# 6.036: Introduction to Machine Learning

Tues 10/19 lecture time → midterm review! We'll continue neural net fun on 10/26.

**Lecture start:** Tuesdays 9:35am

**Who's talking?** Prof. Tamara Broderick

**Questions?** Ask on Piazza: "lecture (week) 6" folder

**Materials:** slides, video will all be available on Canvas

**Live Zoom feed:** https://mit.zoom.us/j/94238622313

## Last Time(s)

I. Linear regression

II. Linear classification

III. Choosing features

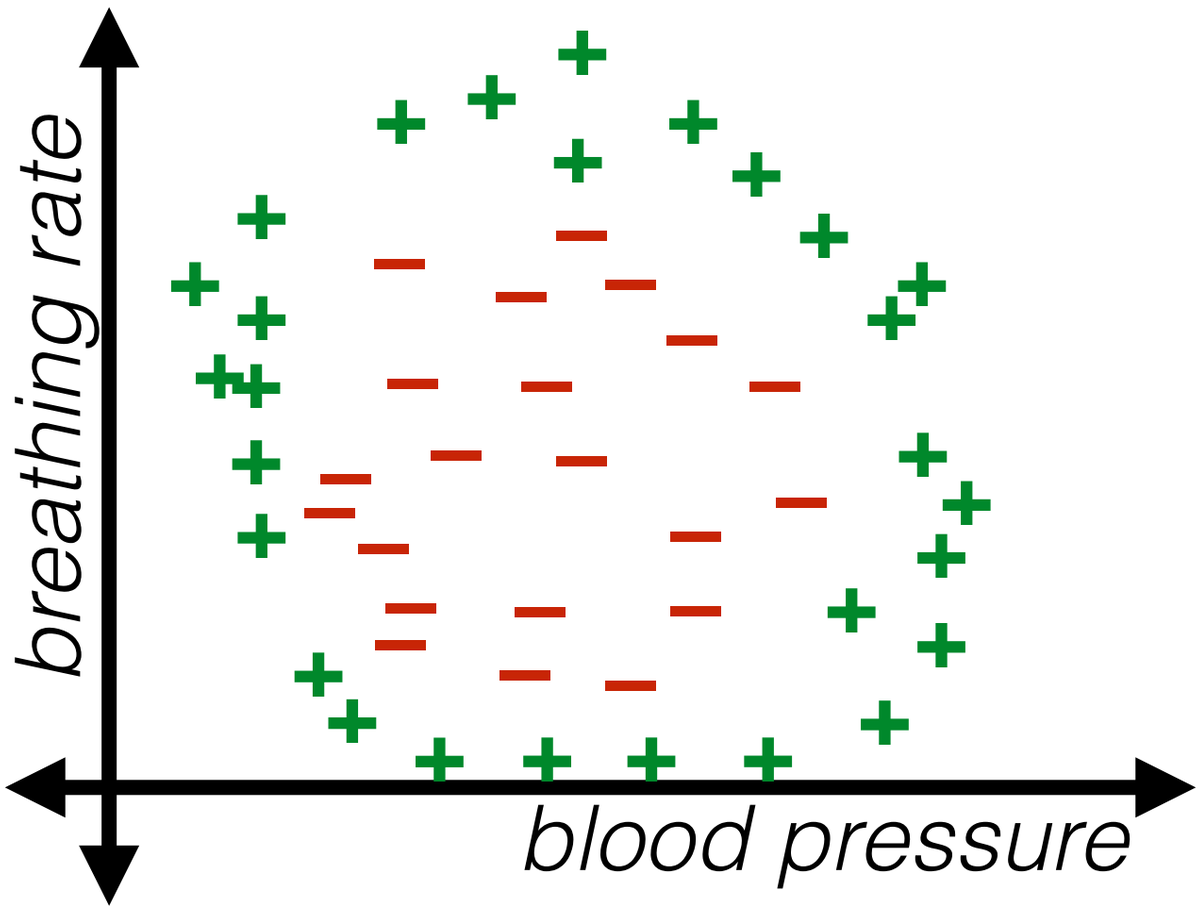
## Today's Plan

I. Polynomial features

II. Step-function features

III. Neural nets: hypothesis class

# Classification boundaries



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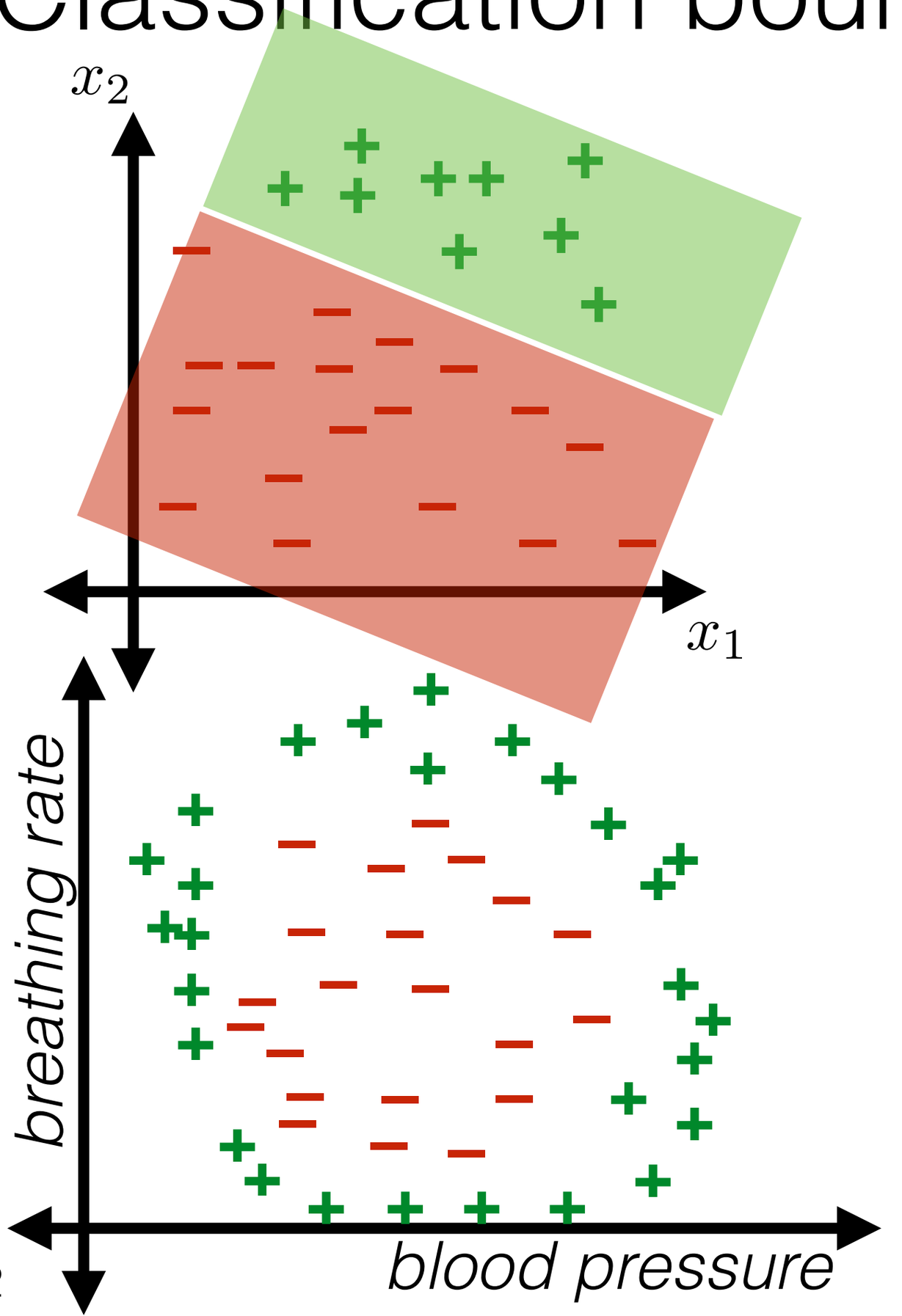

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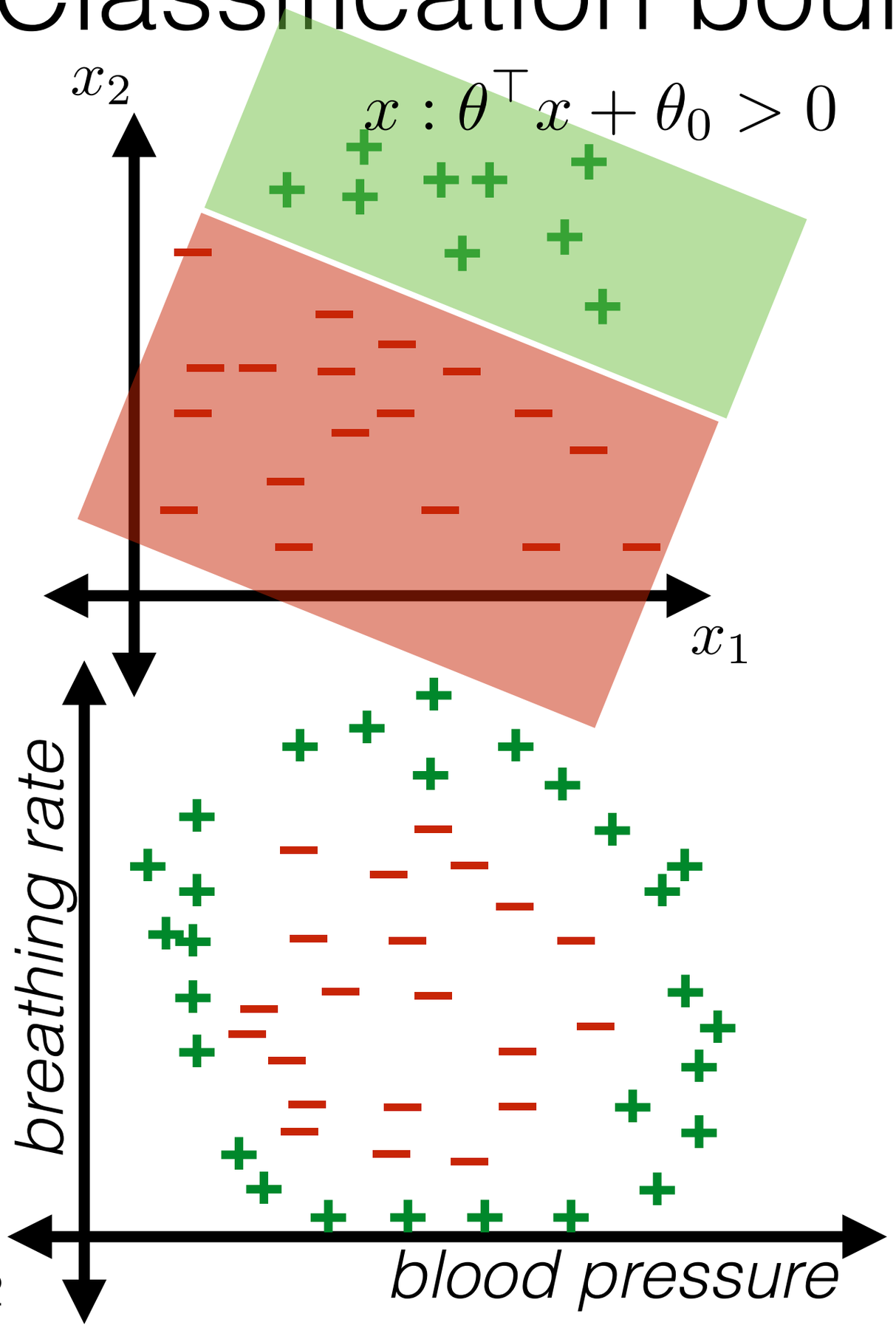

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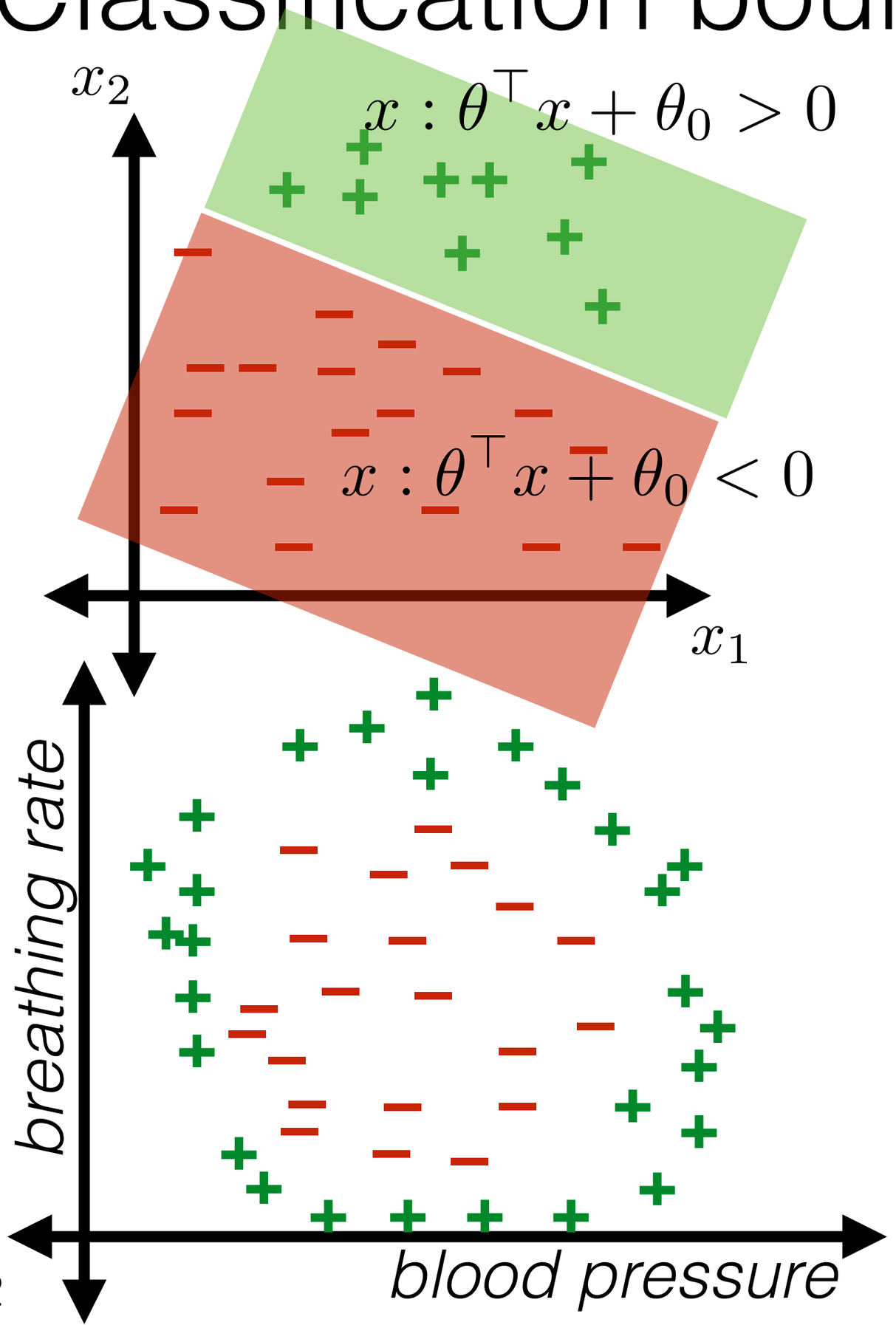

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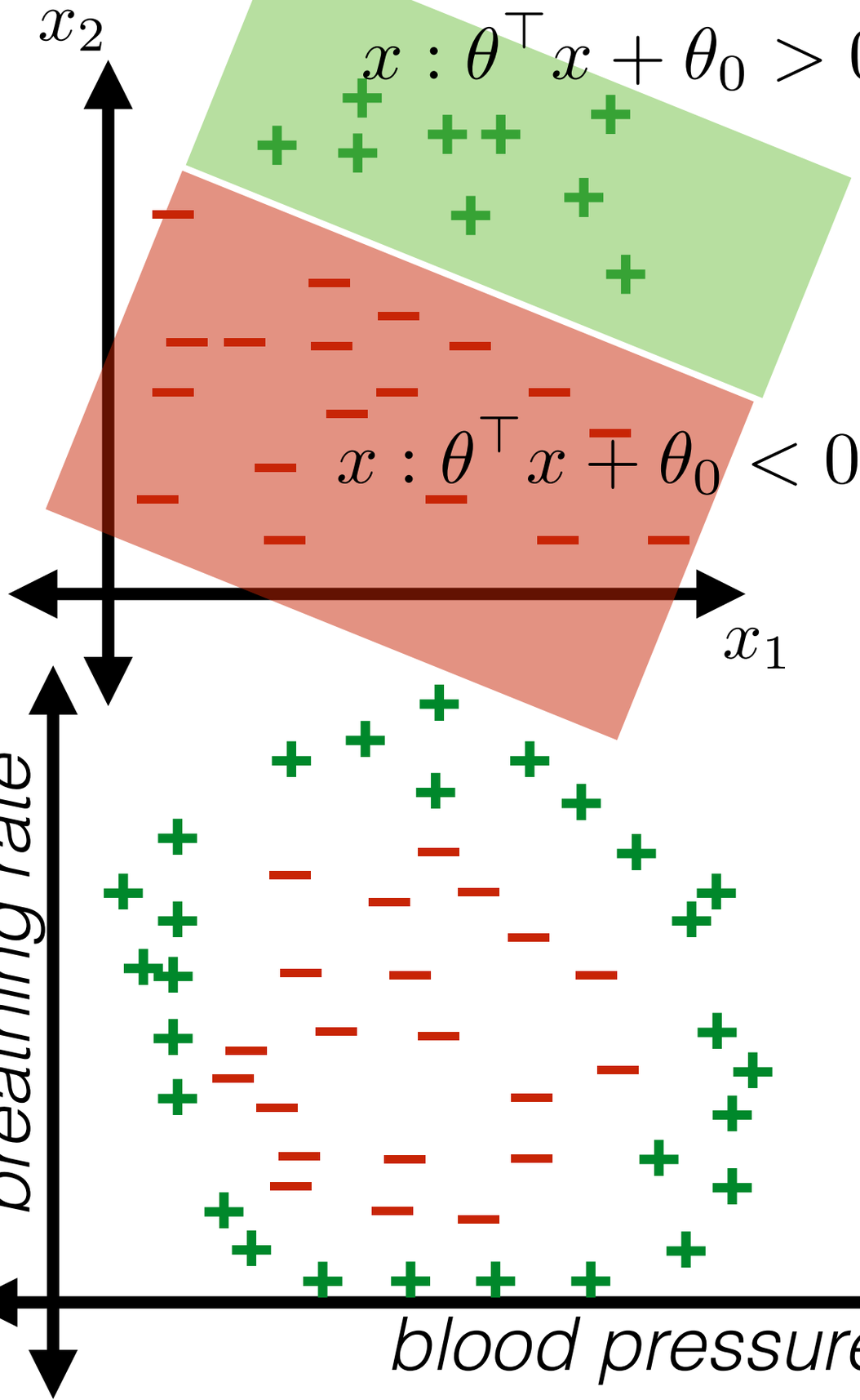


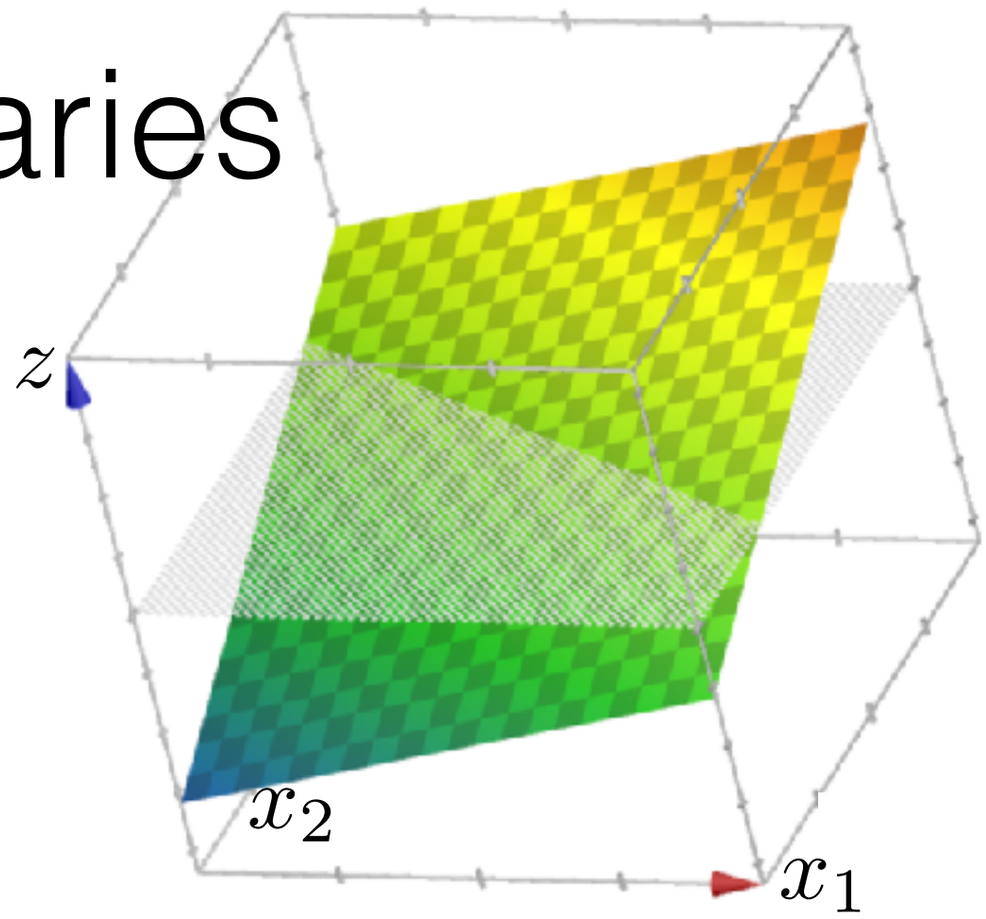

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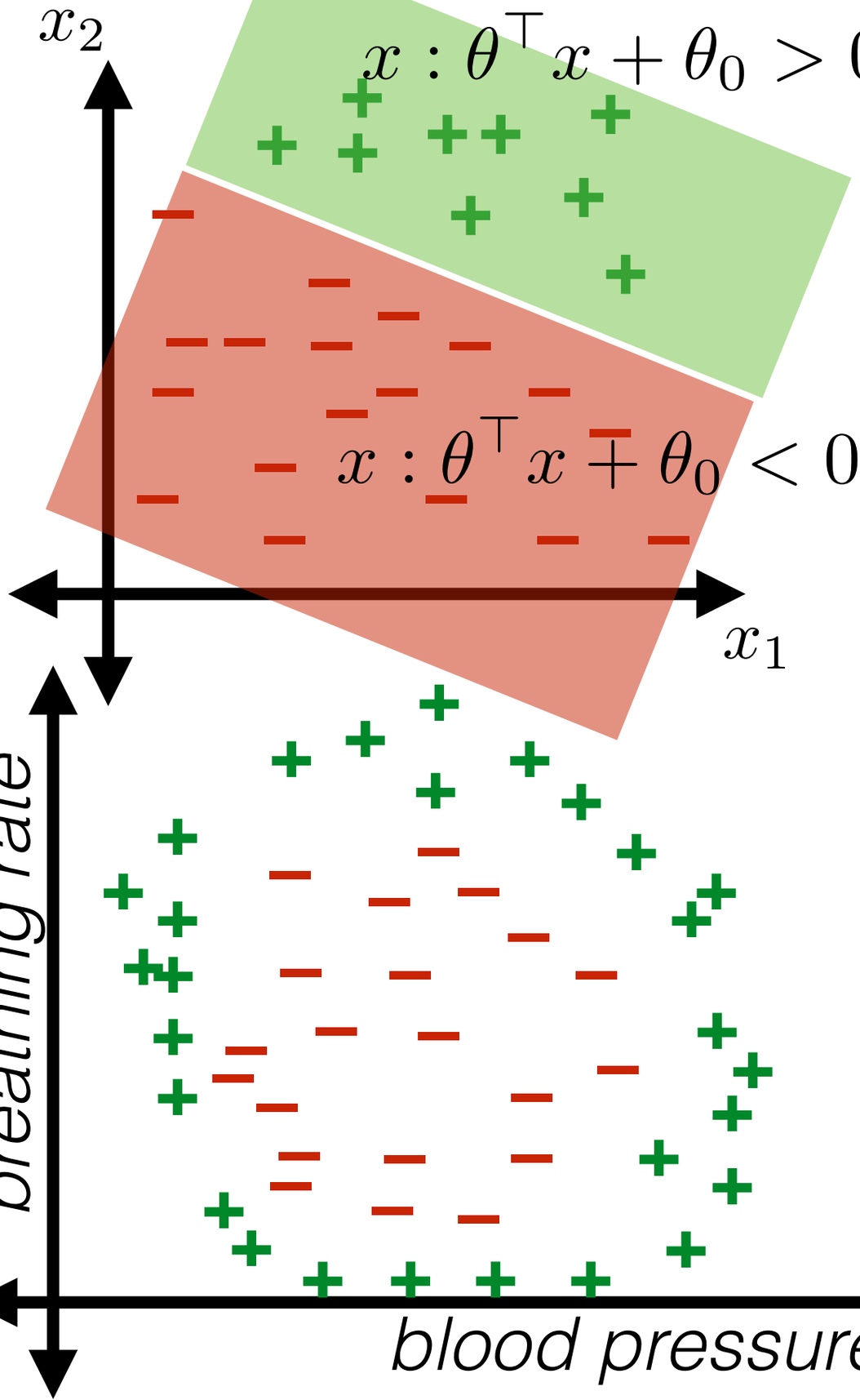


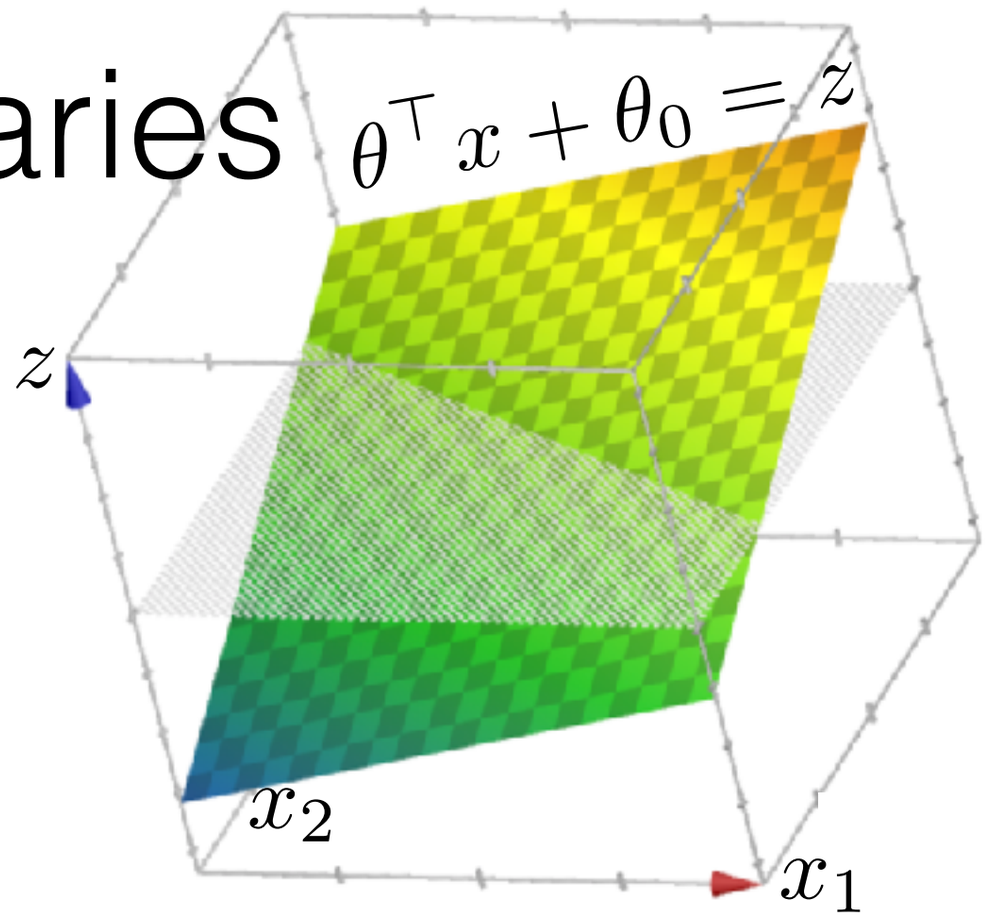

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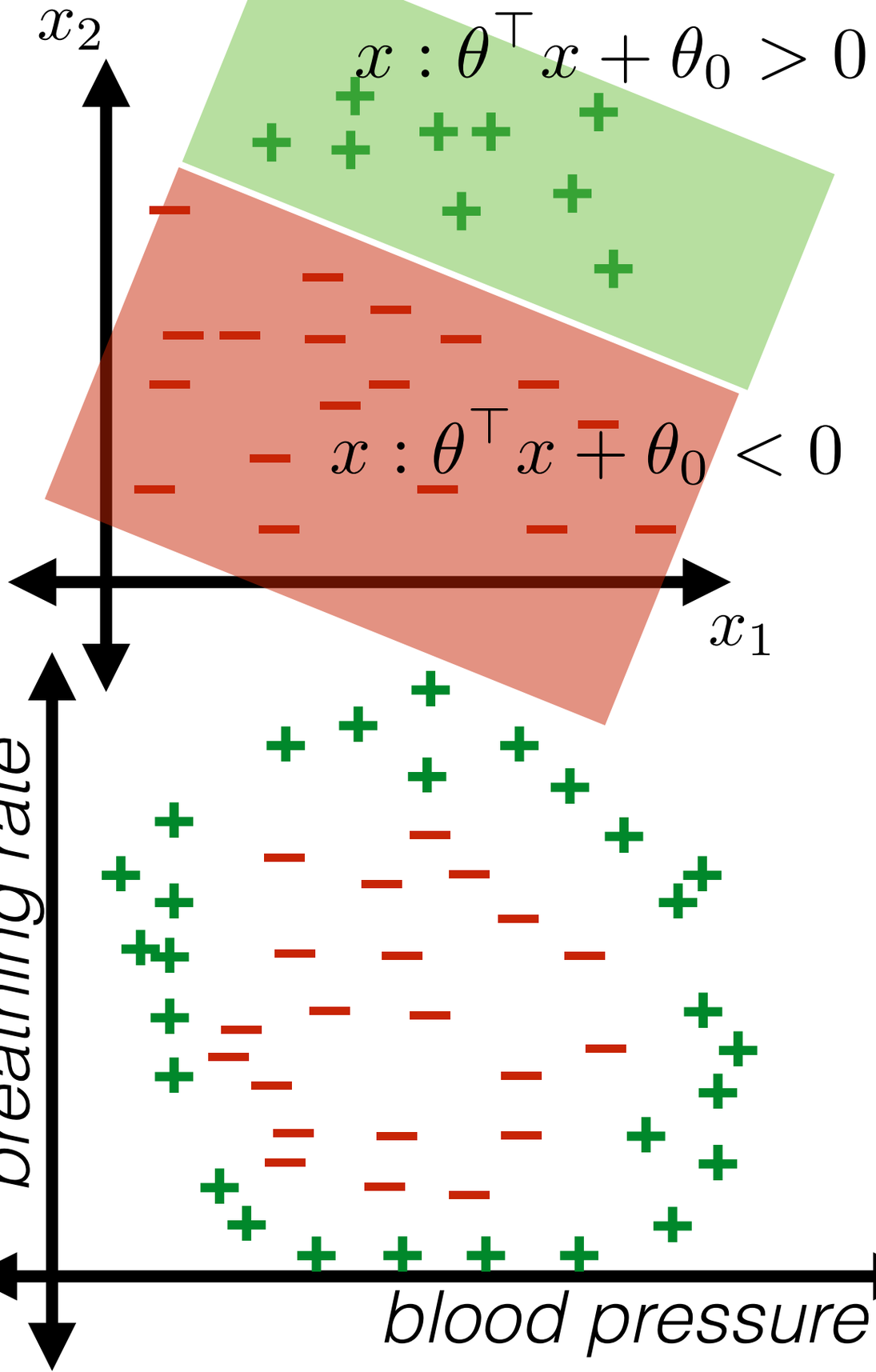


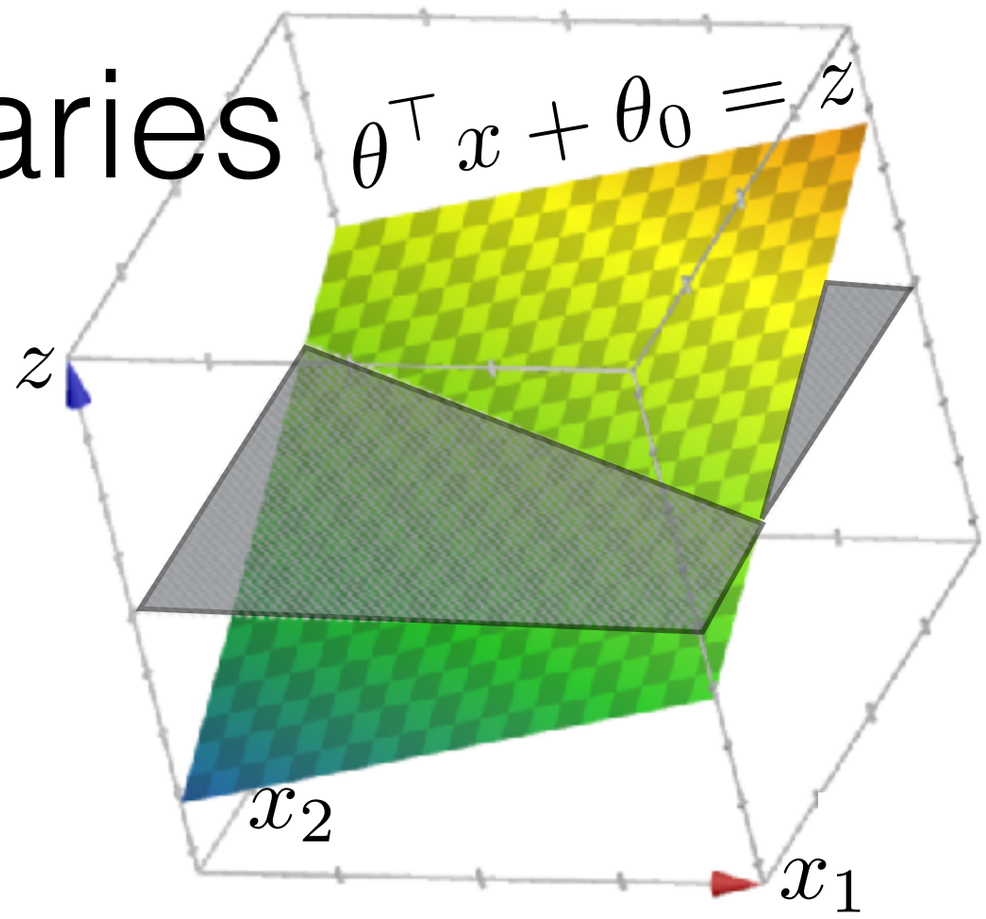

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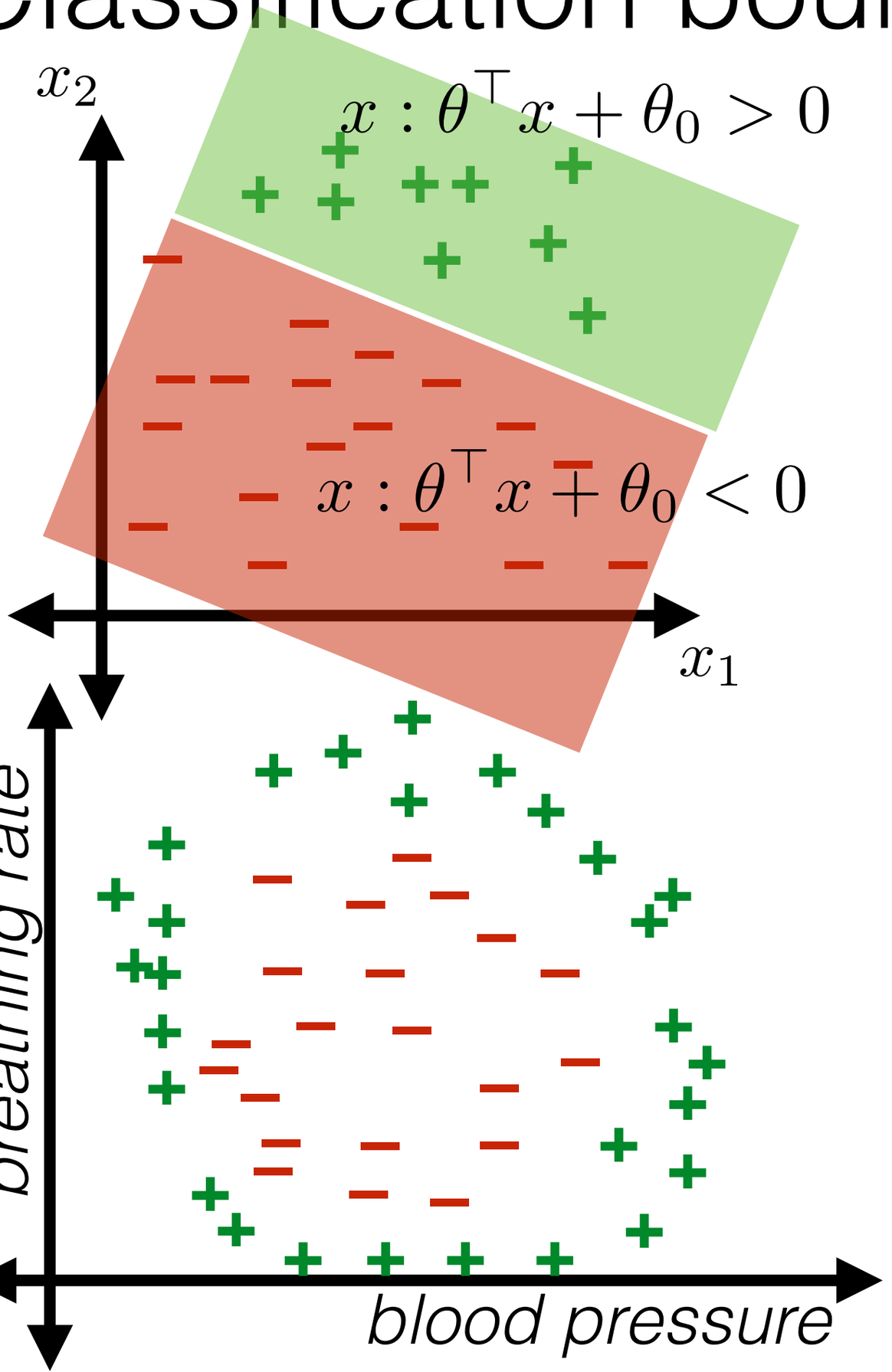


