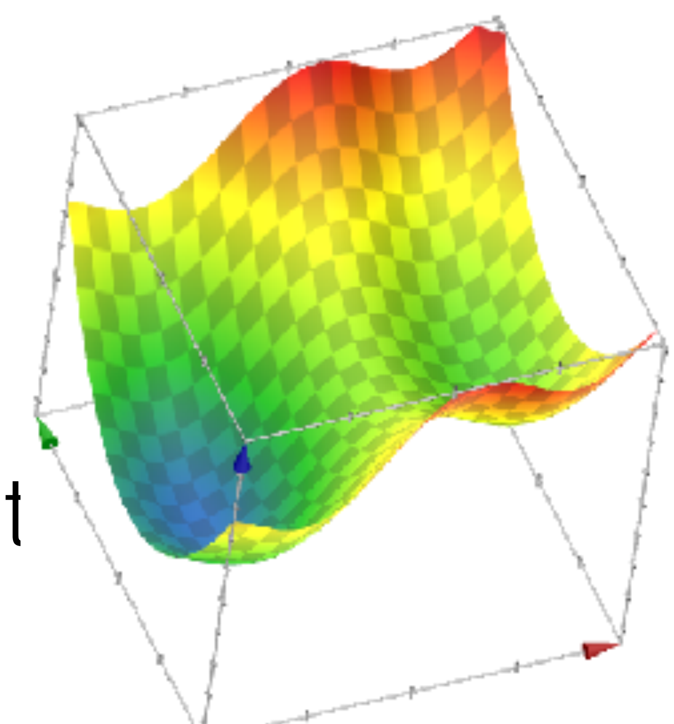

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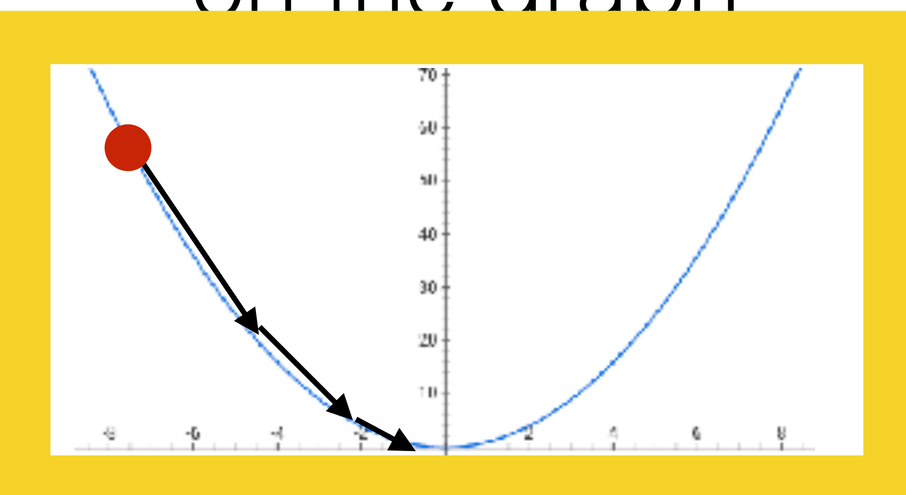
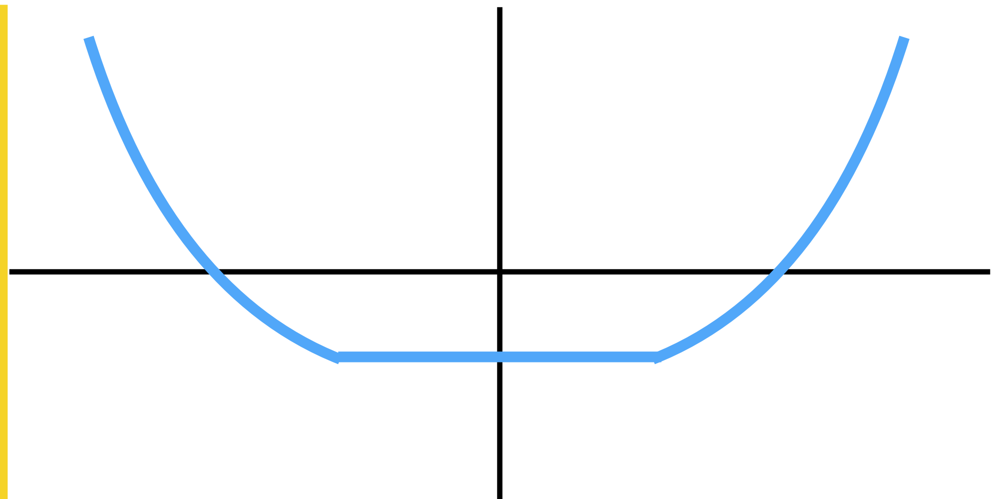
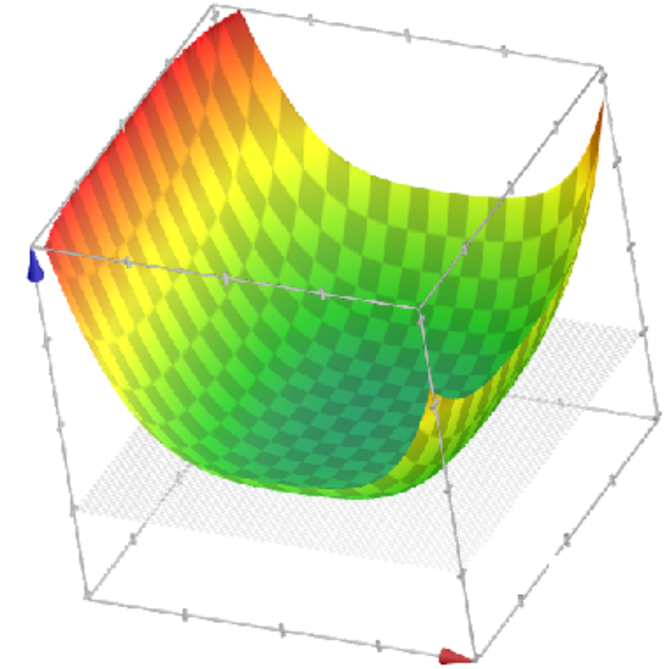
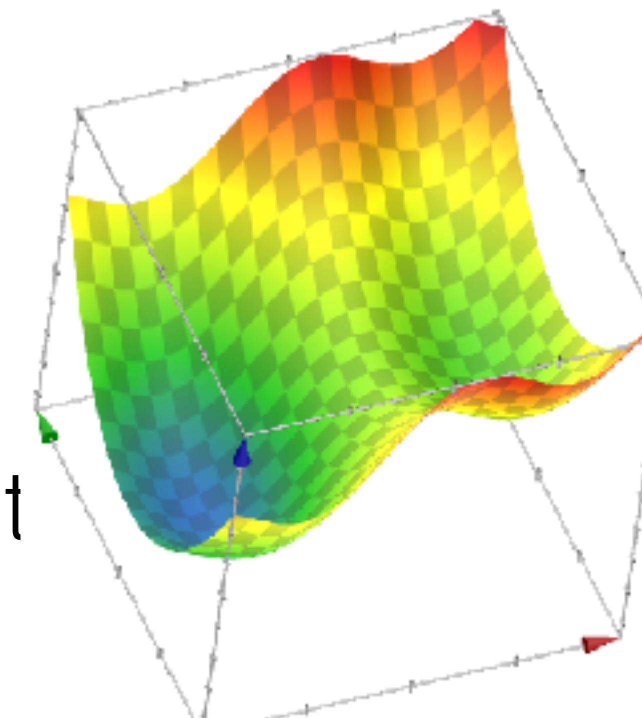






- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is sufficiently "smooth" and convex
    - *f* has at least one global optimum
    - η is sufficiently small
  - **Conclusion**: If run long enough, gradient descent will return a value within $\tilde{\epsilon}$ of a global optimum Θ



Optimizing ridge regression

Optimizing ridge regression

- Gradient descent

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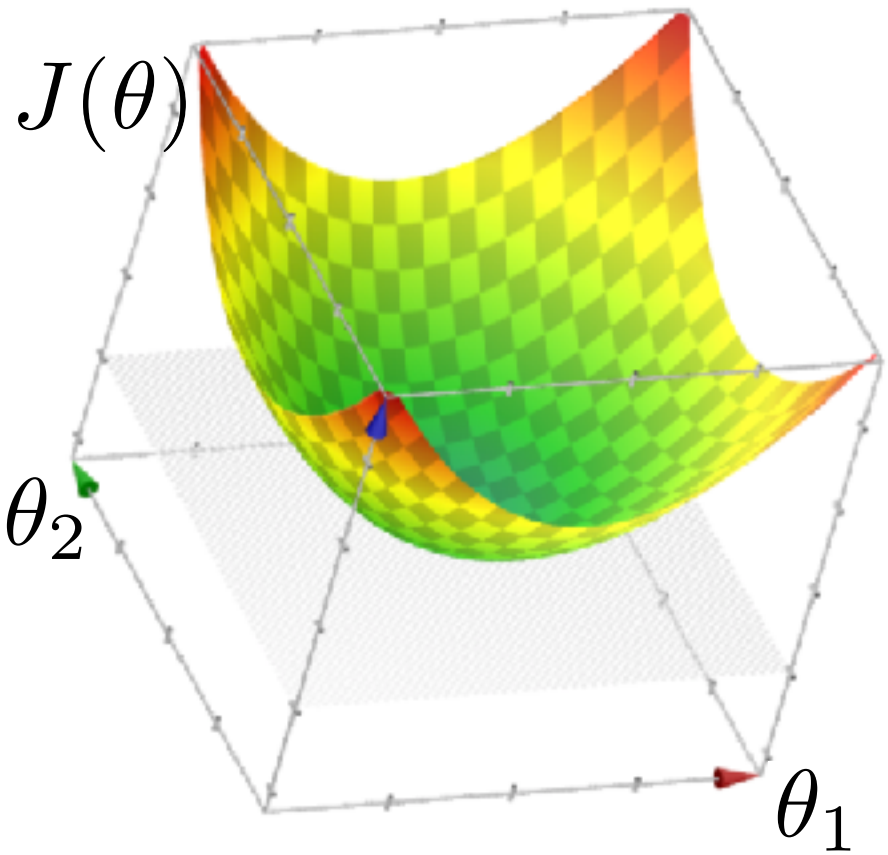
- Gradient descent



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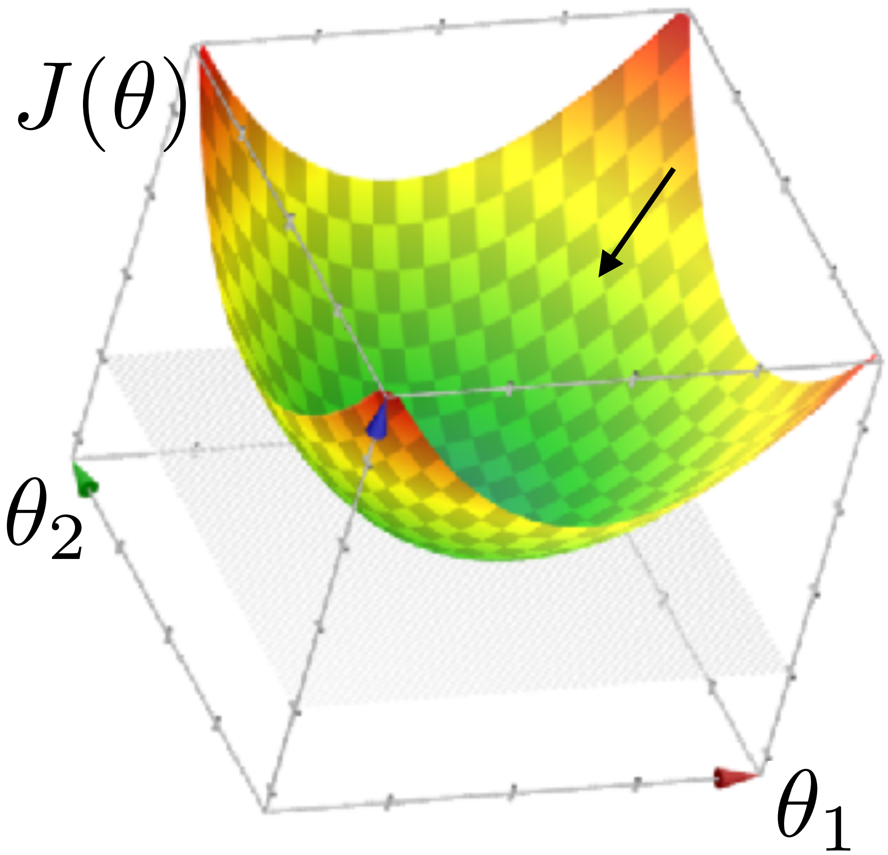

Optimizing ridge regression

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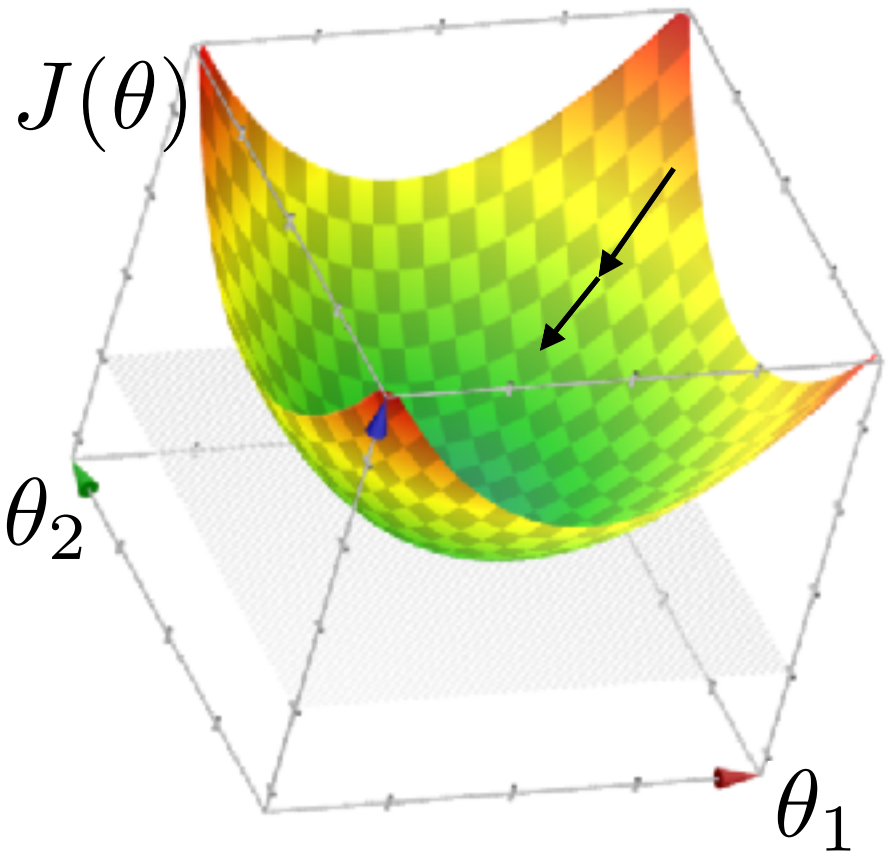

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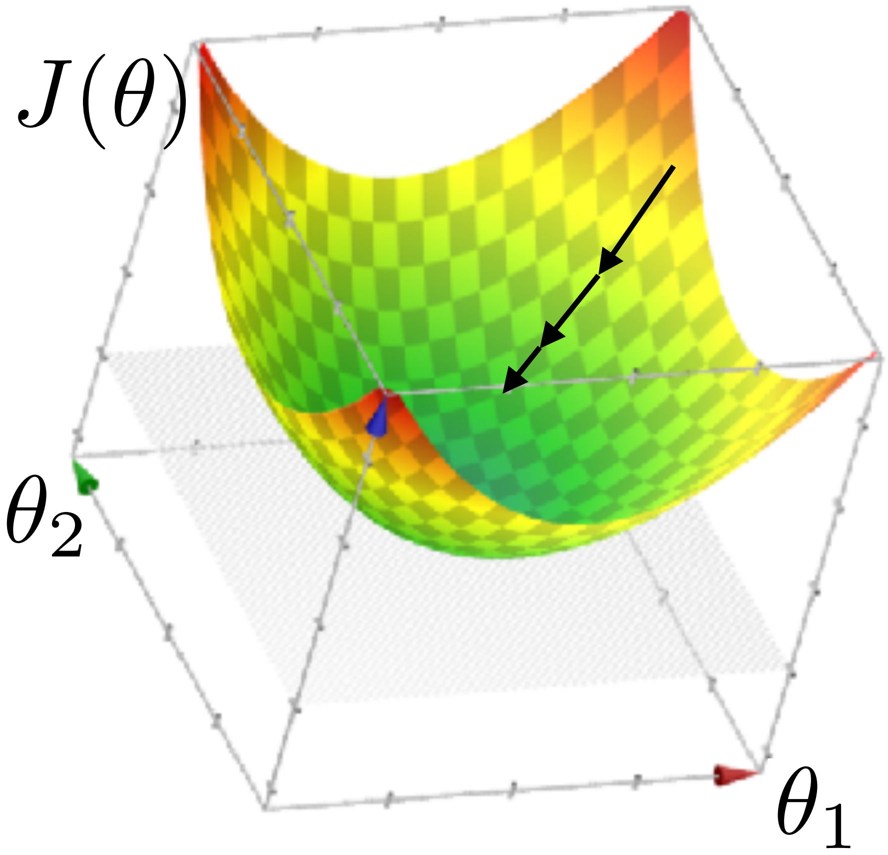

Optimizing ridge regression

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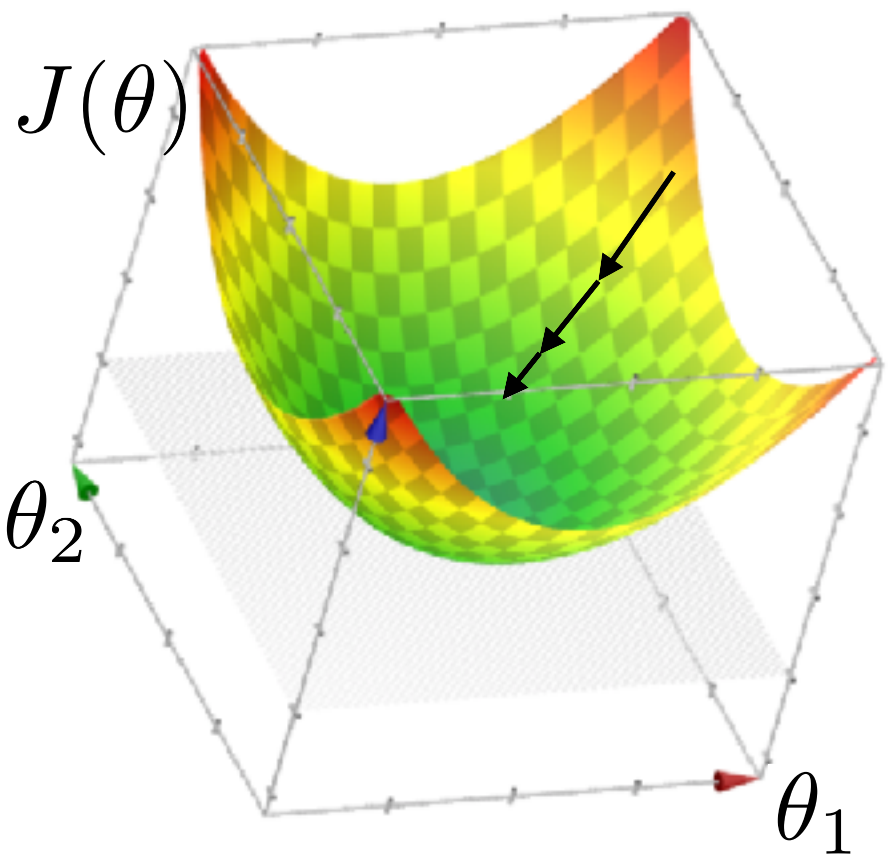
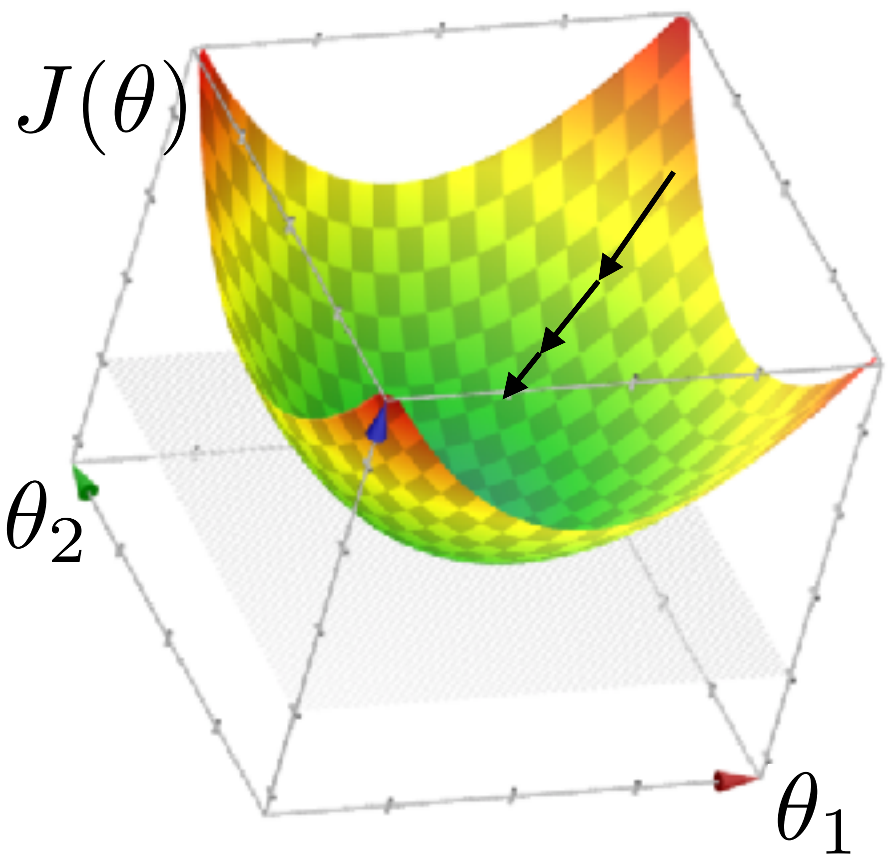