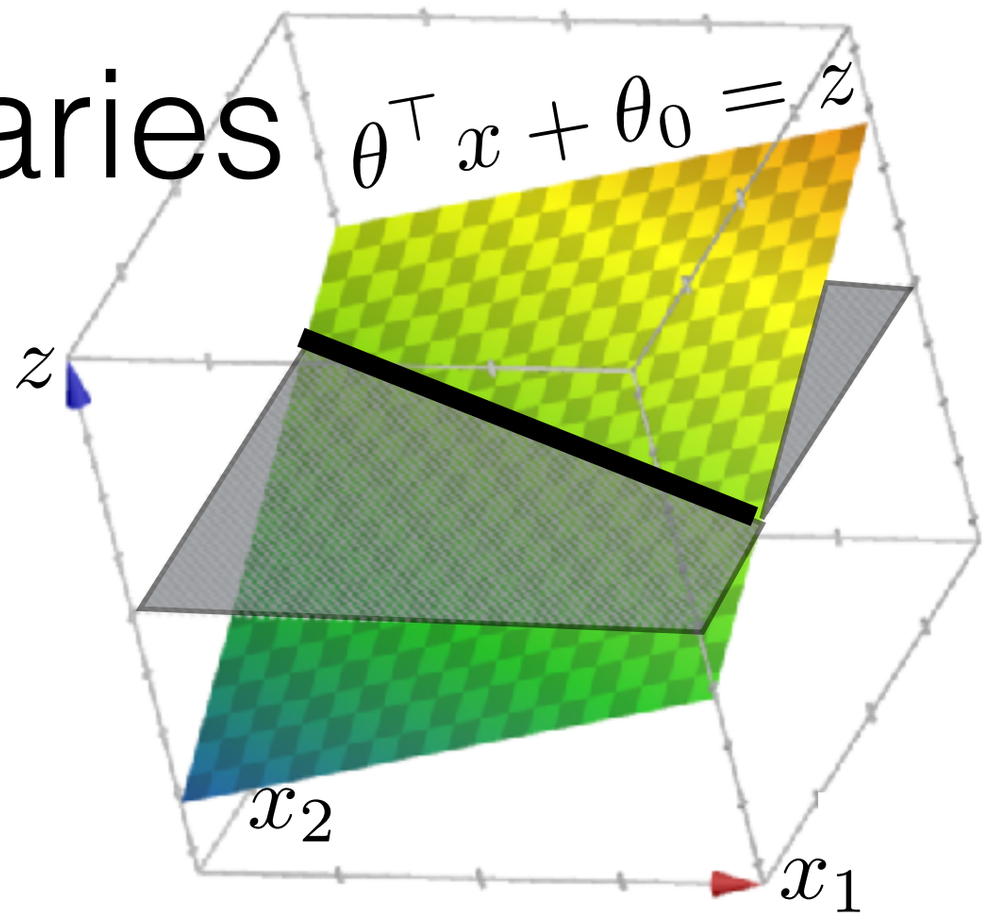

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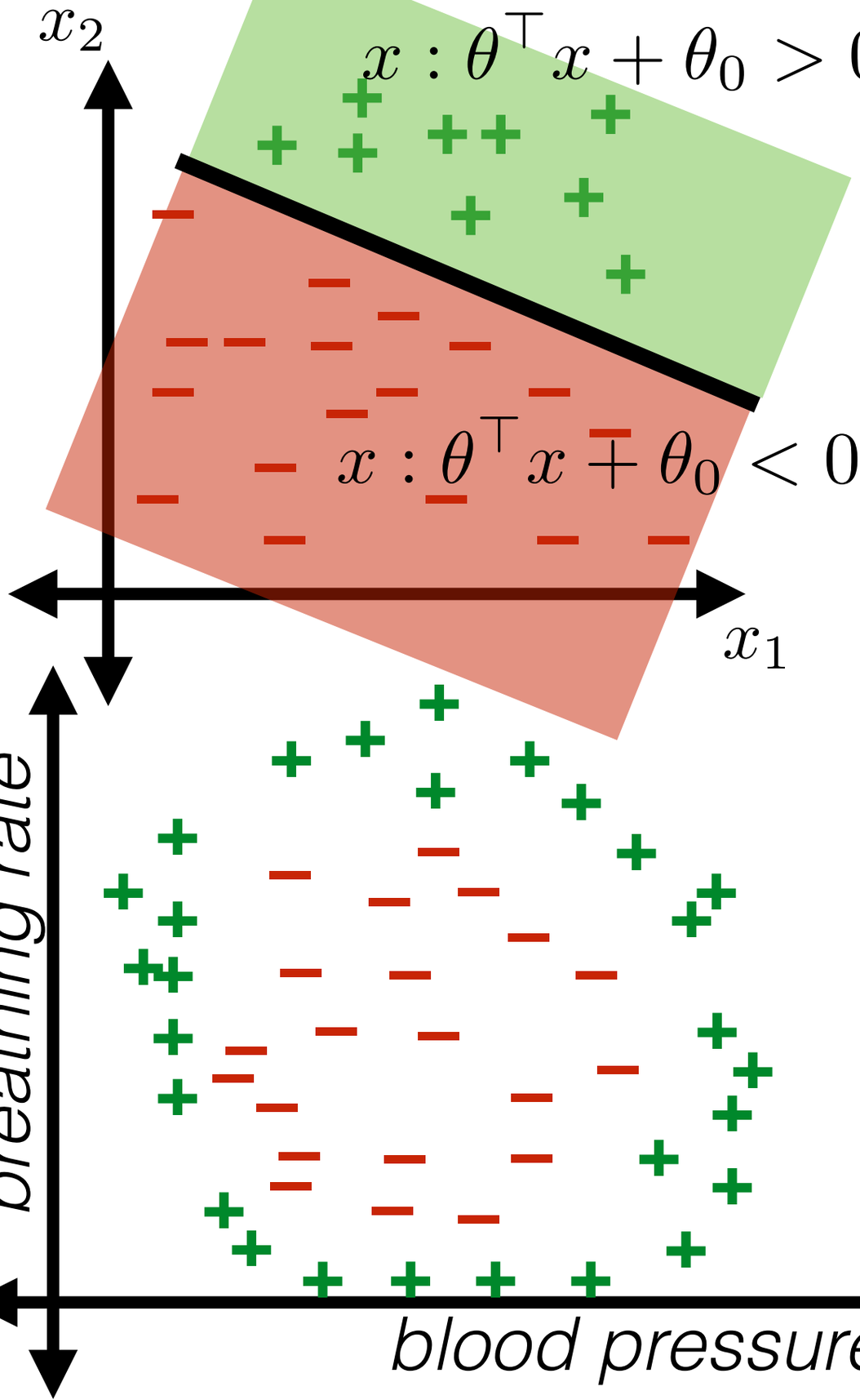


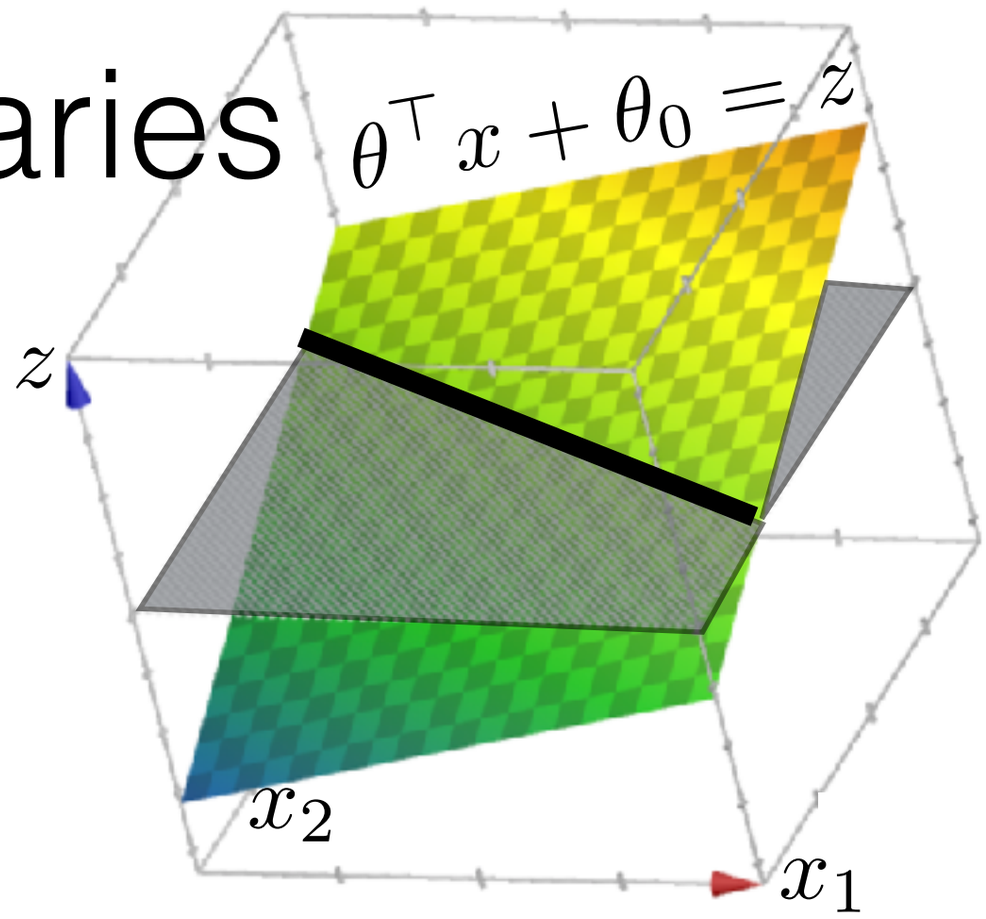

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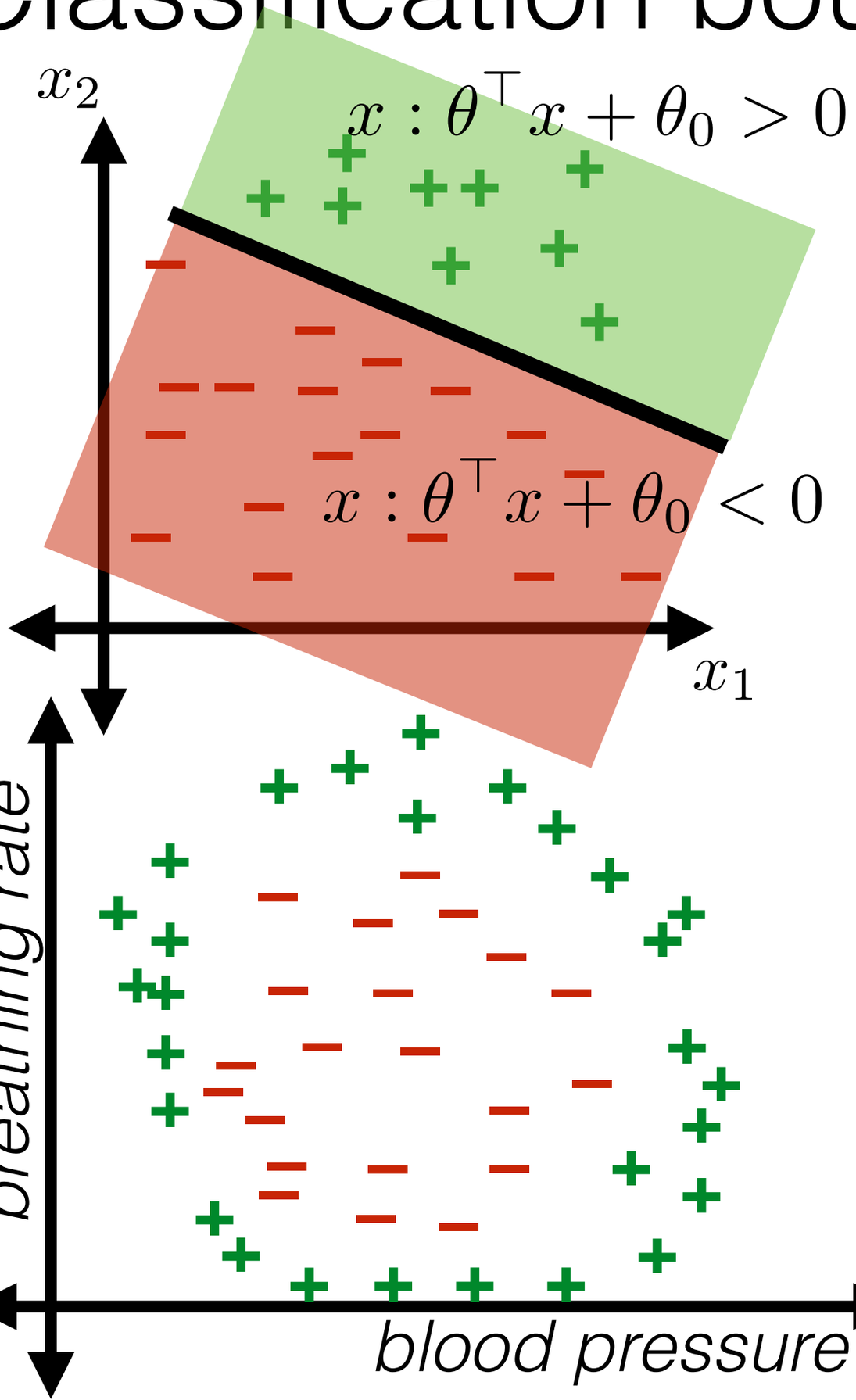


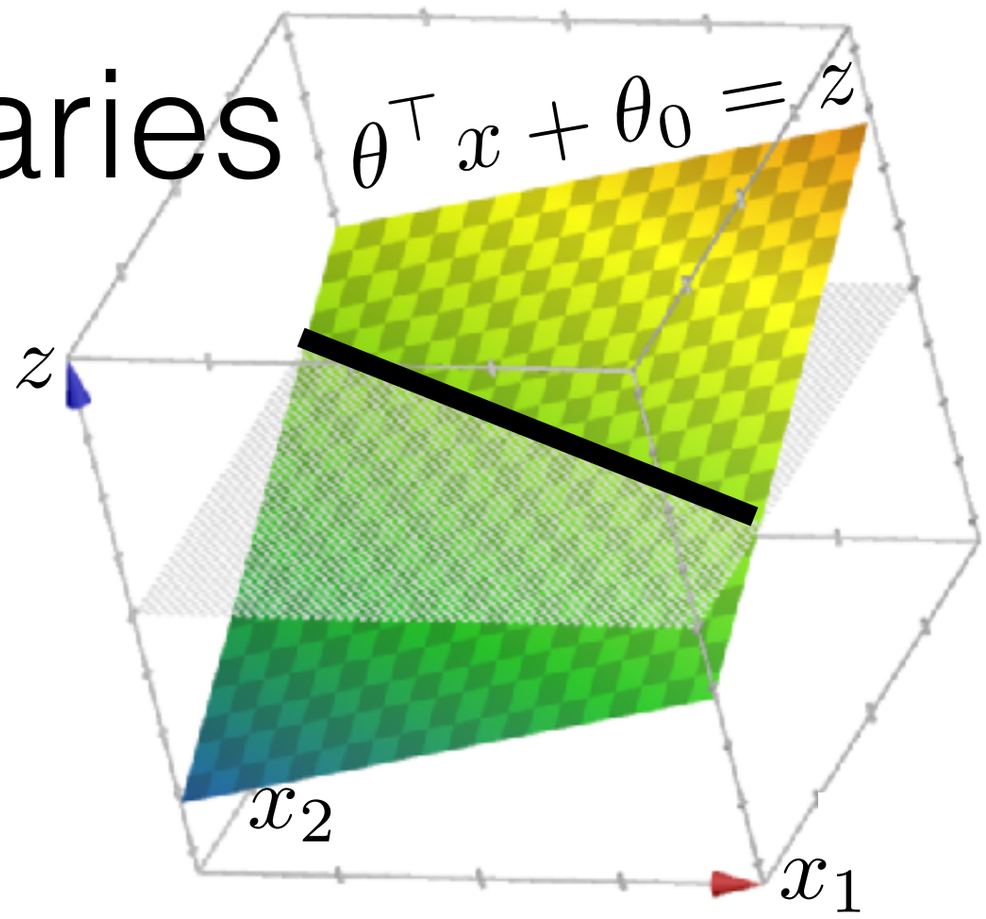

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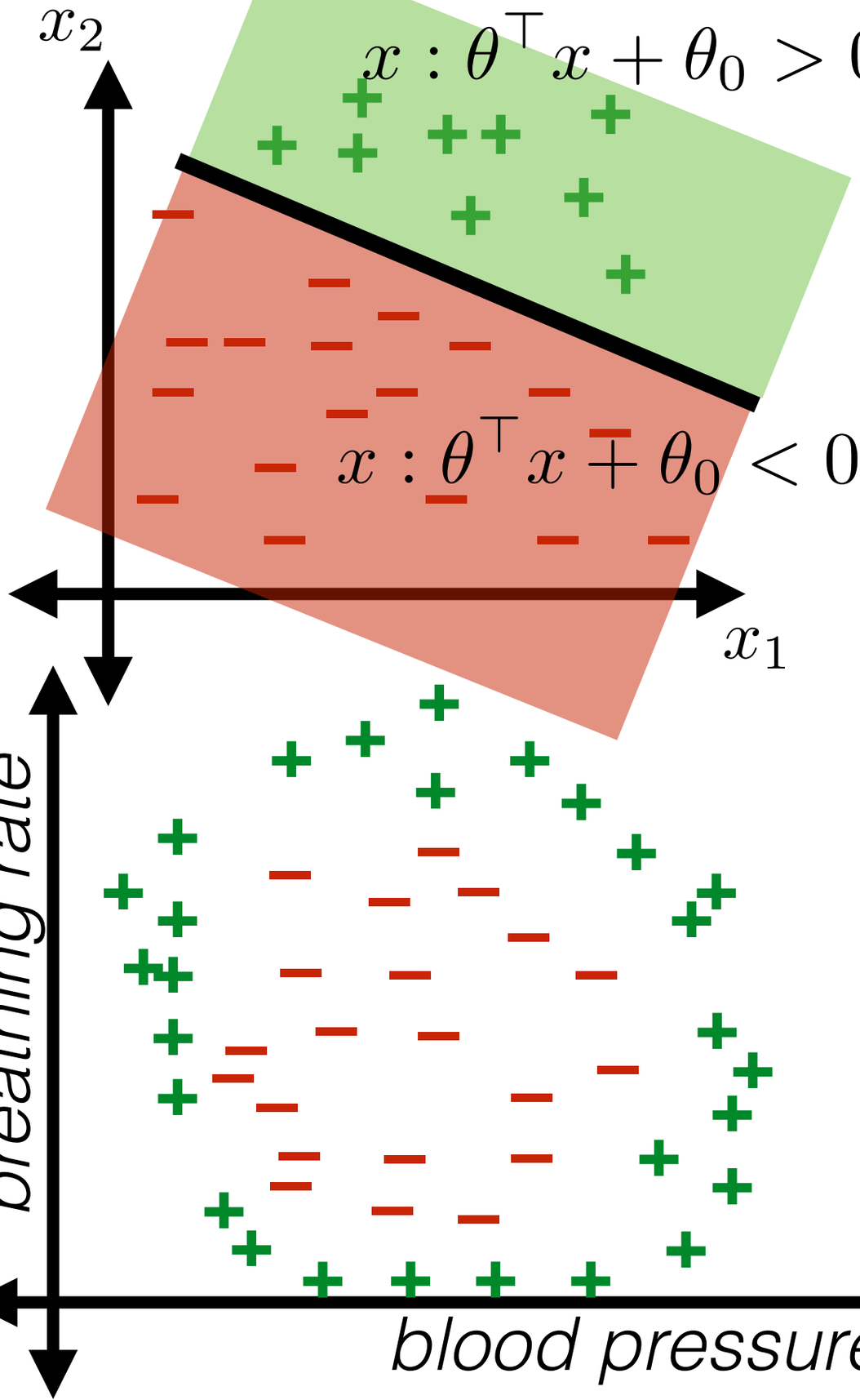


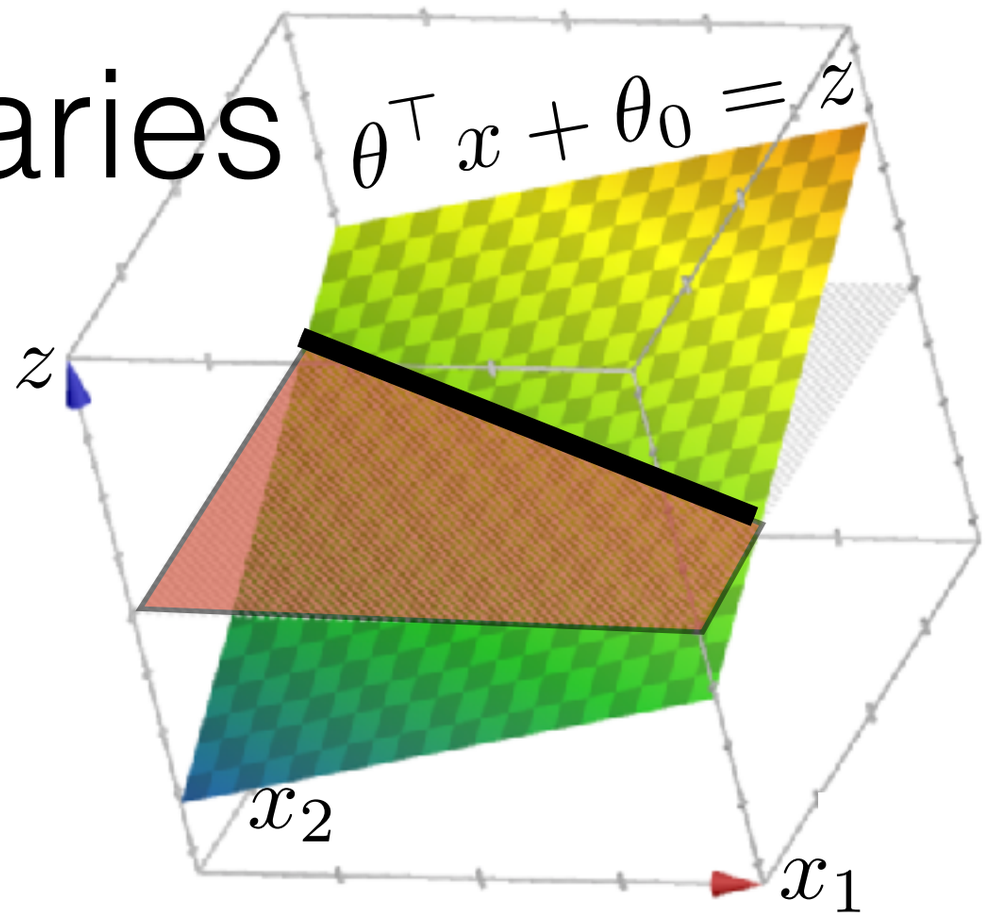

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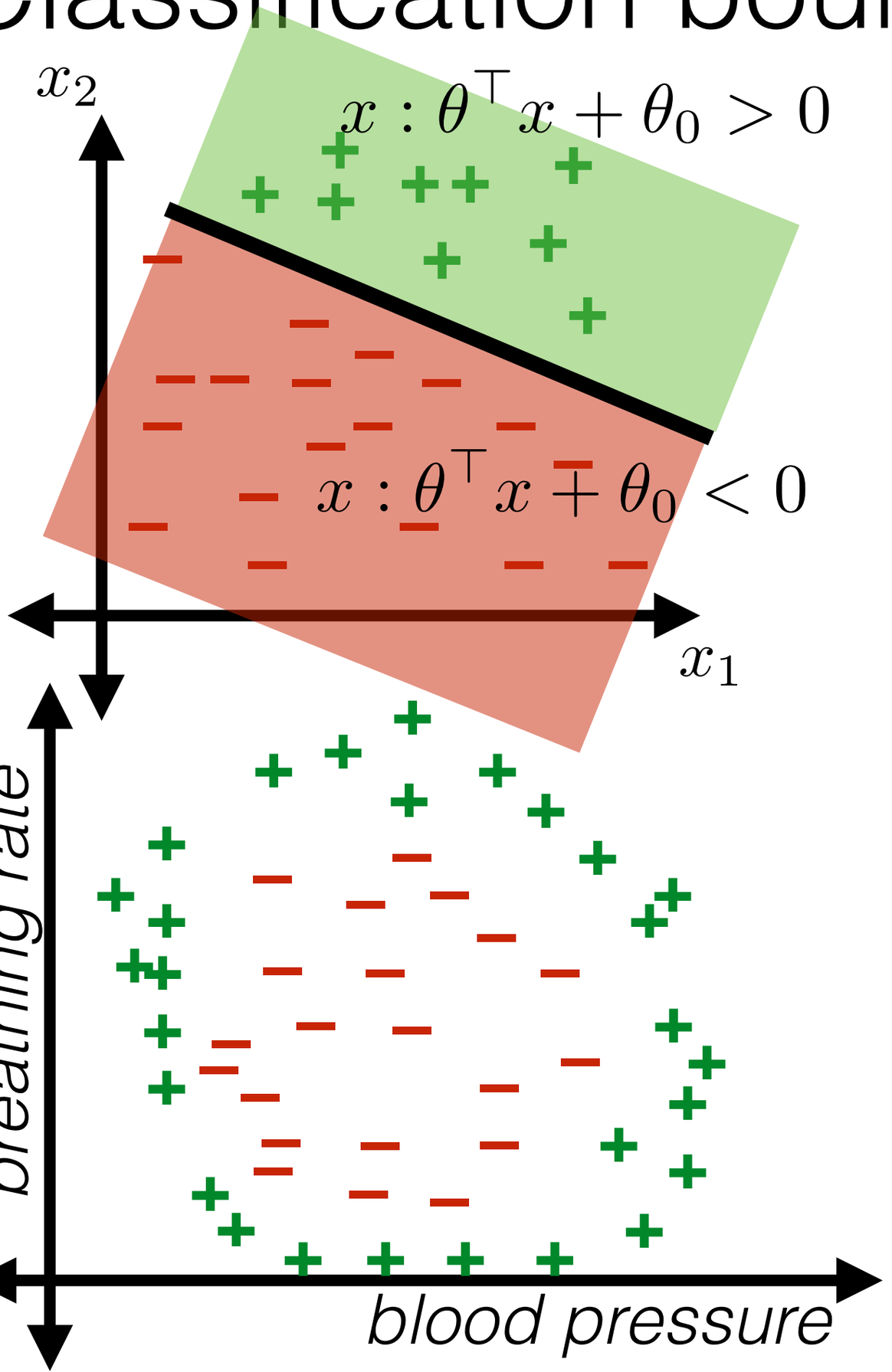


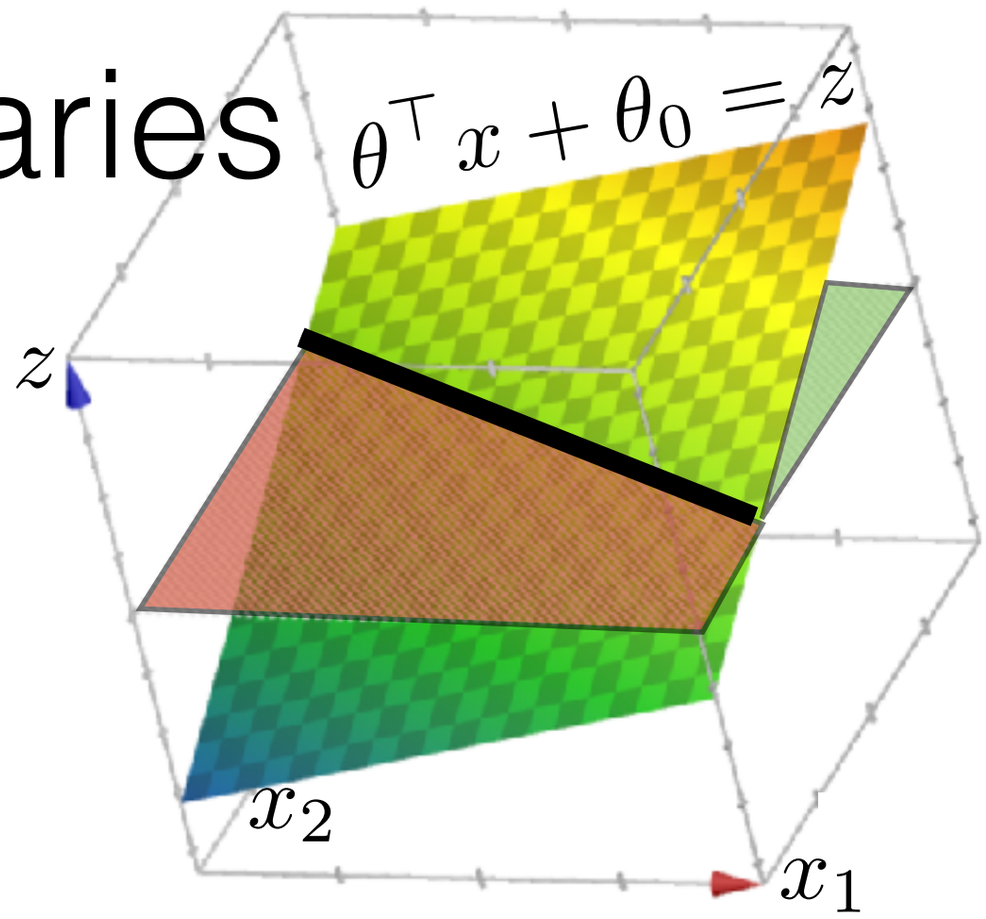

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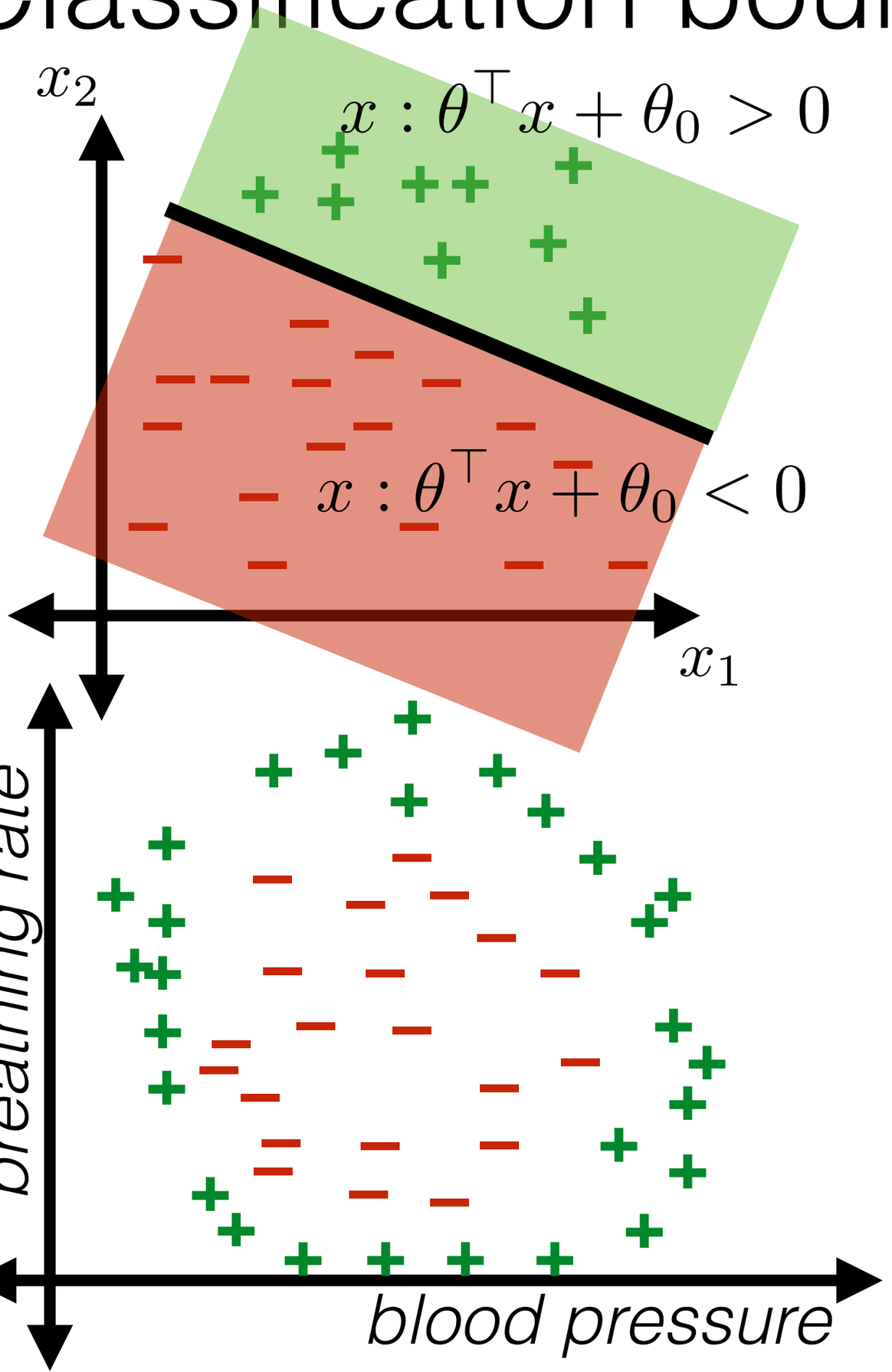