Optimizing ridge regression

- Gradient descent vs. analytical/closed-form/direct solution



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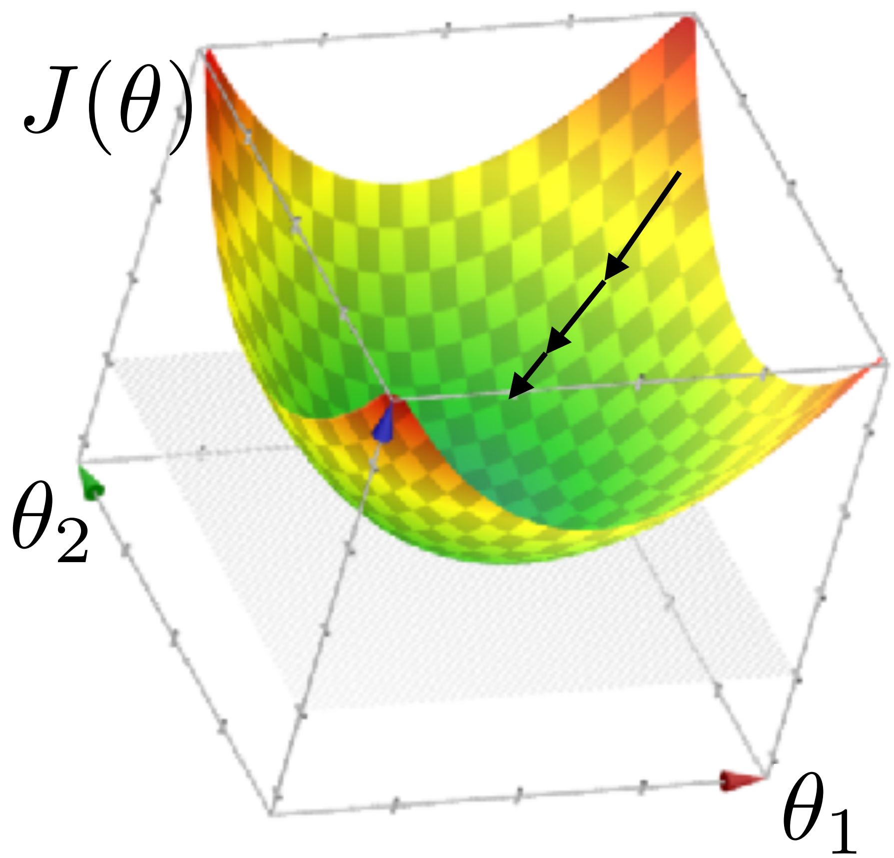

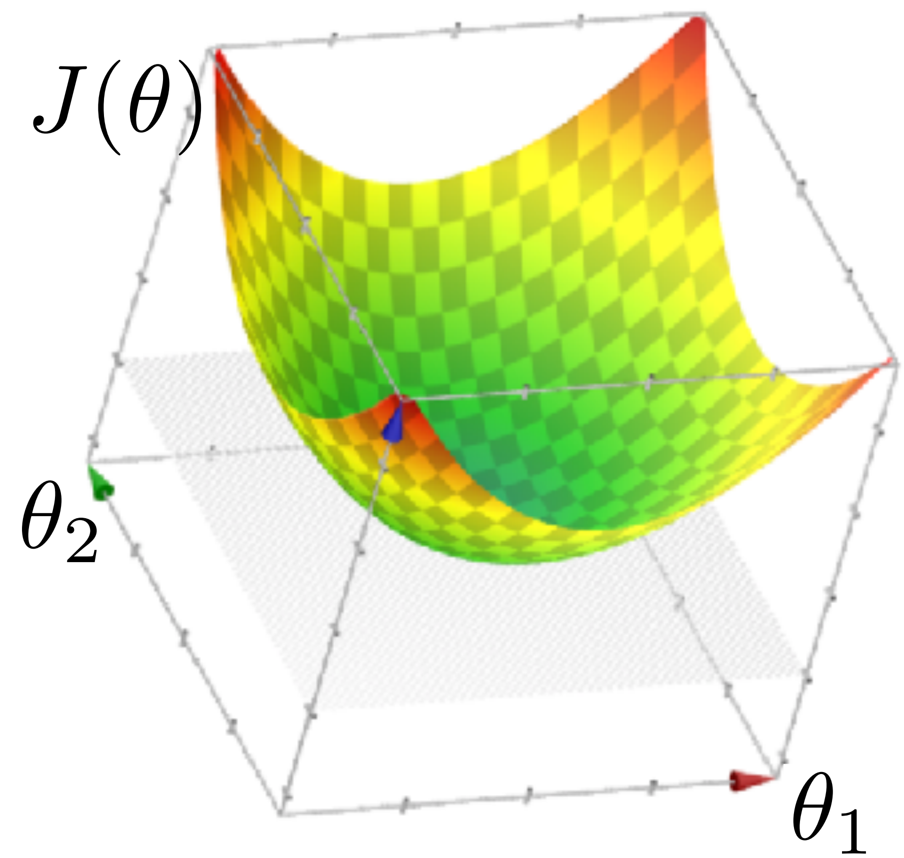

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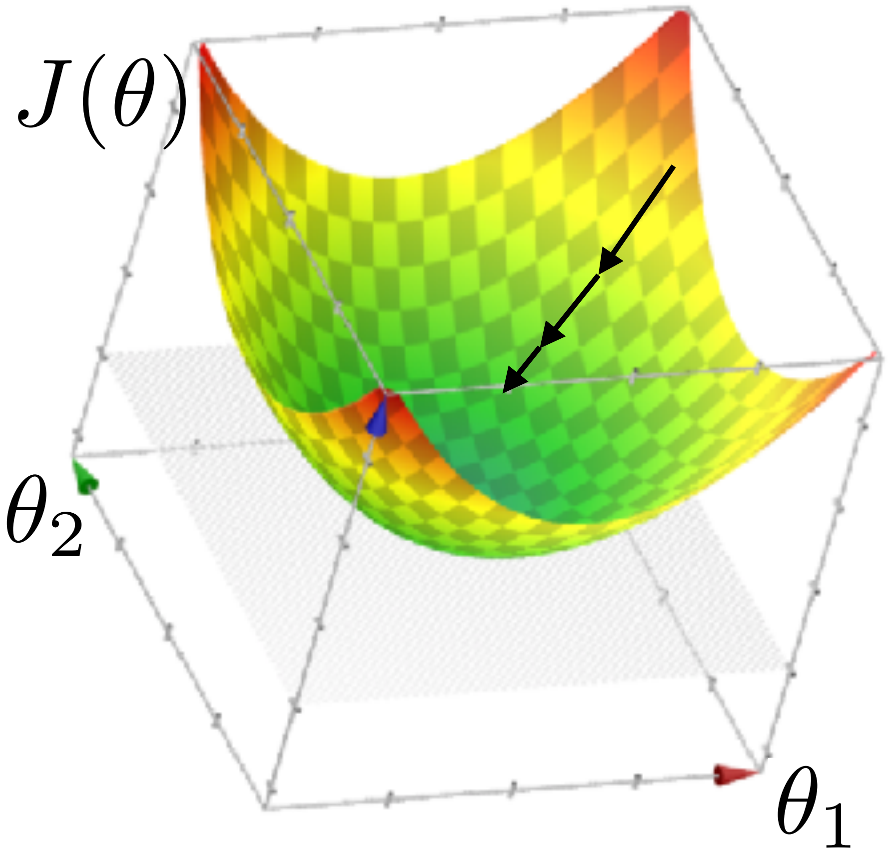

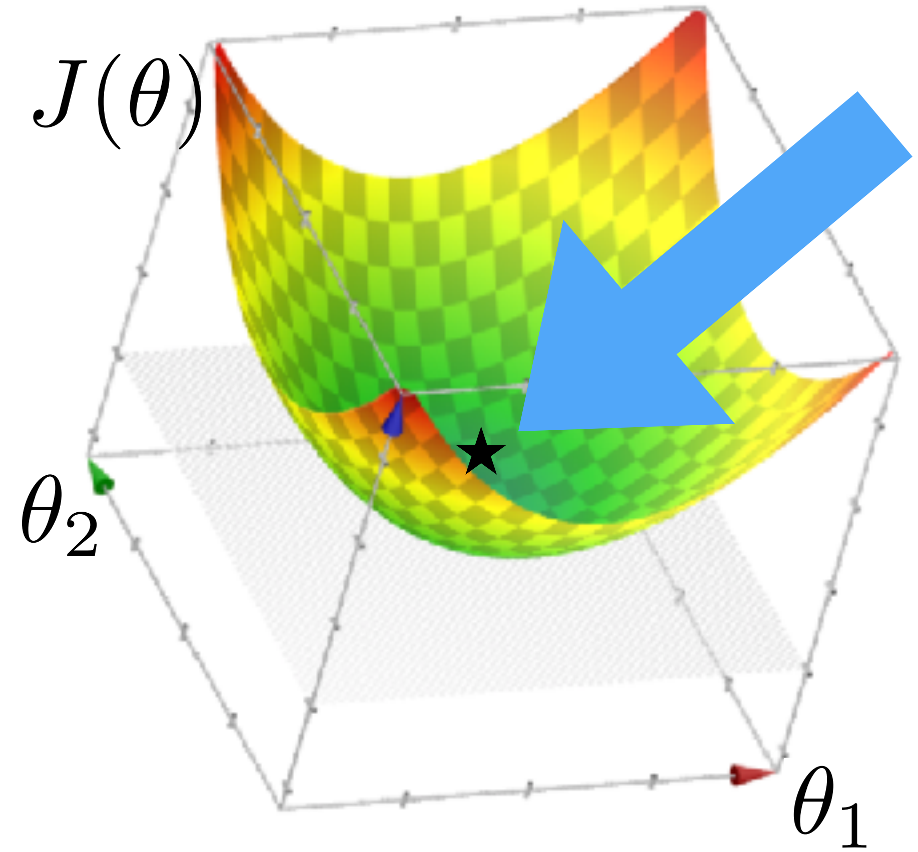

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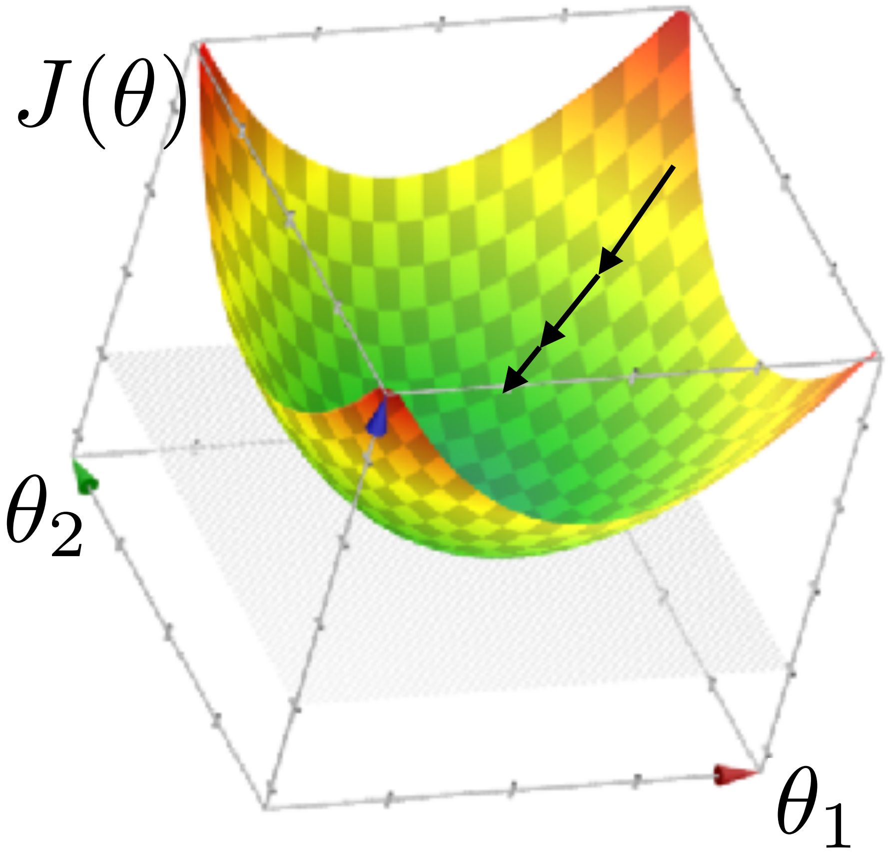

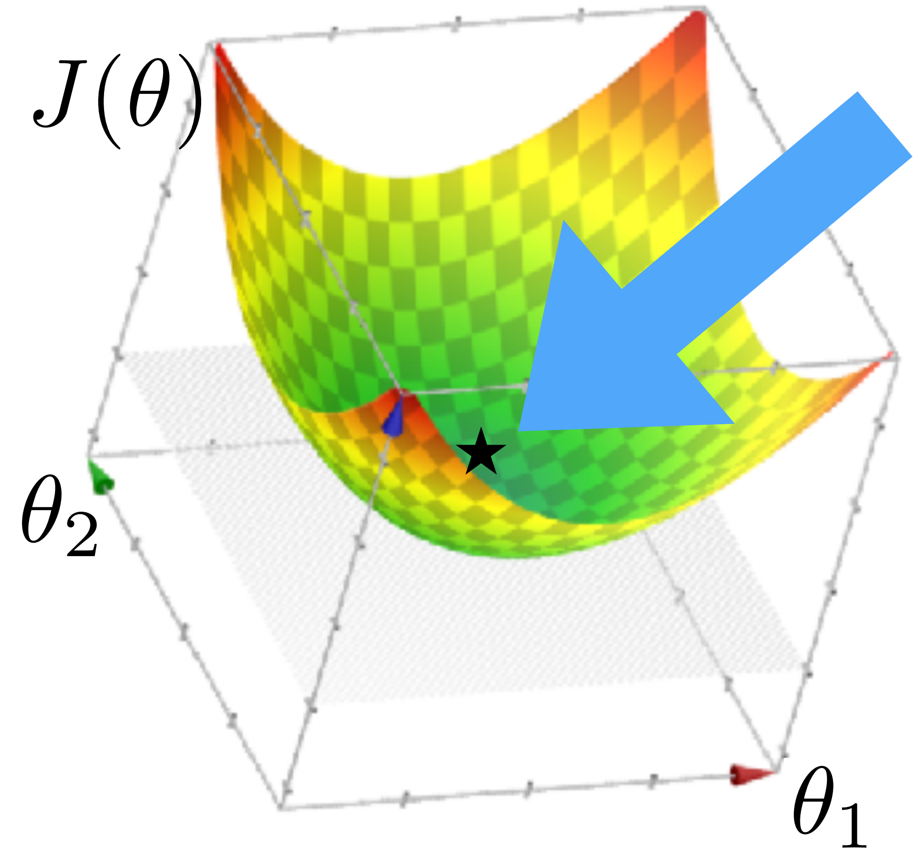

- Accuracy doesn't mean anything without running time

Optimizing ridge regression

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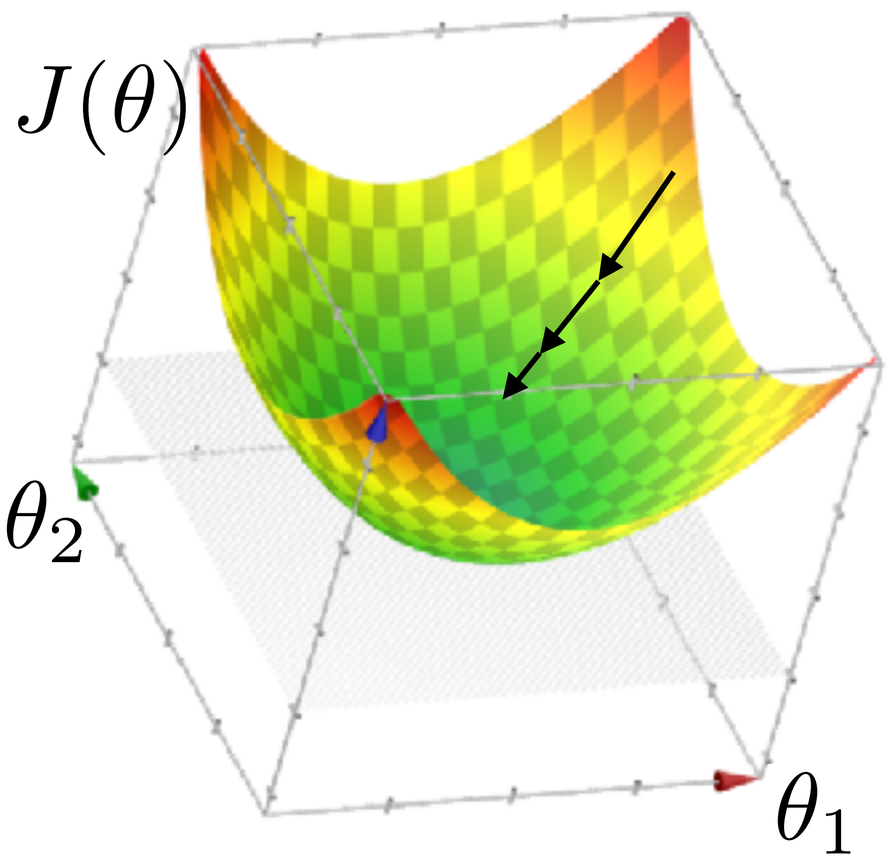

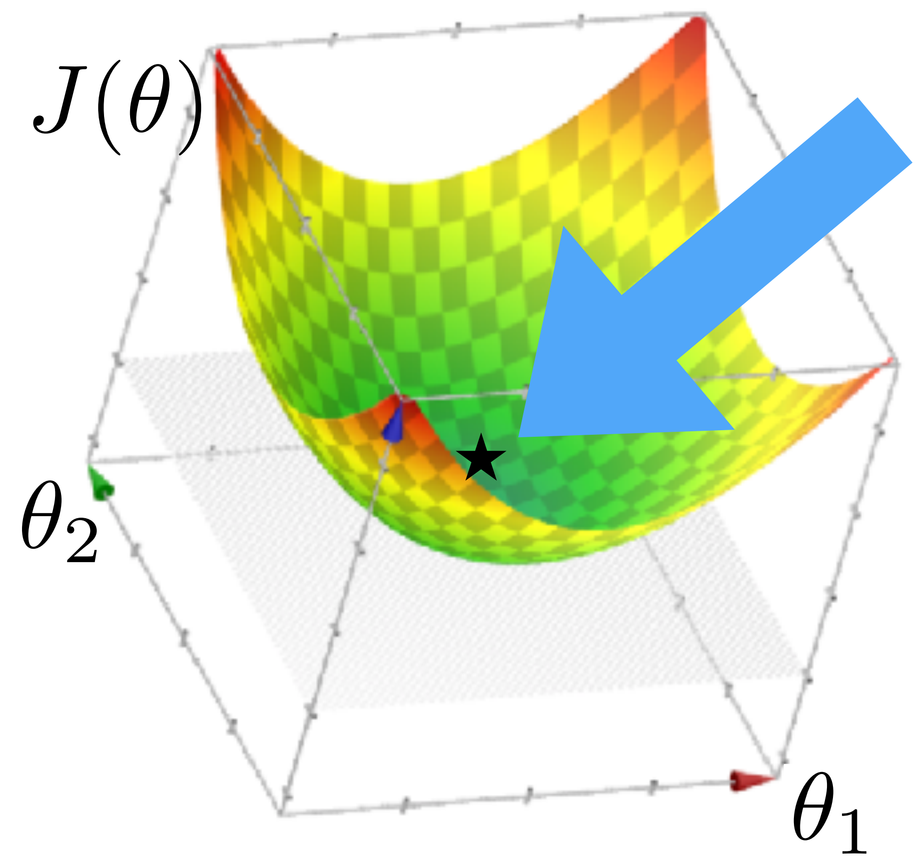

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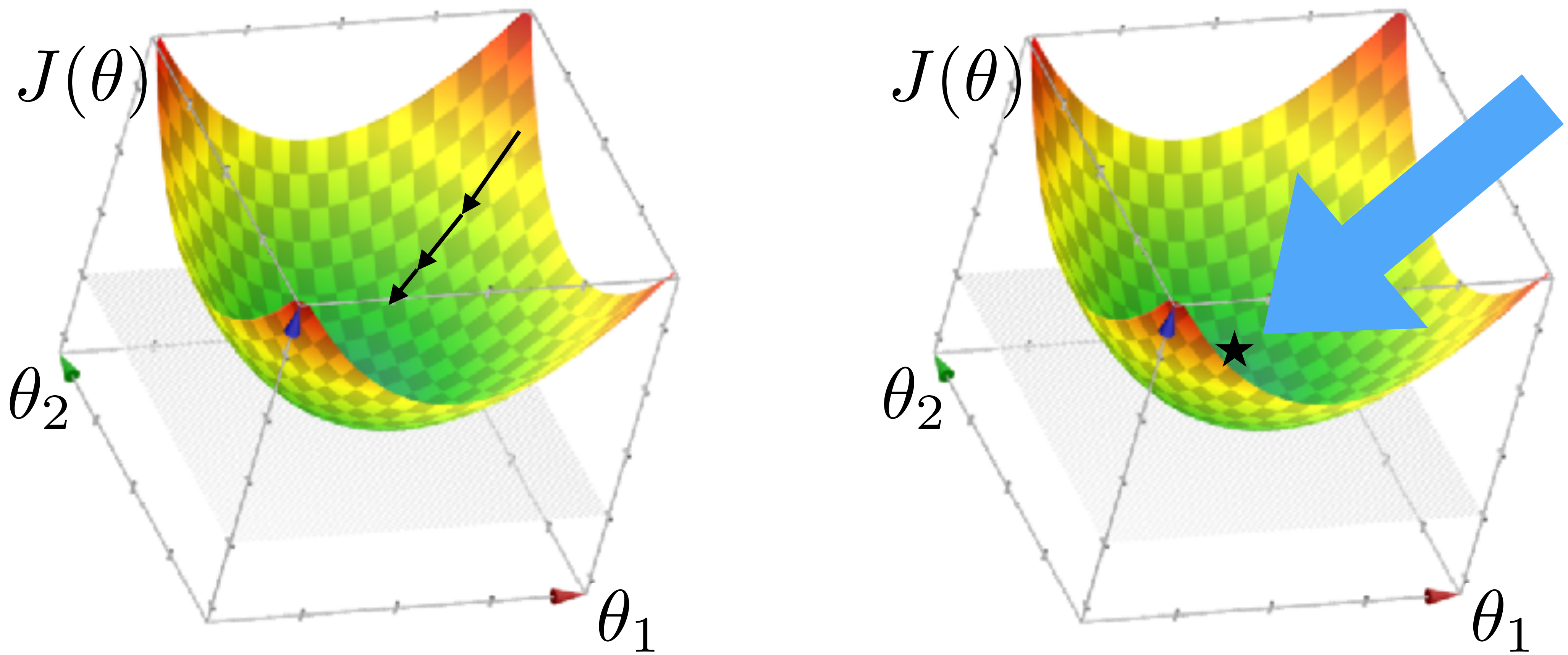

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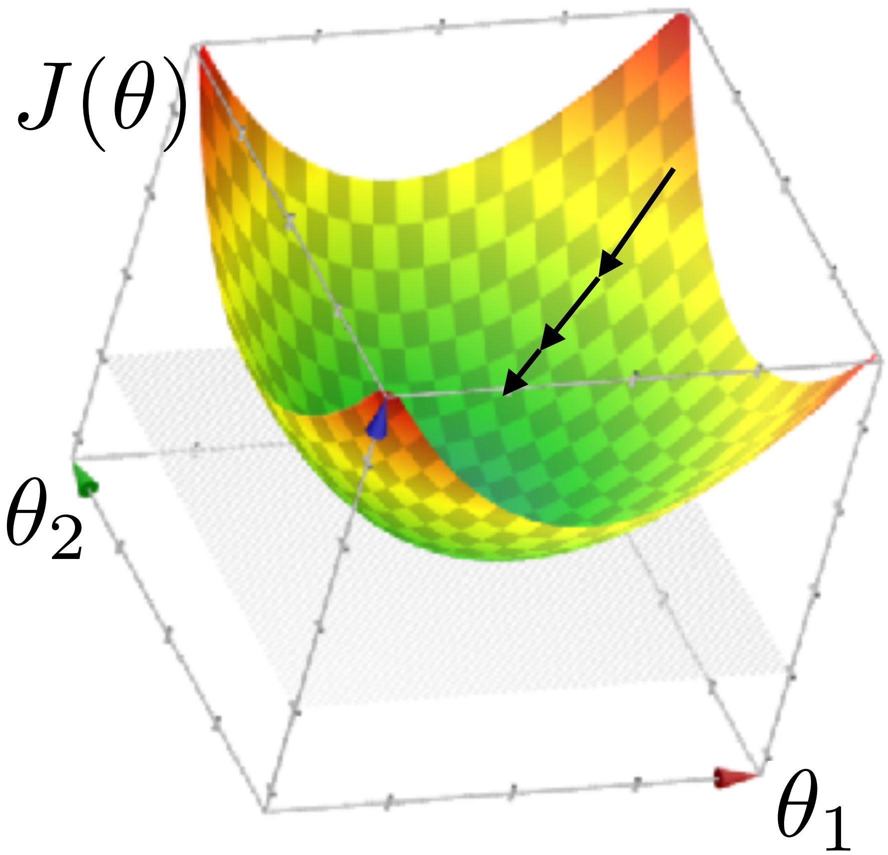

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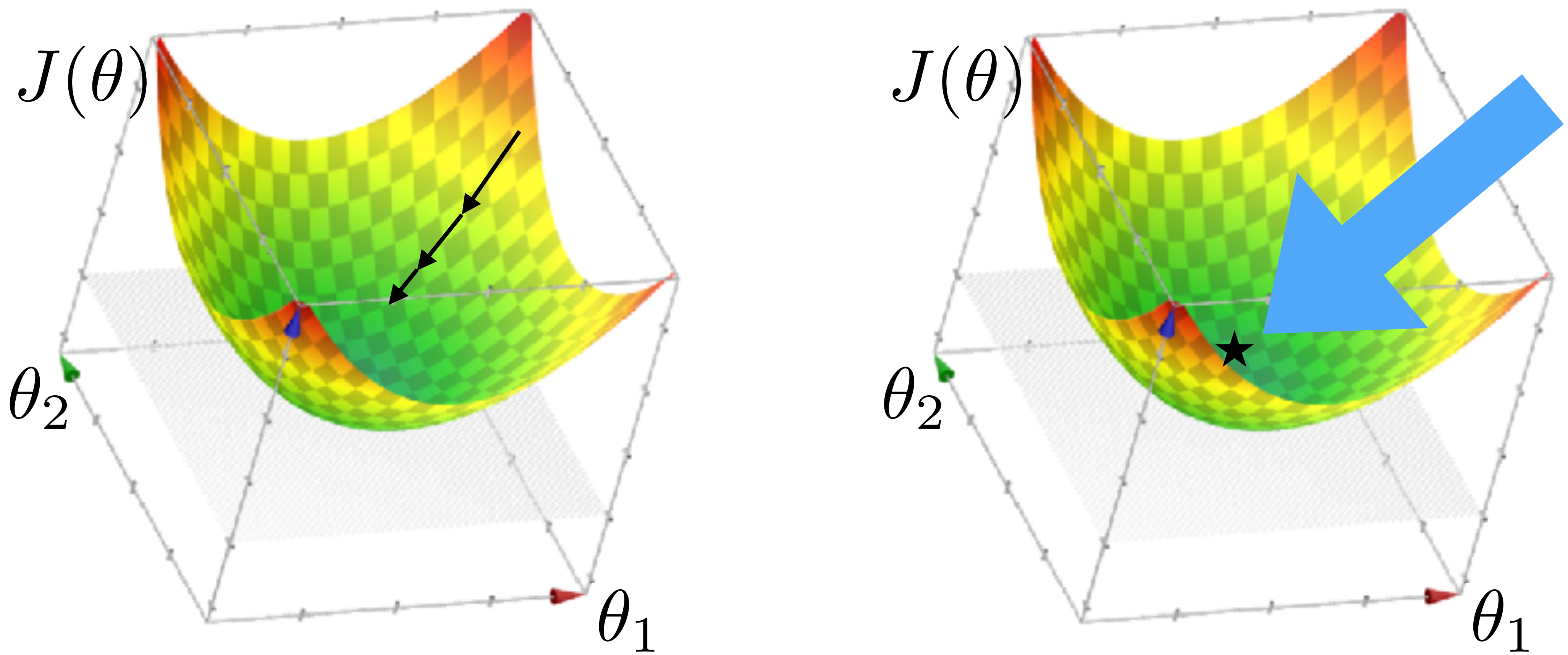

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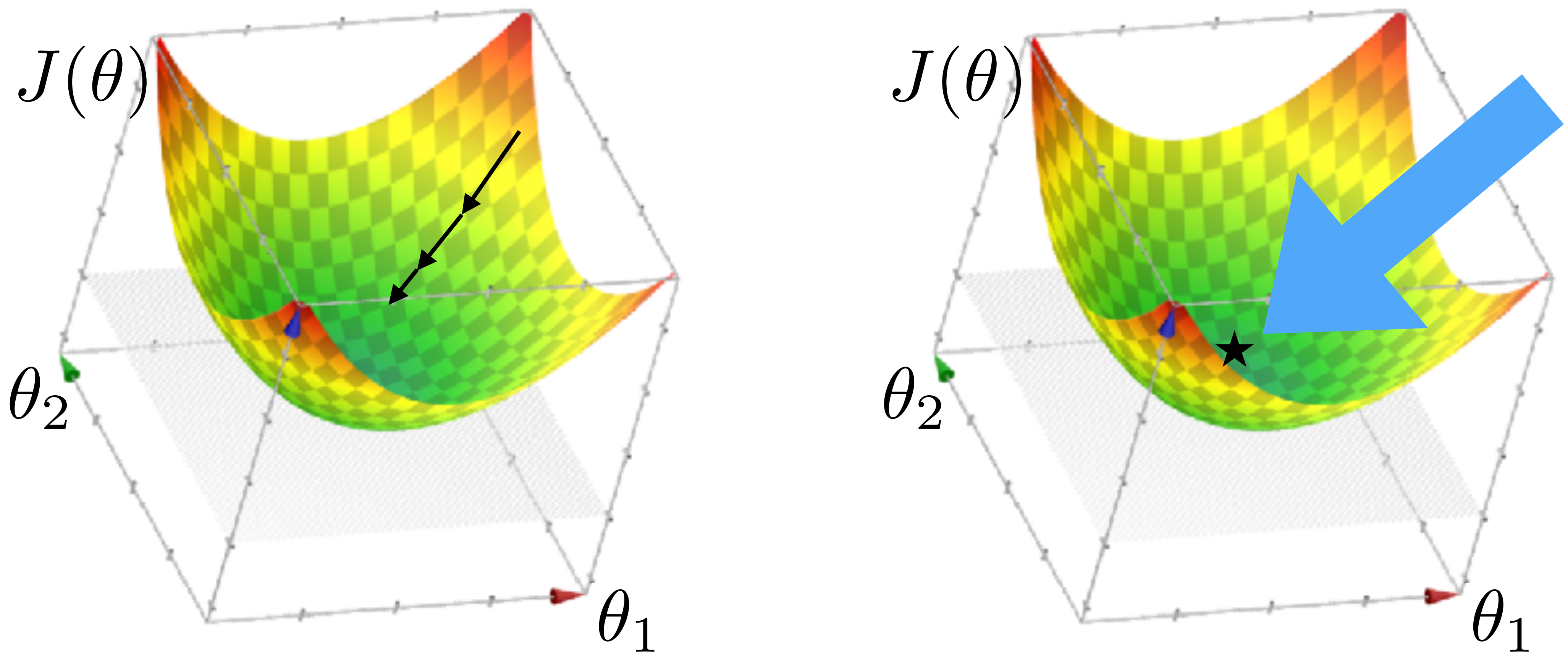

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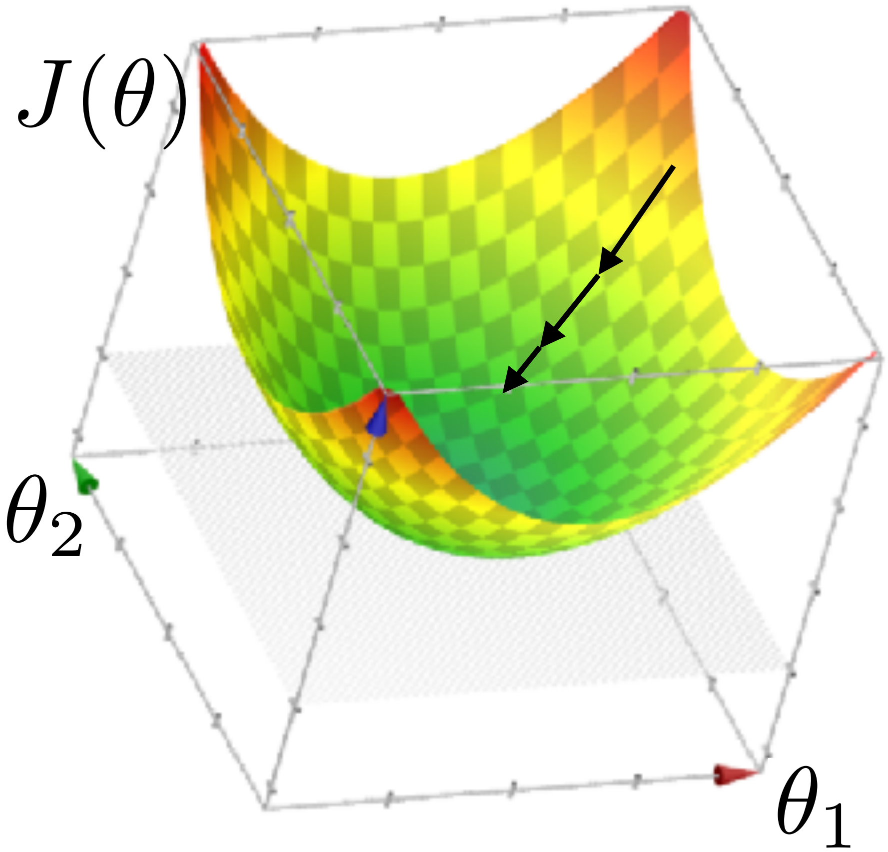

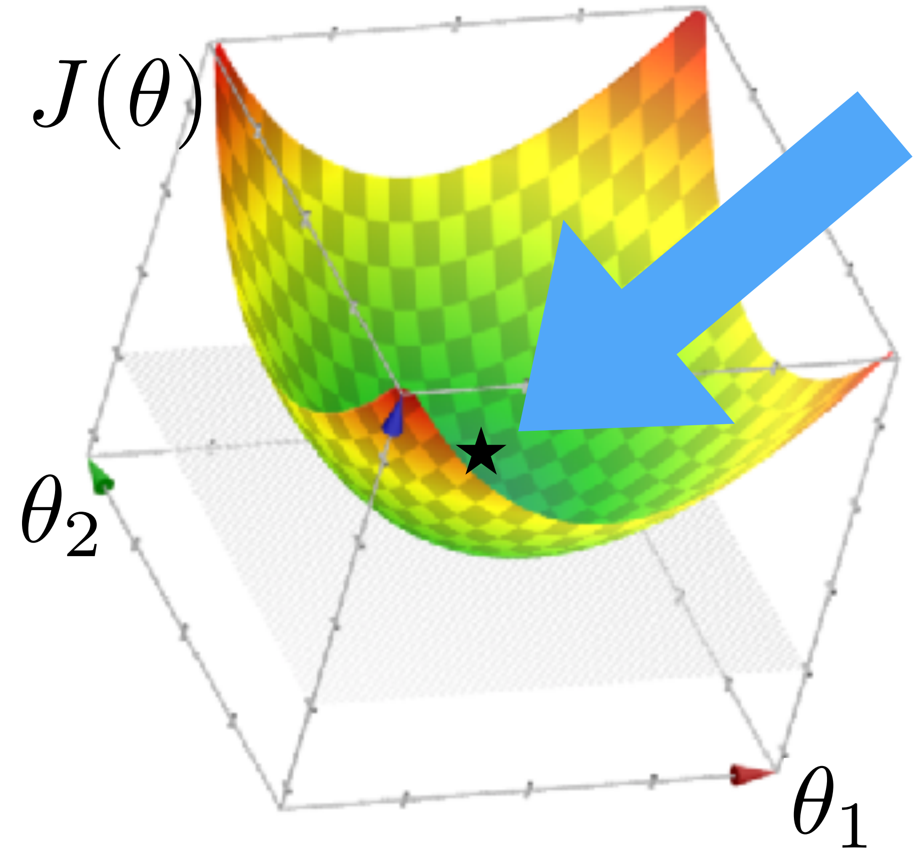

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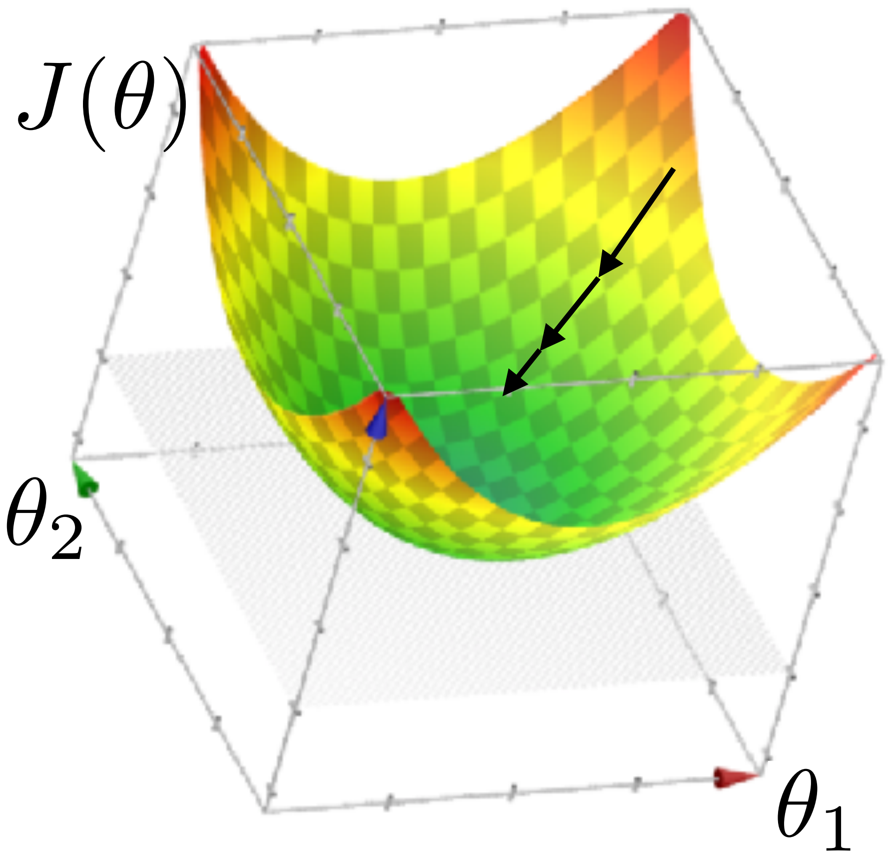