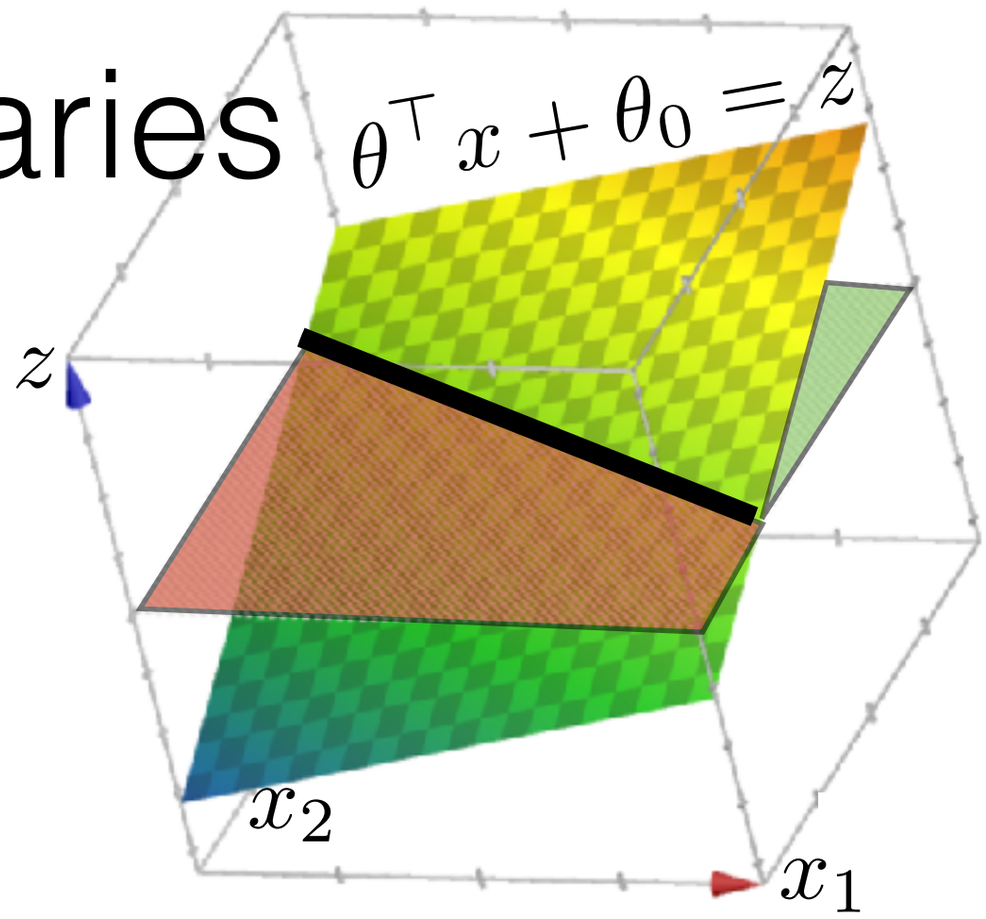

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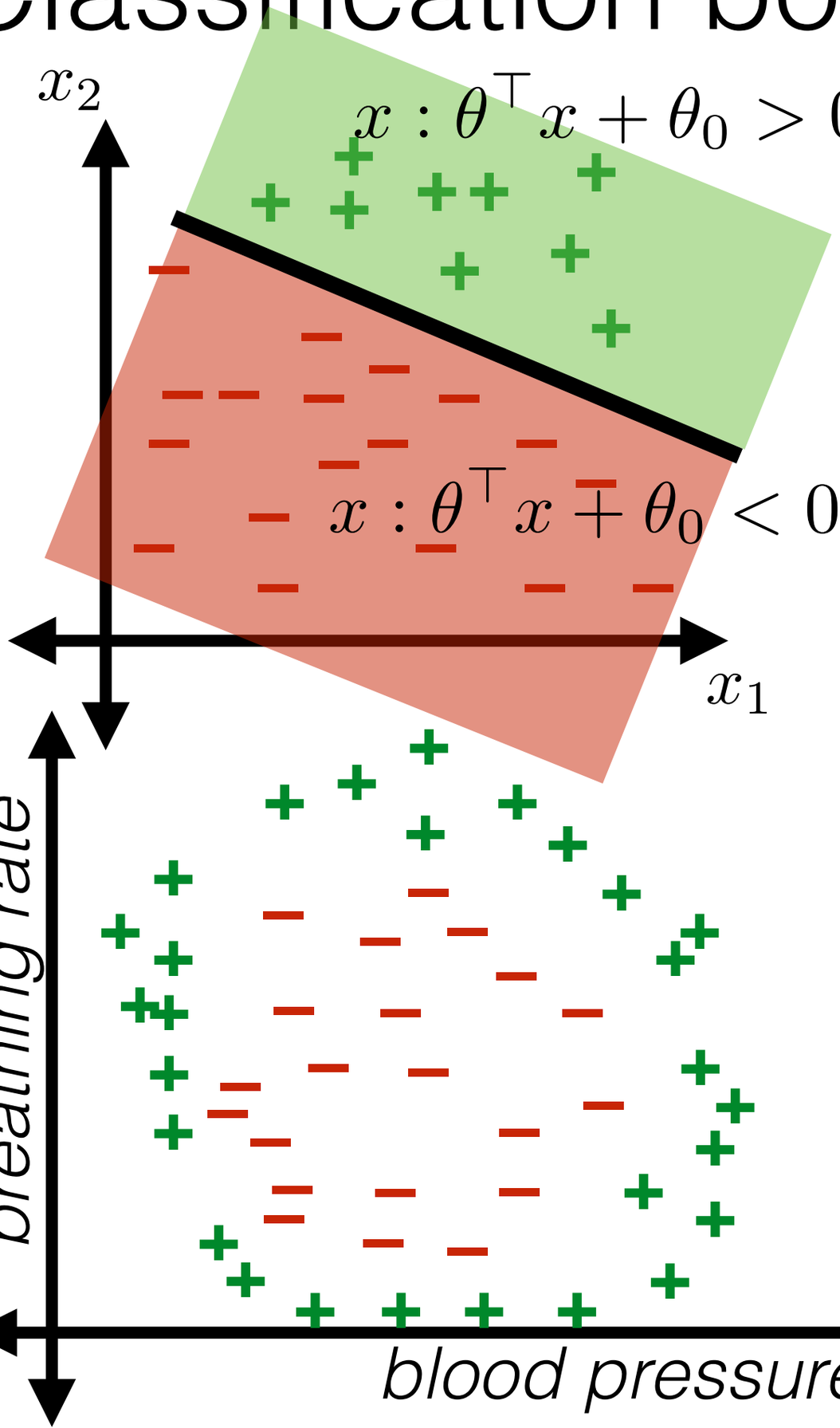


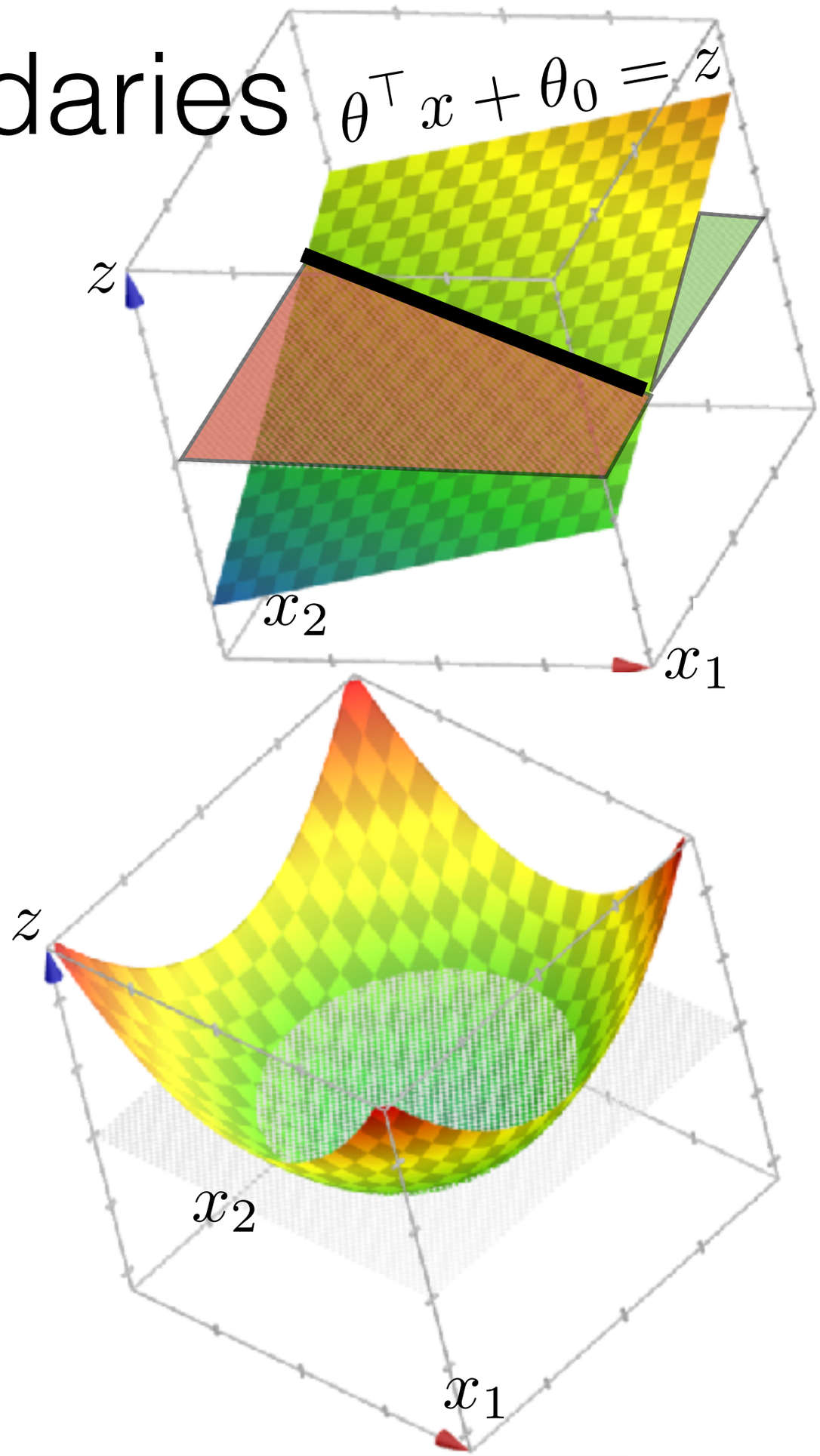

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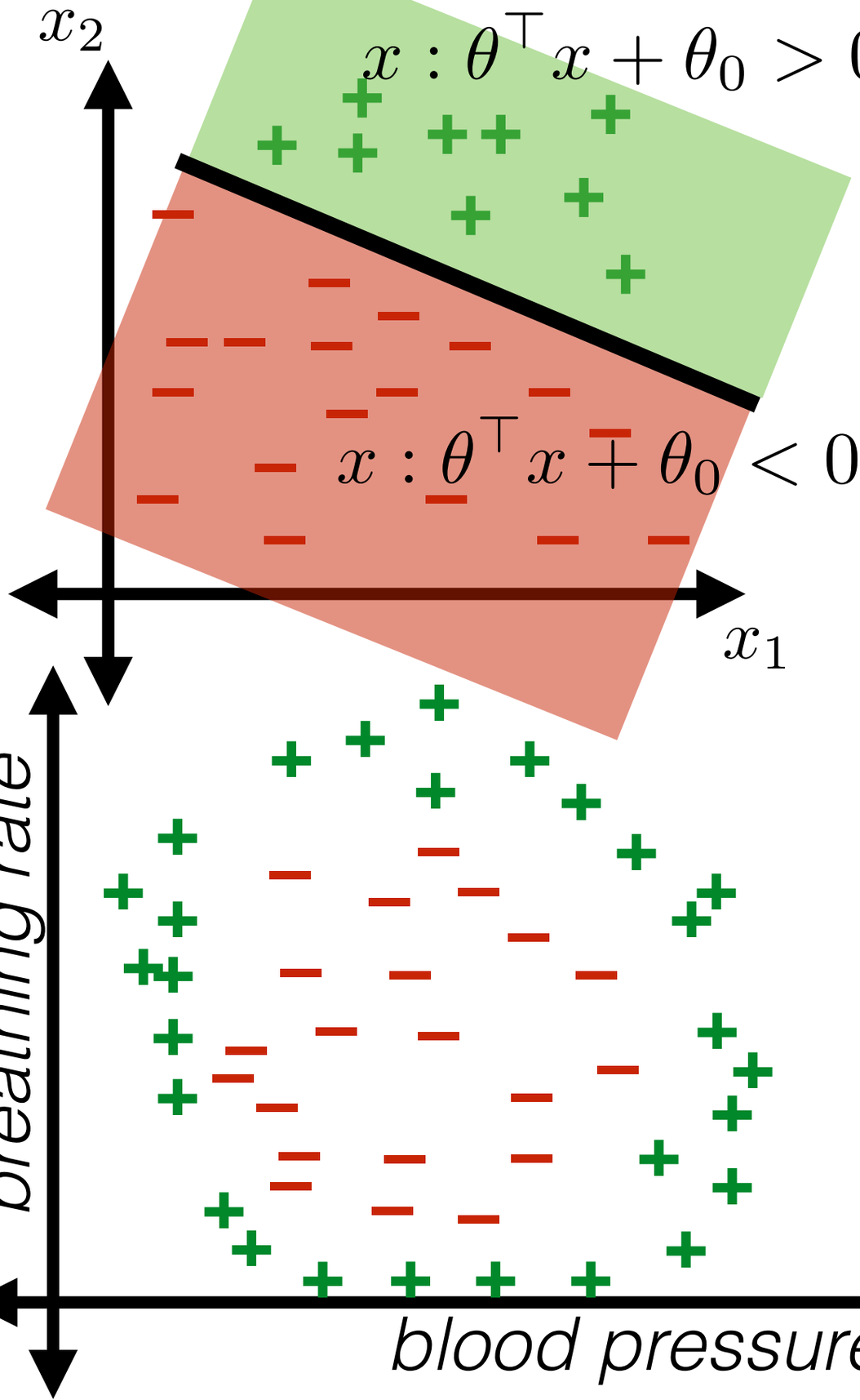


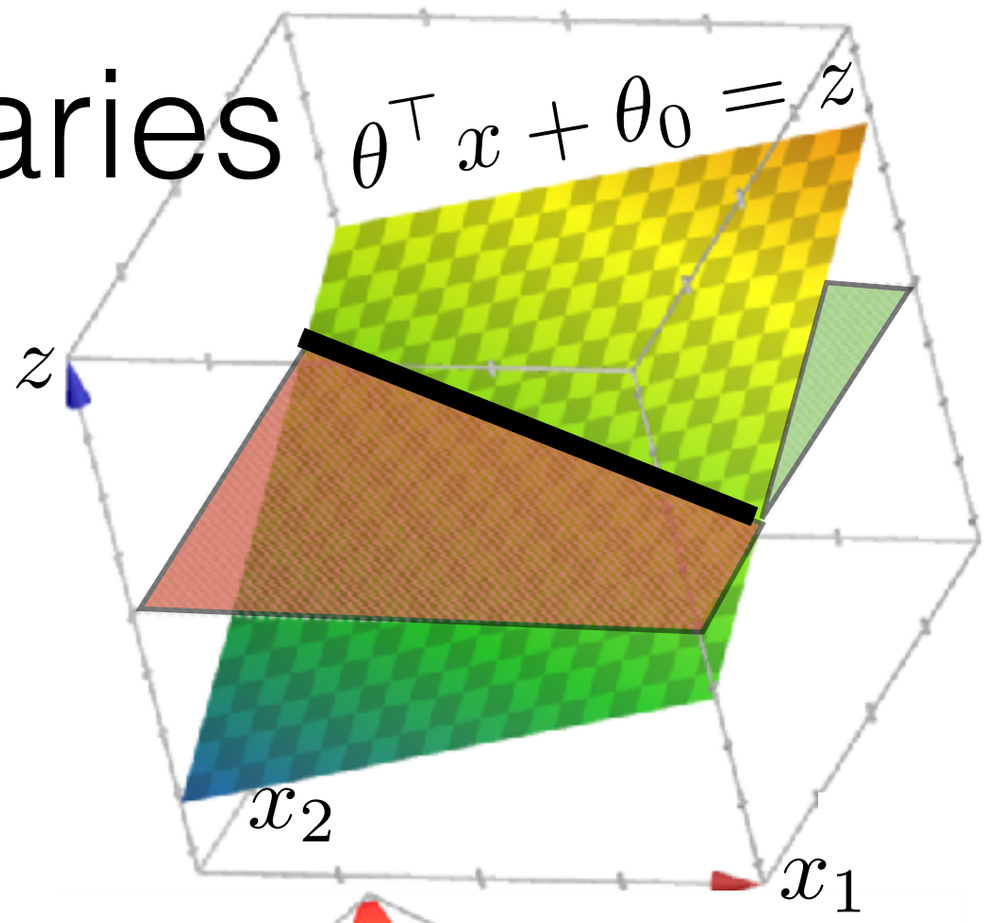


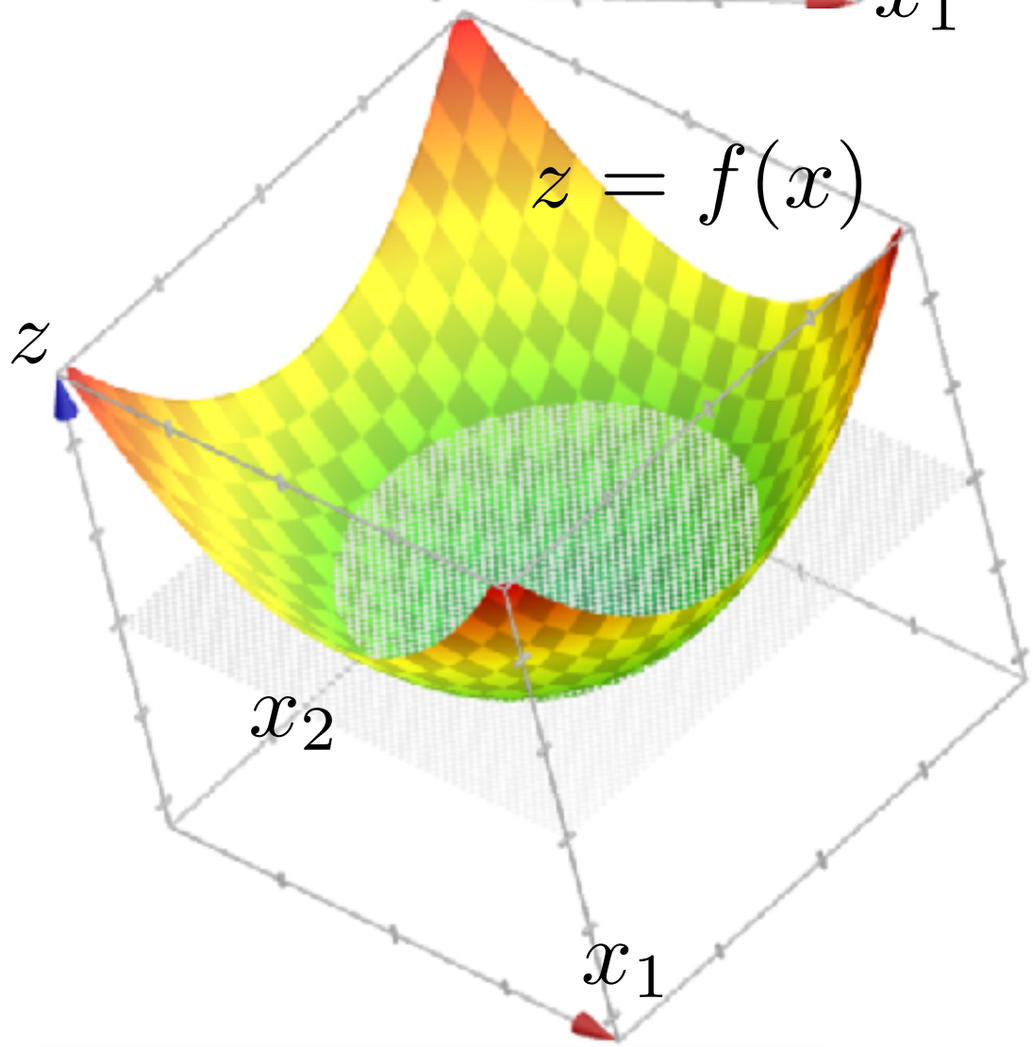

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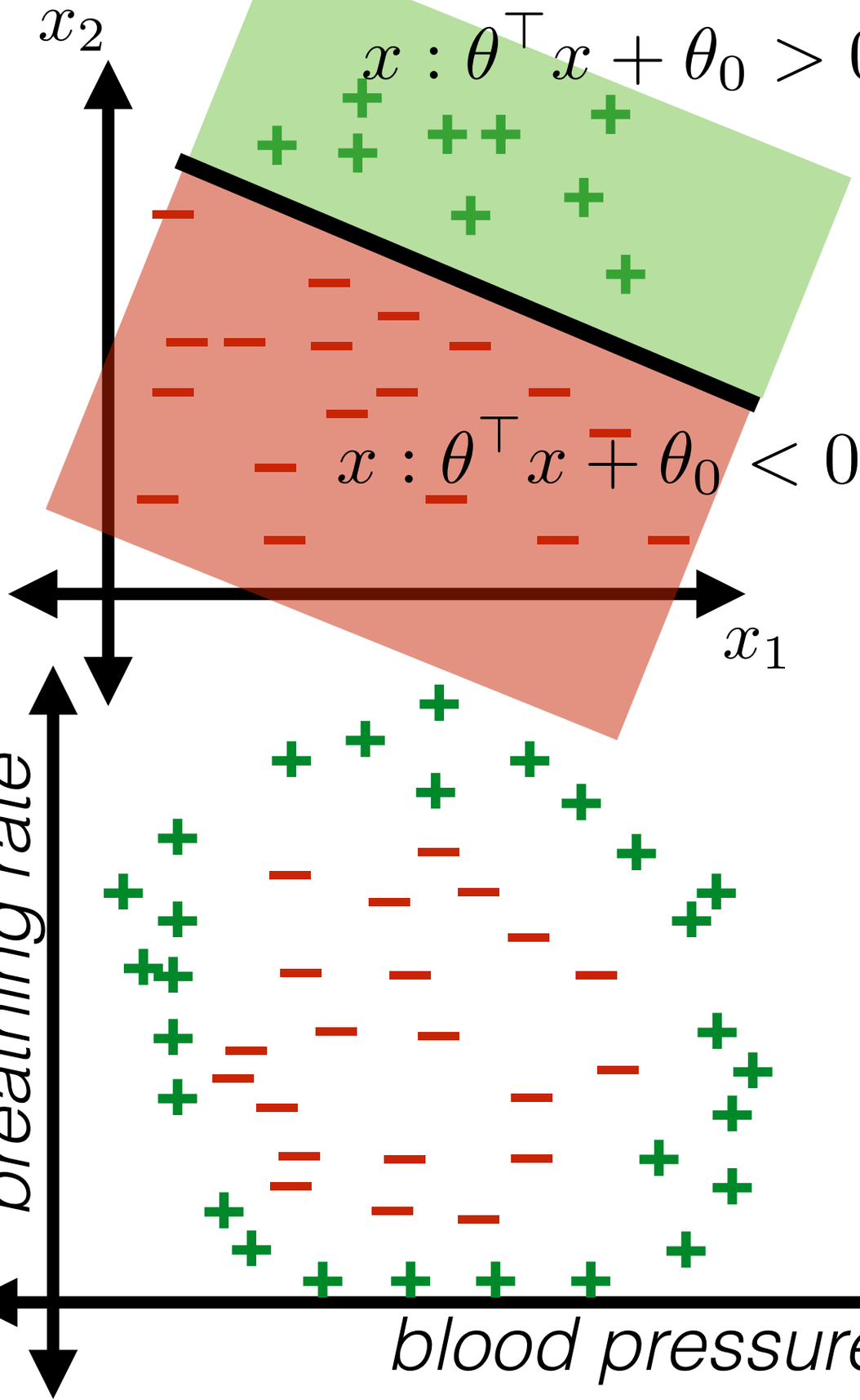


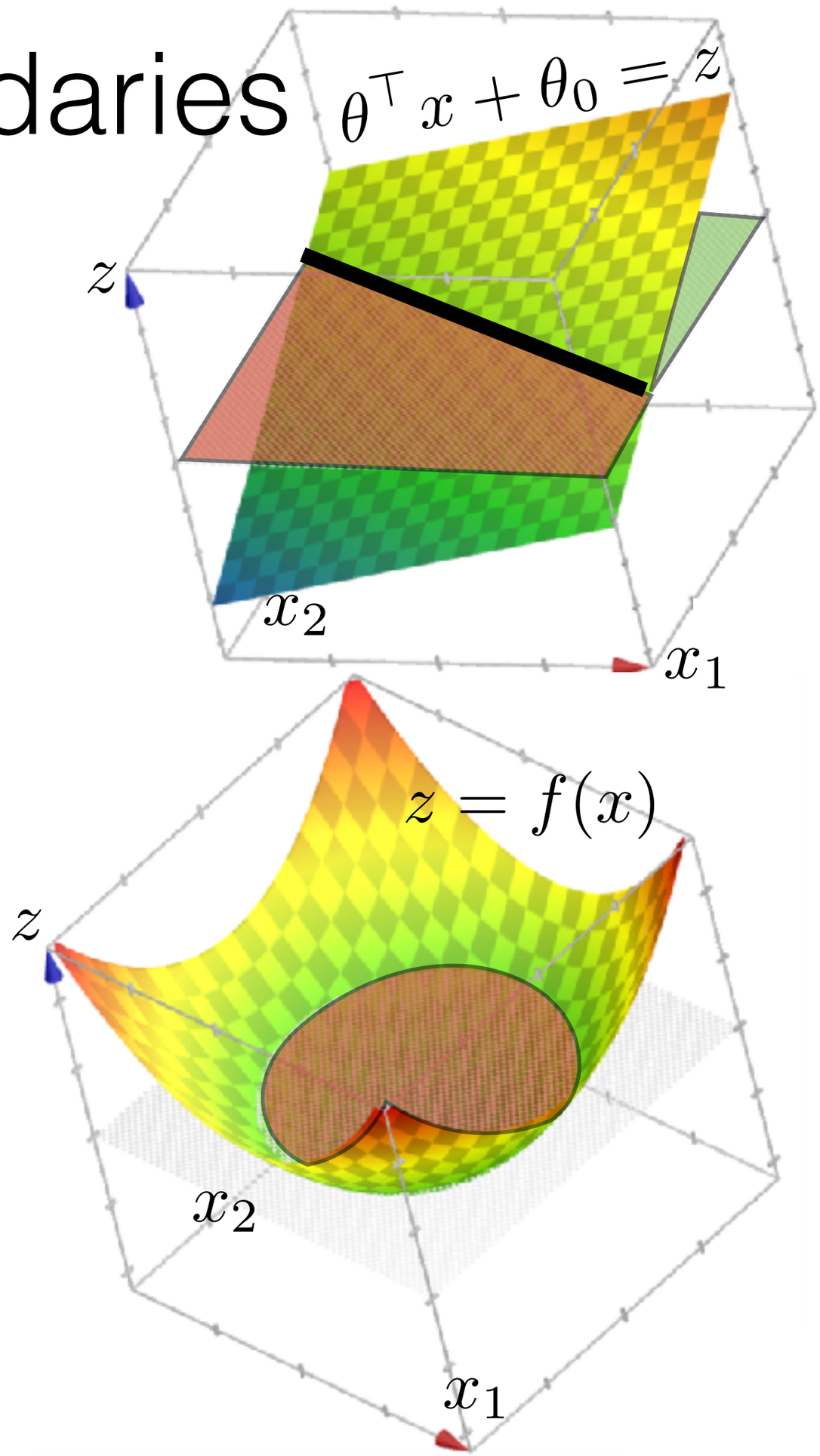

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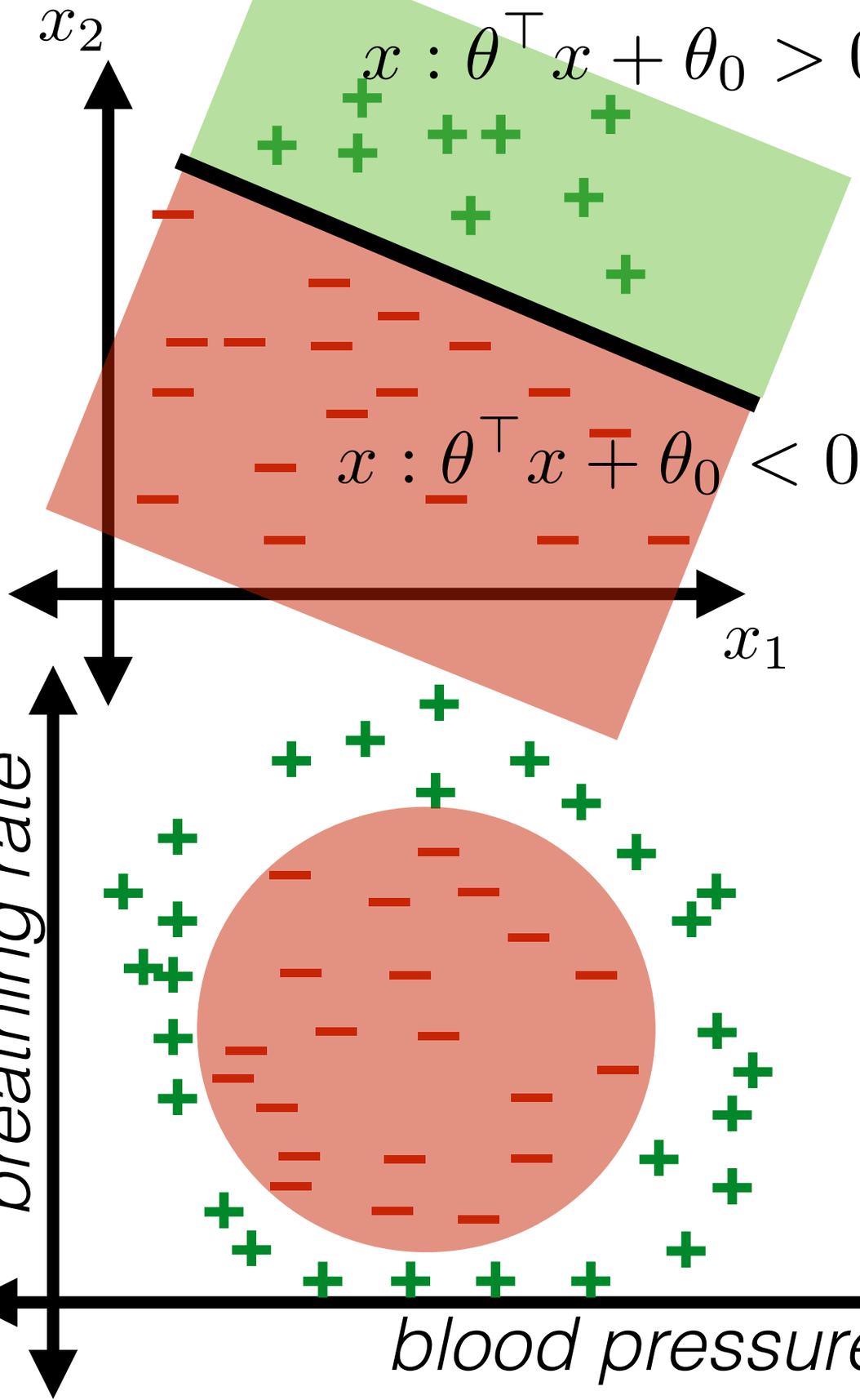


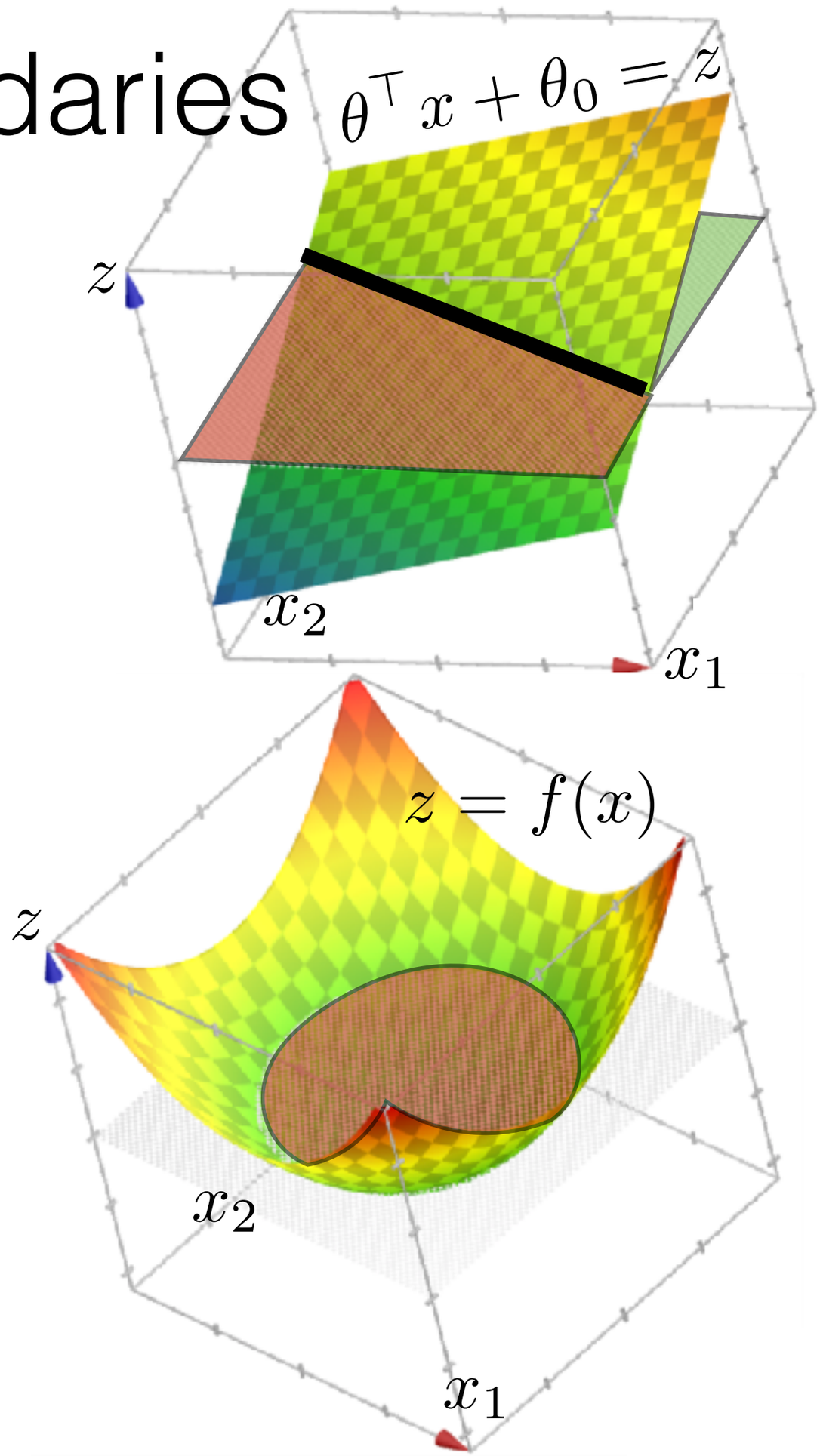

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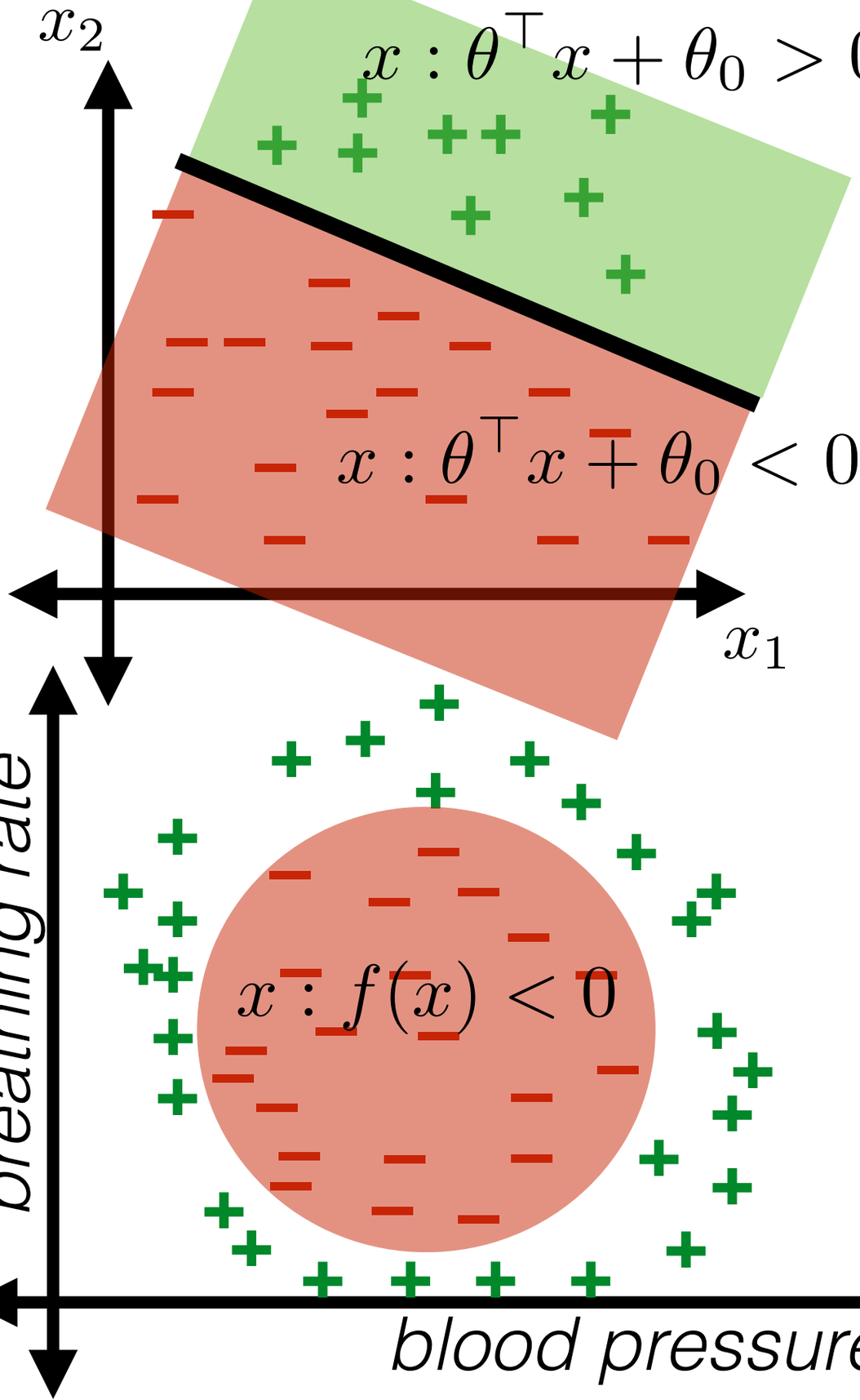