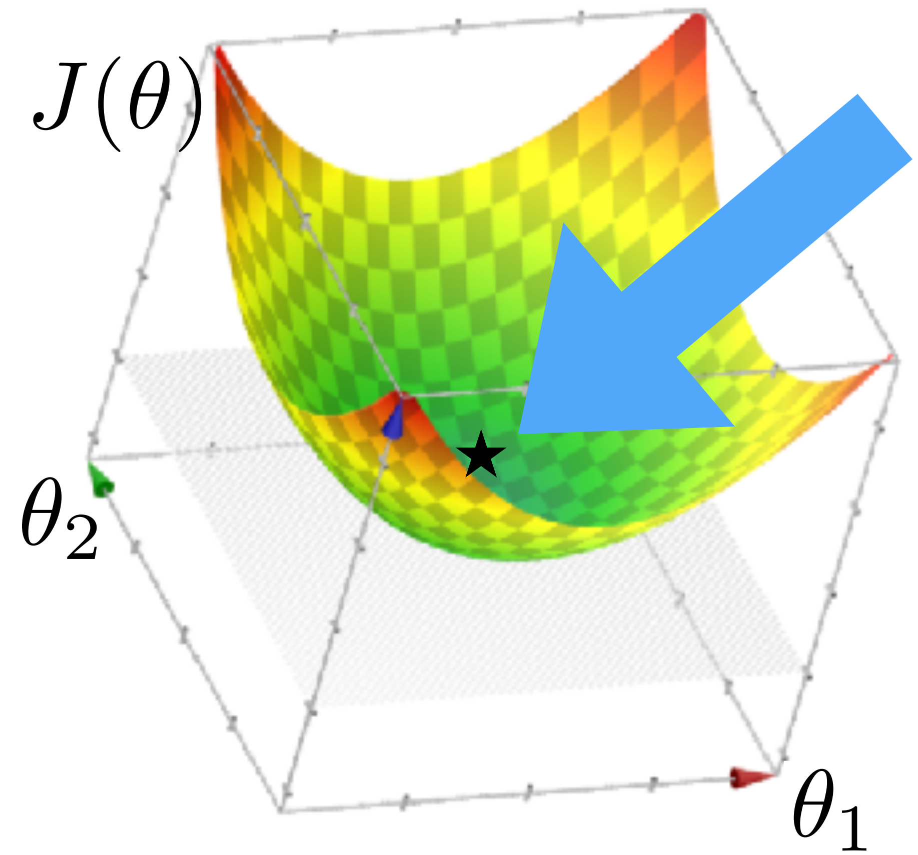

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Matrix inversion running time: $O(d^3)$

Gradient descent for ridge regression

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- Gradient descent with *f* = ridge regression objective

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  - For the moment, assume no offset (can extend)

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Gradient-Descent($\Theta_{\text{init}}, \eta, f, \nabla_\Theta f$)

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  Initialize t = 0

  repeat

   t = t + 1

   $\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

  until stopping criterion

  Return $\Theta^{(t)}$

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Gradient-Descent( Θ_init, η, f, ∇_Θ f )
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Exercise: Check!

 until stopping criterion

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until stopping criterion

Return $\theta^{(t)}$

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Gradient descent for ridge regression

- Gradient descent with *f* = ridge regression objective
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```
RidgeRegression-Gradient-Descent( θ_init, η )
  Initialize θ^(0) = θ_init
  Initialize t = 0
  repeat
    t = t + 1
```

$$\theta^{(t)} = \theta^{(t-1)} - \eta\left\{\frac{1}{n}\sum_{i=1}^{n} 2\left[\theta^{(t-1)\top}x^{(i)} - y^{(i)}\right]x^{(i)} + 2\lambda\theta^{(t-1)}\right\}$$

```
  until stopping criterion
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- No more matrix inversion! (see lab)

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- But have to look at all *n* data points every step

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    θ^(t) = θ^(t-1) − η{ (1/n) Σ_{i=1}^{n} 2[θ^(t-1)⊤ x^(i) − y^(i)] x^(i) + 2λθ^(t-1) }
  until stopping criterion
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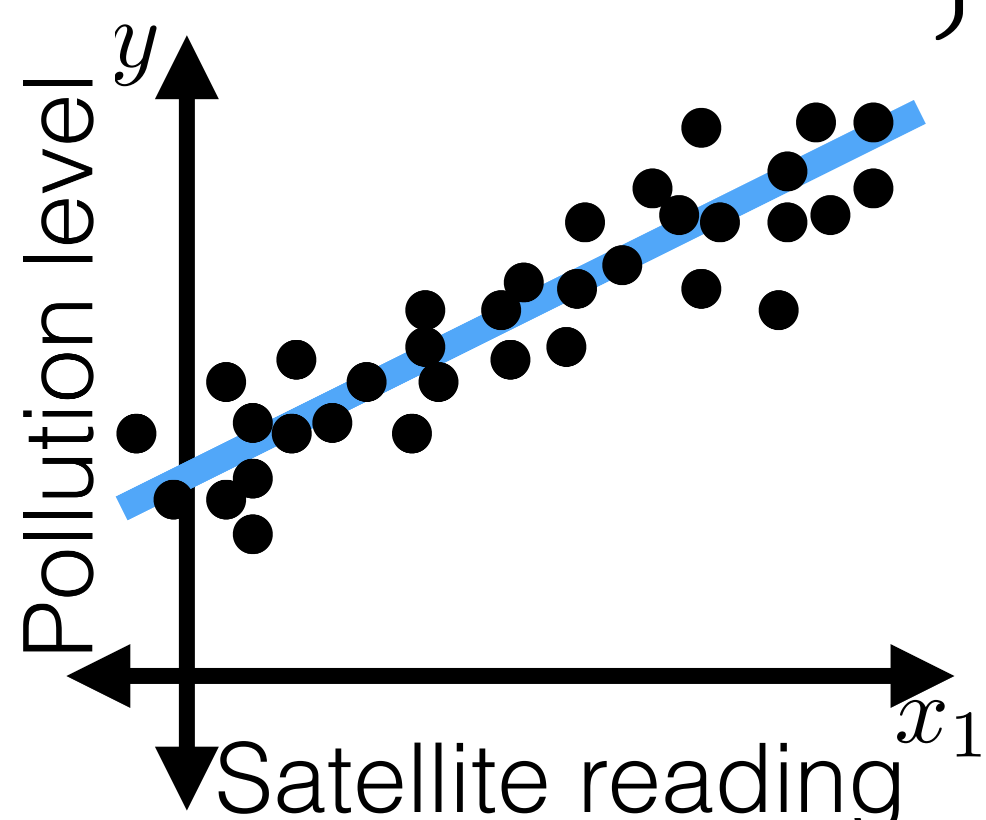

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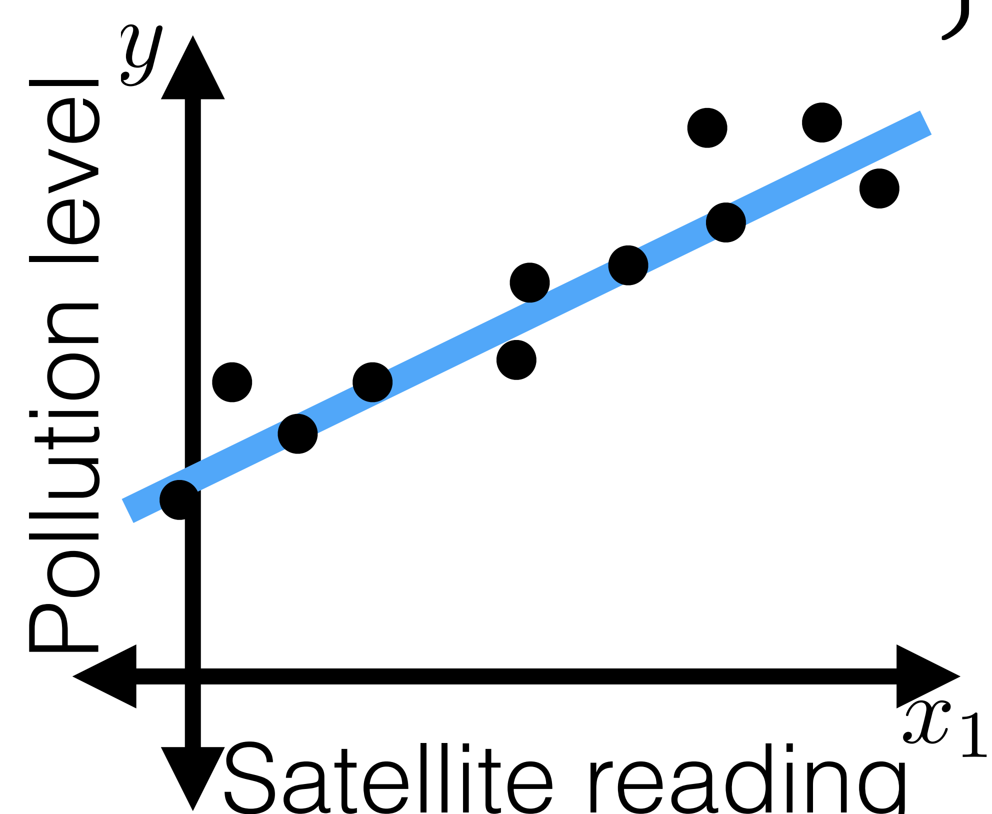

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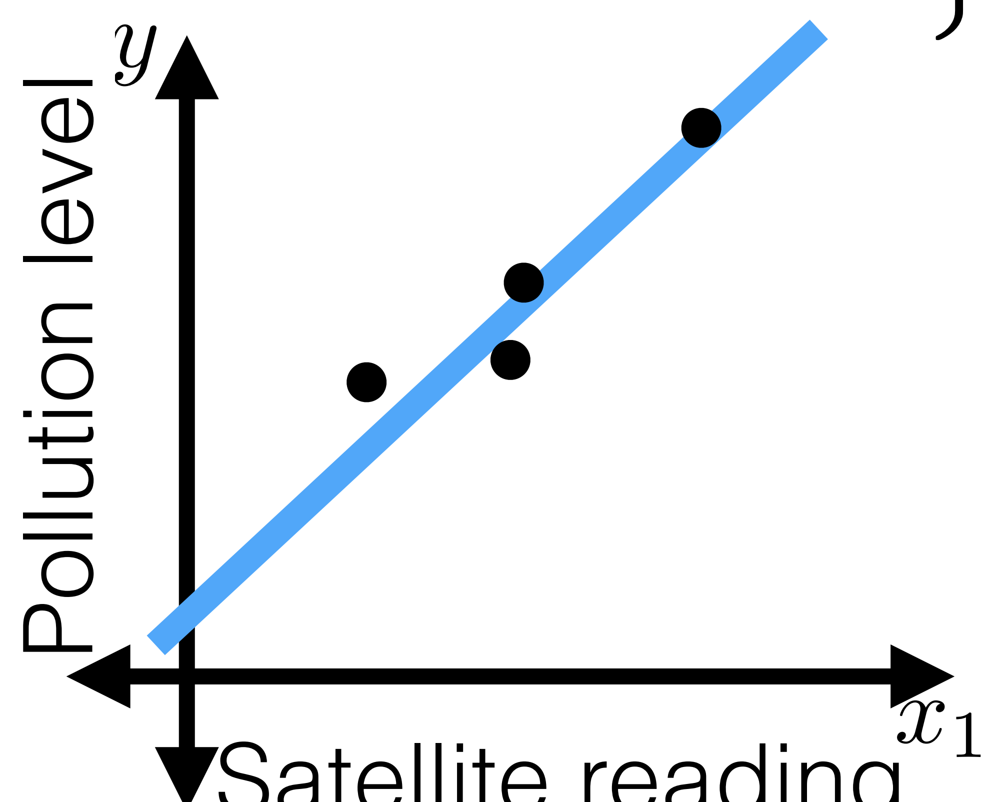
  Return $\theta^{(t)}$

Exercise: Check!

- No more matrix inversion! (see lab)
- But have to look at all *n* data points every step
- How to better handle large *n*?



Stochastic gradient descent

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

$$J_{\text{linreg}}(\Theta) = J_{\text{linreg}}(\theta, \theta_0) = \frac{1}{n}\sum_{i=1}^{n}(\theta^\top x^{(i)} + \theta_0 - y^{(i)})^2$$

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

$$J_{\text{linreg}}(\Theta) = J_{\text{linreg}}(\theta, \theta_0) = \frac{1}{n} \sum_{i=1}^{n} (\theta^\top x^{(i)} + \theta_0 - y^{(i)})^2$$

- A common machine learning objective:

$$f(\Theta) = \frac{1}{n} \sum_{i=1}^{n} f_i(\Theta)$$

Stochastic gradient descent

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- Stay tuned for more examples

Stochastic gradient descent

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`Stochastic-Gradient-Descent(`$\Theta_{\text{init}}, \eta, T$`)`

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

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- A common machine learning objective:

$$f(\Theta) = \frac{1}{n}\sum_{i=1}^{n} f_i(\Theta)$$

  - Stay tuned for more examples
  - Compare to training error defn.

Stochastic-Gradient-Descent($\Theta_{\text{init}}, \eta, T$)

Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

$$J_{\text{linreg}}(\Theta) = J_{\text{linreg}}(\theta, \theta_0) = \frac{1}{n}\sum_{i=1}^{n}(\theta^\top x^{(i)} + \theta_0 - y^{(i)})^2$$

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- Compare to training error defn.

Stochastic-Gradient-Descent($\Theta_{\text{init}}, \eta, T$)

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  for t = 1 **to** T

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

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- Compare to training error defn.

Stochastic-Gradient-Descent($\Theta_{\text{init}}, \eta, T$)

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

  for t = 1 **to** T

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Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

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Stochastic-Gradient-Descent($\Theta_{\text{init}}, \eta, T$)

  Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

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Stochastic gradient descent

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- Stay tuned for more examples
- Compare to training error defn.

```
Stochastic-Gradient-Descent(Θ_init, η, T)
  Initialize Θ^(0) = Θ_init
  for t = 1 to T
    randomly select i from {1,…,n} (with equal probability)
    Θ^(t) = Θ^(t-1) - η(t) ∇_Θ f_i(Θ^(t-1))
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Stochastic-Gradient-Descent($\Theta_{\text{init}}, \eta, T$)

Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

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Compare to gradient descent update:

$$\Theta^{(t)} = \Theta^{(t-1)} - \eta\nabla_\Theta f(\Theta^{(t-1)})$$

Stochastic gradient descent

- Linear regression objective (with $\lambda = 0$):

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$$f(\Theta) = \frac{1}{n}\sum_{i=1}^{n} f_i(\Theta)$$

- Stay tuned for more examples
- Compare to training error defn.

```
Stochastic-Gradient-Descent(Θ_init, η, T)
  Initialize Θ^(0) = Θ_init
  for t = 1 to T
    randomly select i from {1,…,n} (with equal probability)
    Θ^(t) = Θ^(t−1) − η(t) ∇_Θ f_i(Θ^(t−1))
```

$\Theta_{\text{init}}, \eta, T$; $\Theta^{(0)} = \Theta_{\text{init}}$

$$\Theta^{(t)} = \Theta^{(t-1)} - \eta(t)\nabla_\Theta f_i(\Theta^{(t-1)})$$

Compare to gradient descent update:

$$\Theta^{(t)} = \Theta^{(t-1)} - \eta\nabla_\Theta f(\Theta^{(t-1)})$$

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  Initialize Θ^(0) = Θ_init
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    Θ^(t) = Θ^(t-1) − η(t) ∇_Θ f_i(Θ^(t-1))
```

$\text{Stochastic-Gradient-Descent}(\Theta_{\text{init}}, \eta, T)$

Initialize $\Theta^{(0)} = \Theta_{\text{init}}$

for t = 1 **to** T

randomly select i from {1,…,n} (with equal probability)

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Compare to gradient descent update:

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Stochastic gradient descent

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$\Theta^{(t)} = \Theta^{(t-1)} - \eta\nabla_\Theta f(\Theta^{(t-1)})$

Stochastic gradient descent

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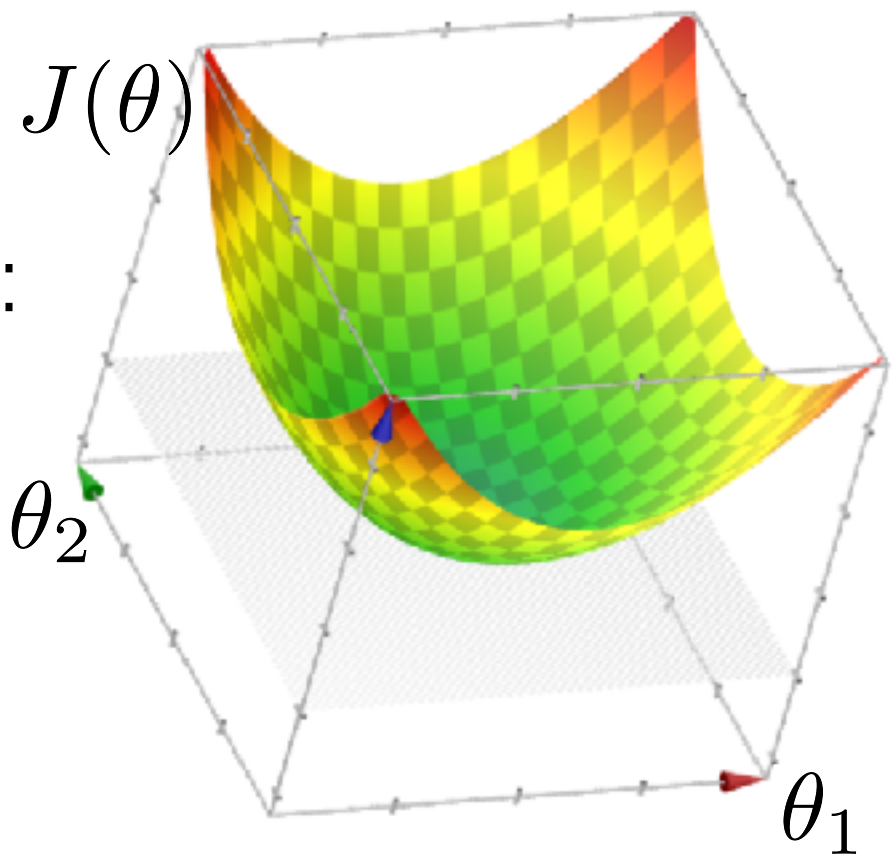
$\Theta^{(t)} = \Theta^{(t-1)} - \eta \nabla_\Theta f(\Theta^{(t-1)})$

- Commonly used with “minibatches”

Stochastic gradient descent (SGD) properties

- GD:



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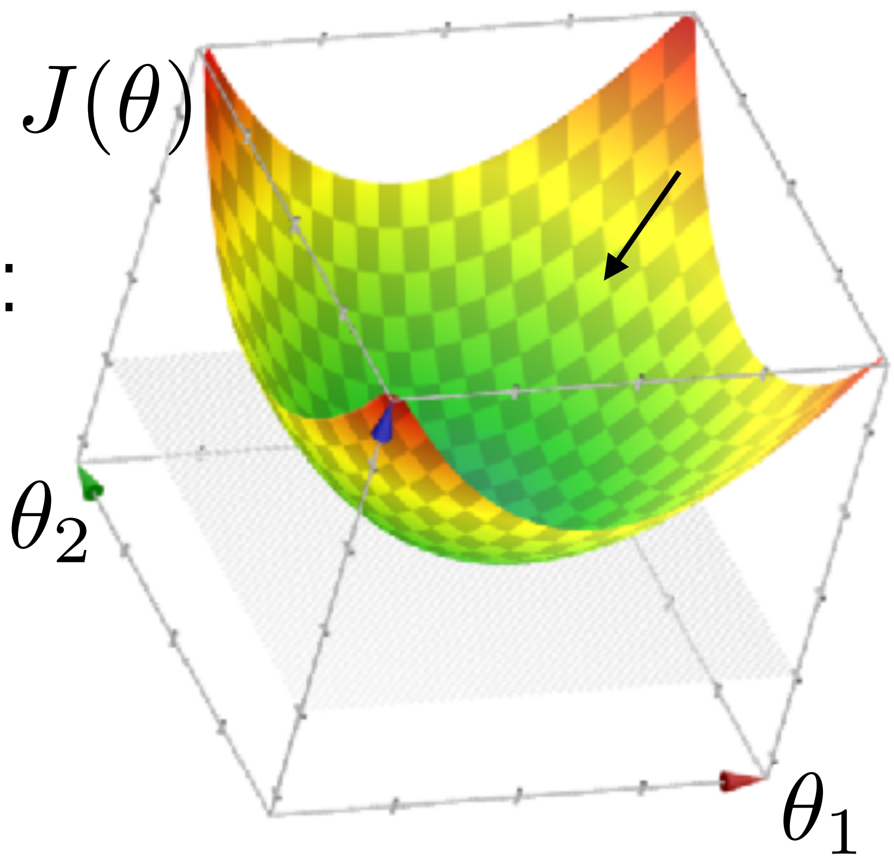