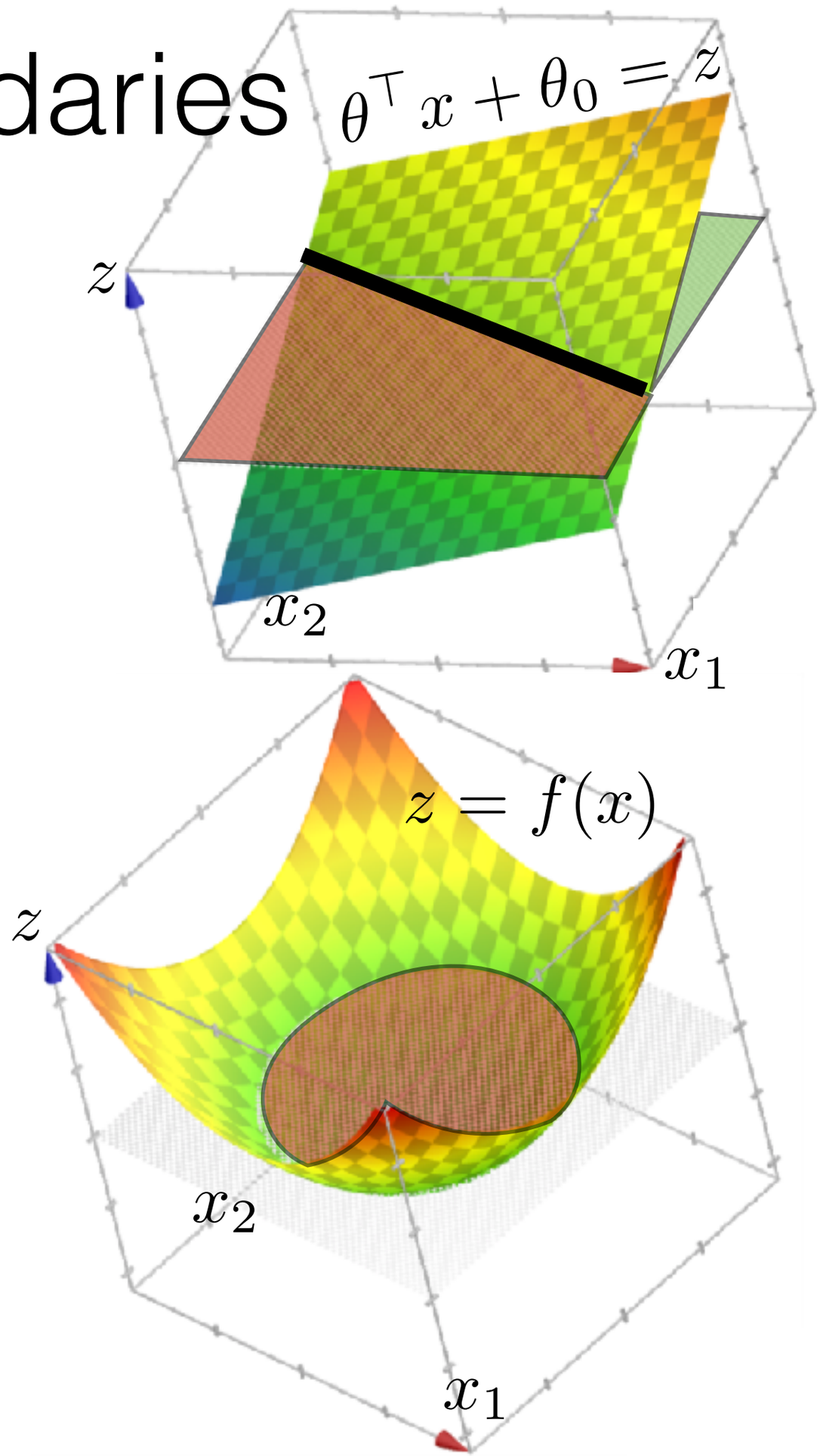

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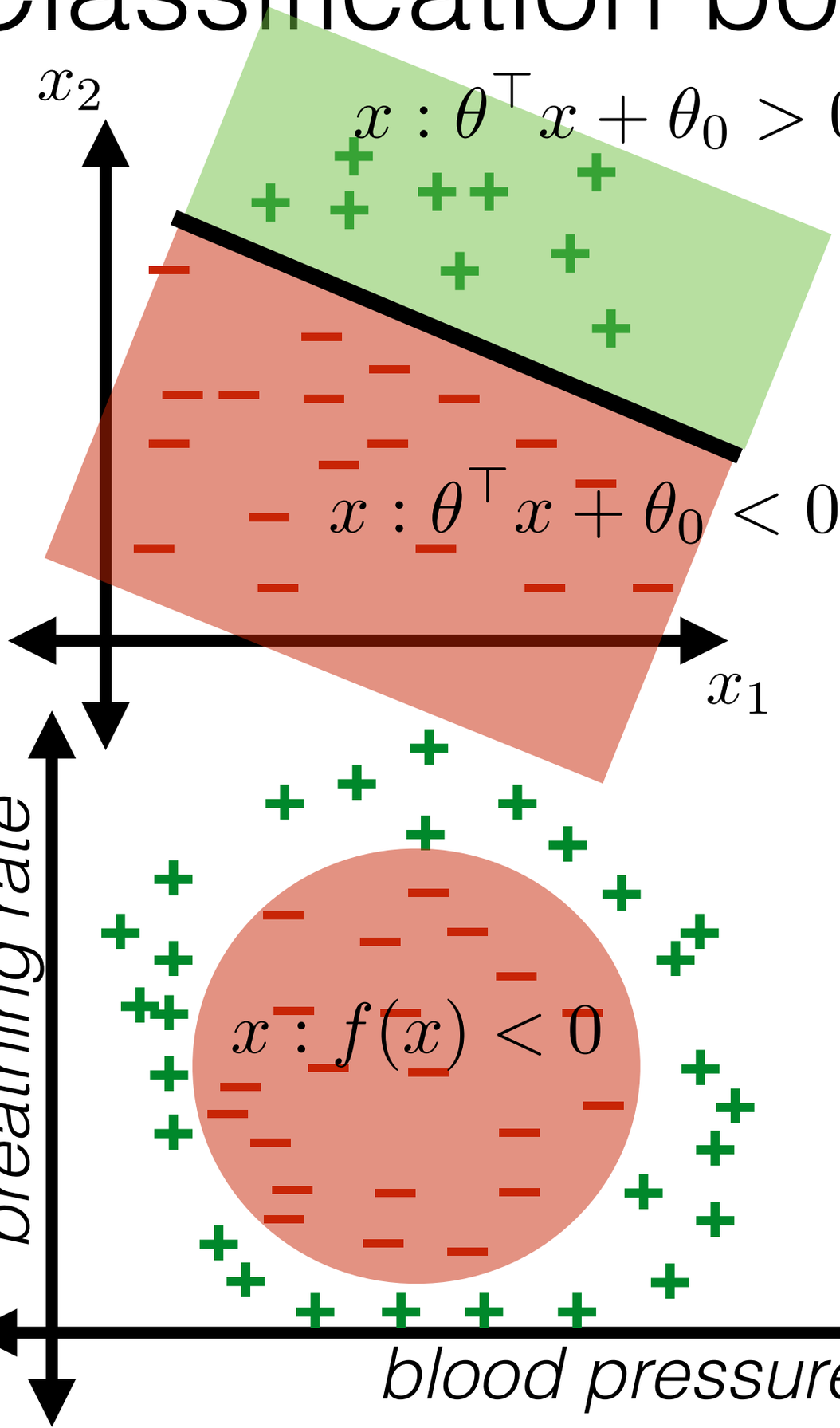


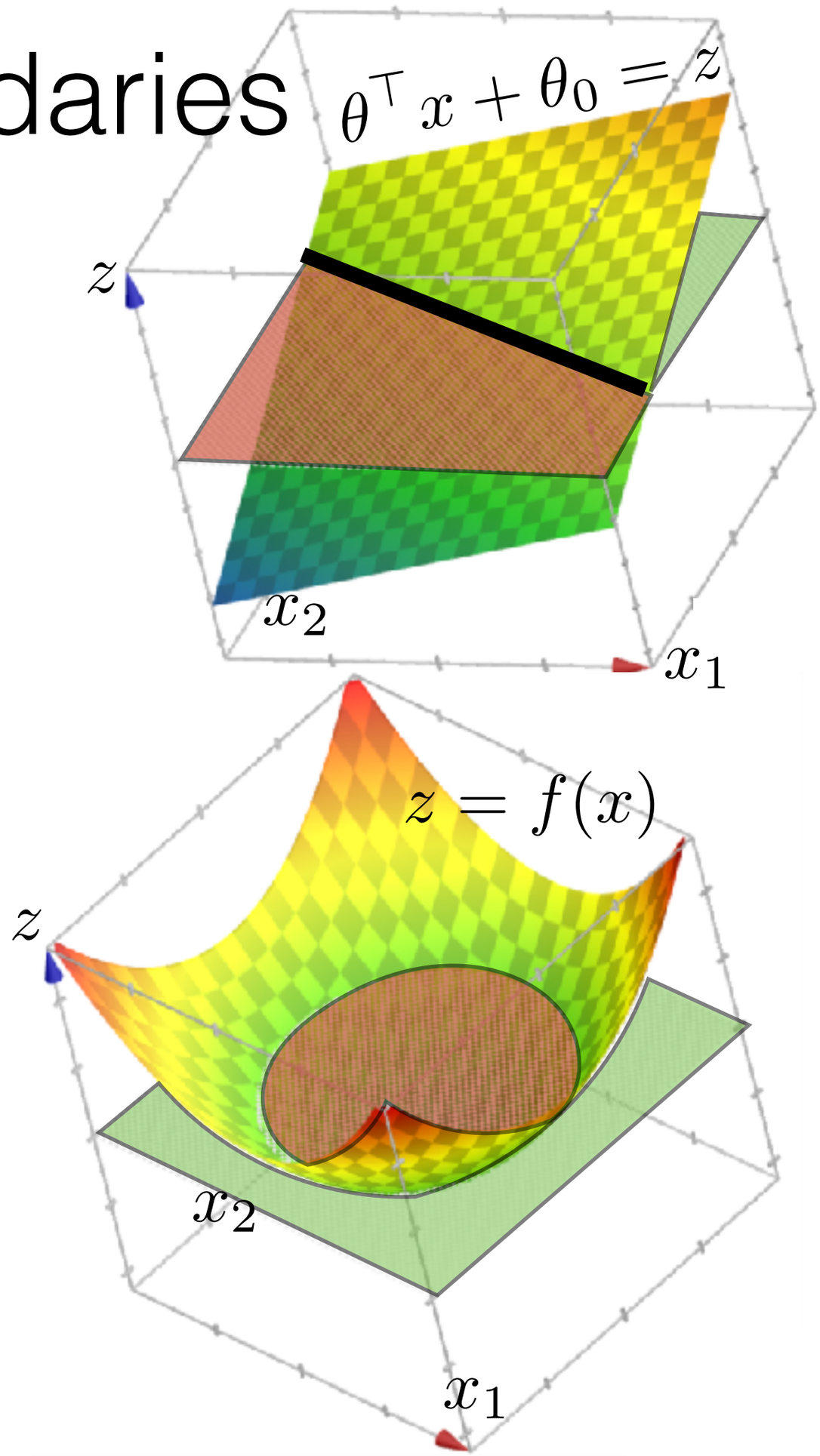

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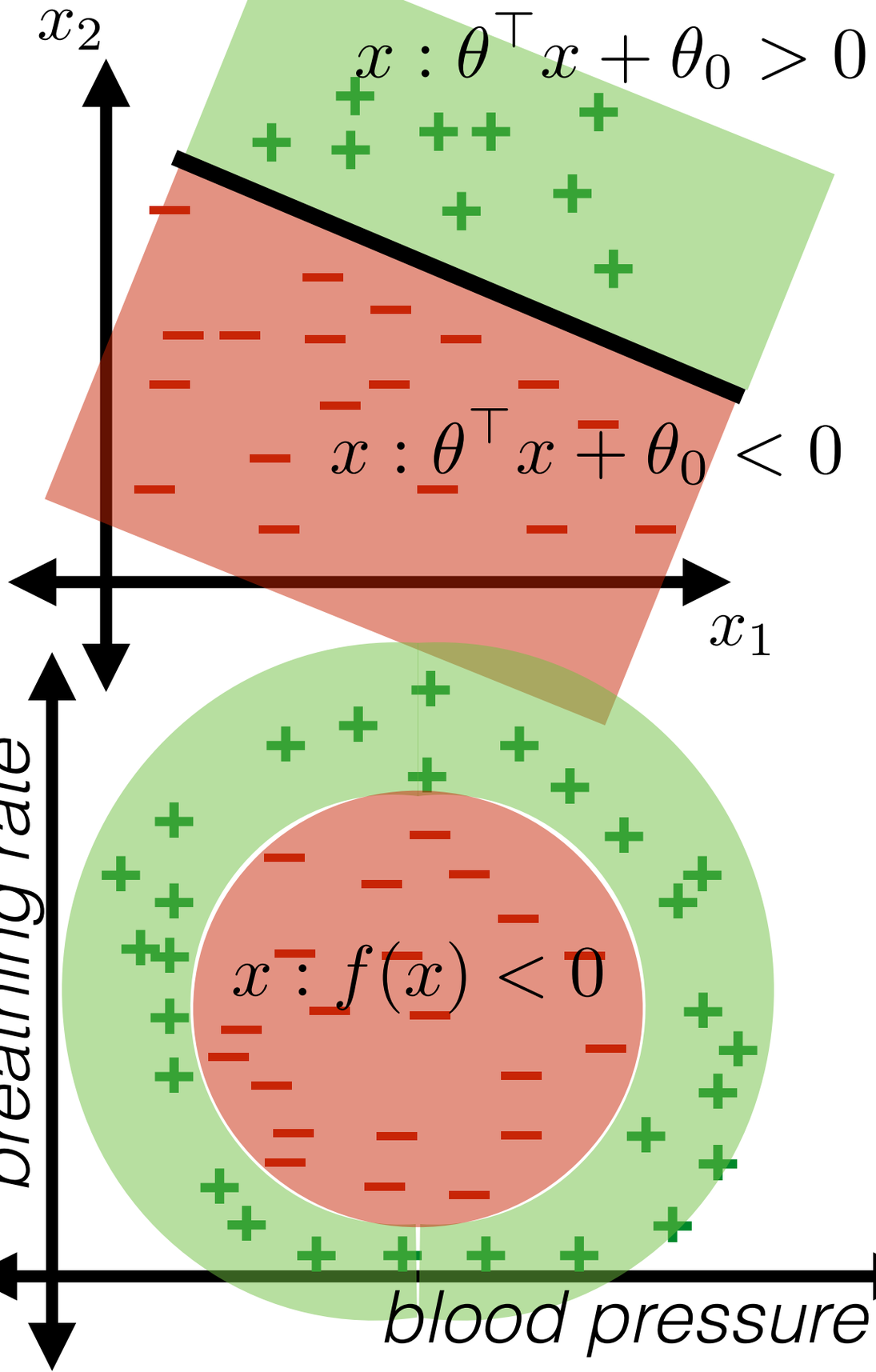


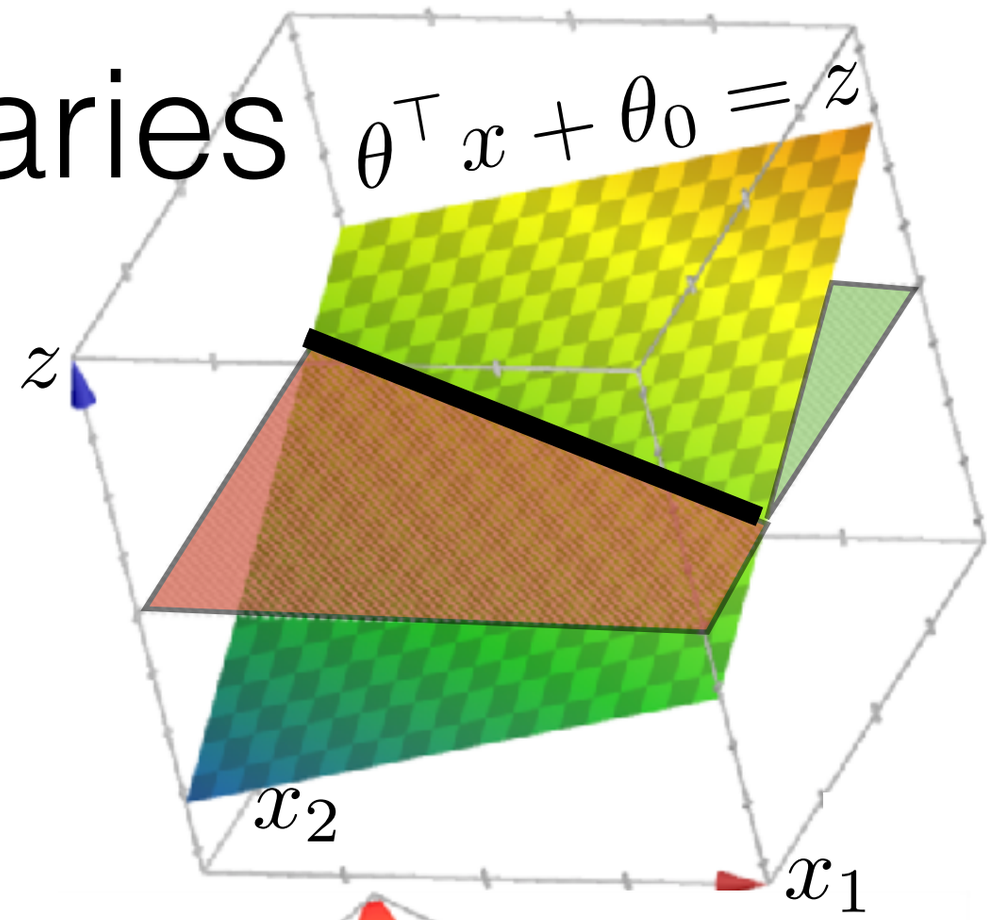


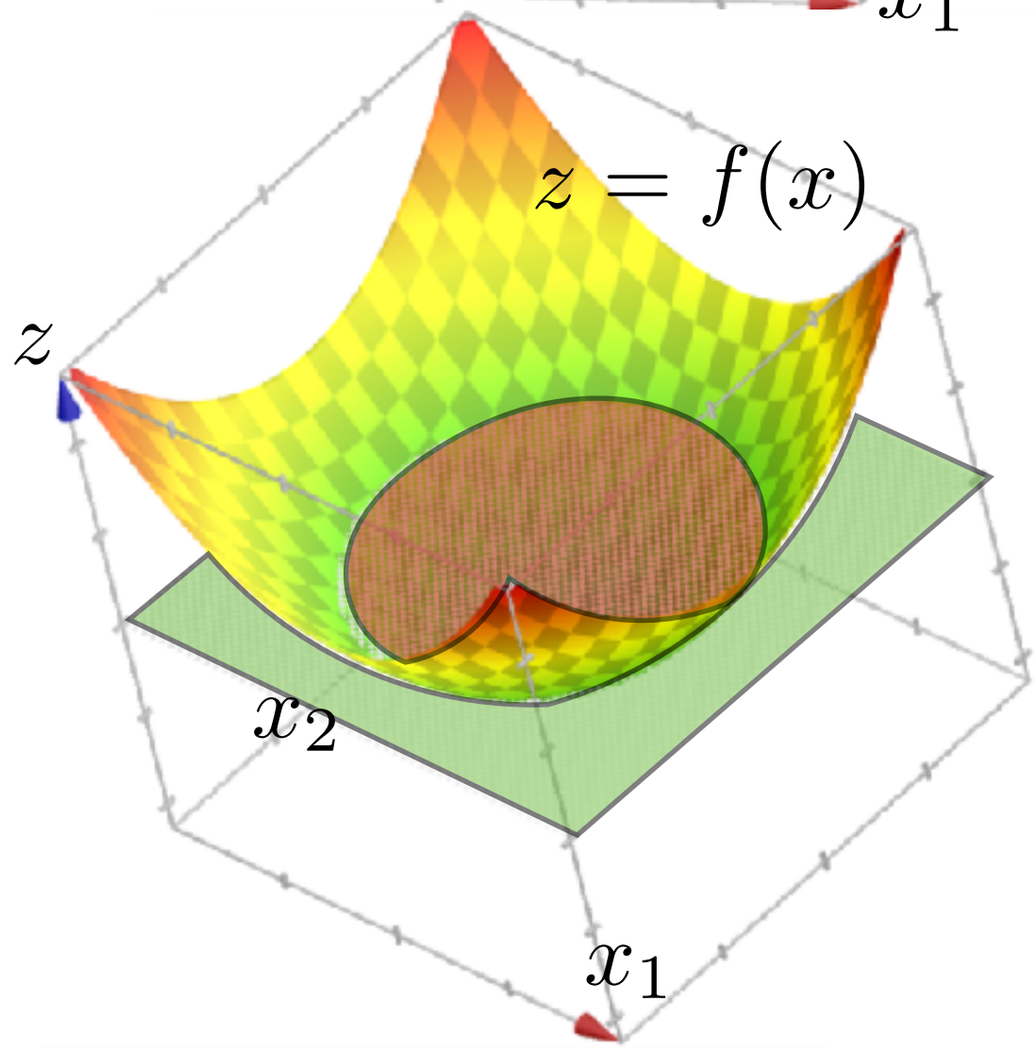

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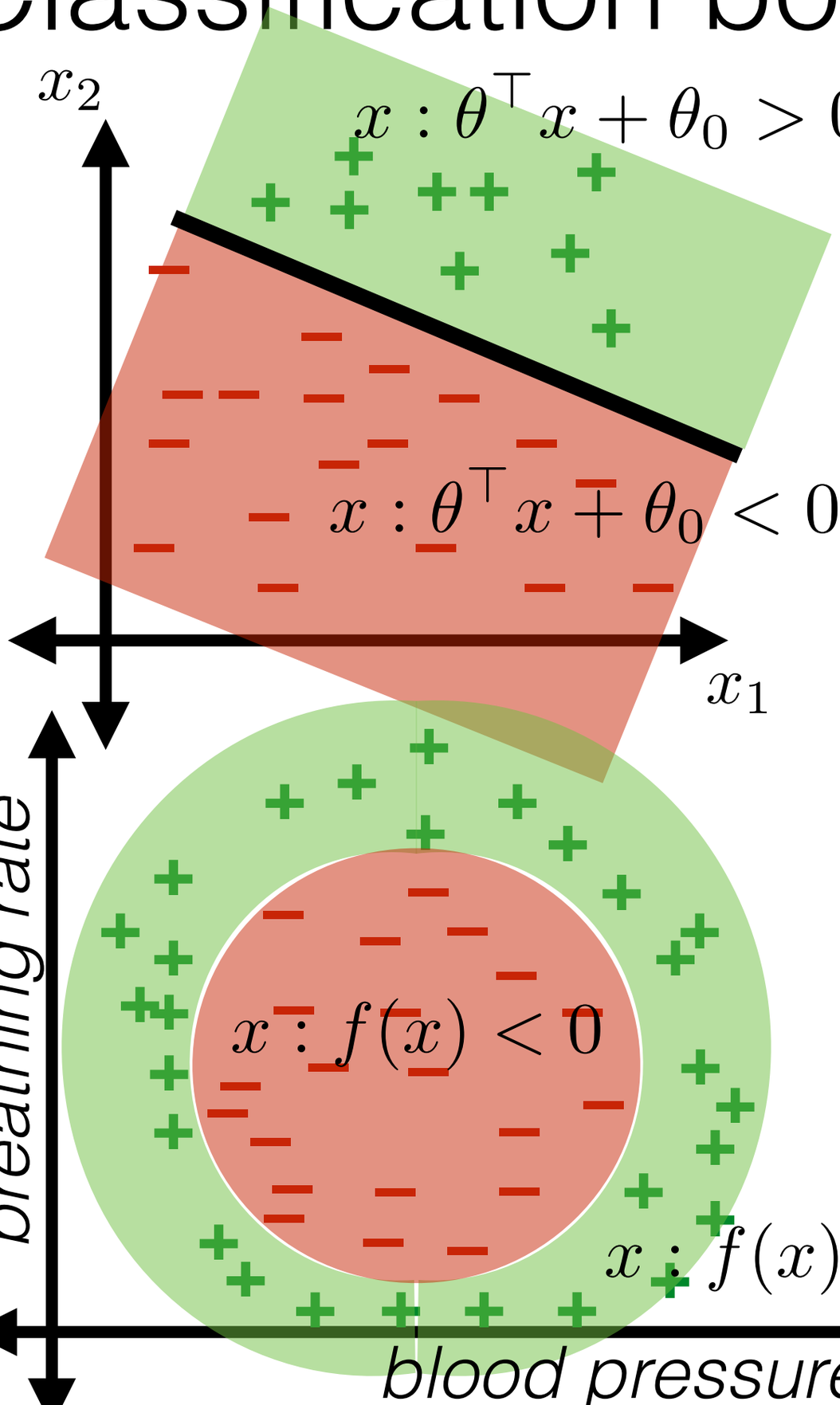


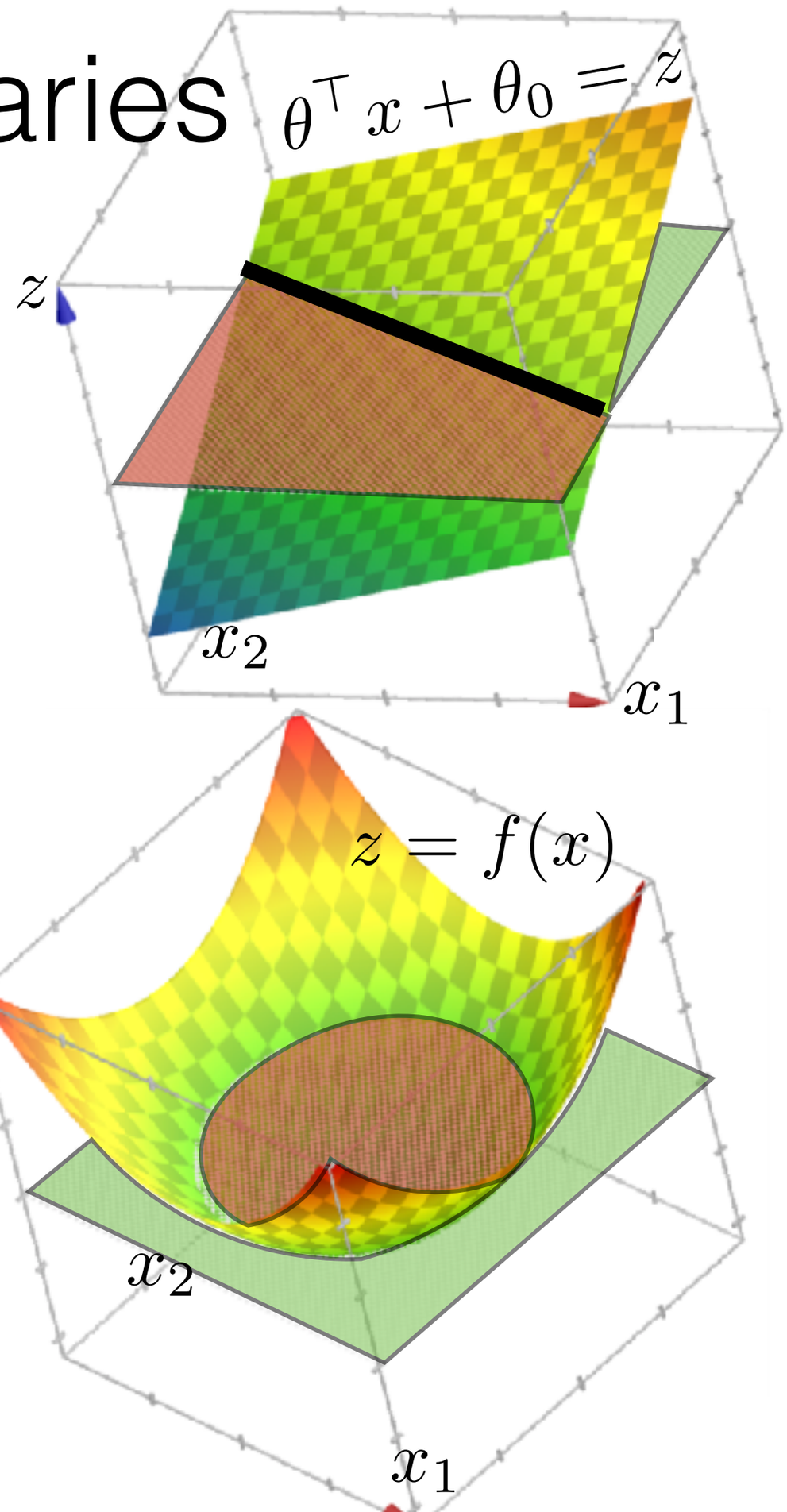

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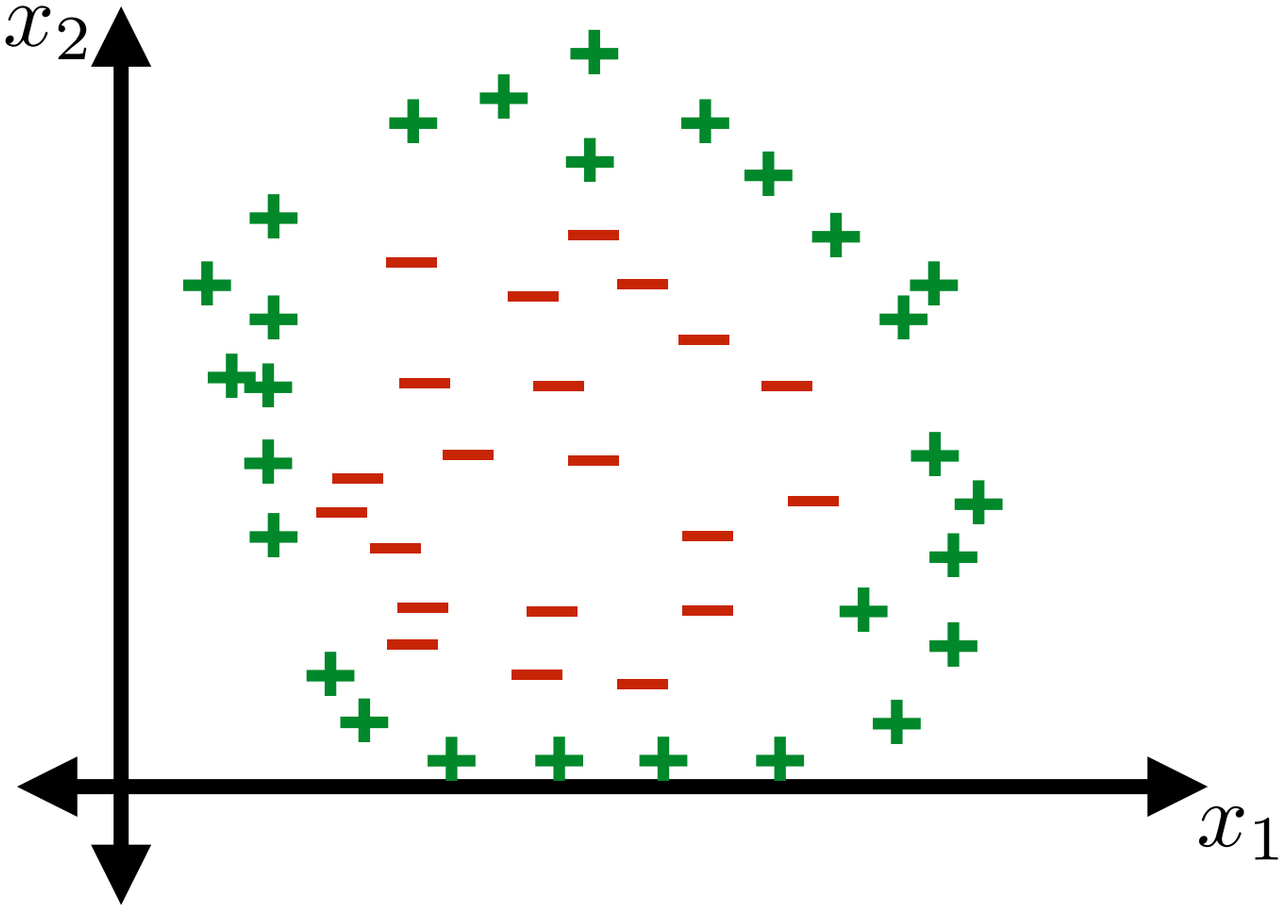

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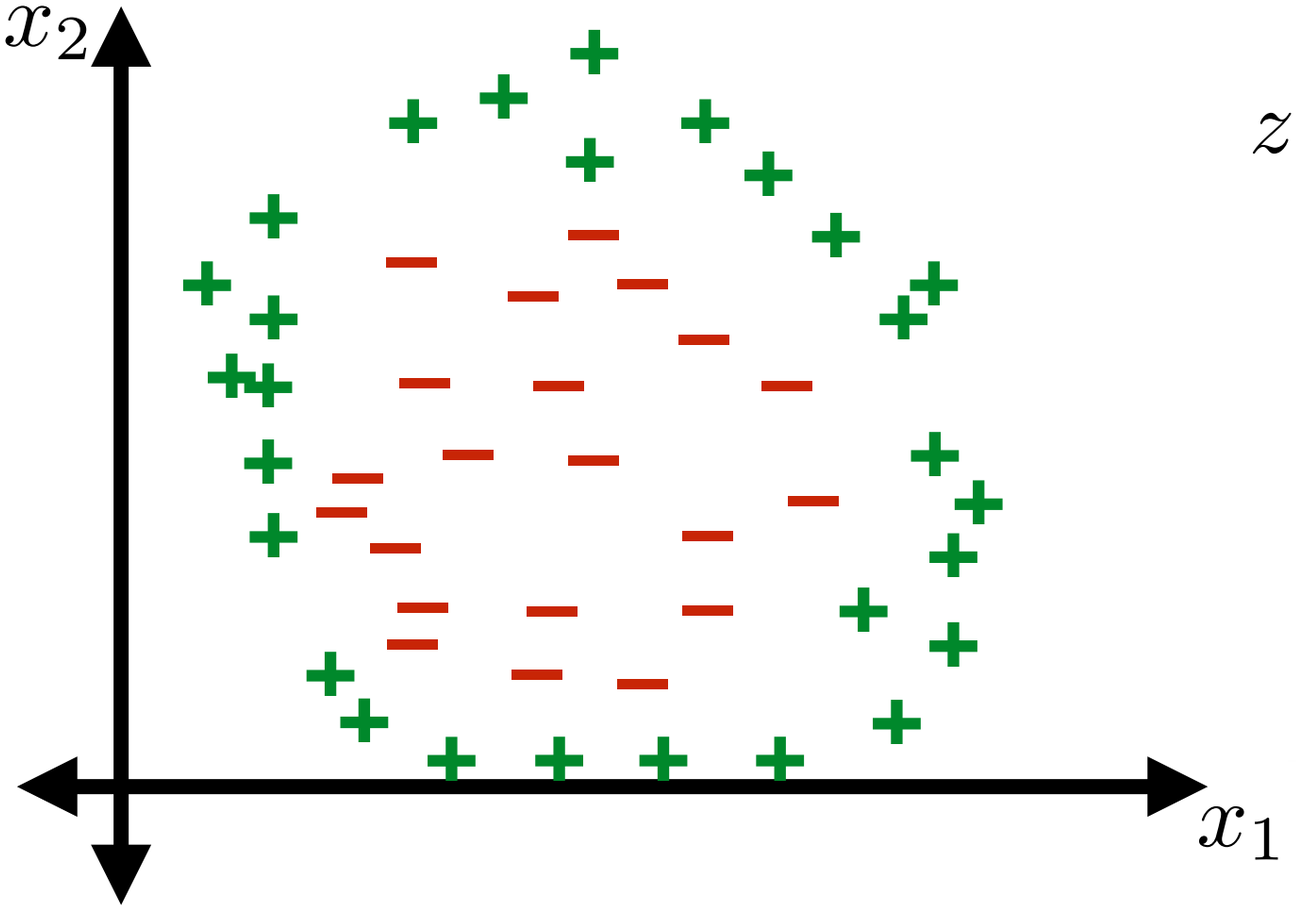


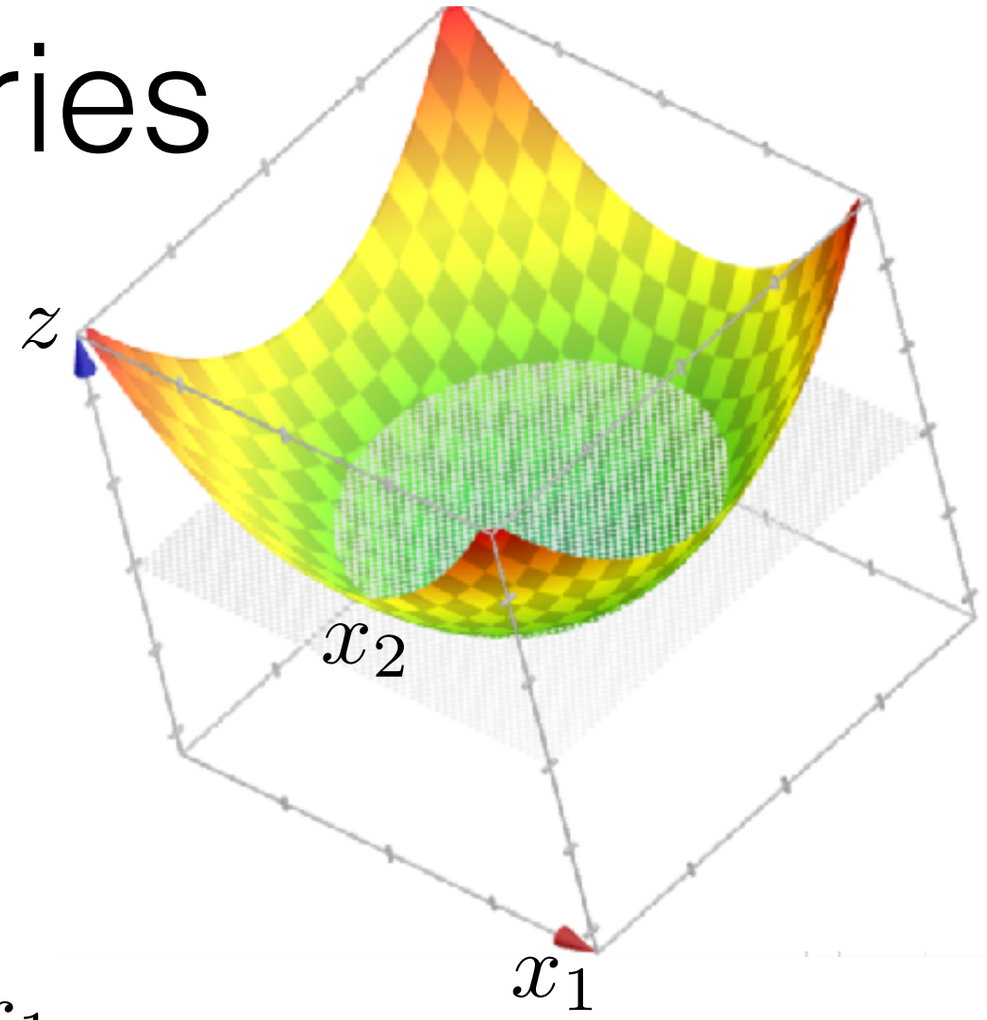

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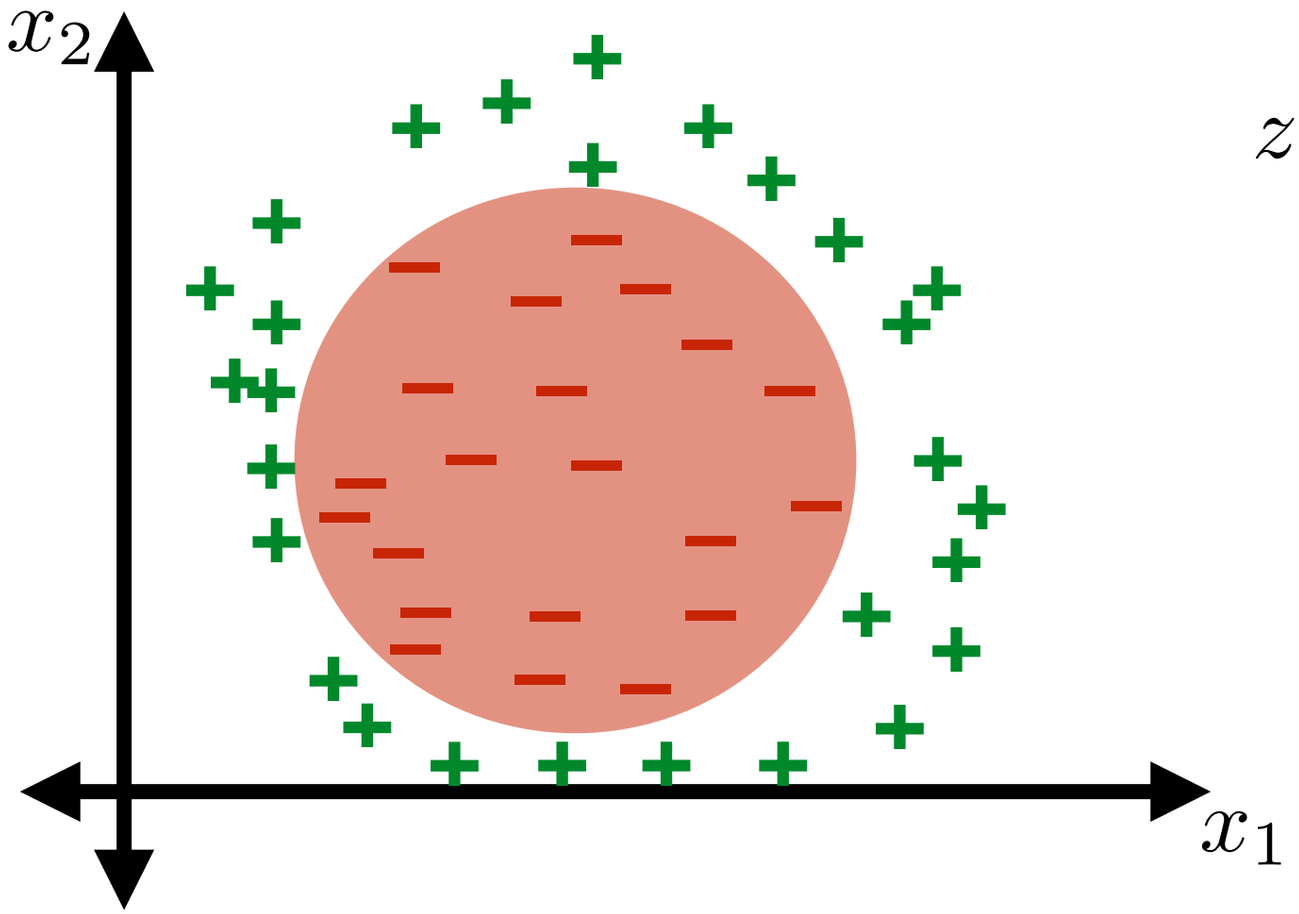


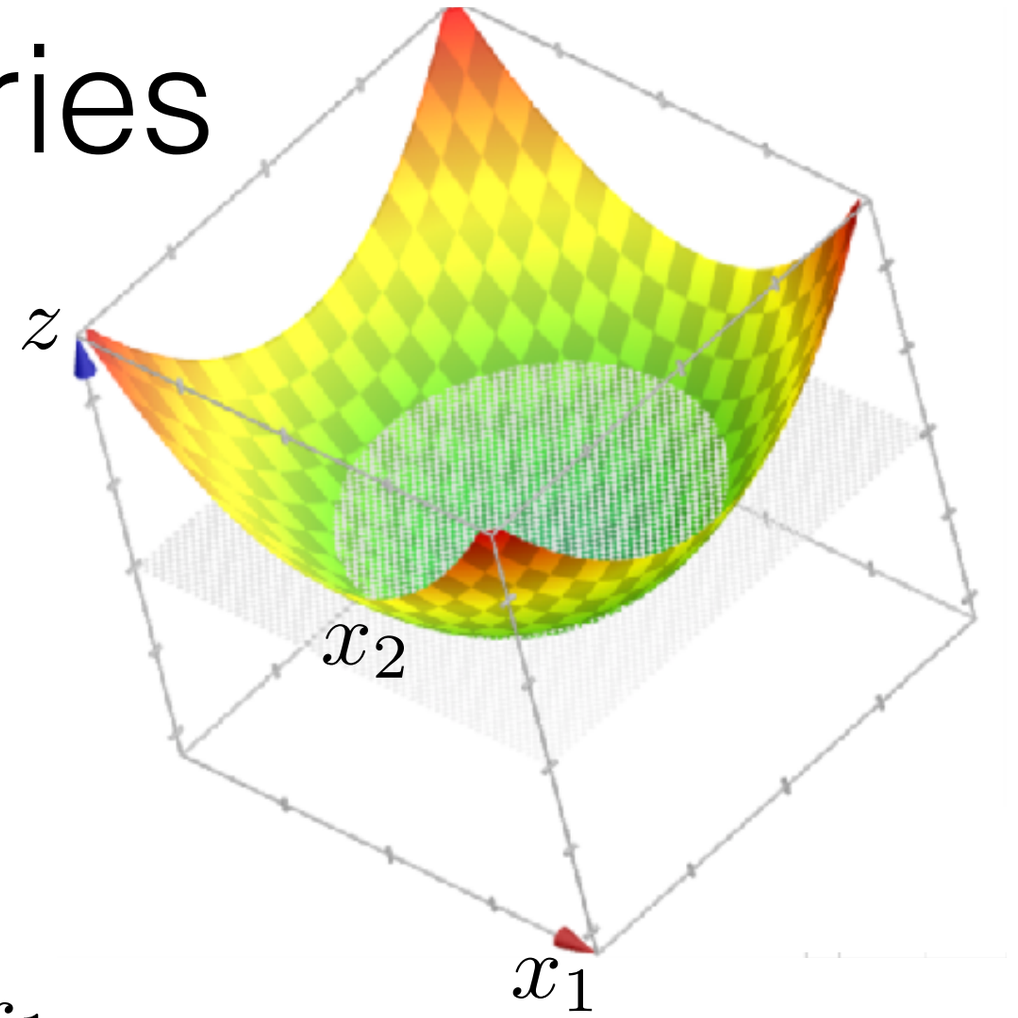

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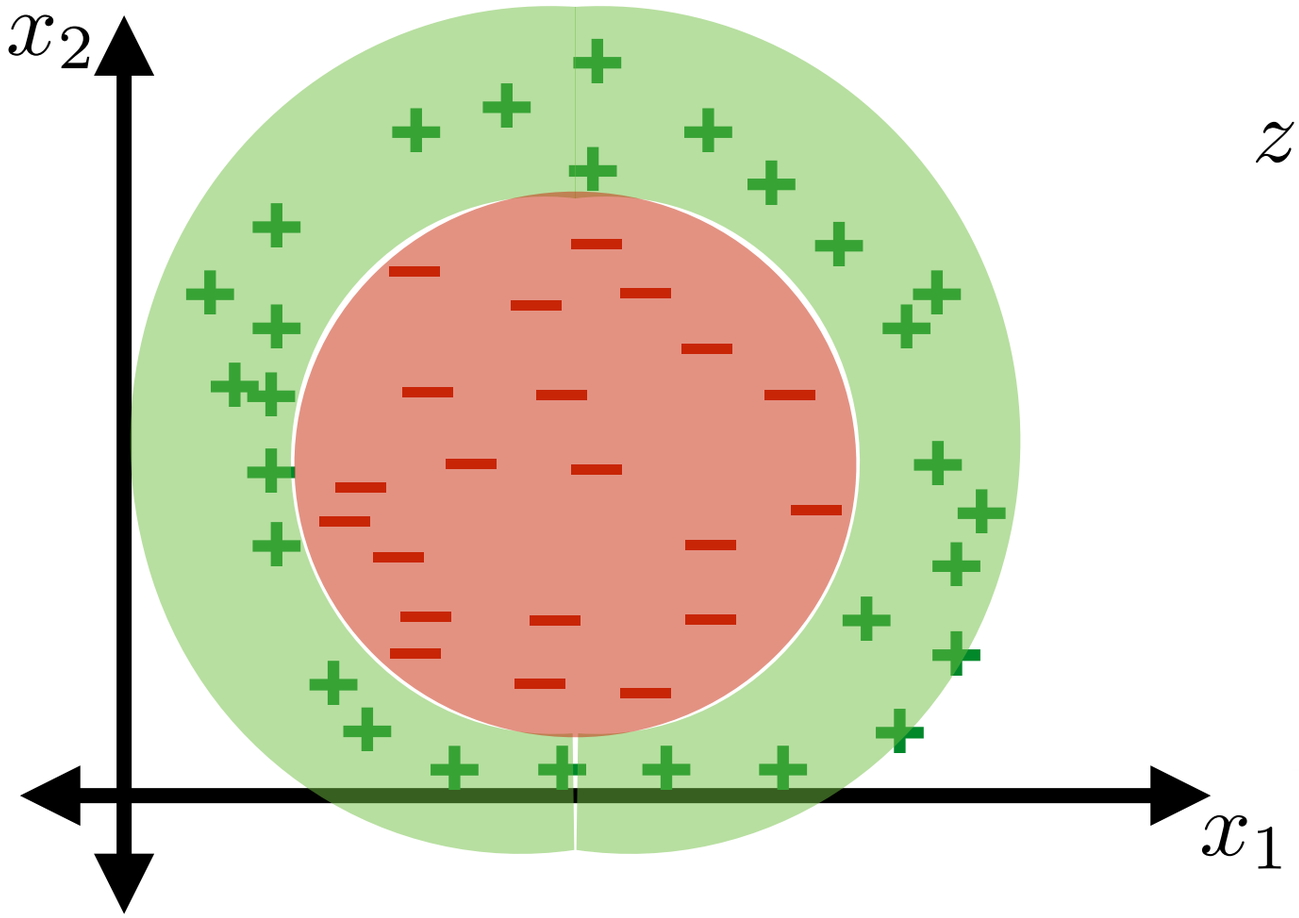


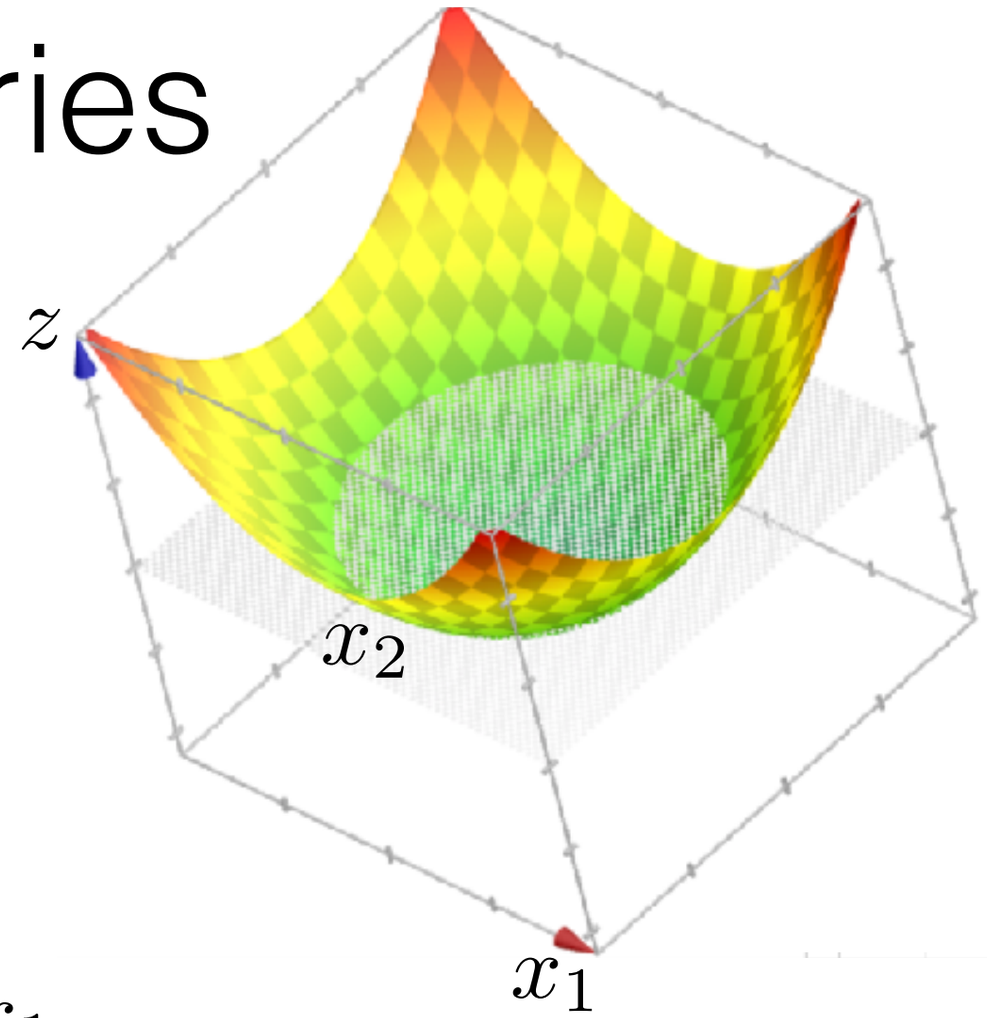

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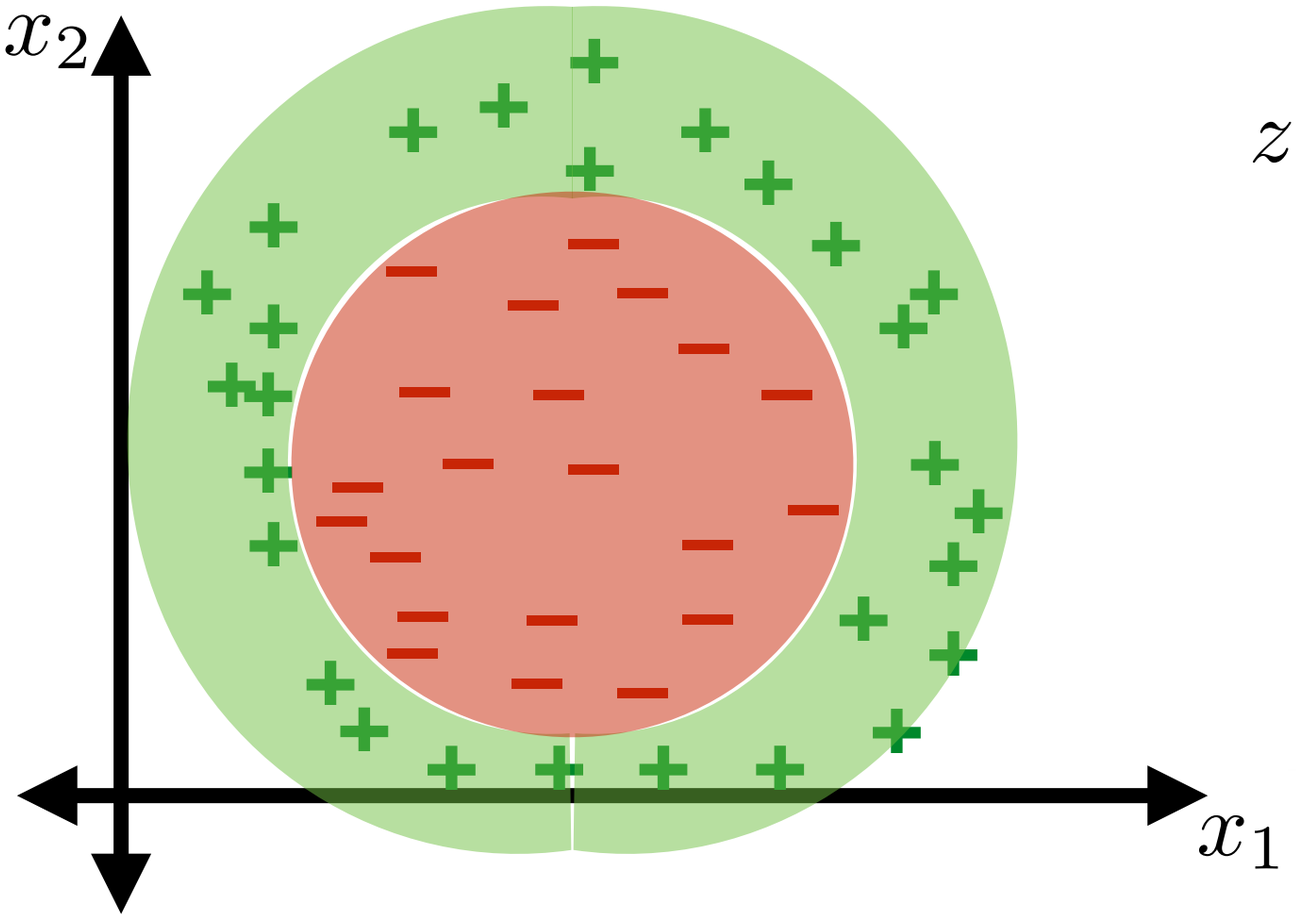


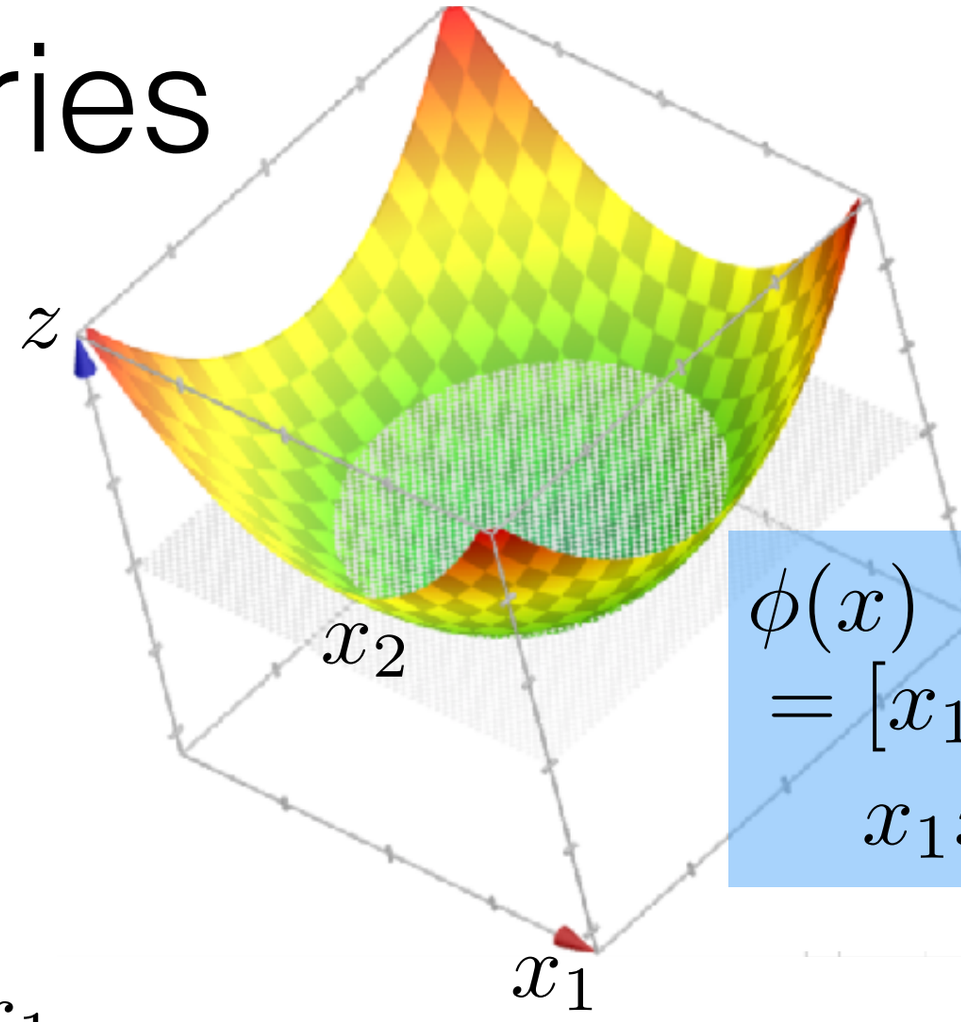


$$\phi(x) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2]^\top$$

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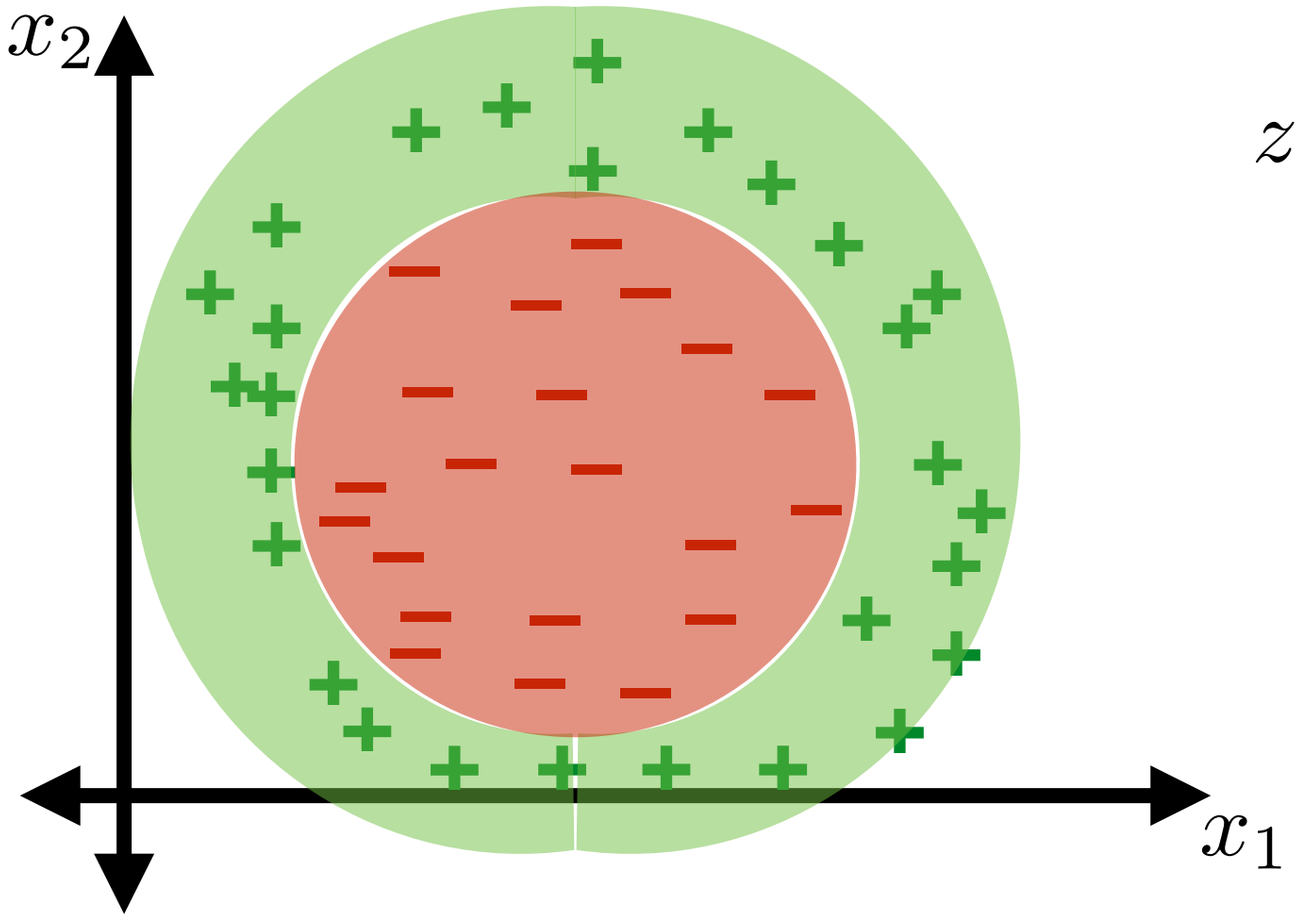
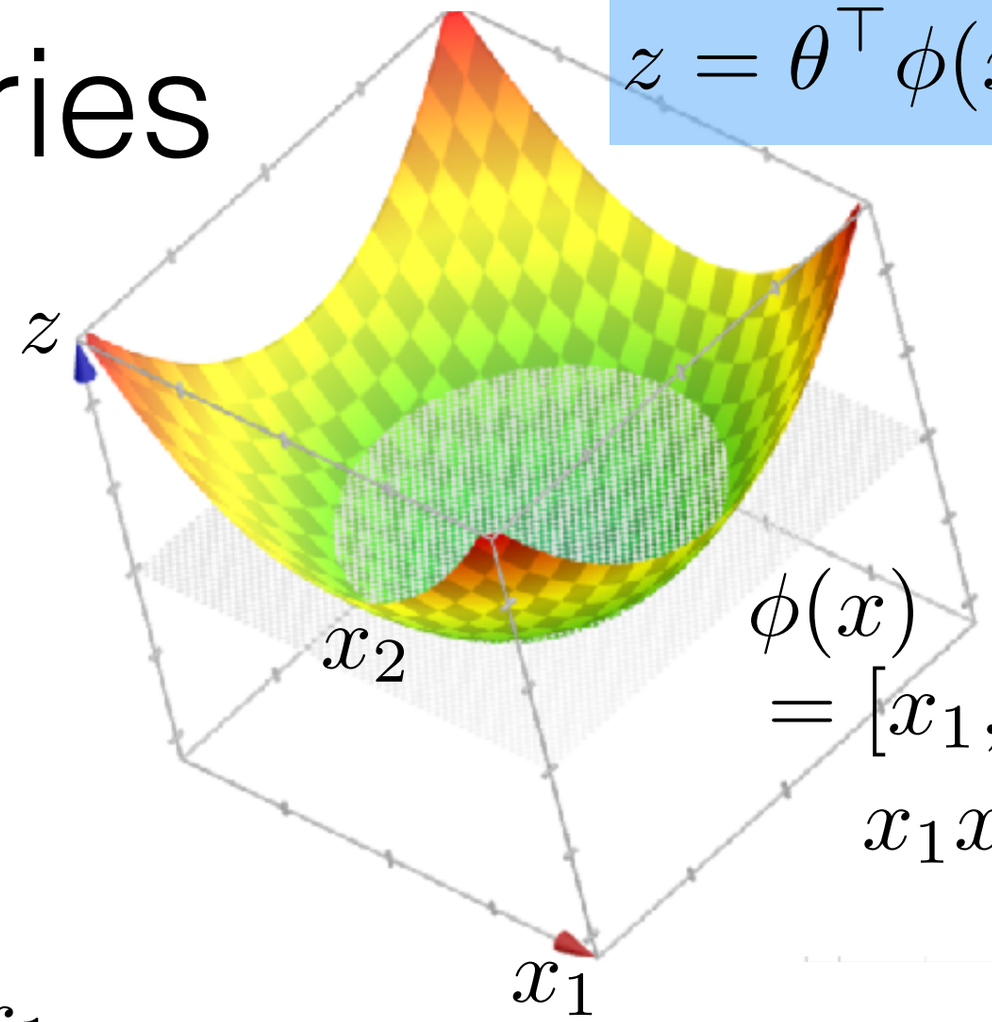



$$z = \theta^\top \phi(x) + \theta_0$$

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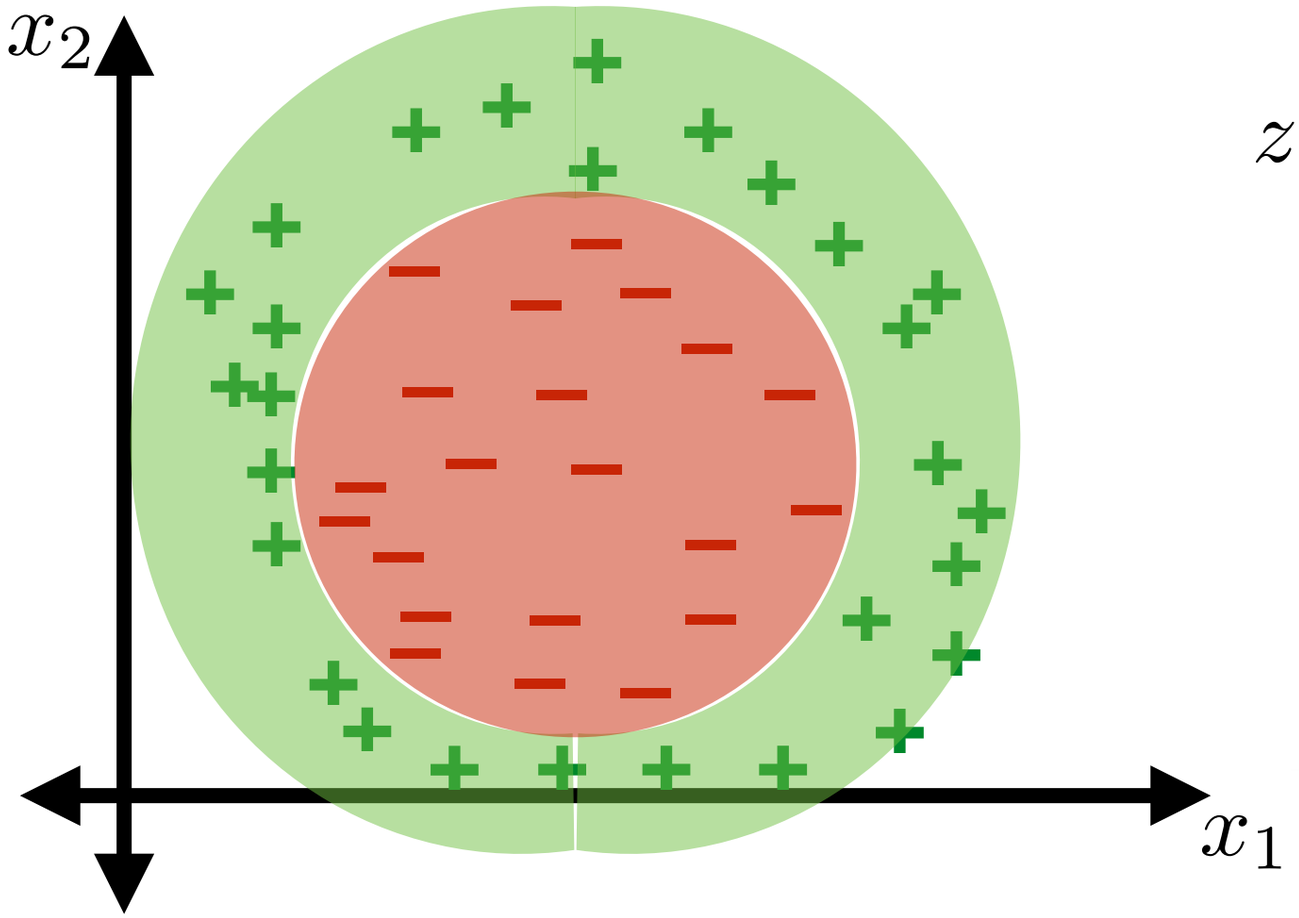