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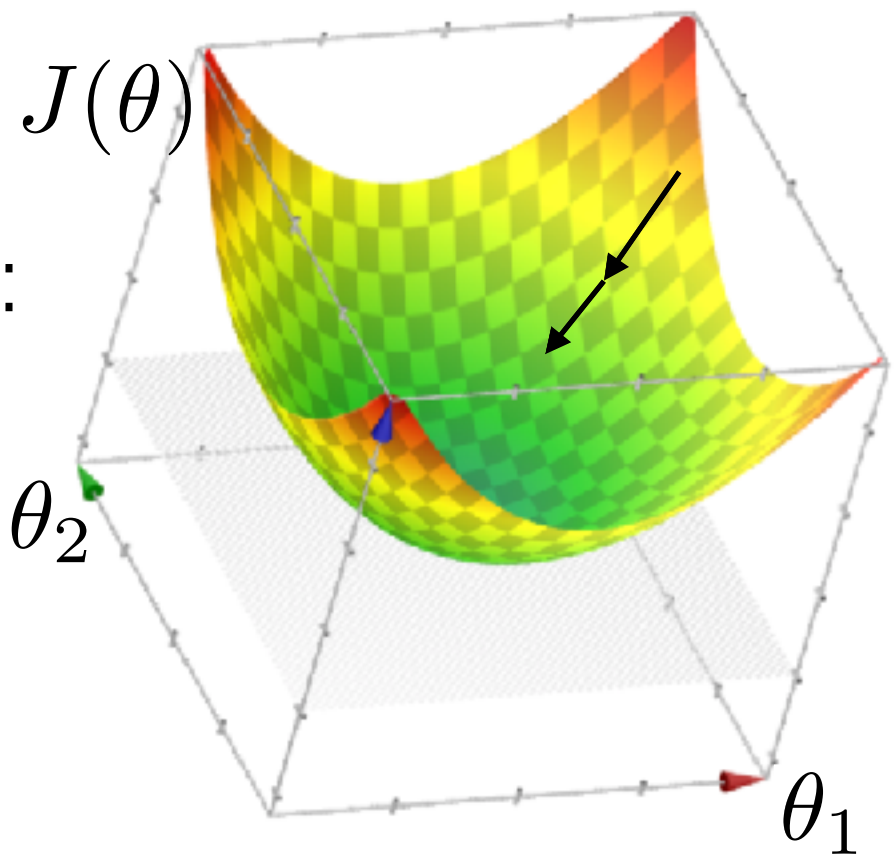

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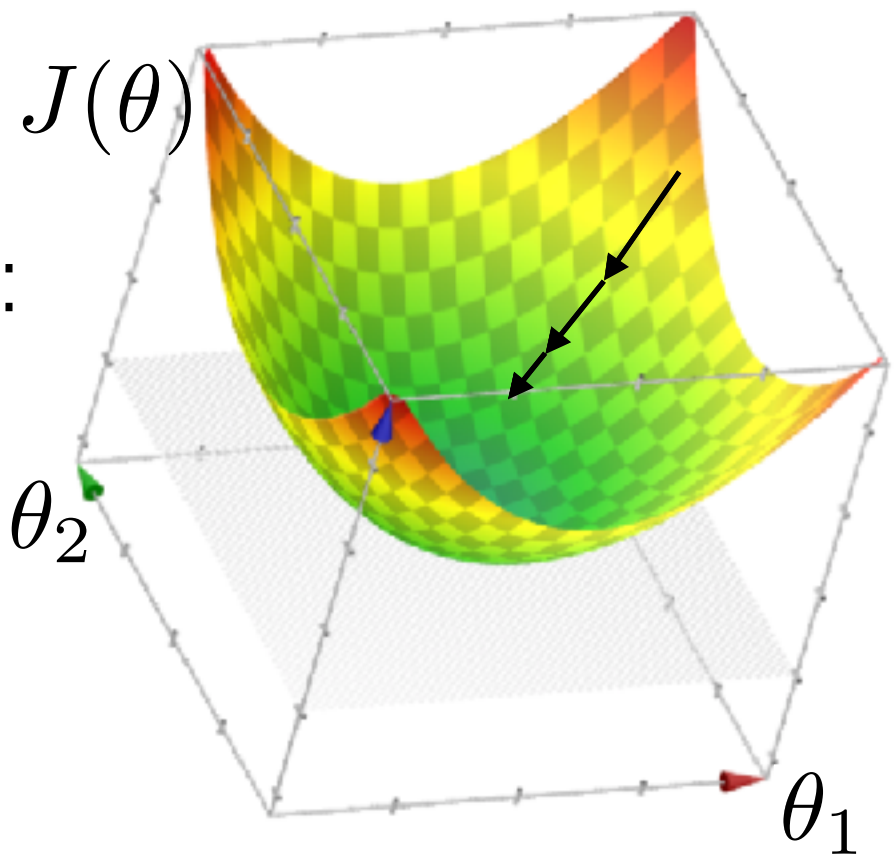

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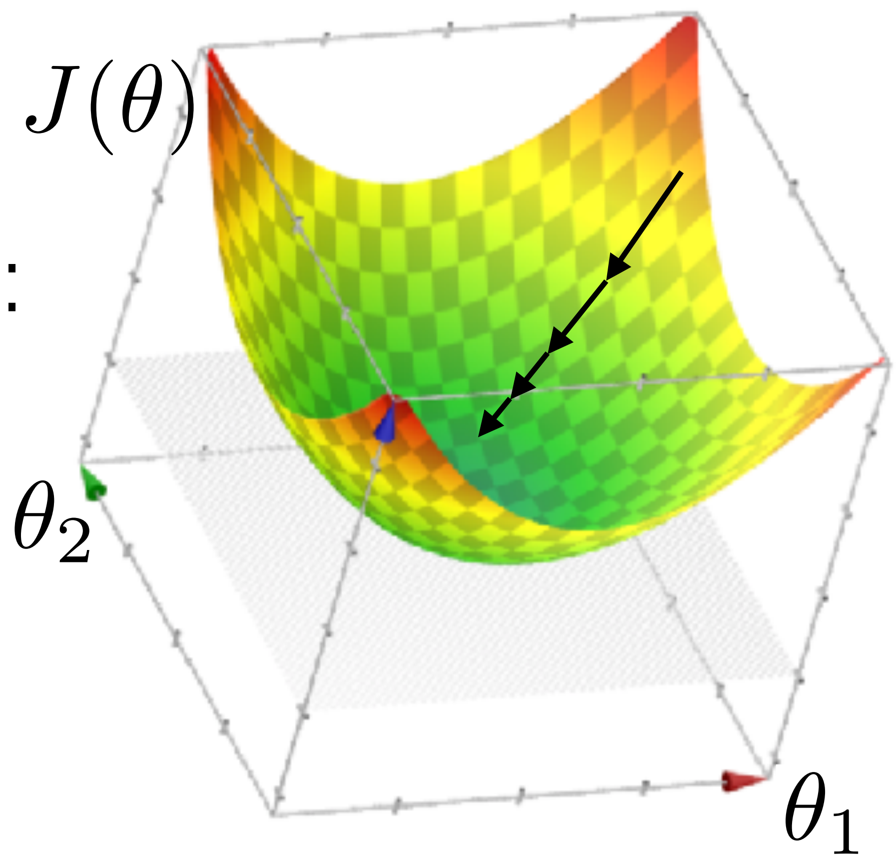

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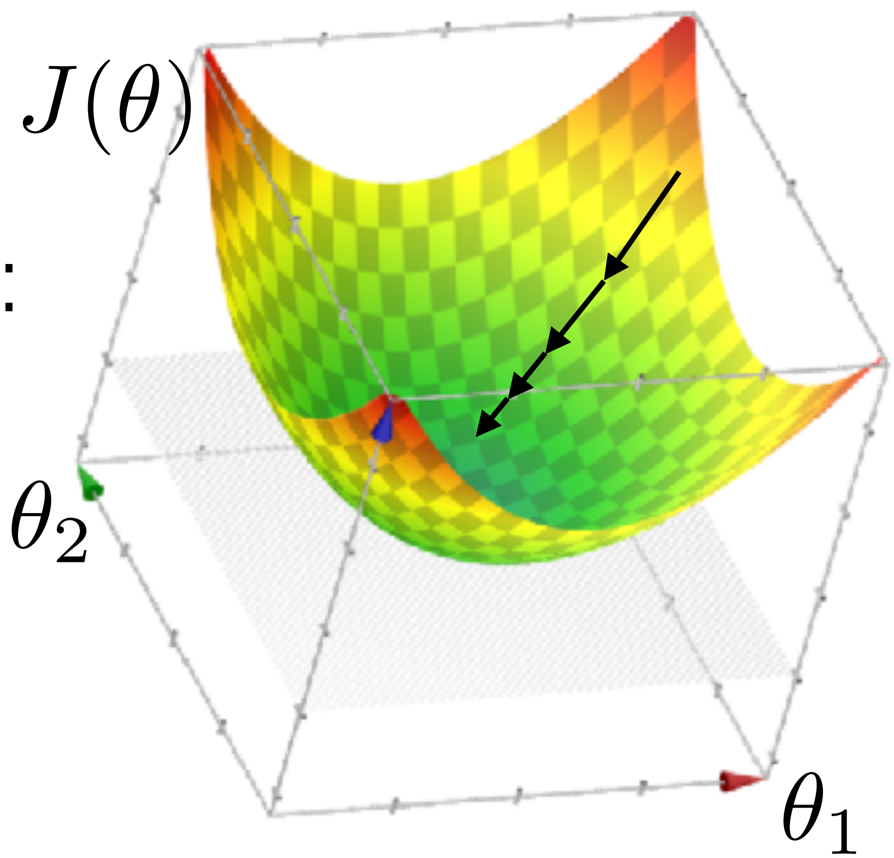

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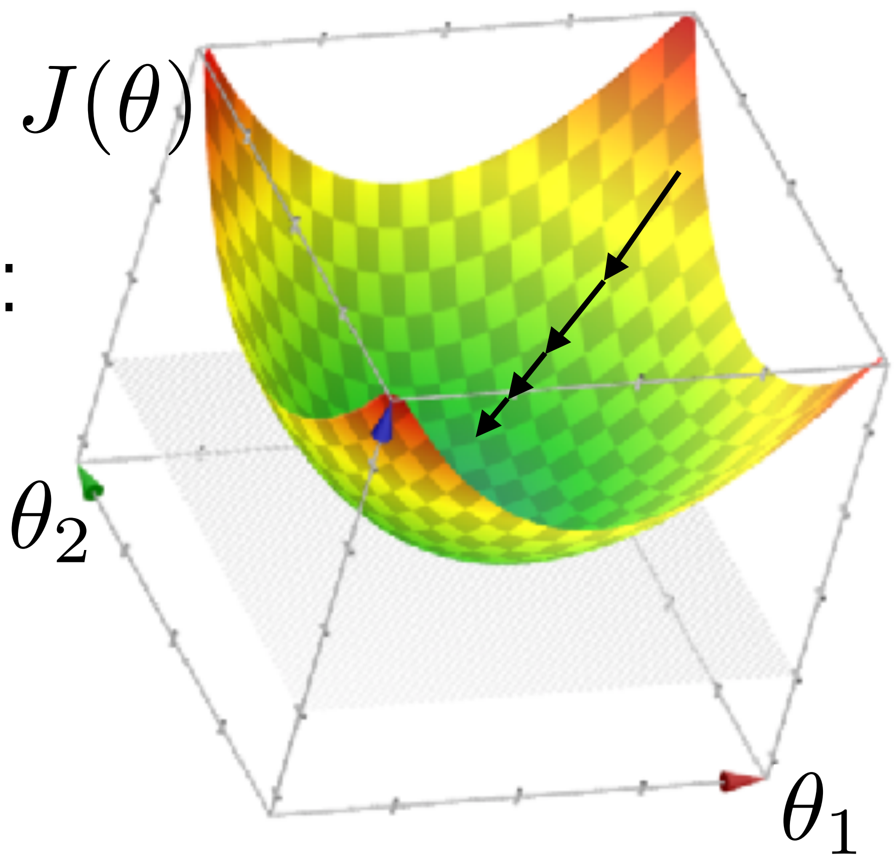

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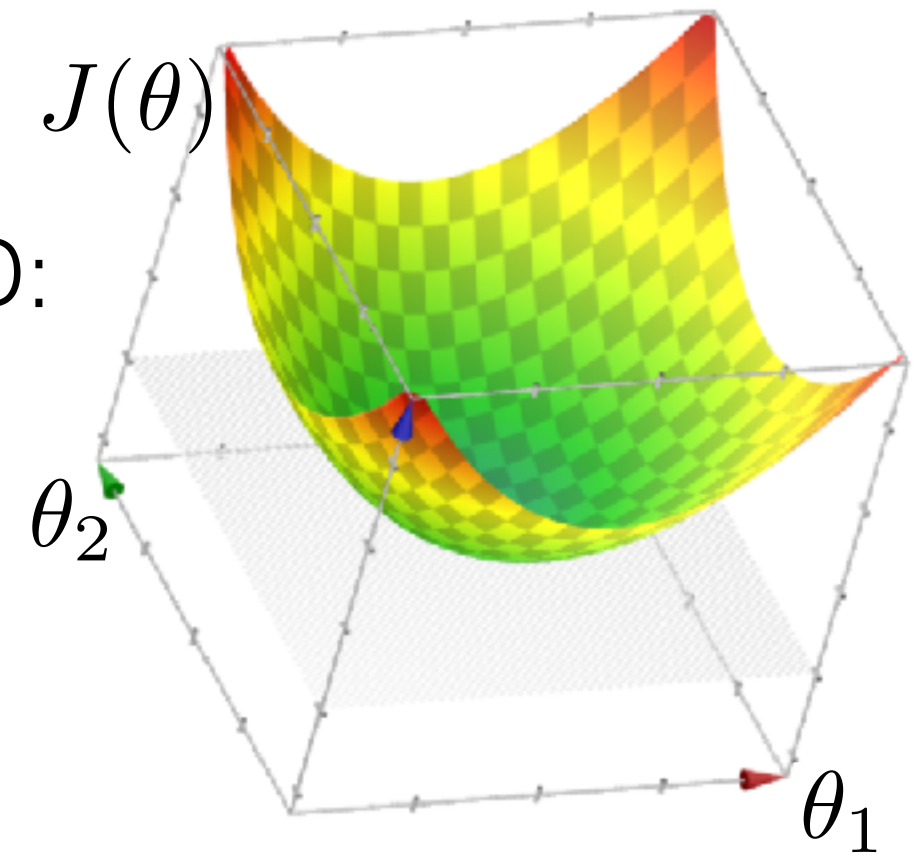

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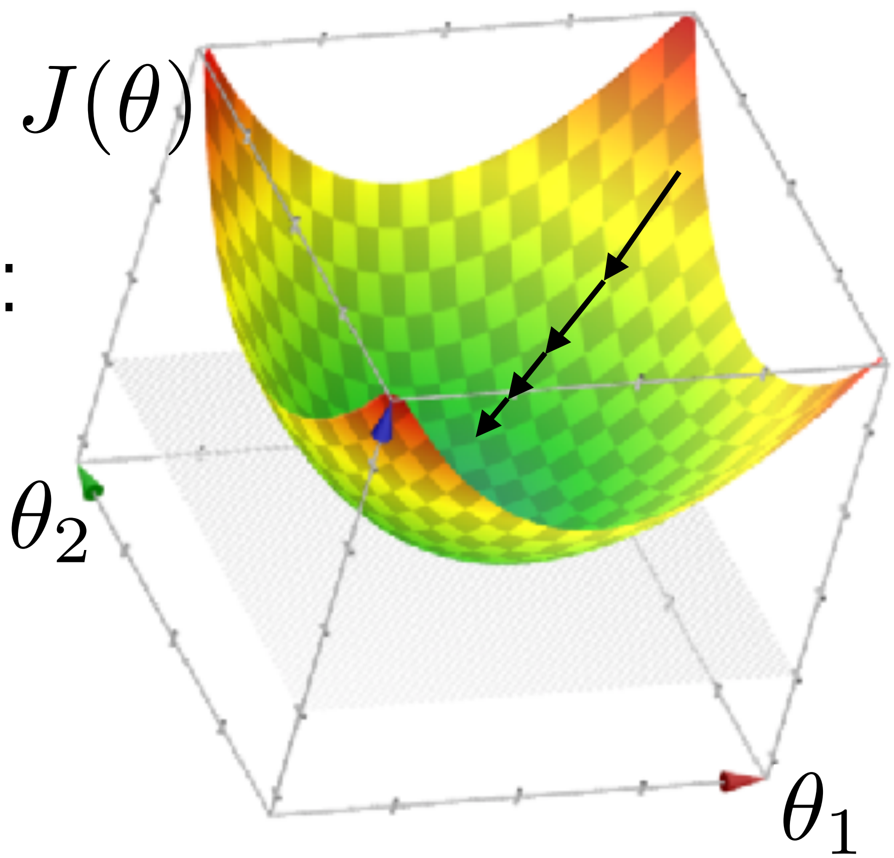

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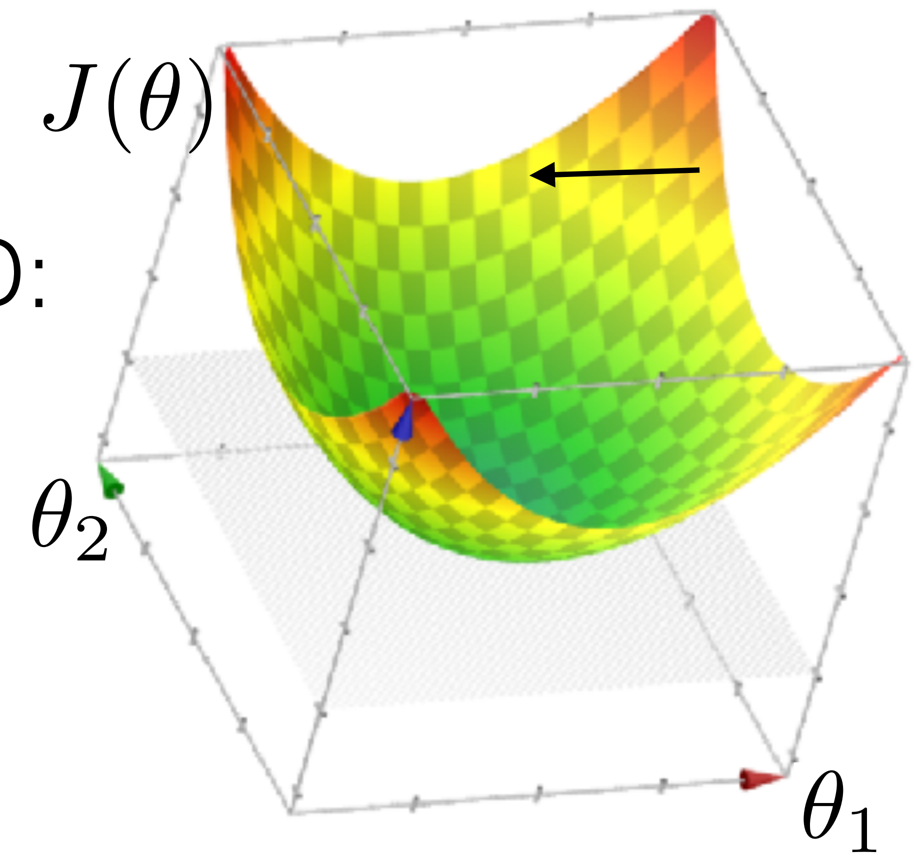

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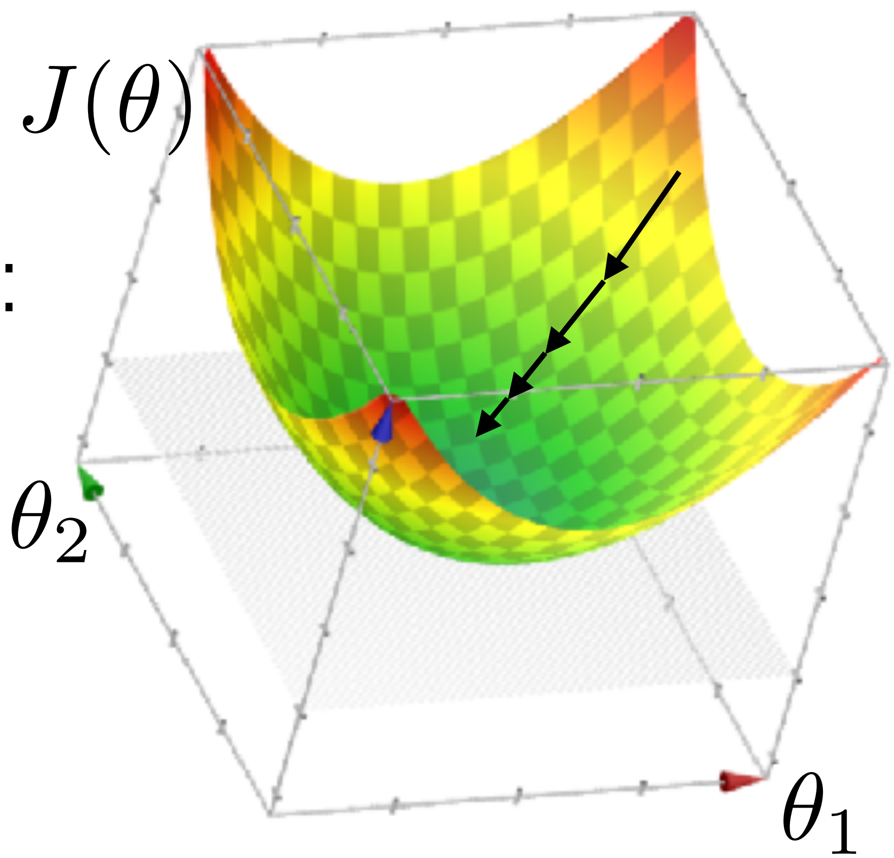

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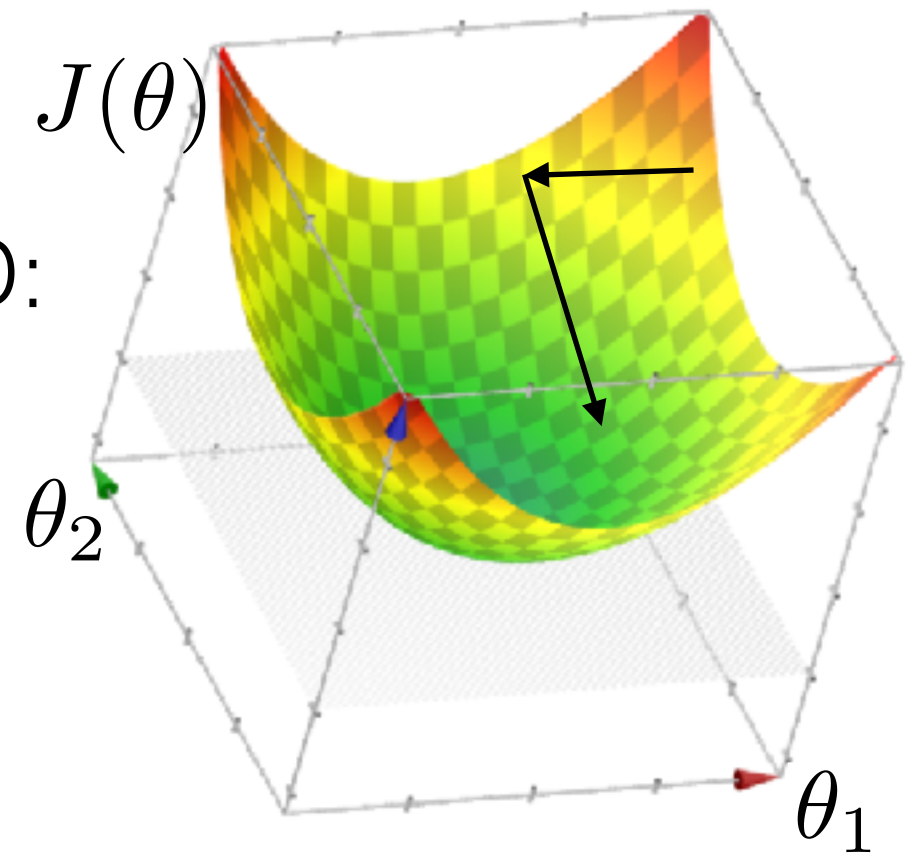

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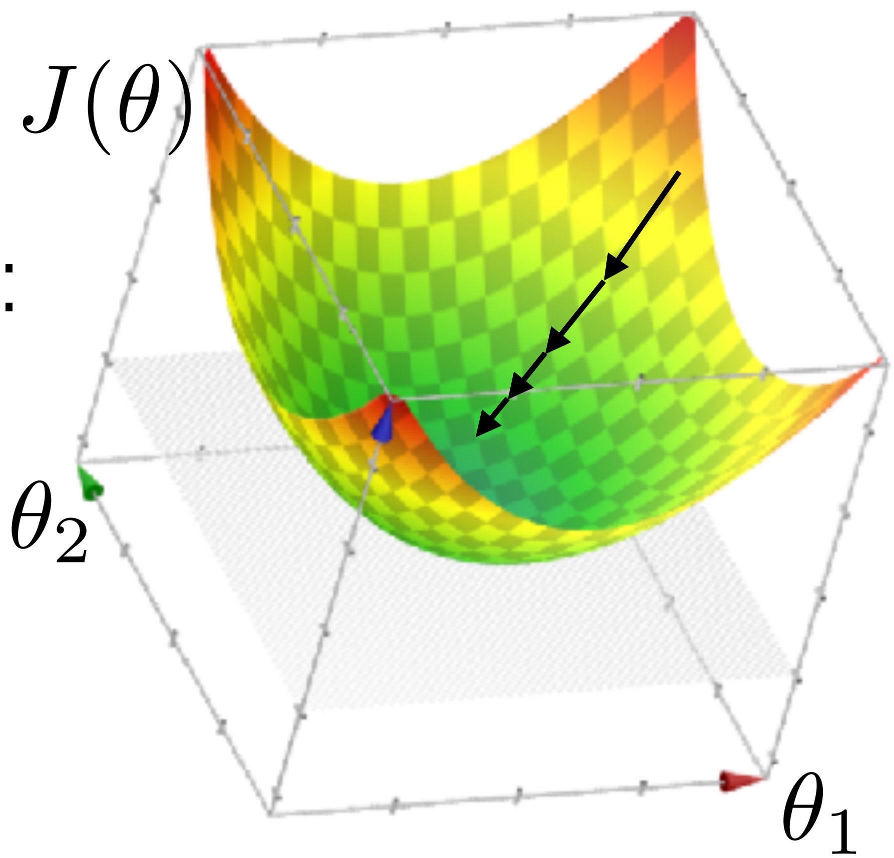

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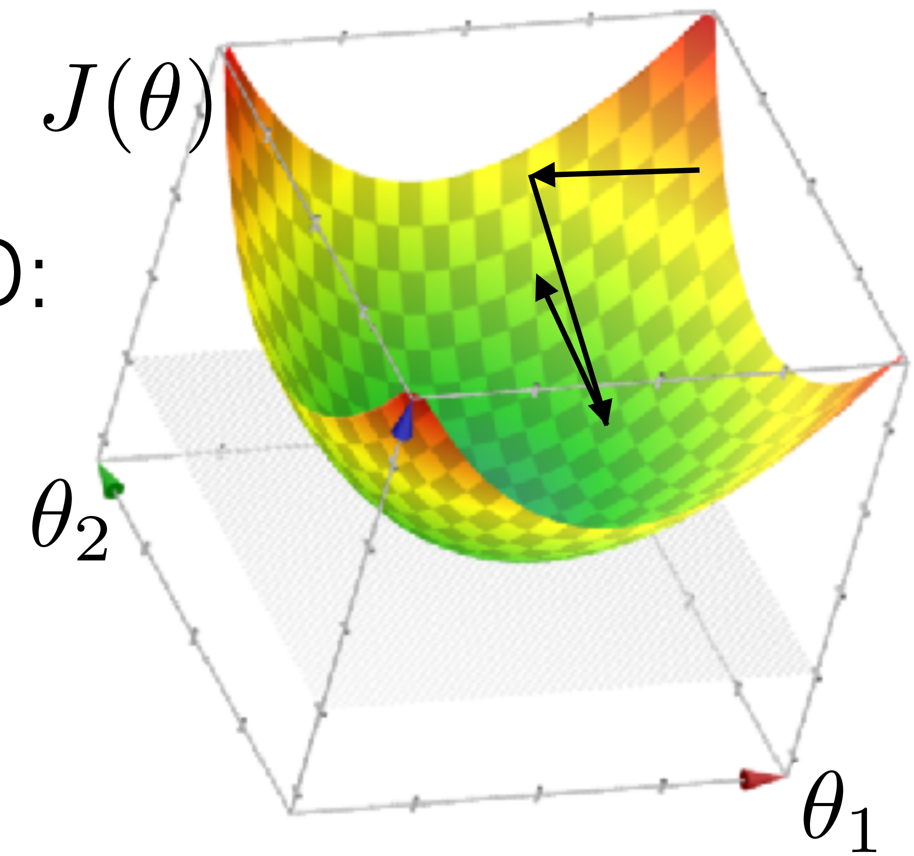

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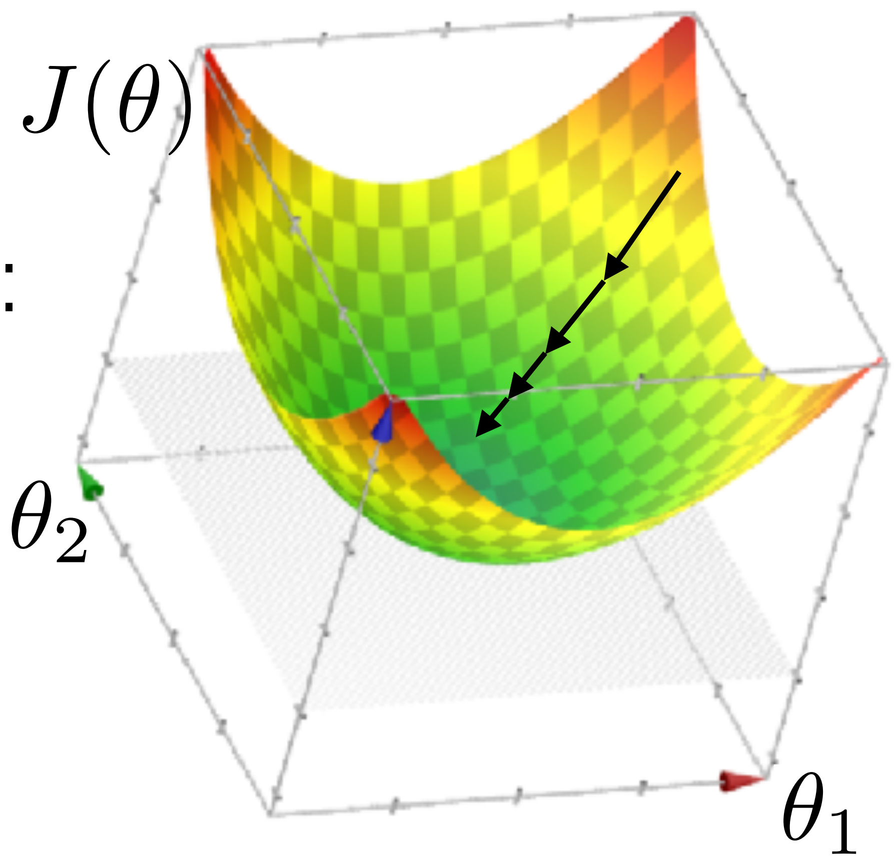

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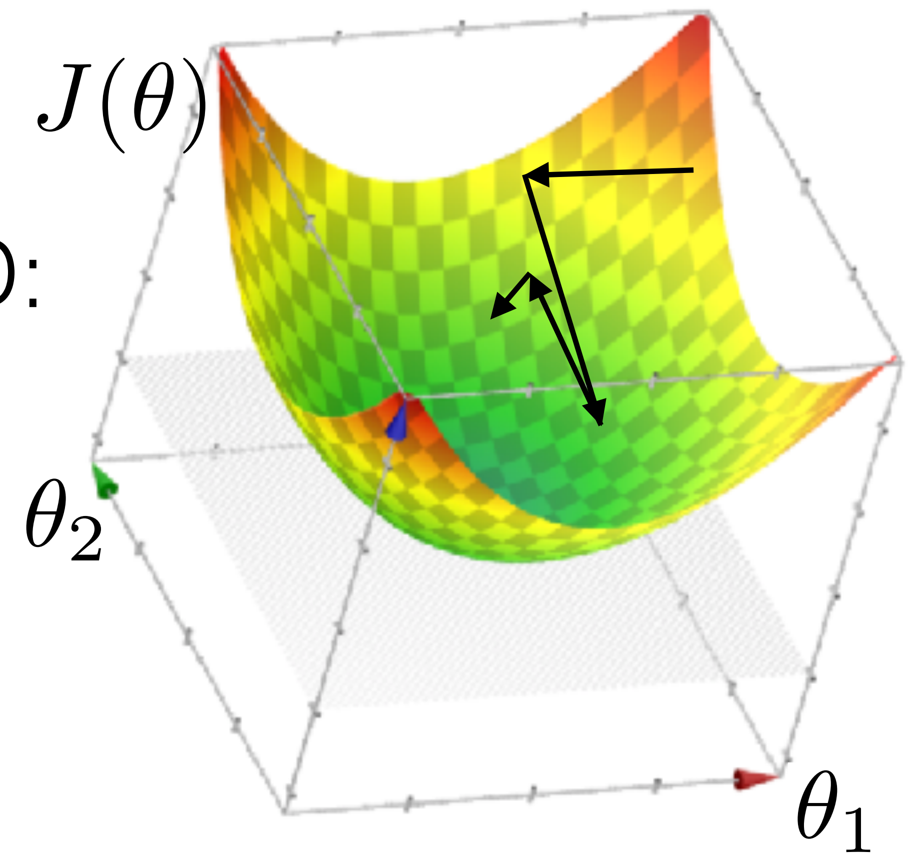

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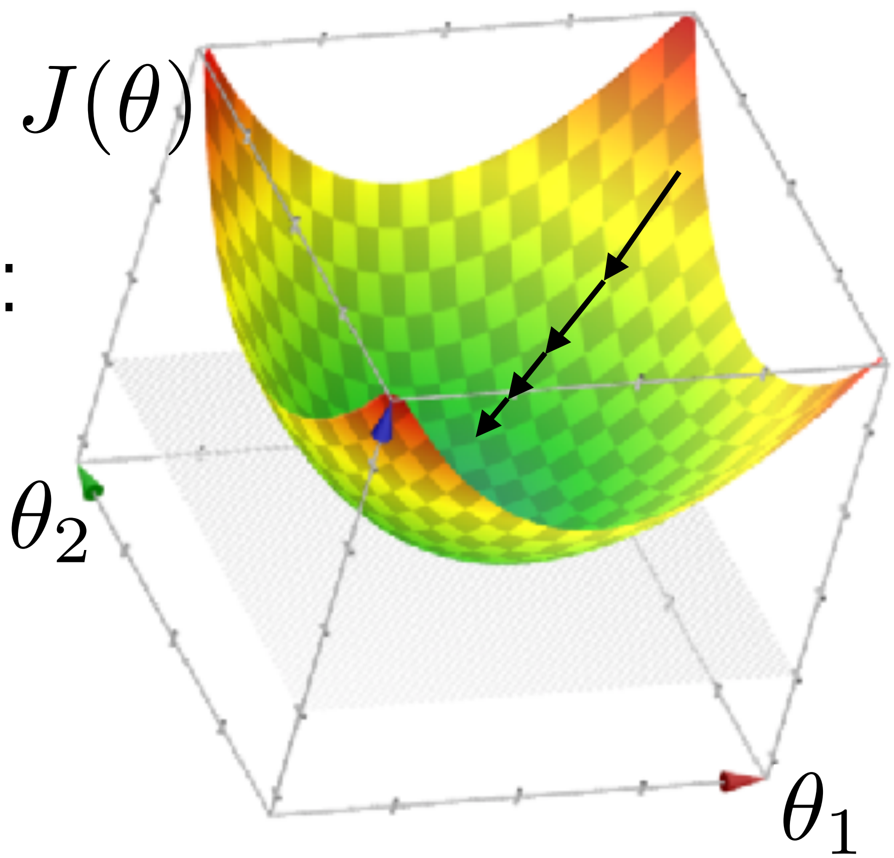

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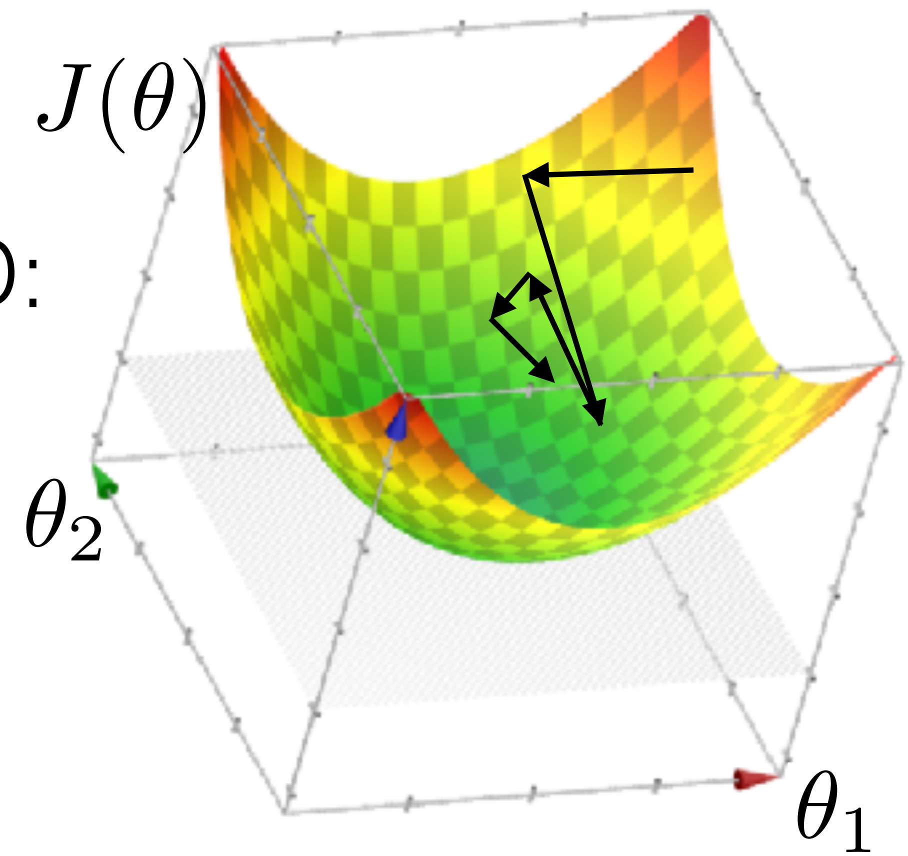

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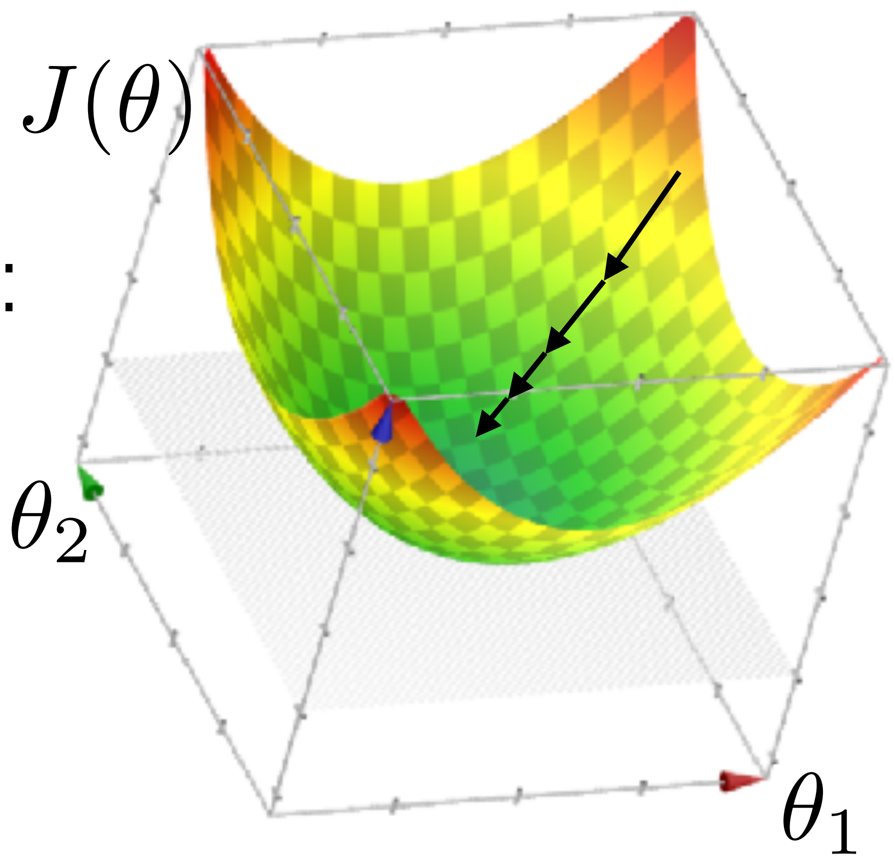