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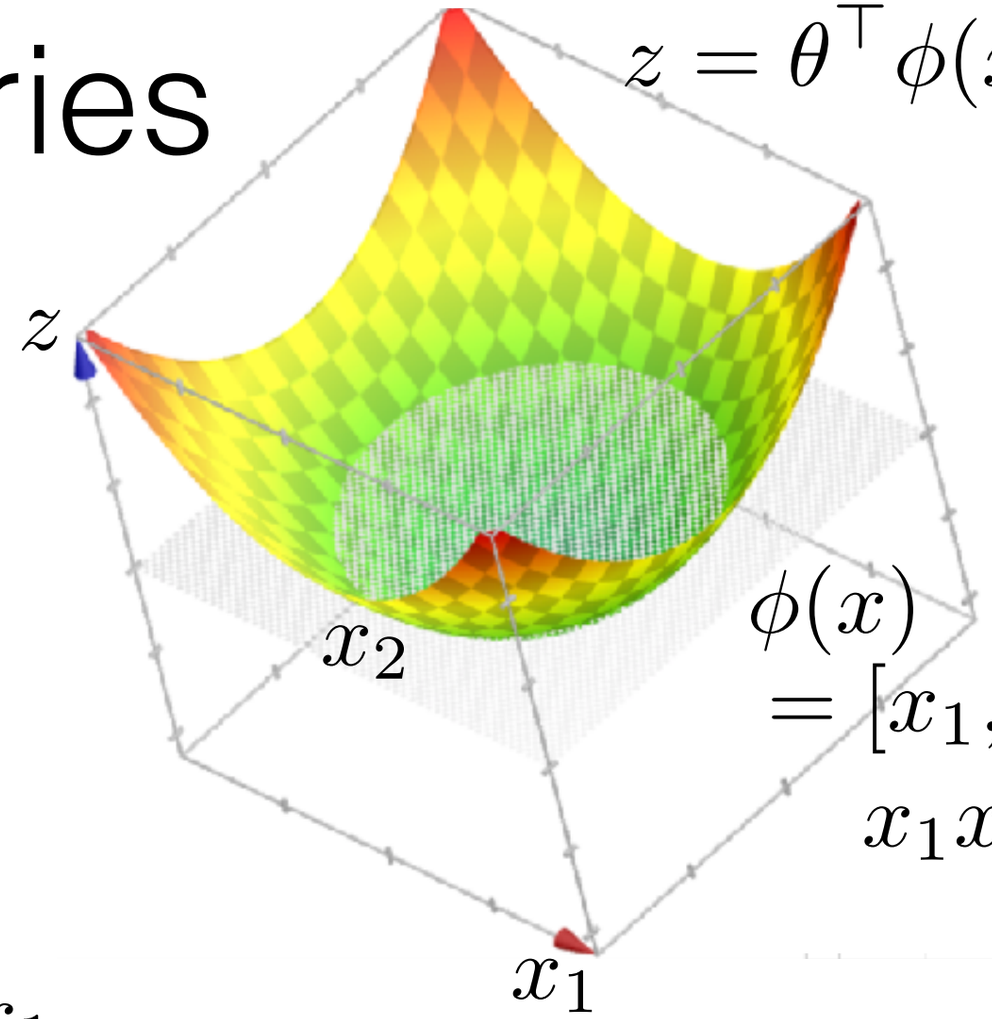


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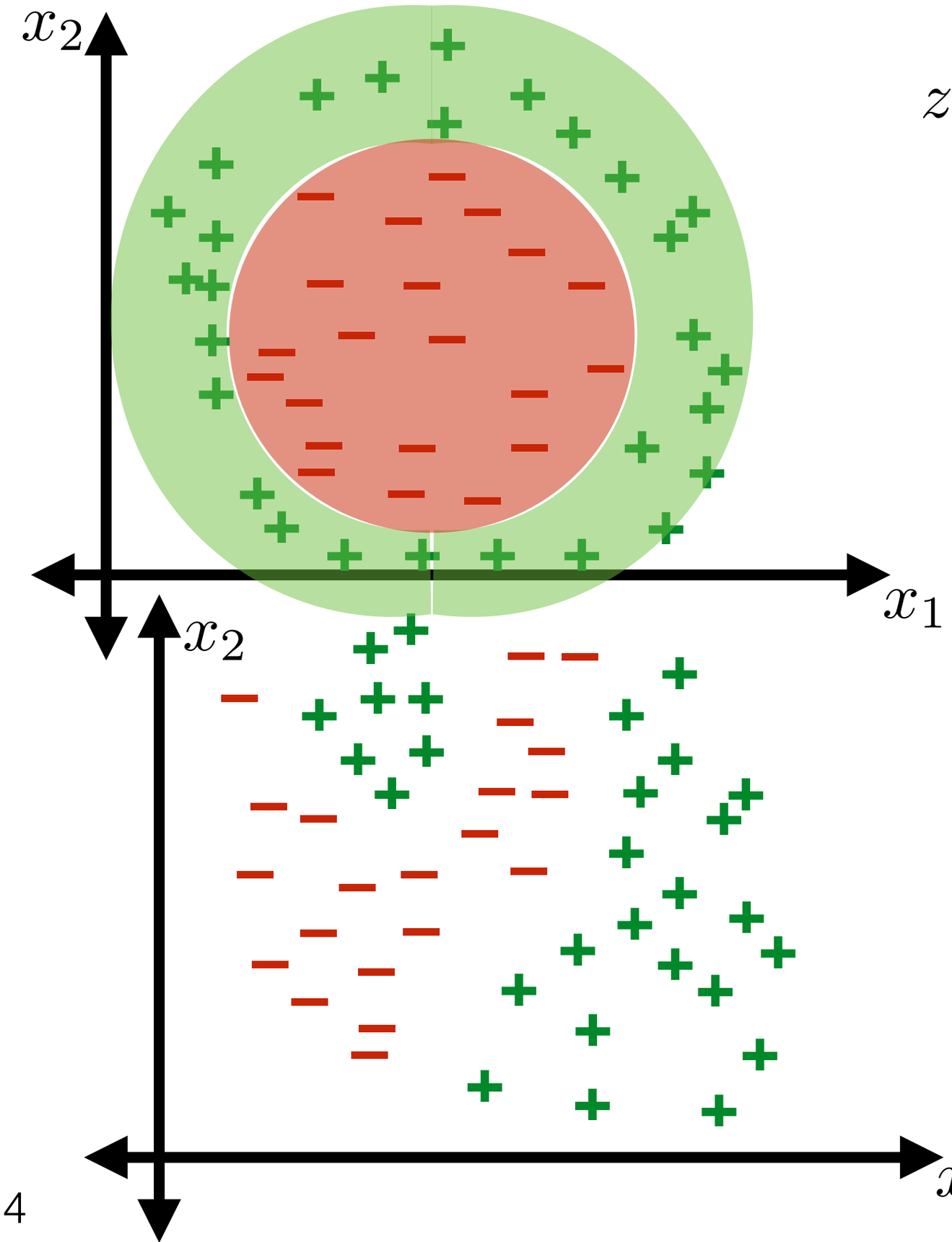


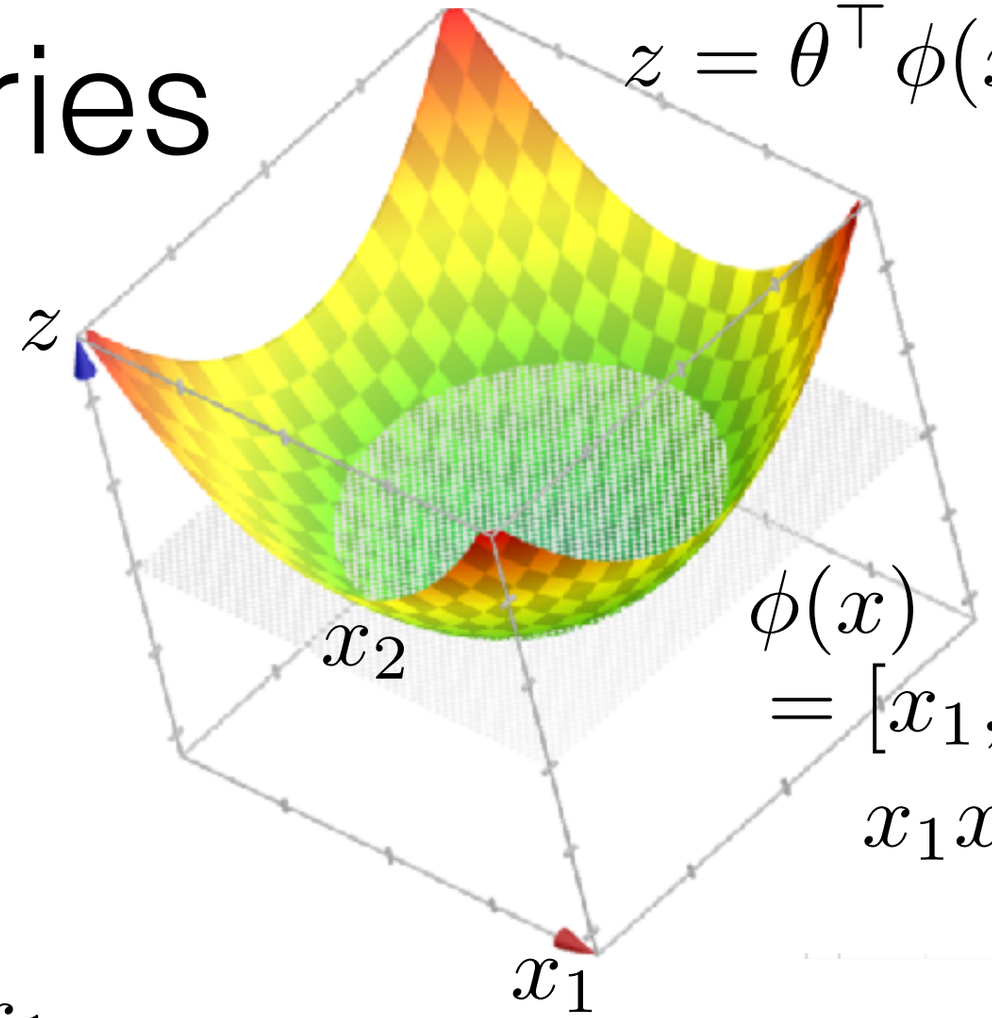


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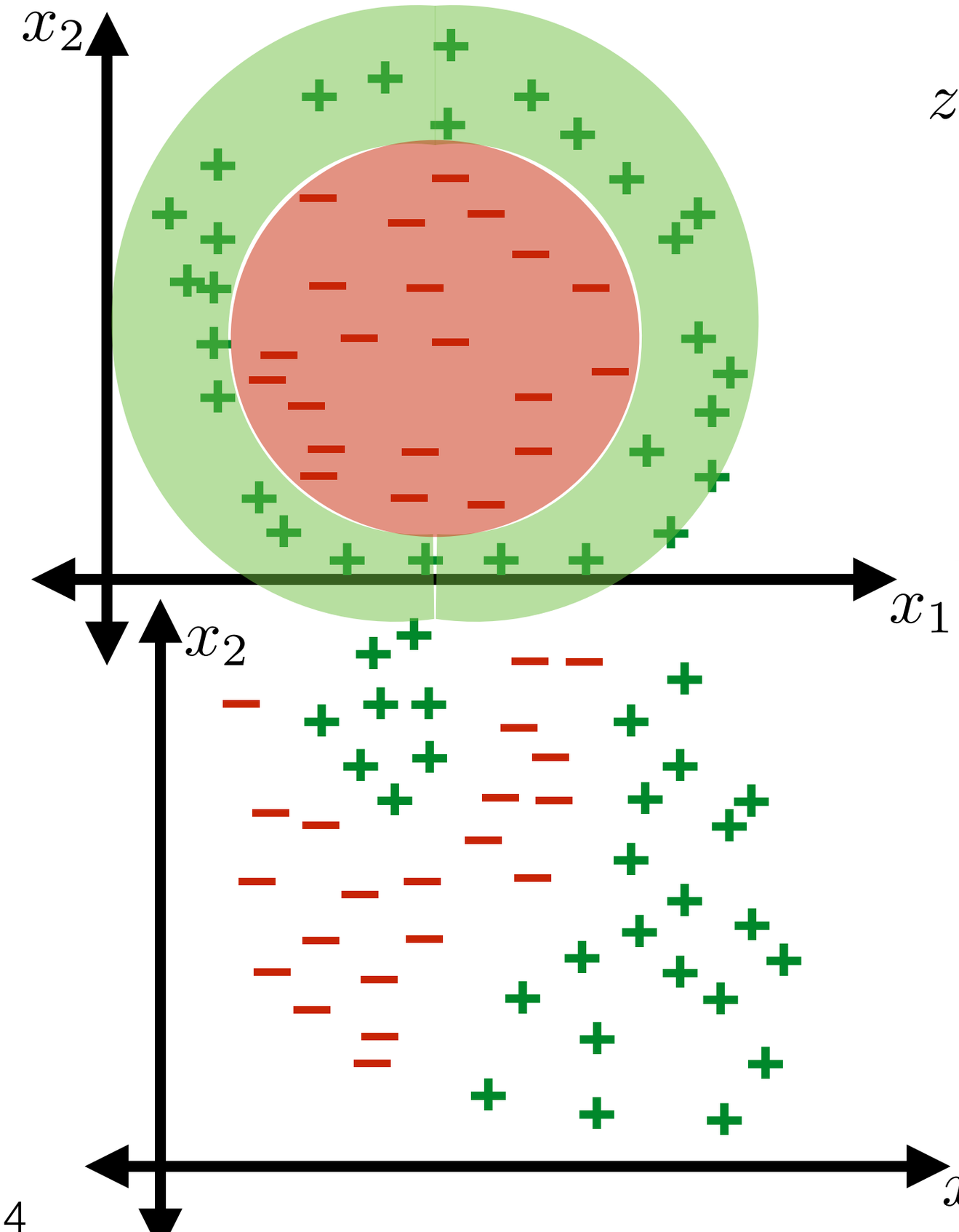


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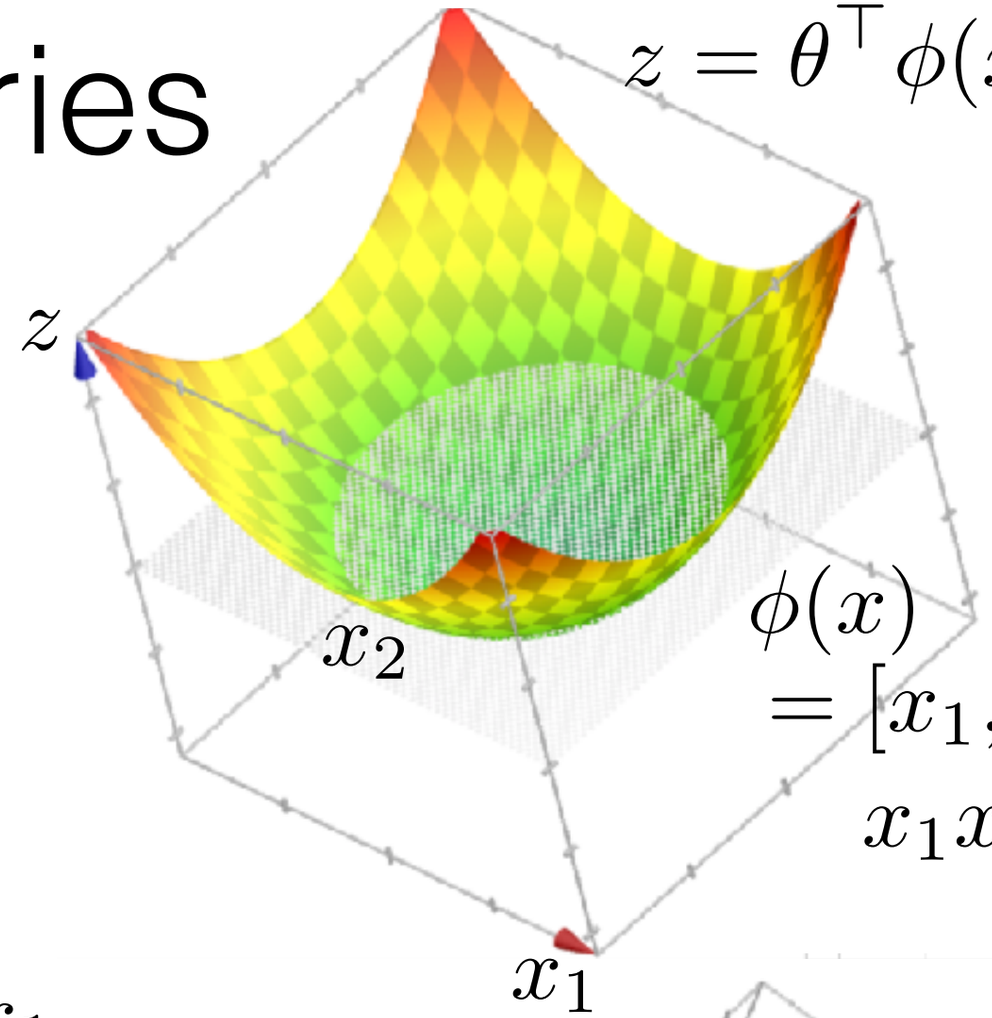


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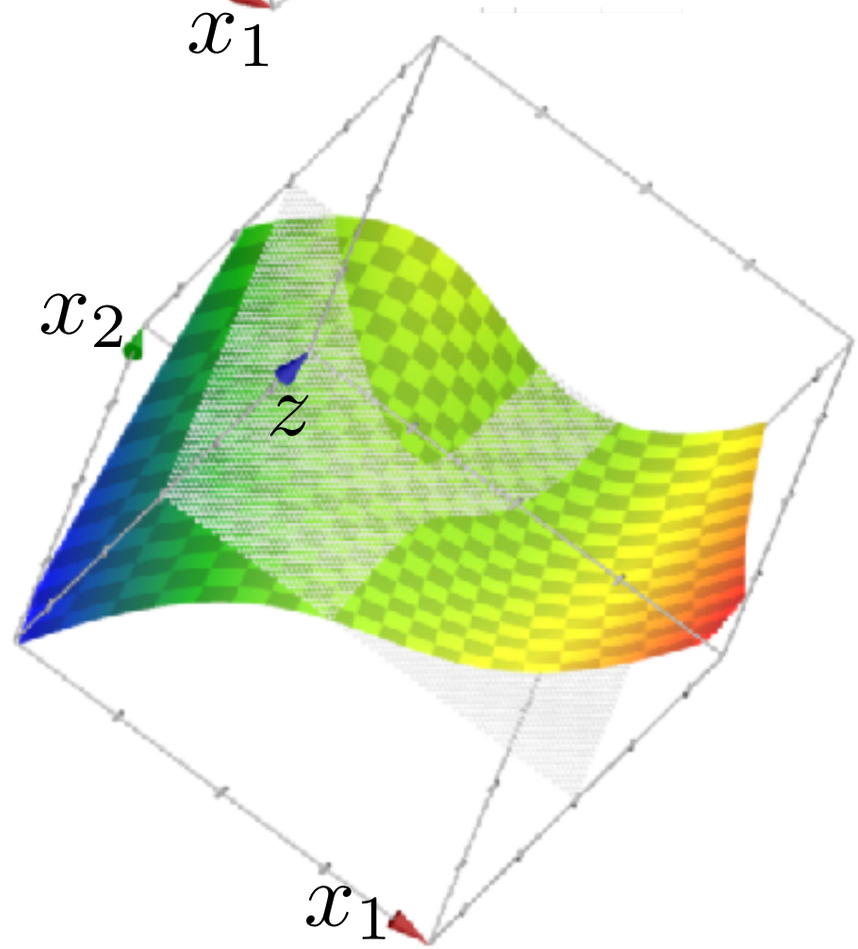

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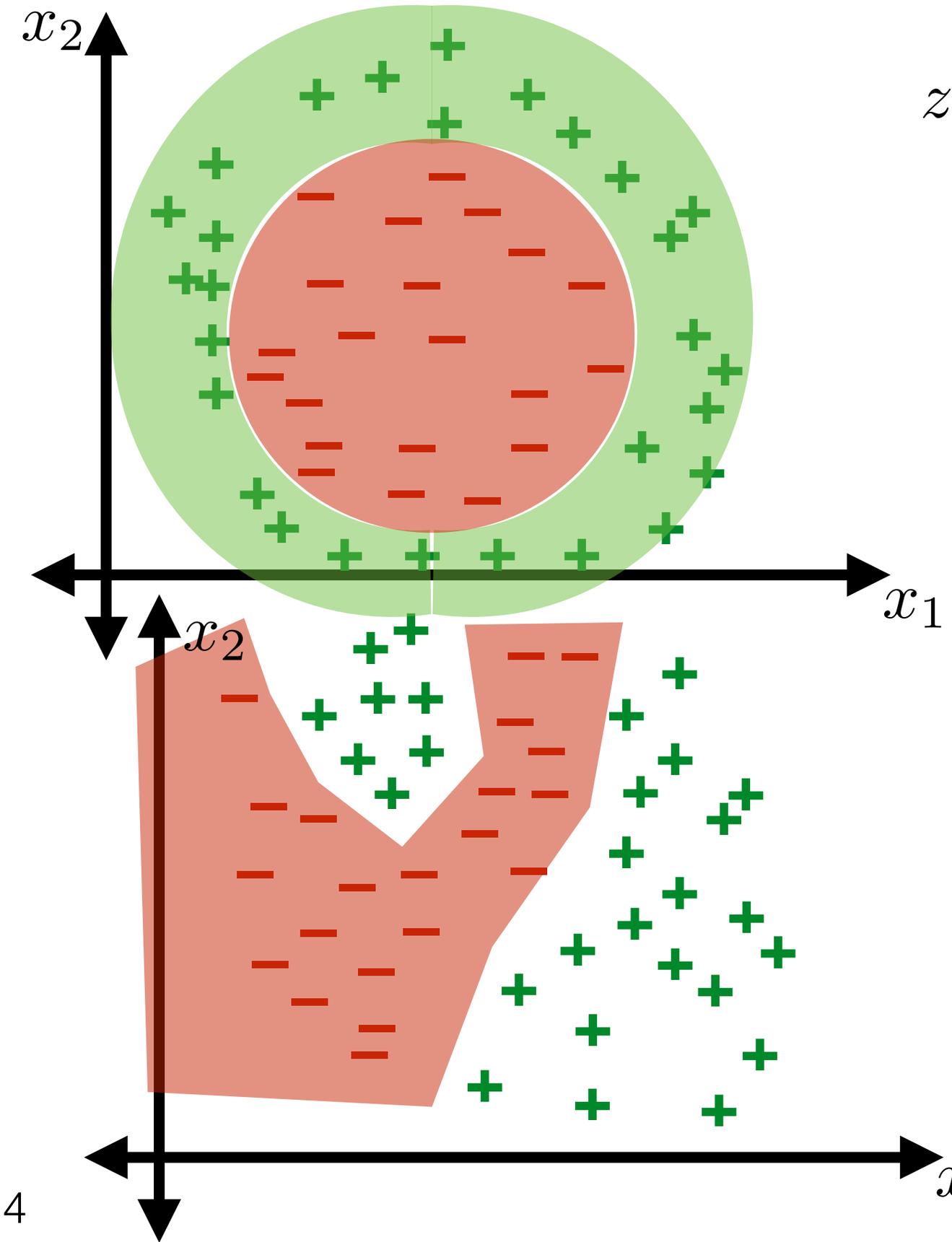


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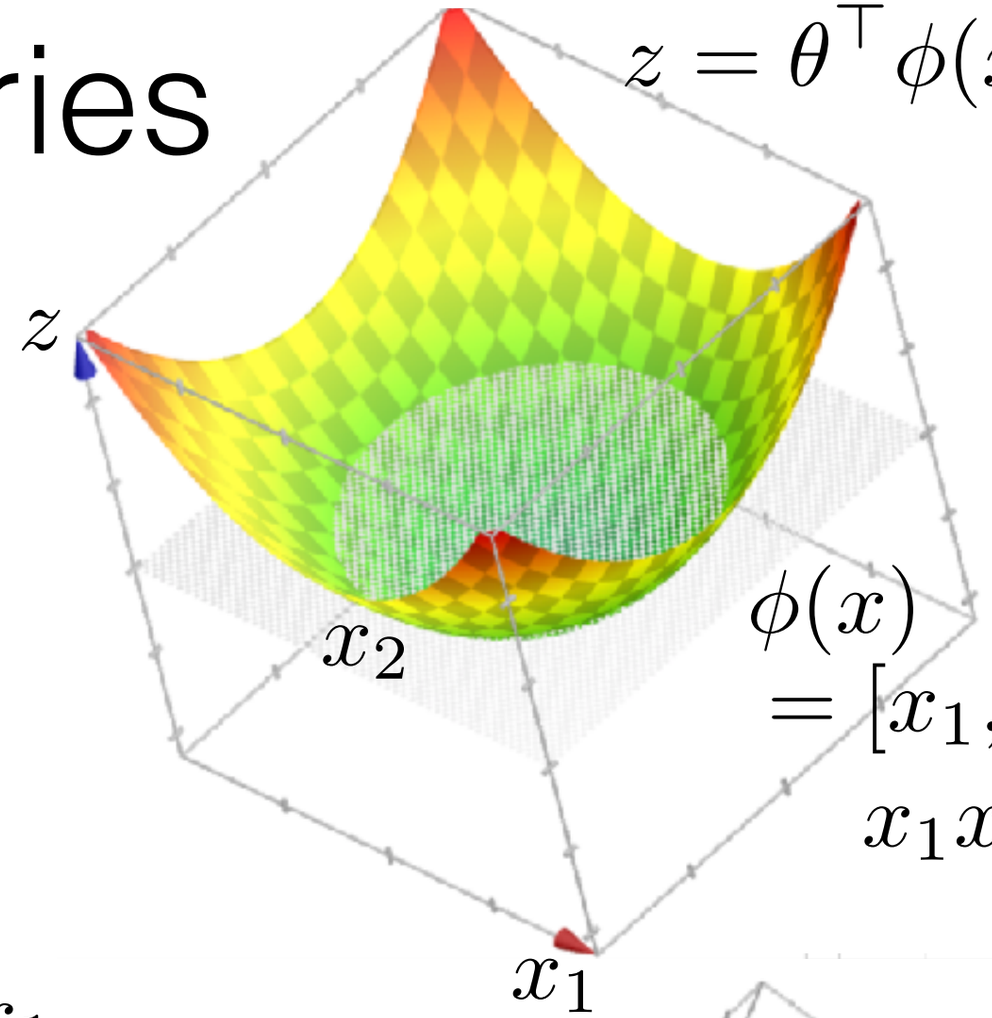


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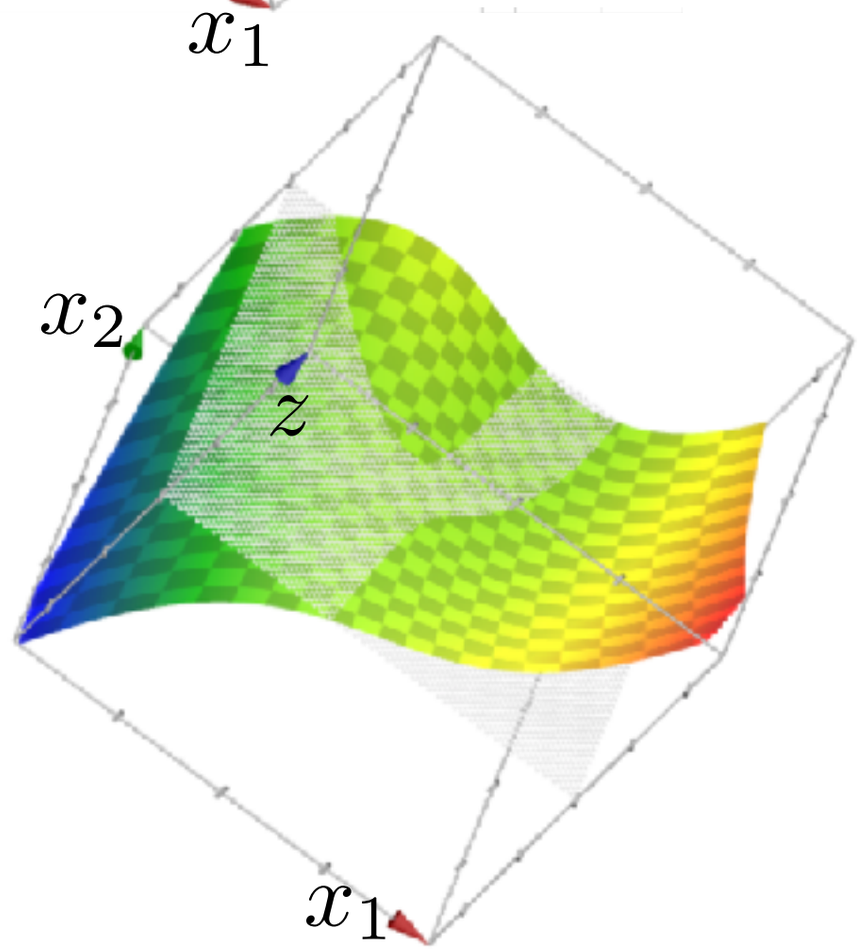

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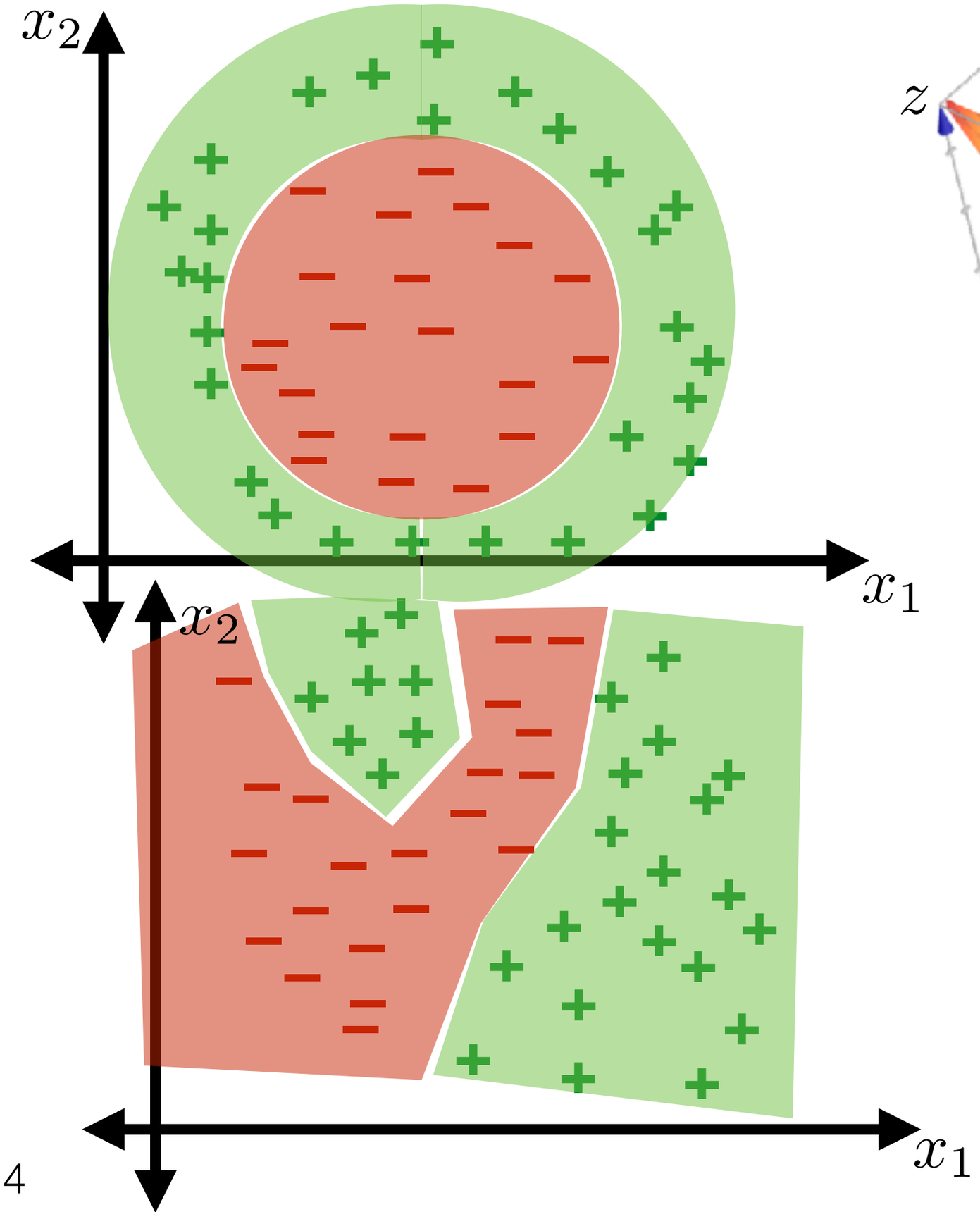


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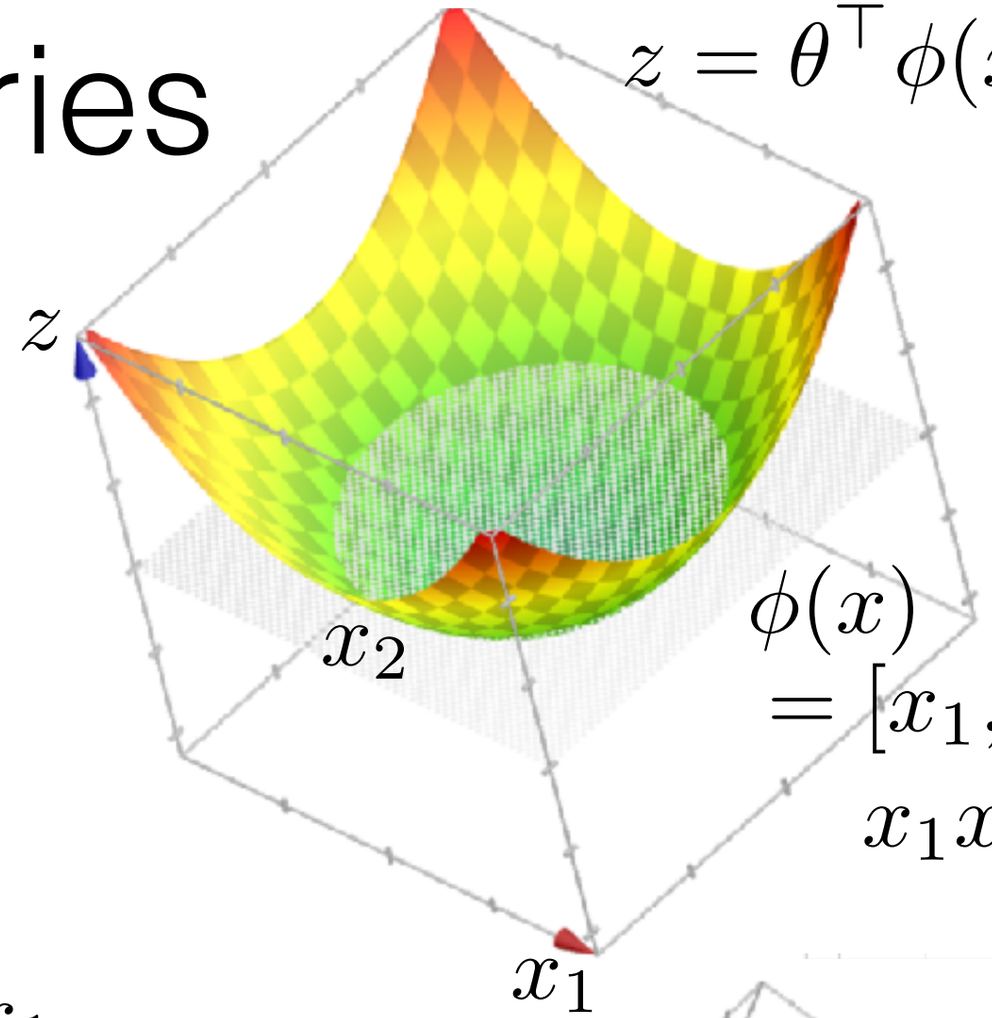


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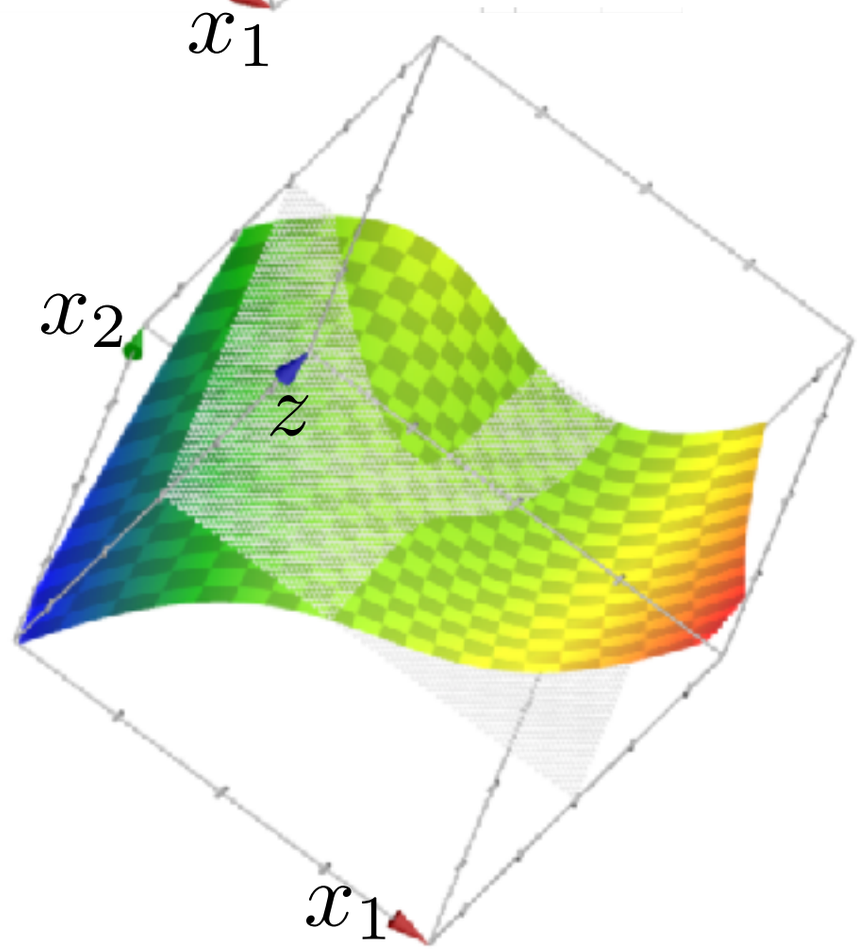

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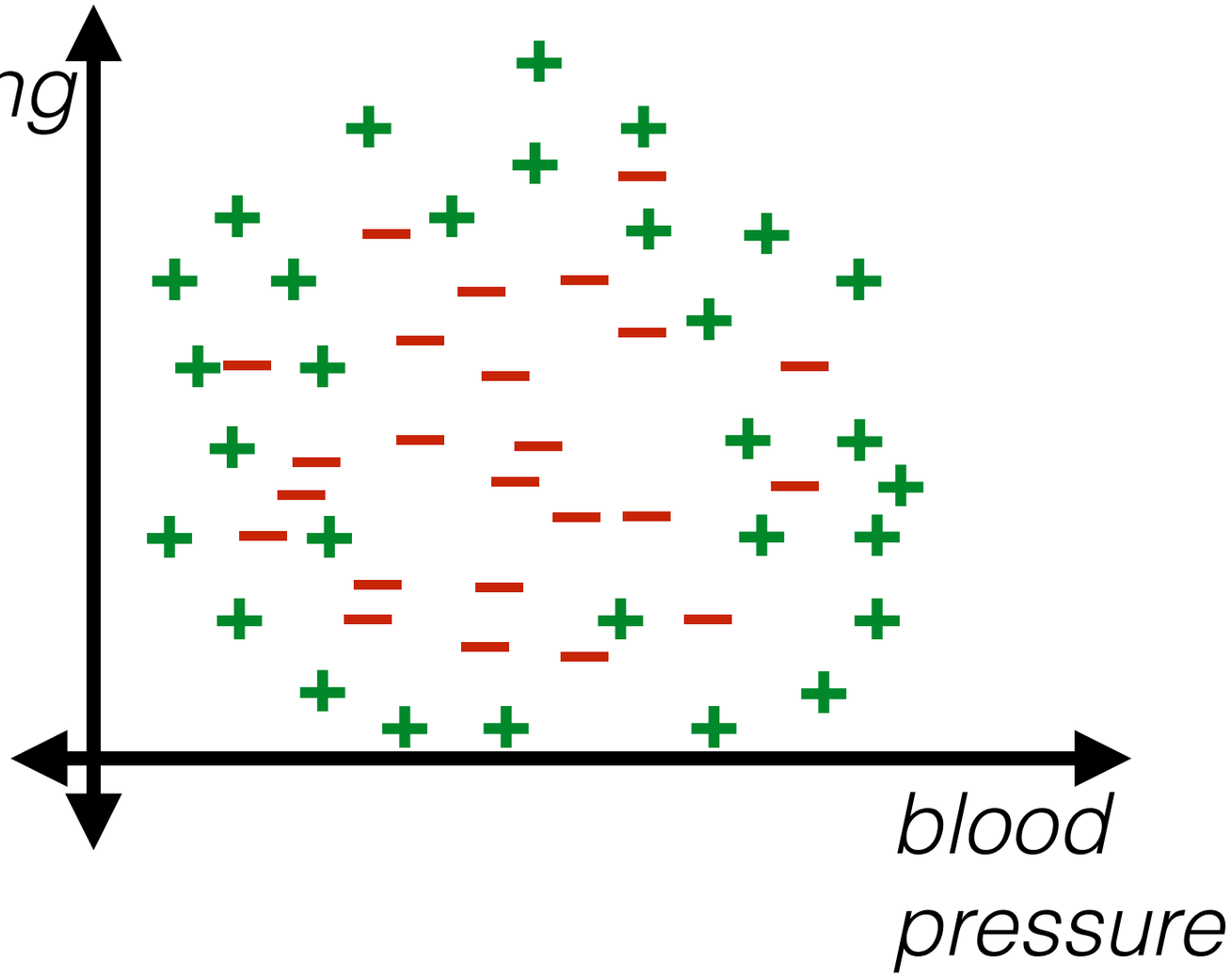

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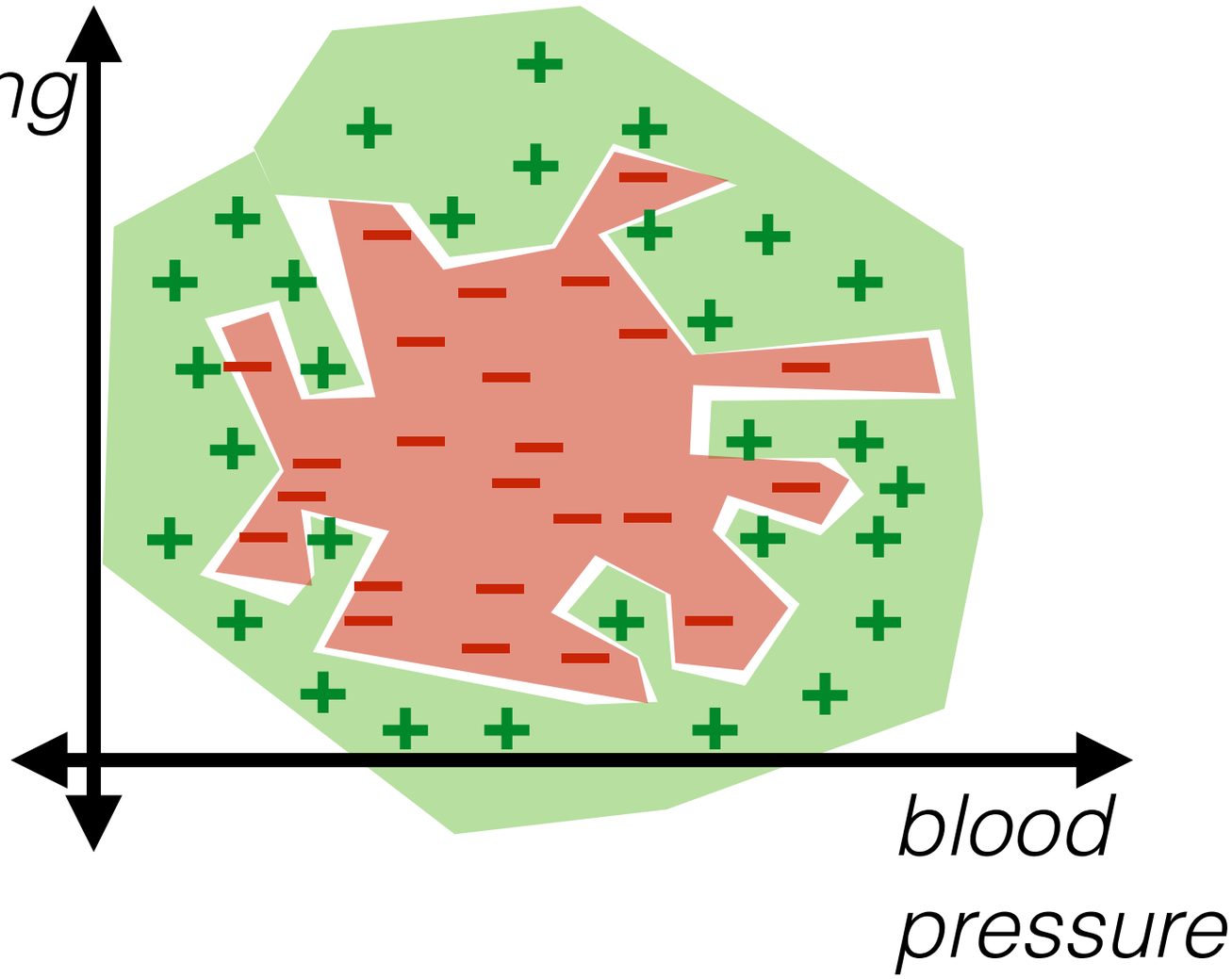

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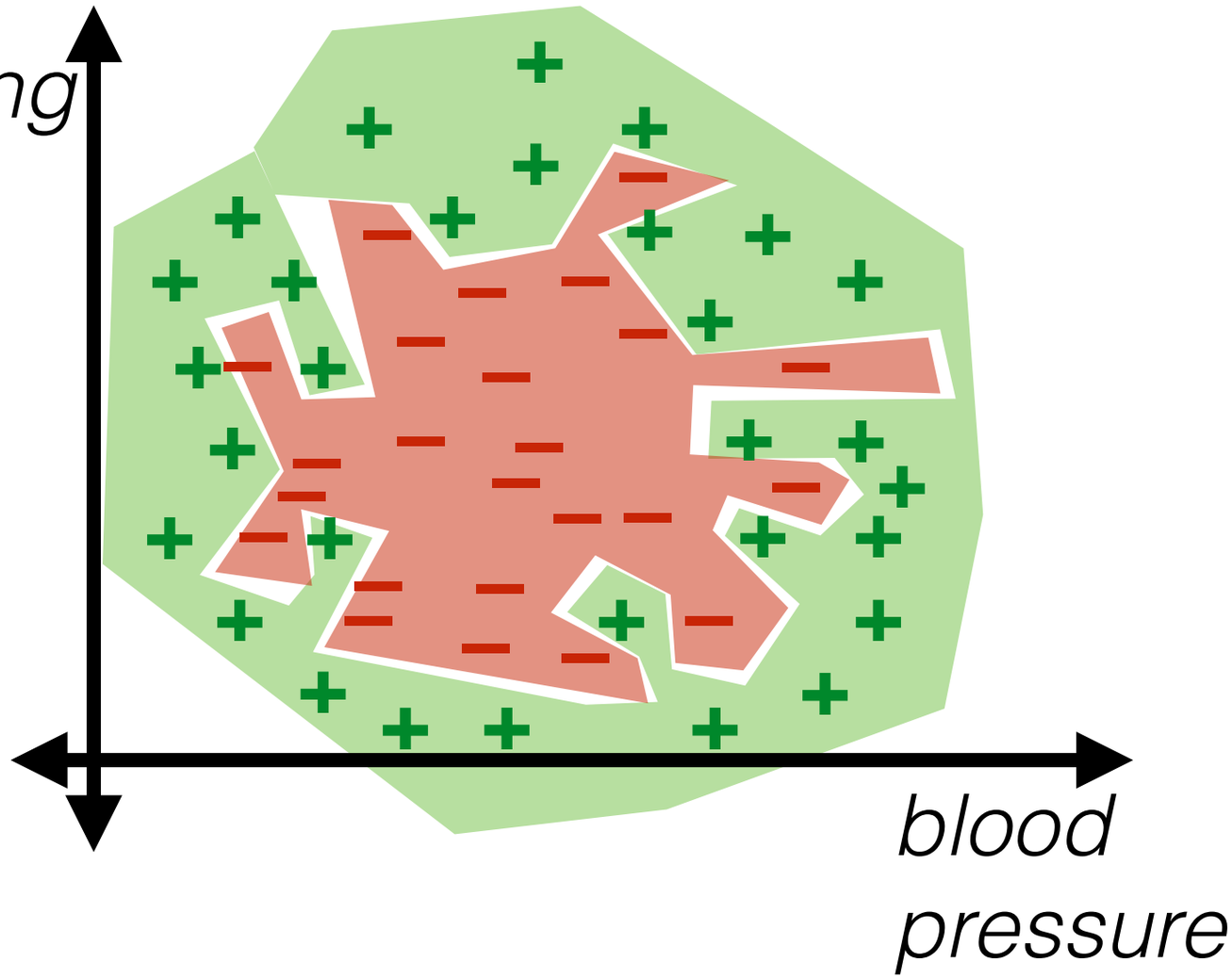


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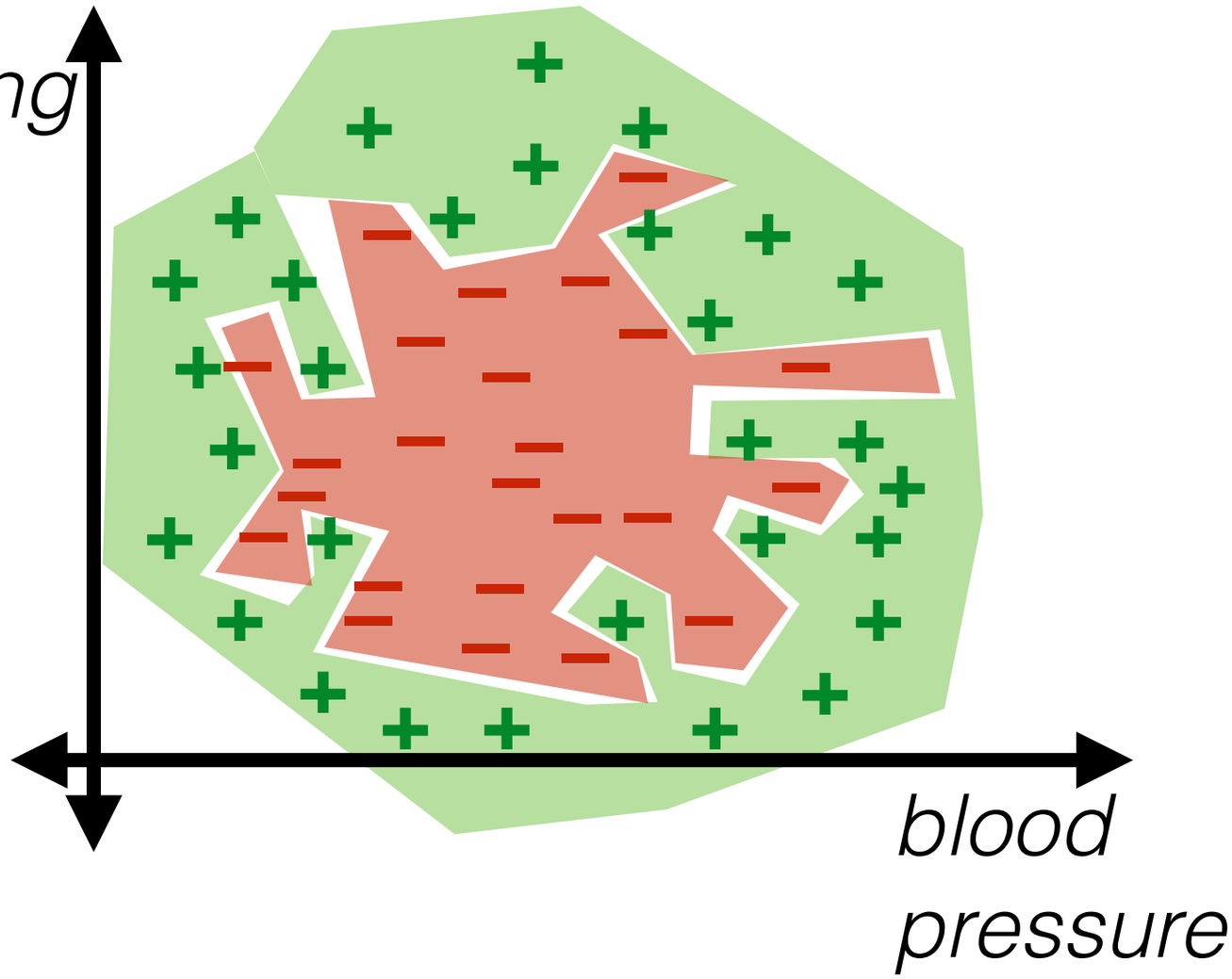


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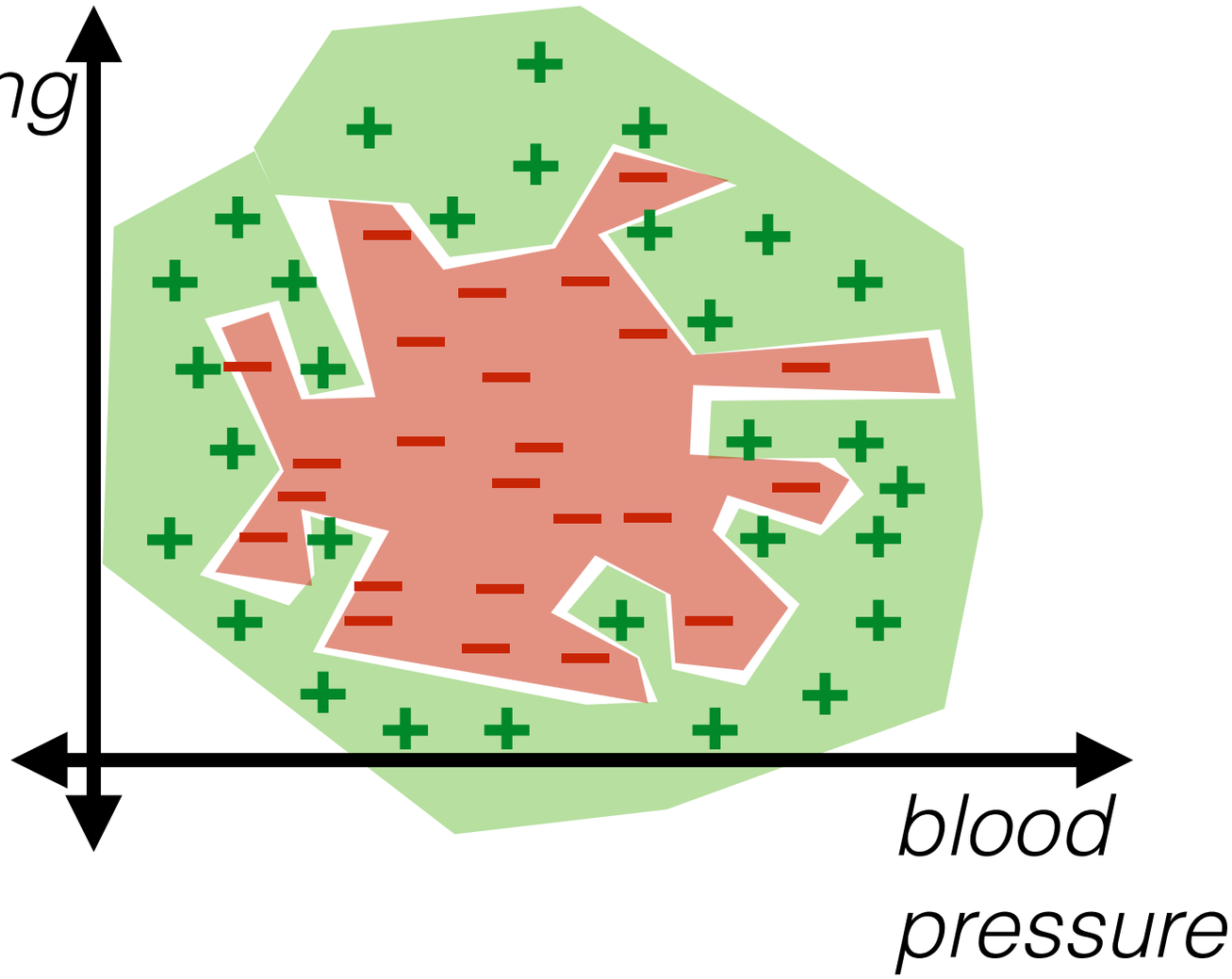


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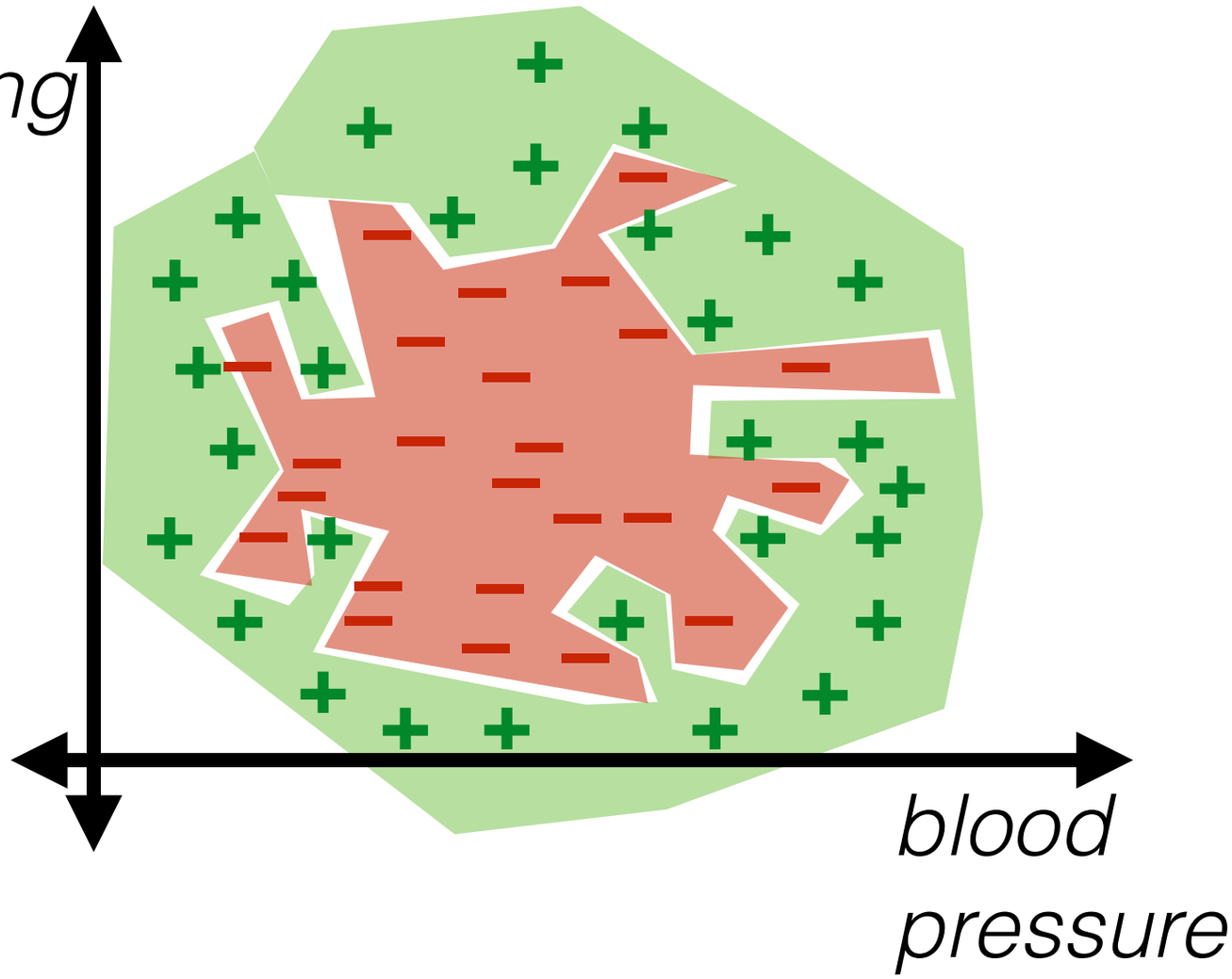


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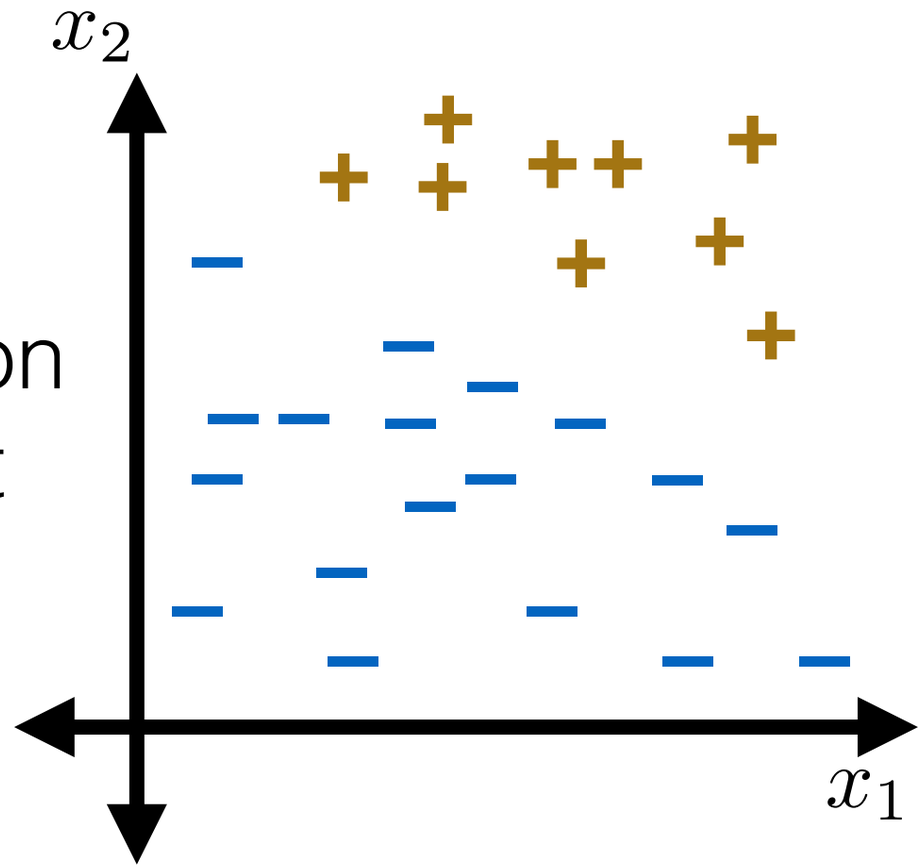

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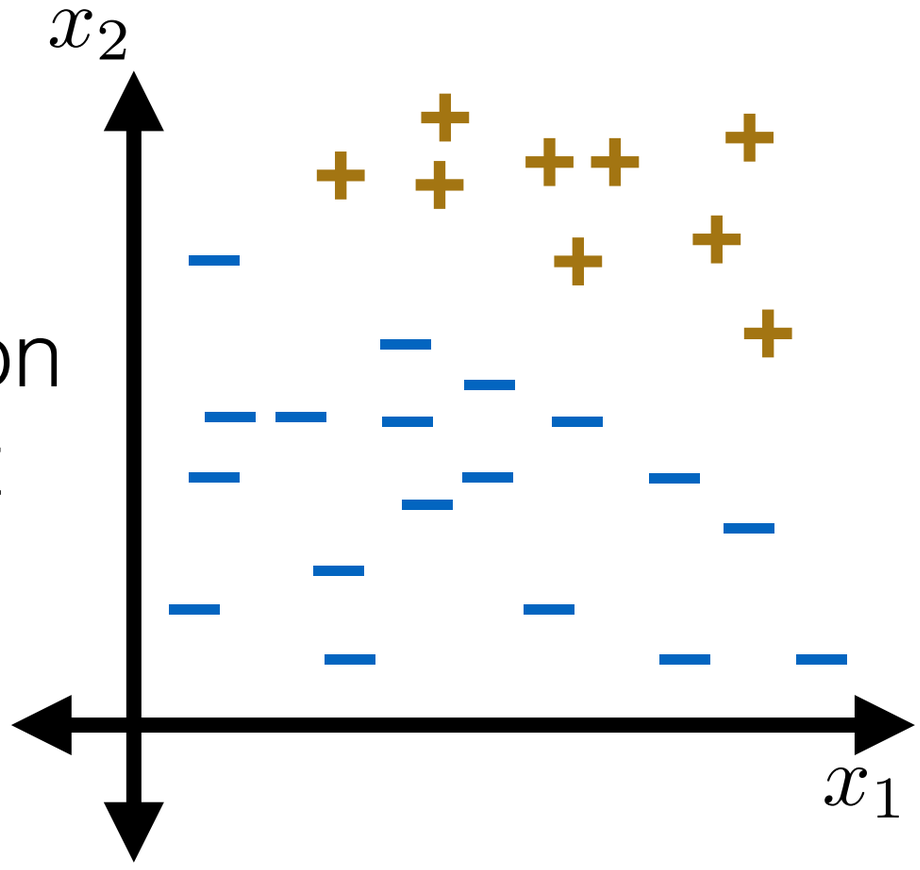


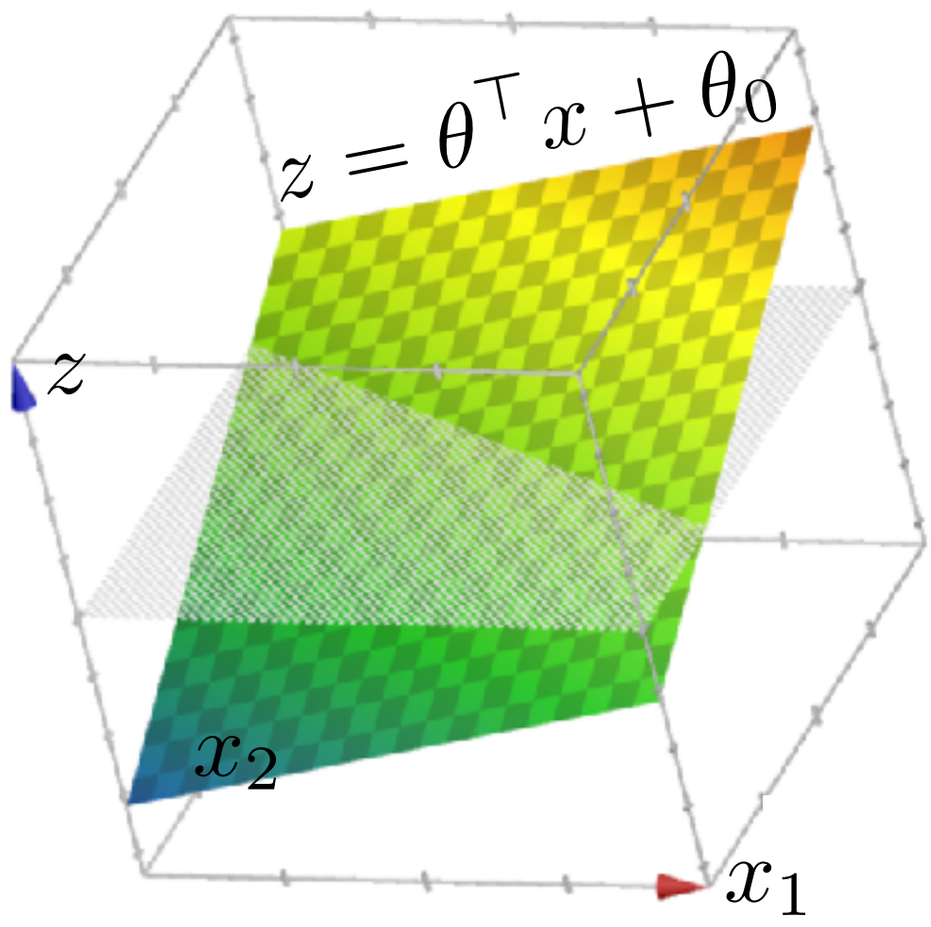

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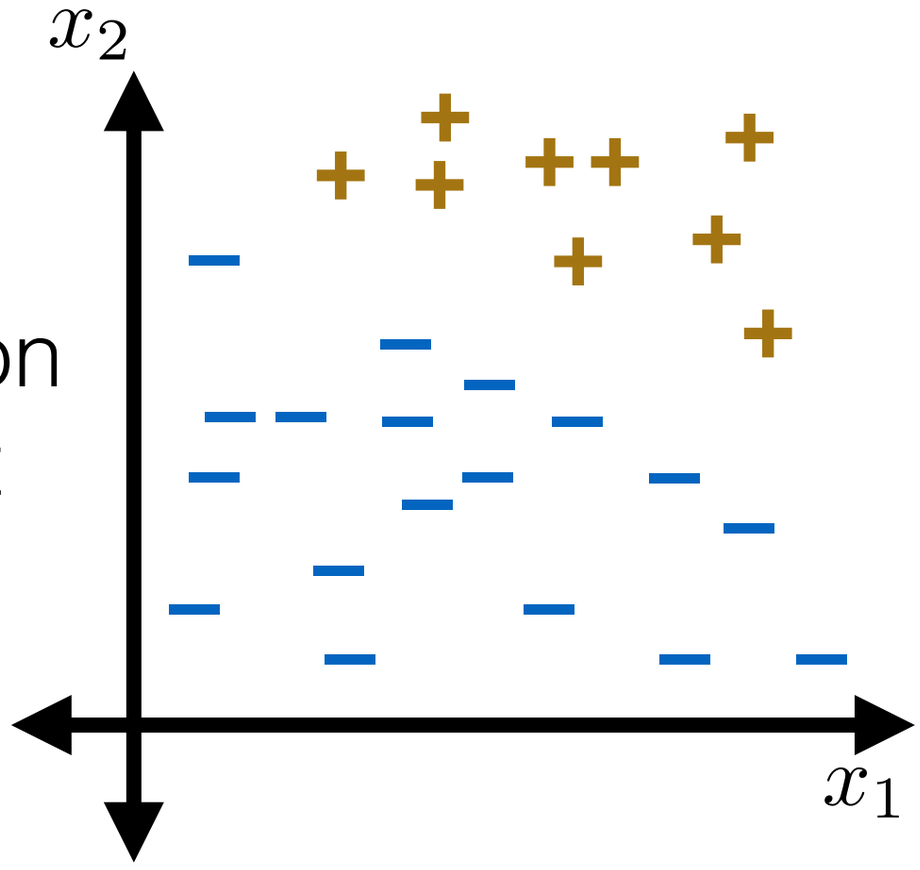