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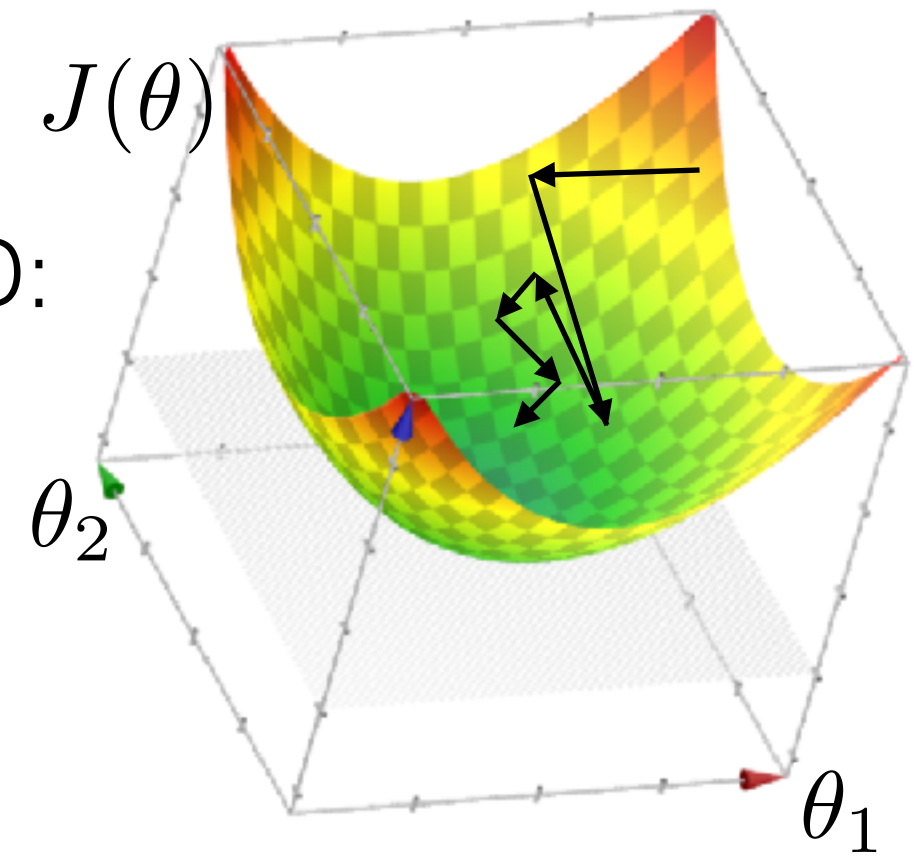

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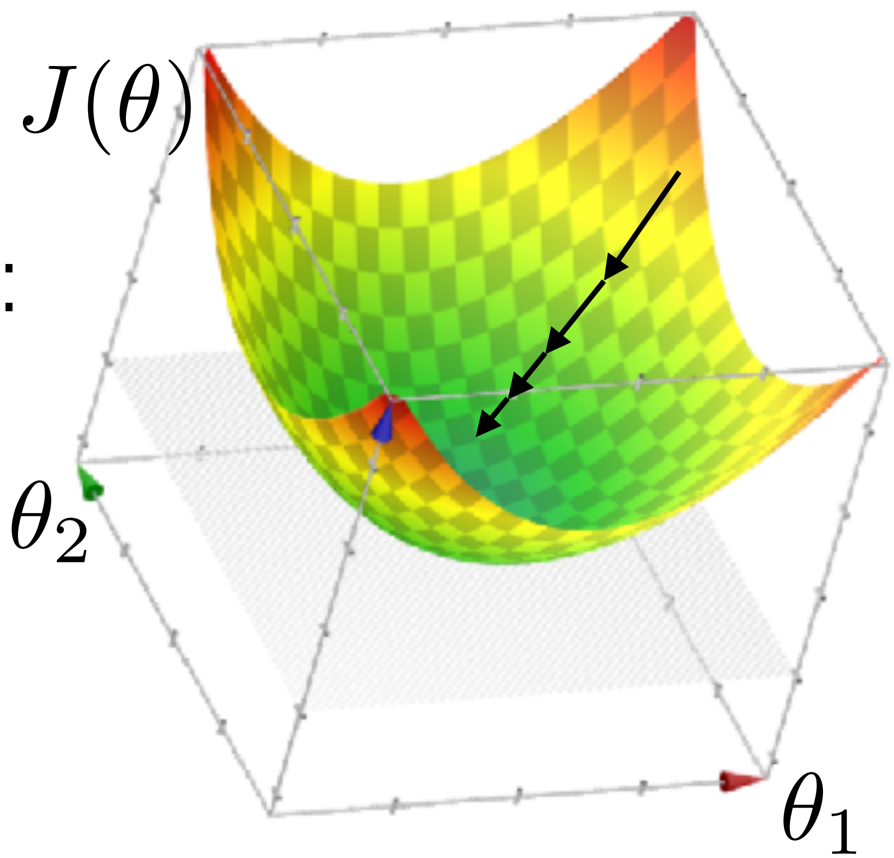

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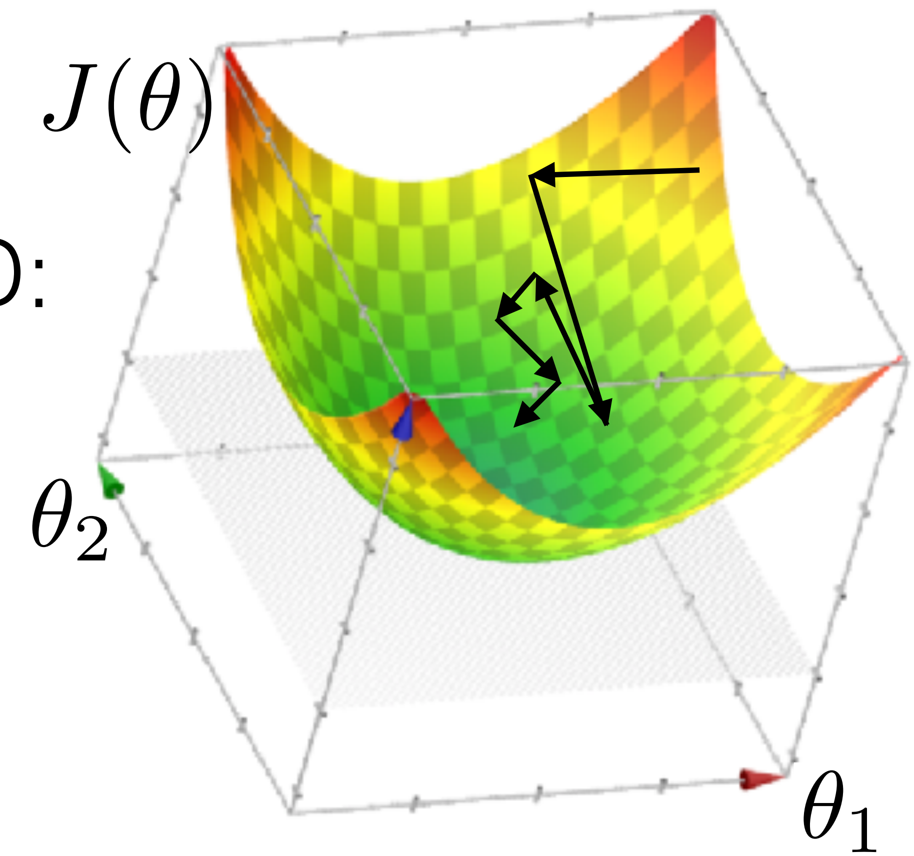

Stochastic gradient descent (SGD) properties

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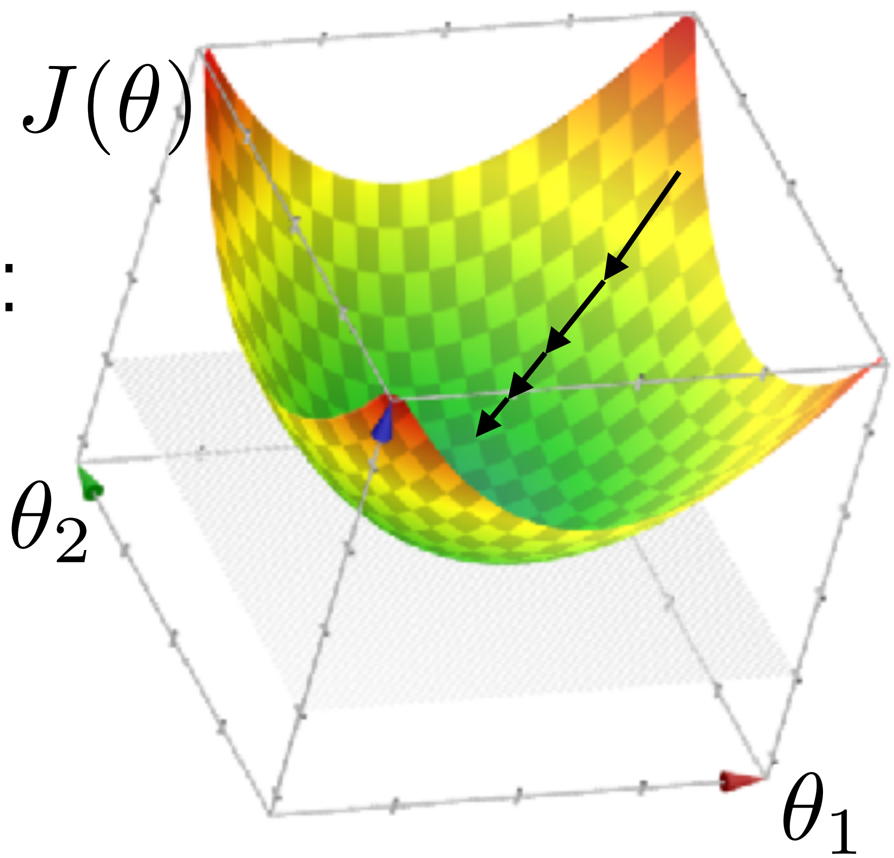

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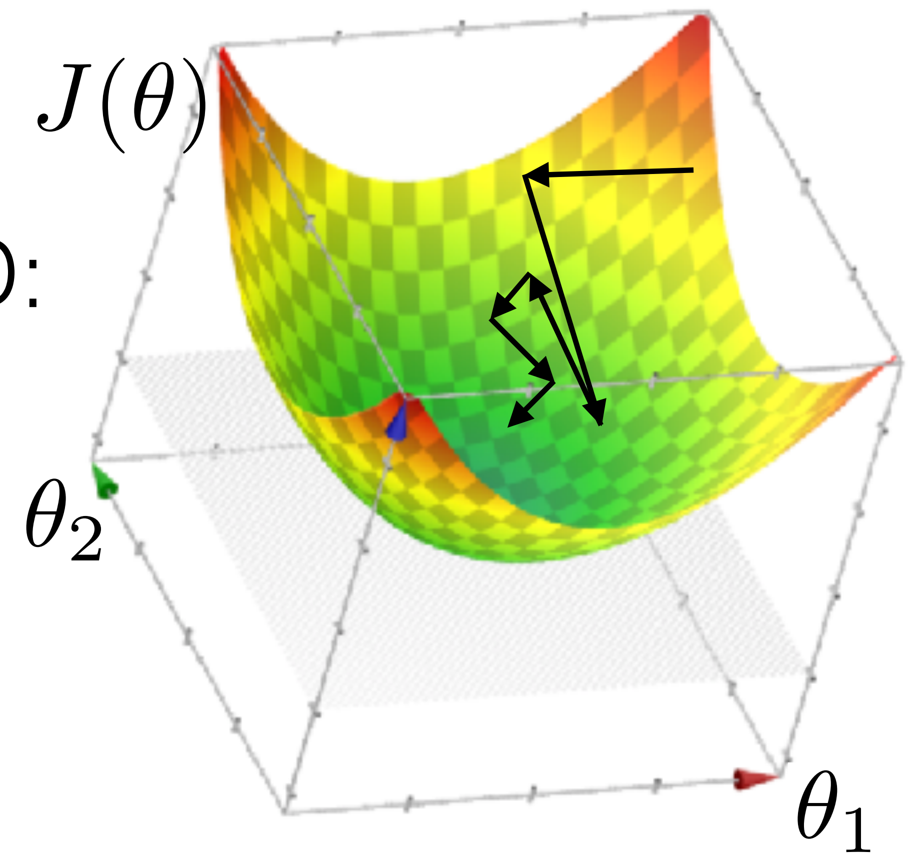
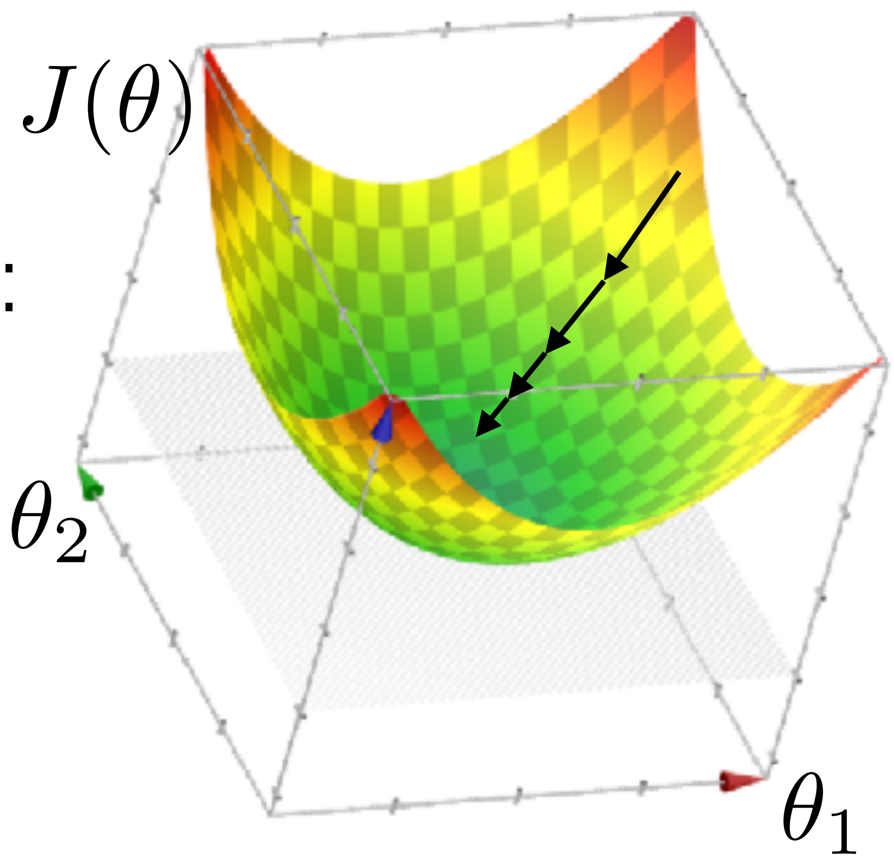


- **Theorem**: SGD performance

Stochastic gradient descent (SGD) properties

- GD:



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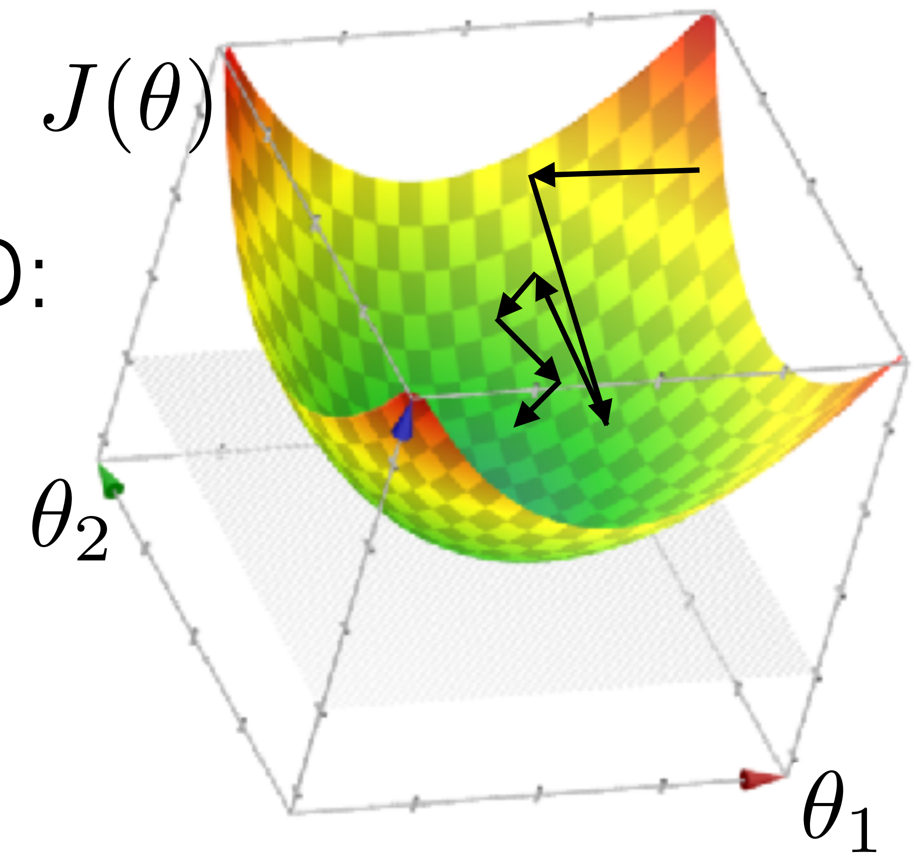
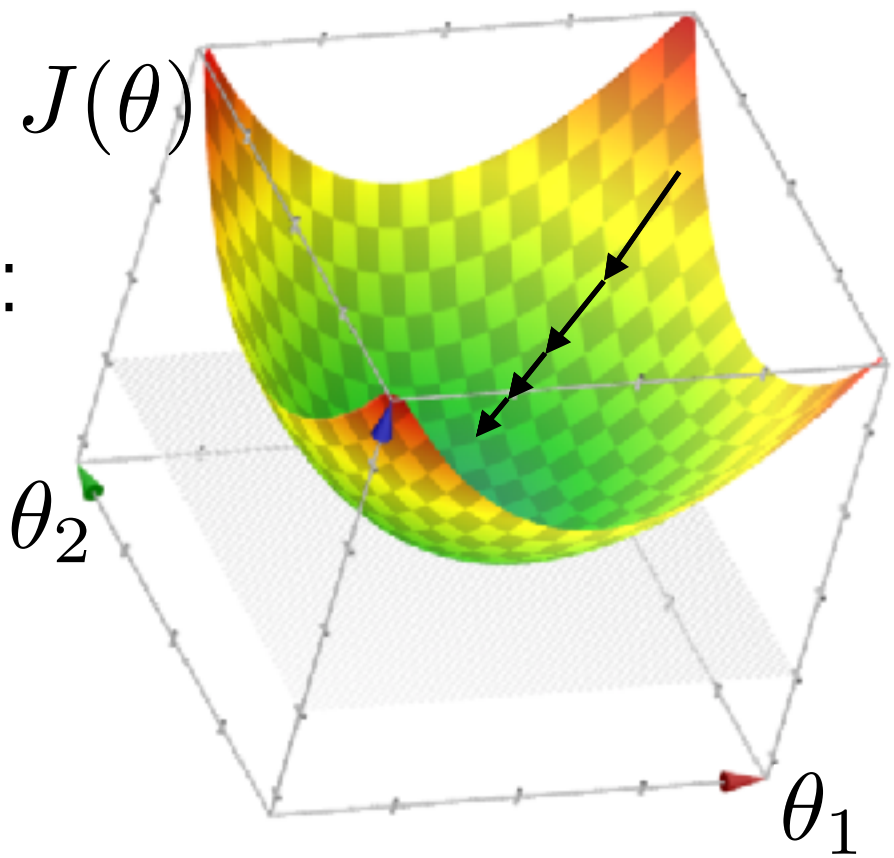


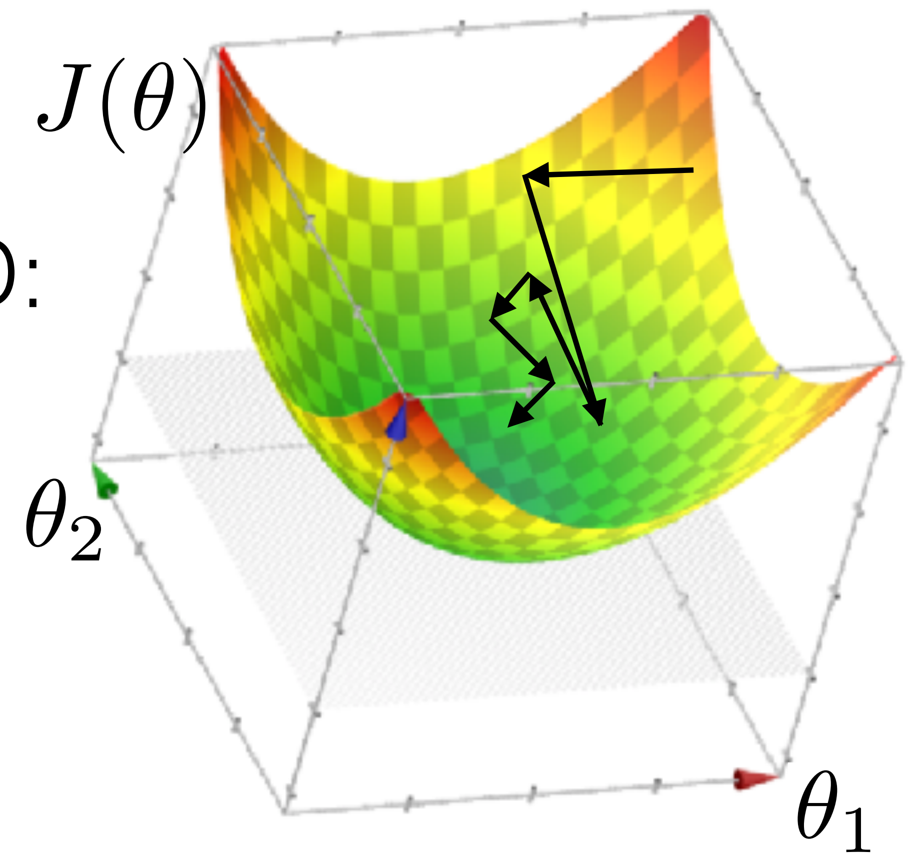
- **Theorem**: SGD performance
  - **Assumptions**:

Stochastic gradient descent (SGD) properties

- GD:



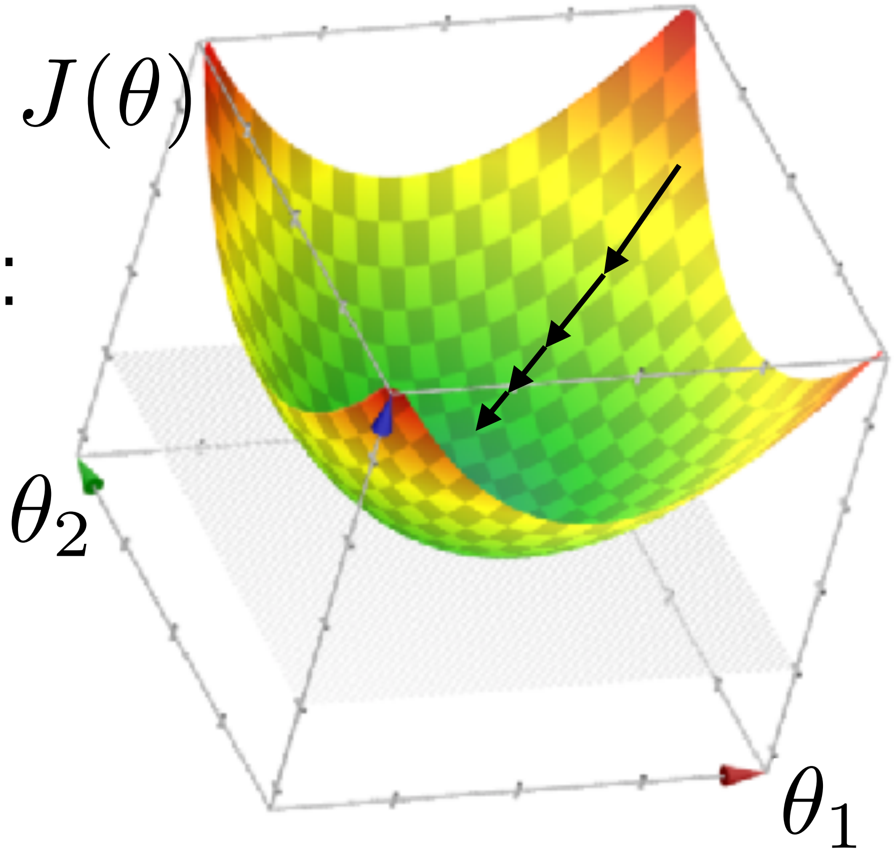
- SGD:



- **Theorem**: SGD performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)

Stochastic gradient descent (SGD) properties

- GD:



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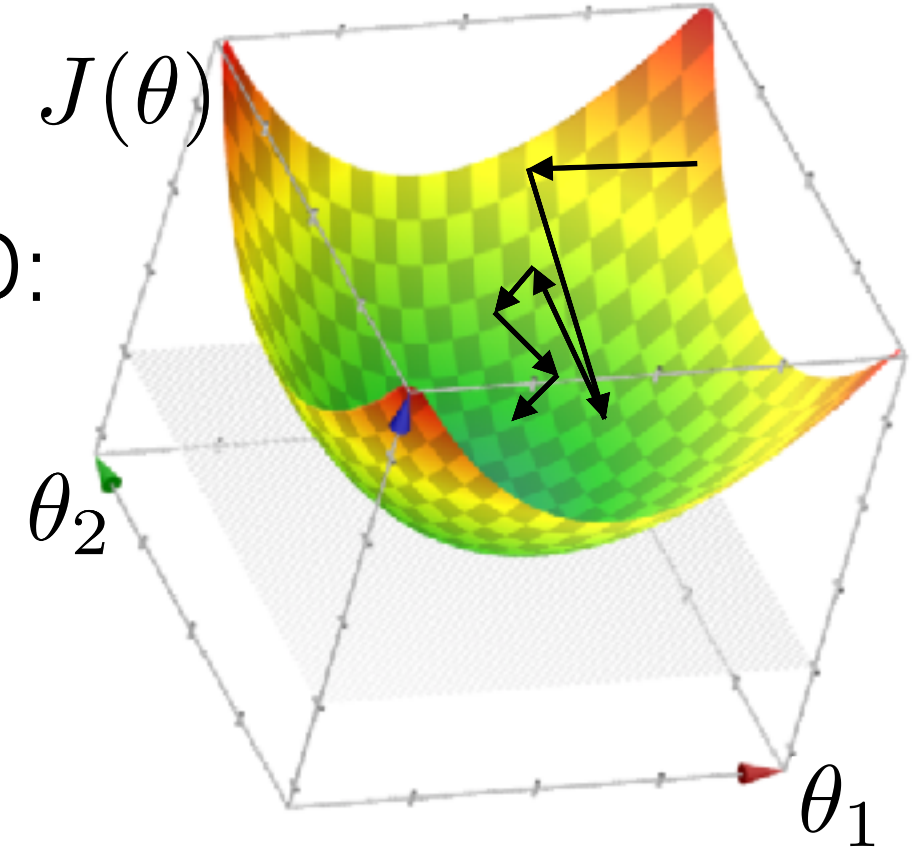

- **Theorem**: SGD performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is “nice” & convex, has a unique global minimizer

Stochastic gradient descent (SGD) properties

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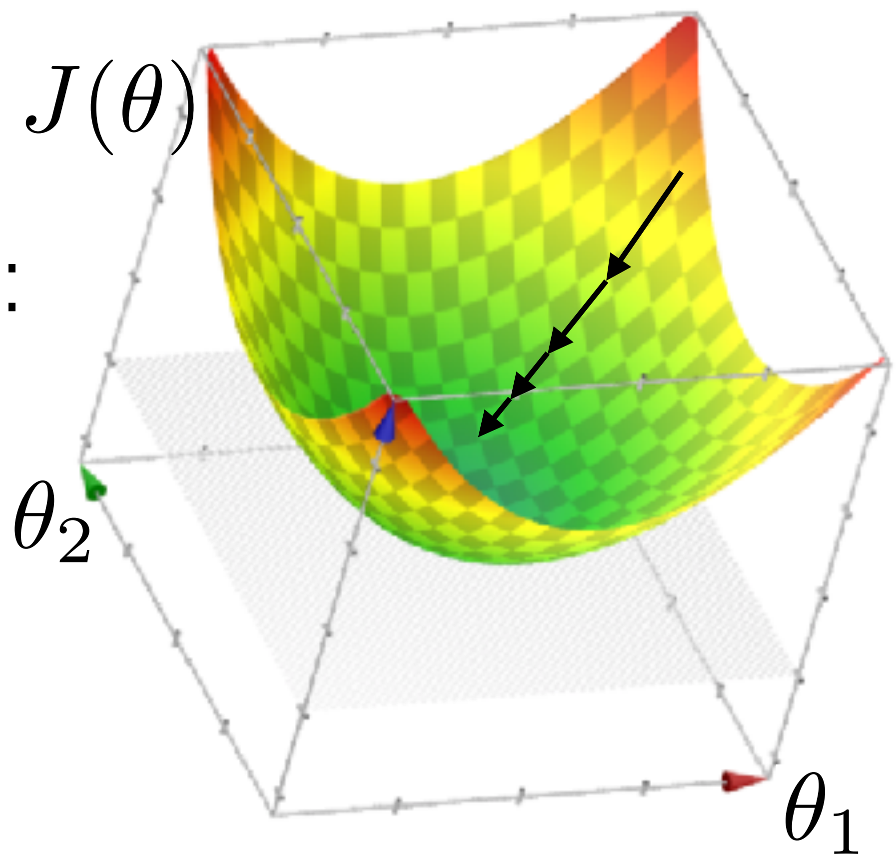

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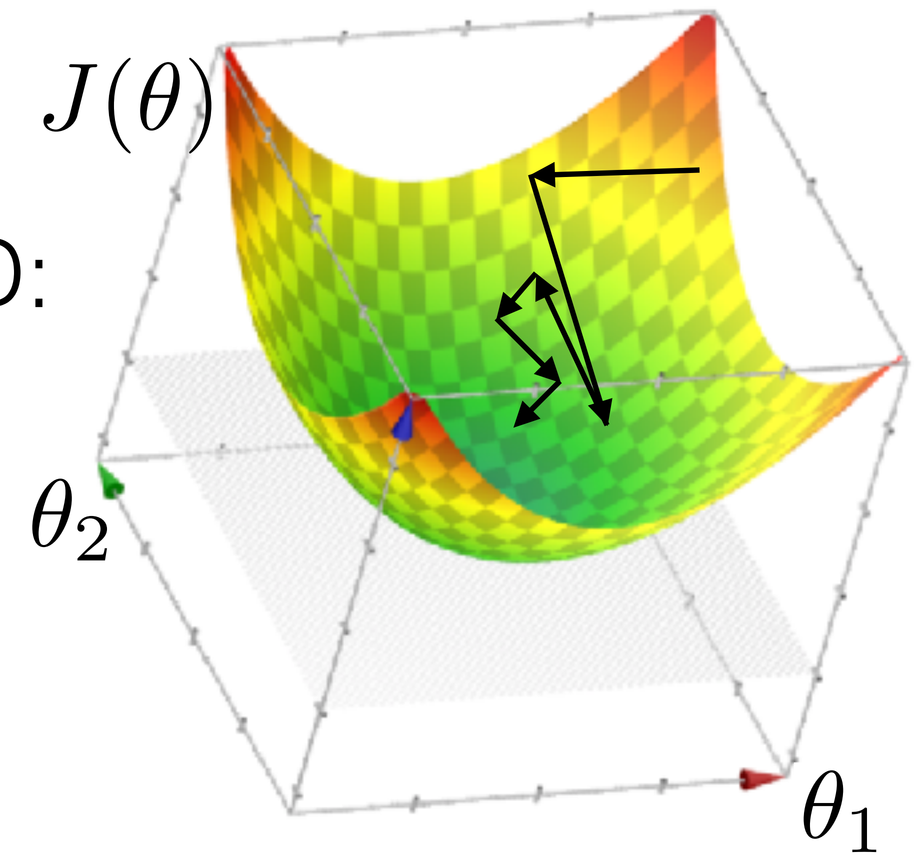
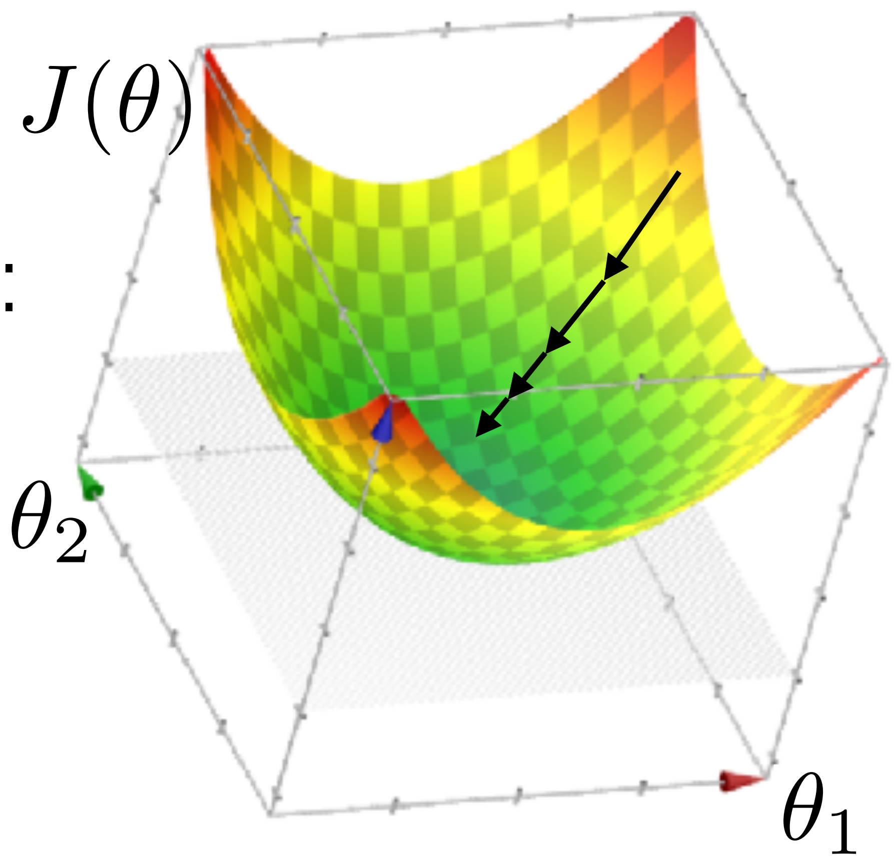


- **Theorem**: SGD performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is "nice" & convex, has a unique global minimizer
    - $\sum_{t=1}^{\infty} \eta(t) = \infty, \sum_{t=1}^{\infty} (\eta(t))^2 < \infty$

Stochastic gradient descent (SGD) properties

- GD:



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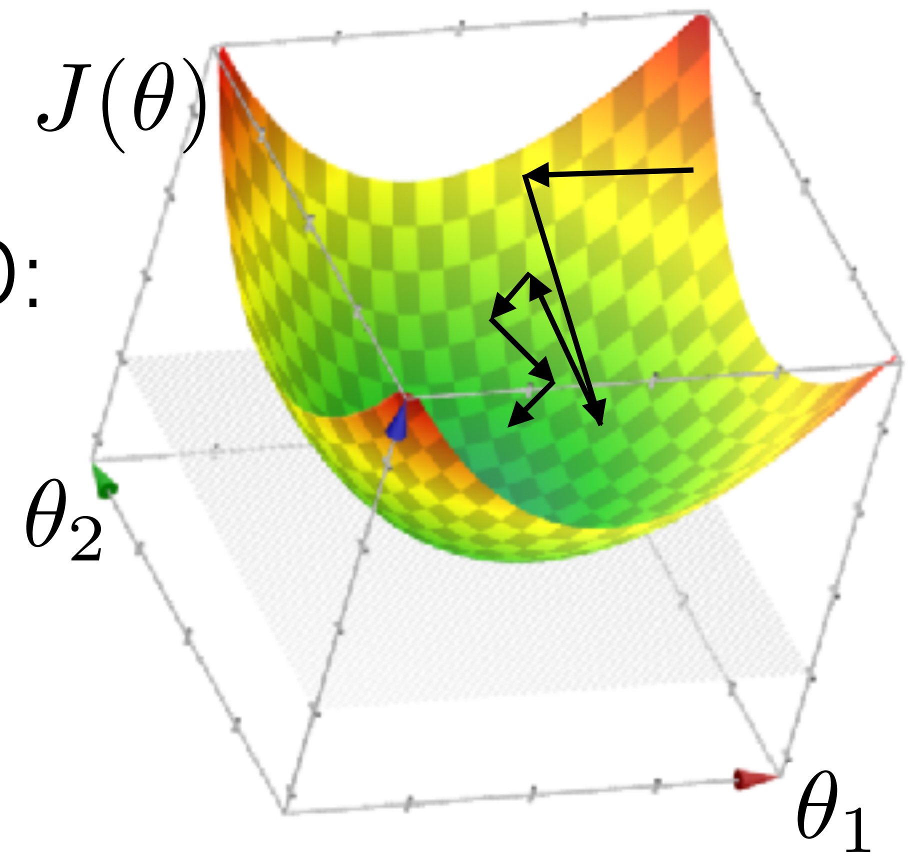

- **Theorem**: SGD performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is "nice" & convex, has a unique global minimizer
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      - e.g. $\eta(t) = \alpha(\tau_0 + t)^{-\kappa} (\kappa \in (0.5, 1])$

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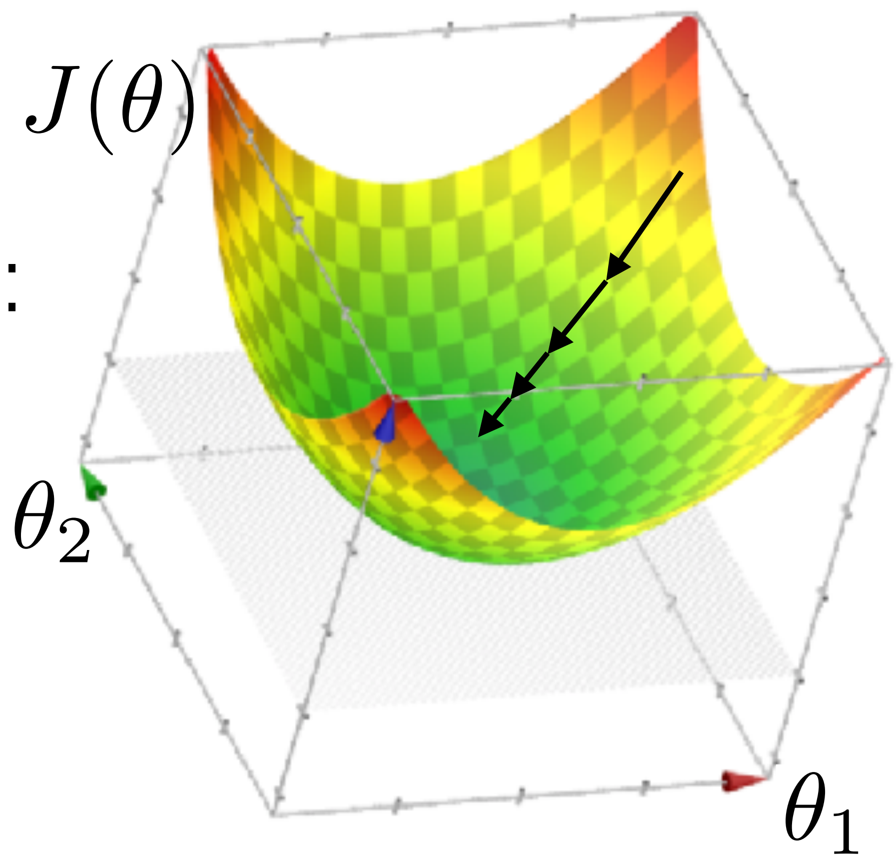

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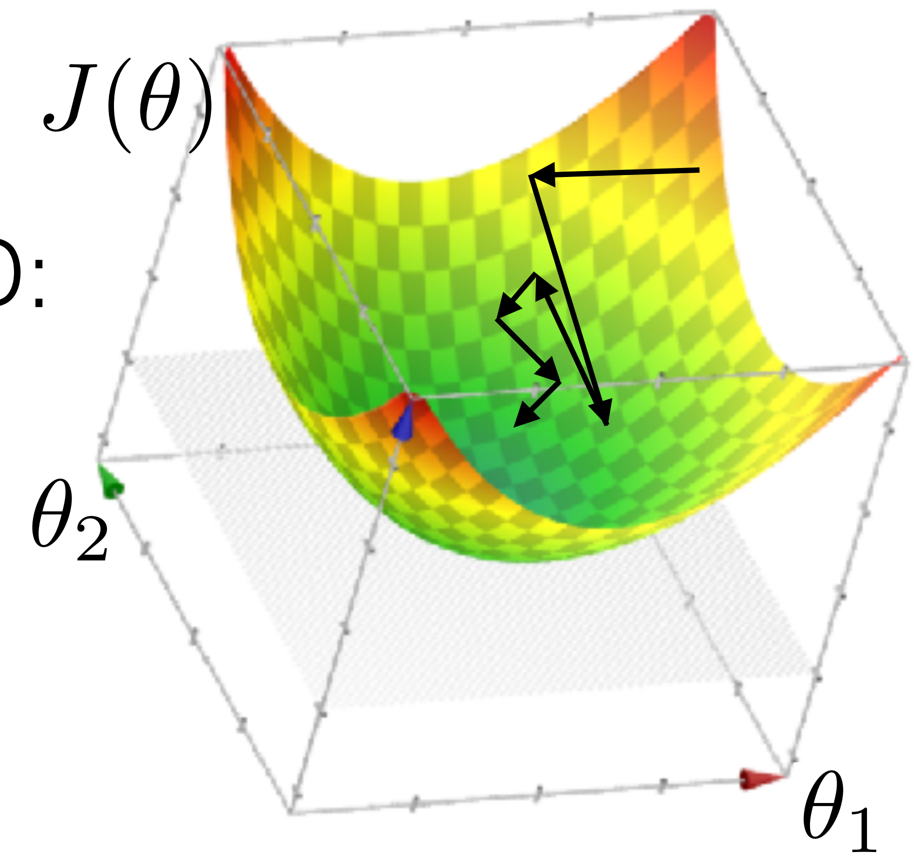

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  - **Conclusion**: If run long enough, stochastic gradient descent will return a value within $\tilde{\epsilon}$ of the global minimizer