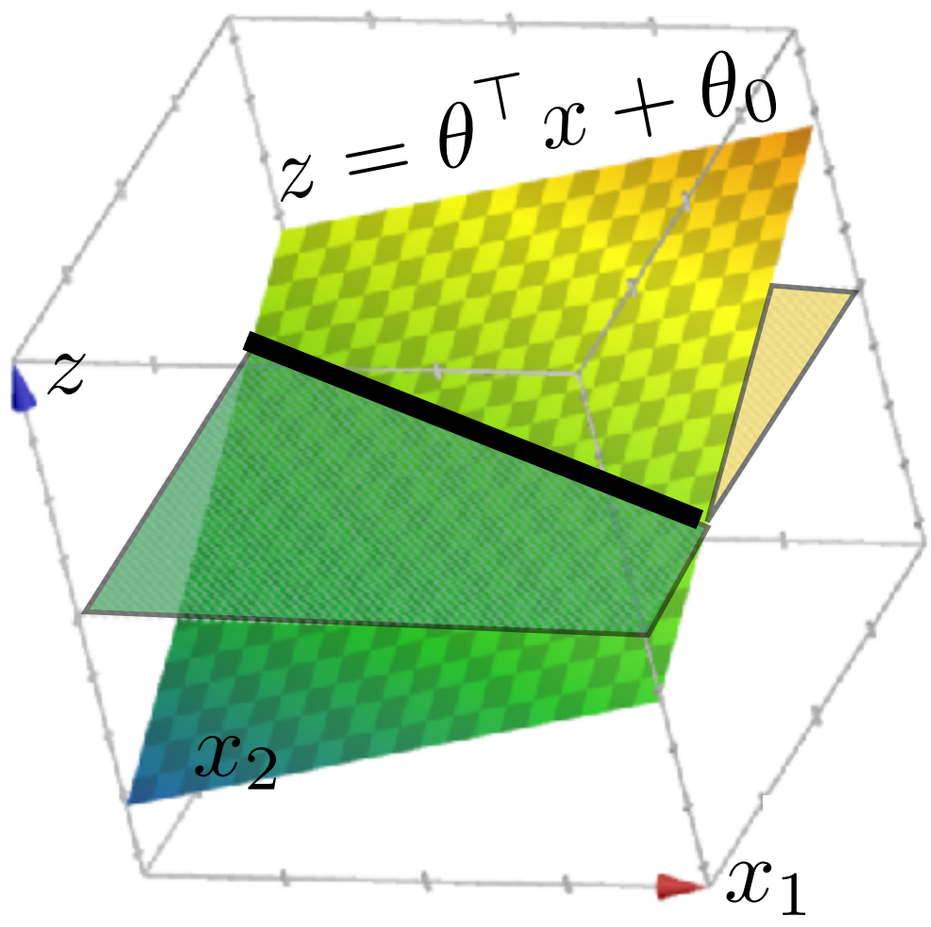

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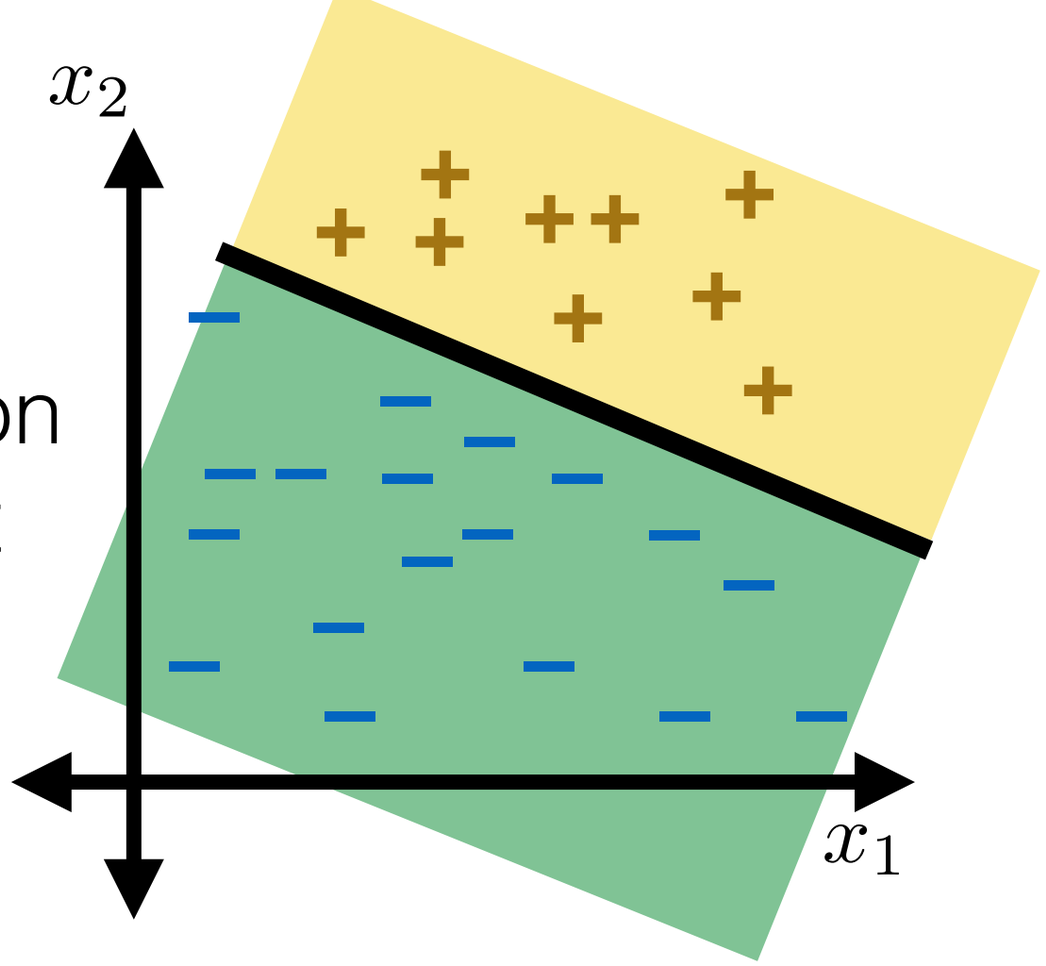


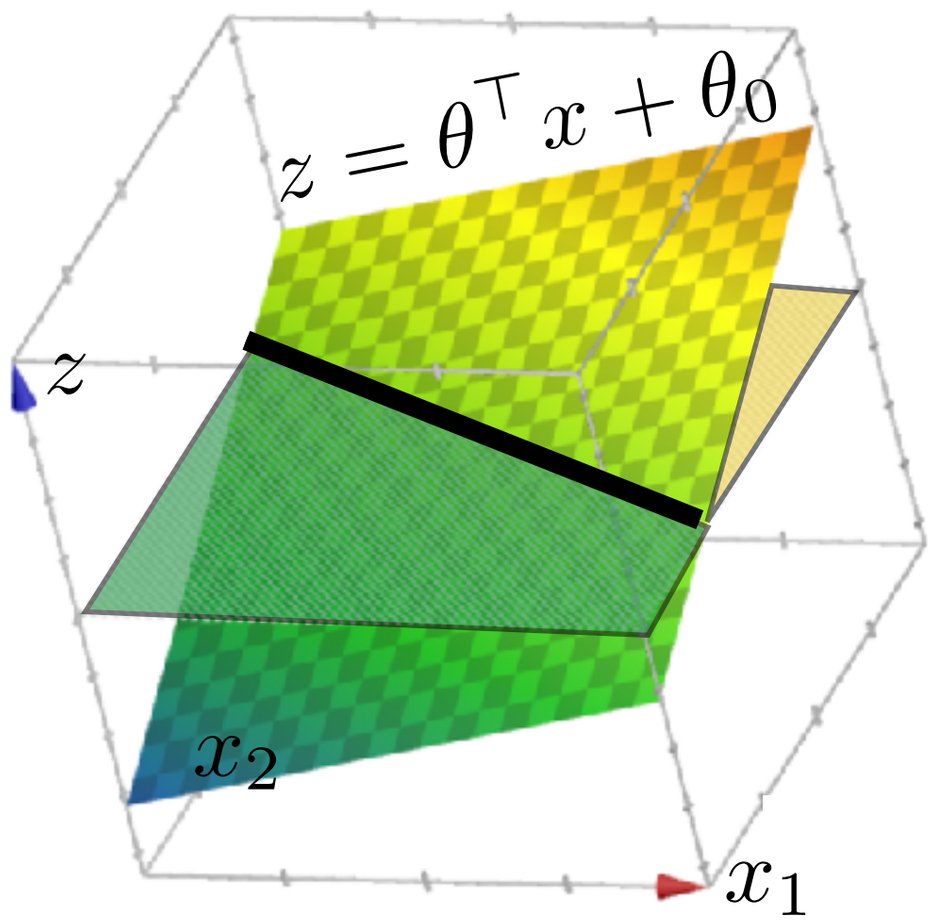

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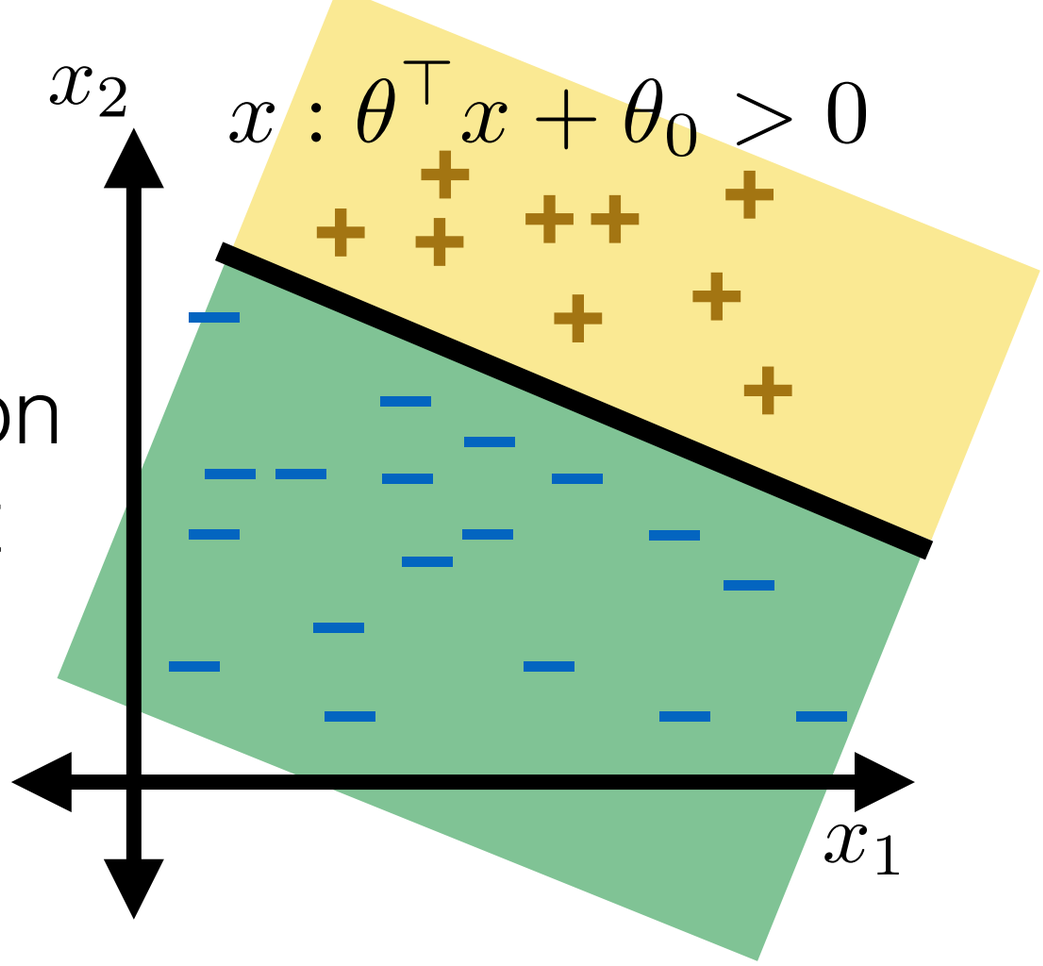


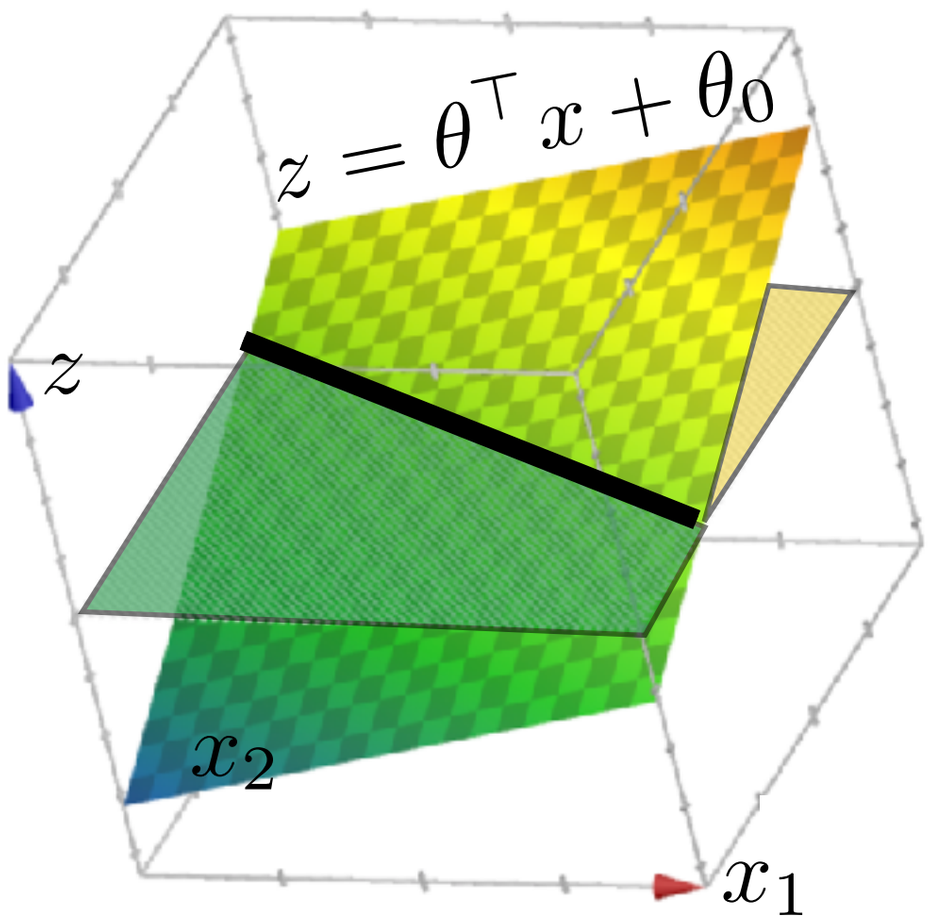

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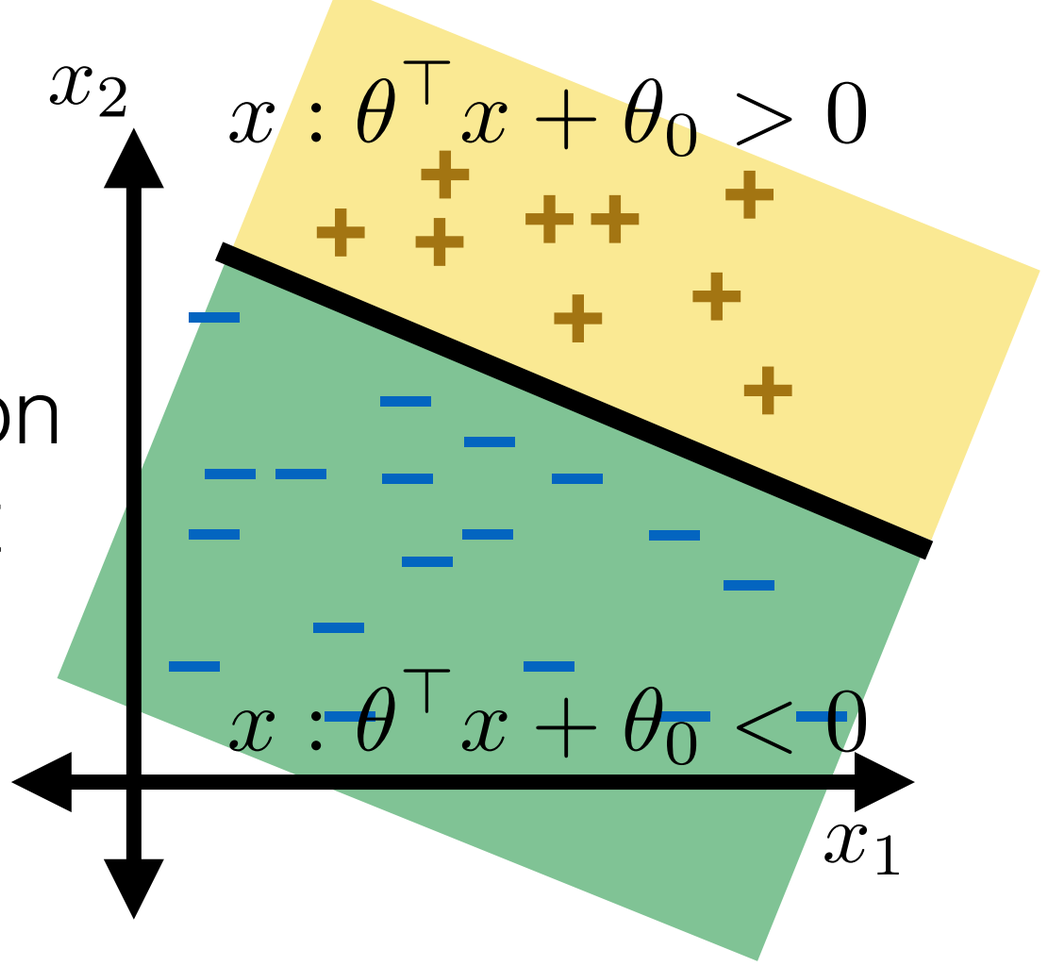


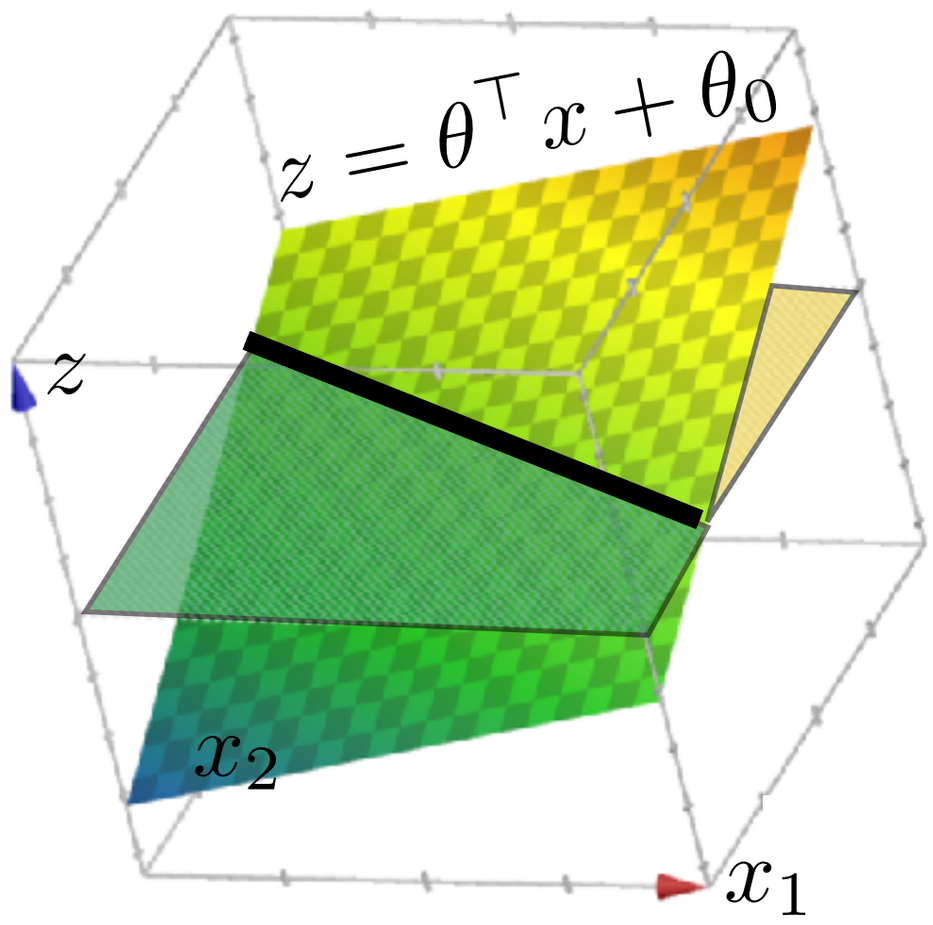

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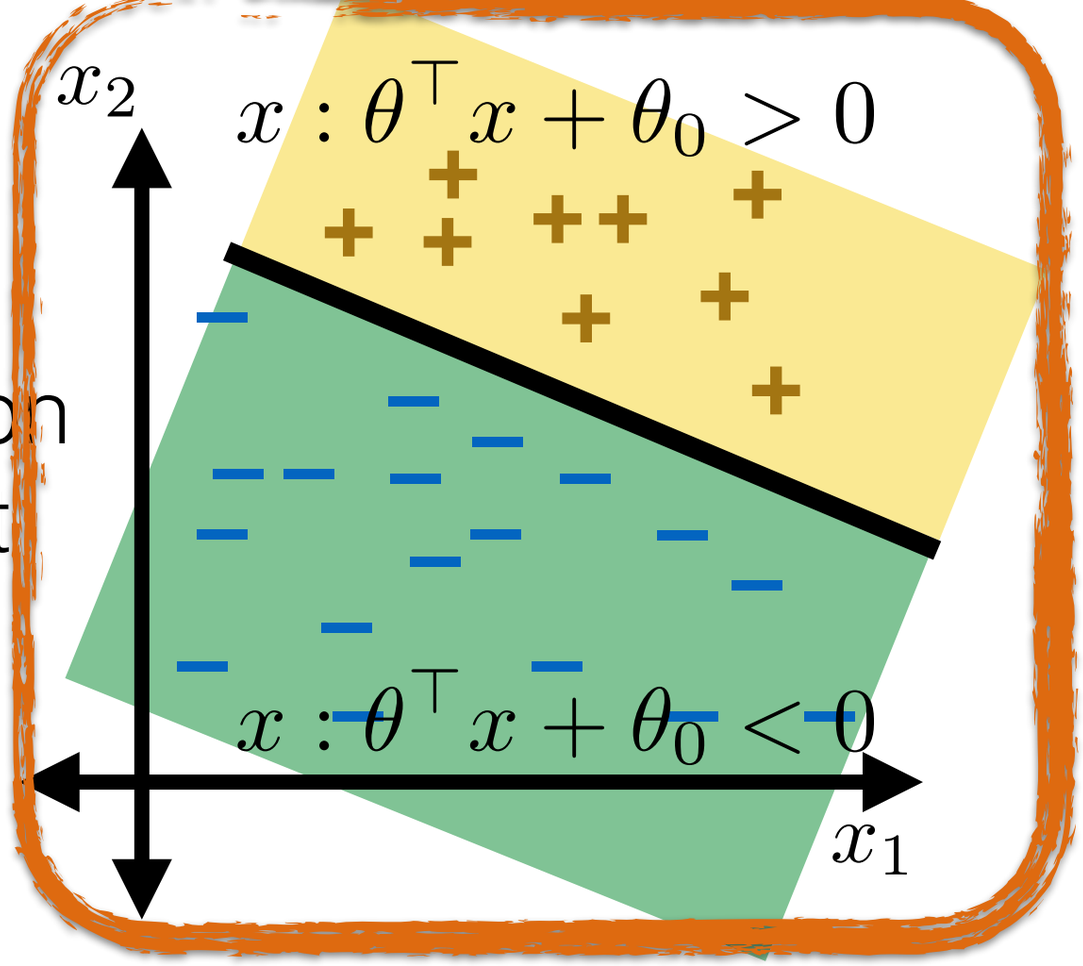


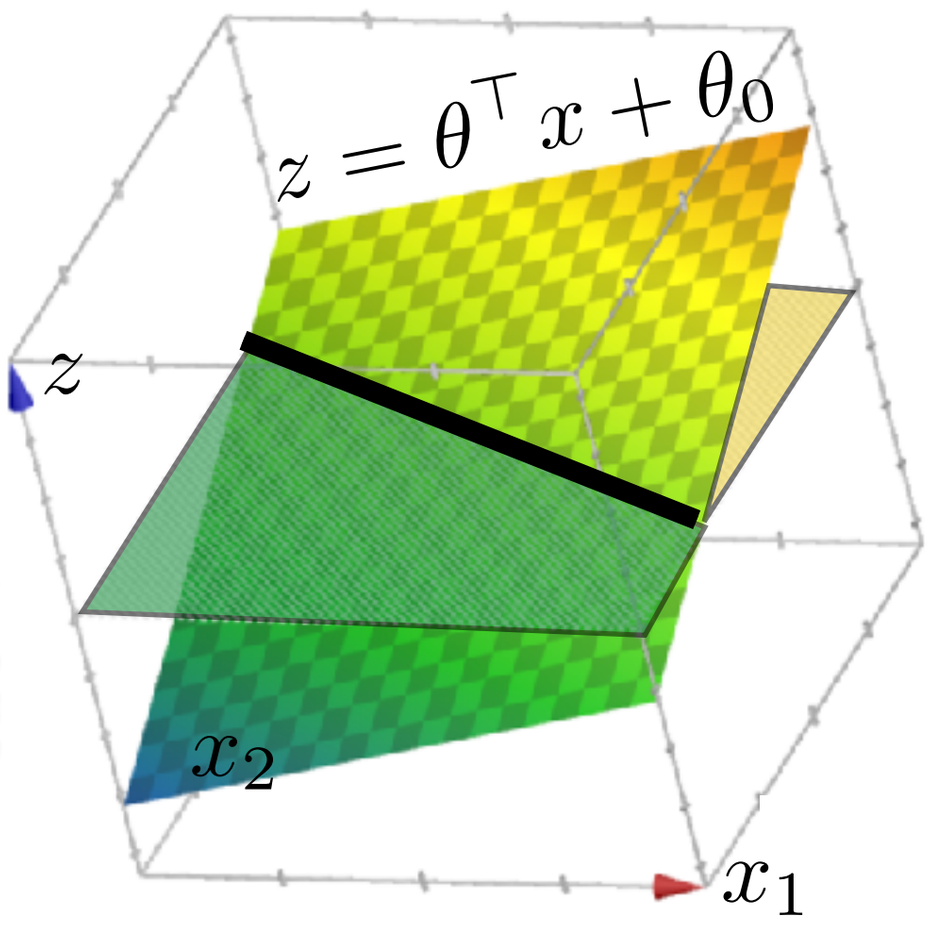

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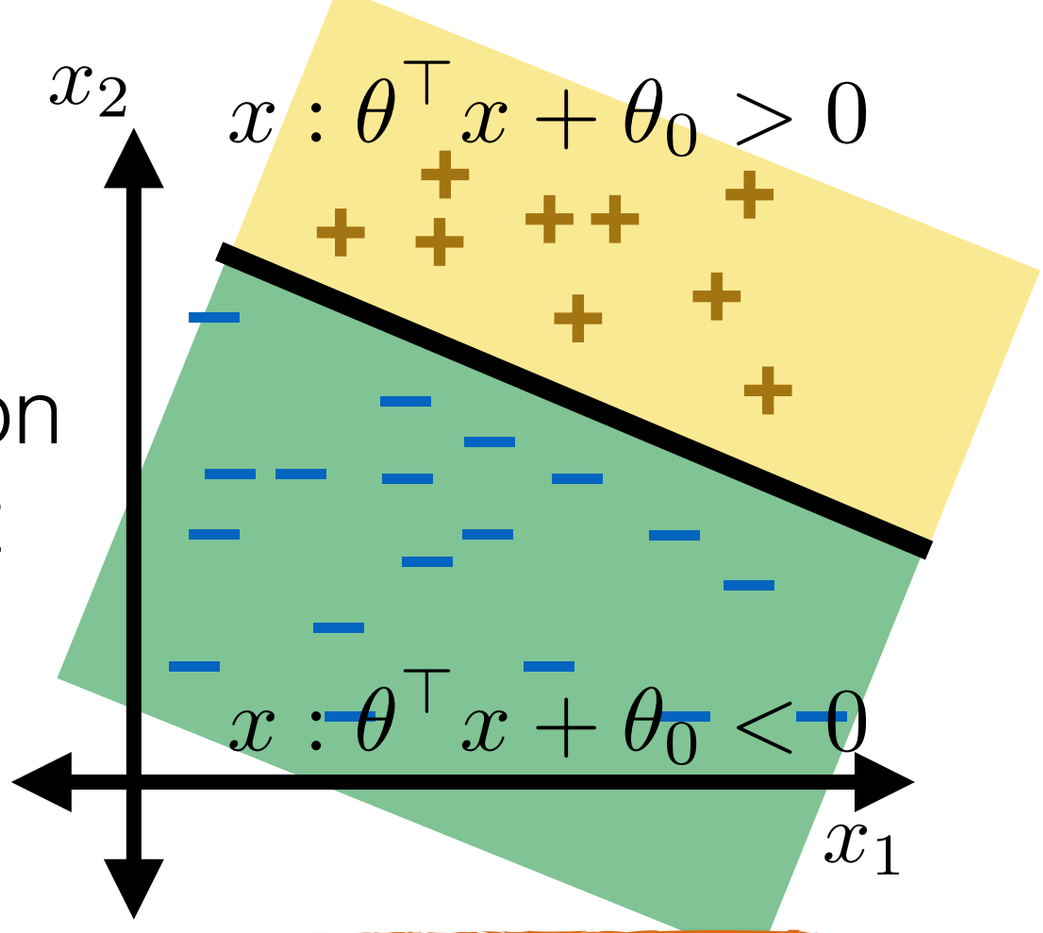


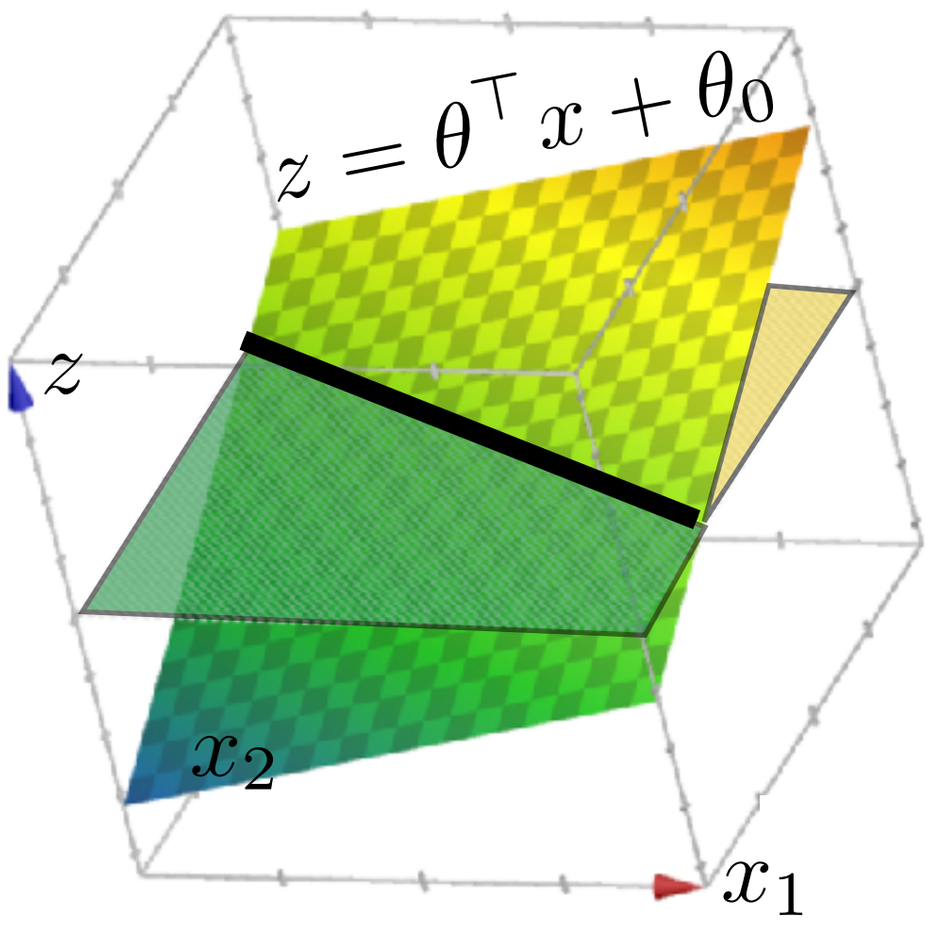


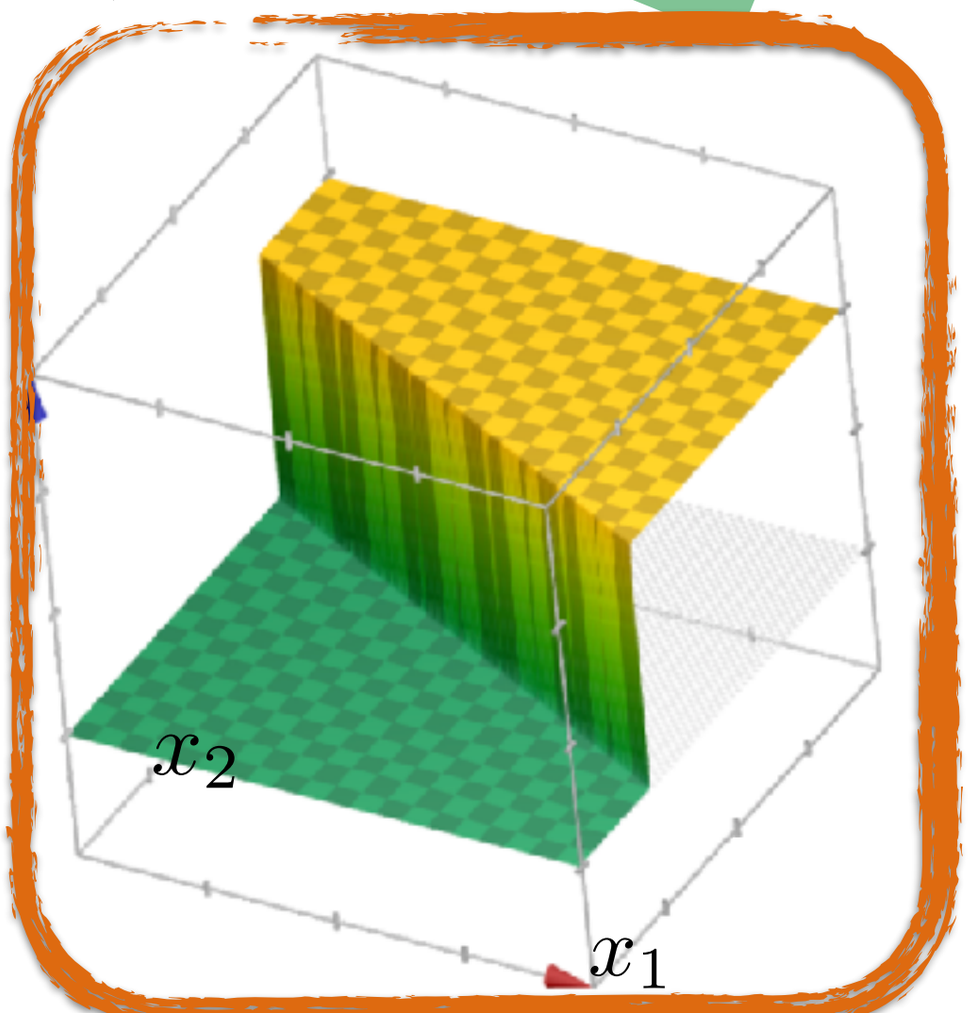

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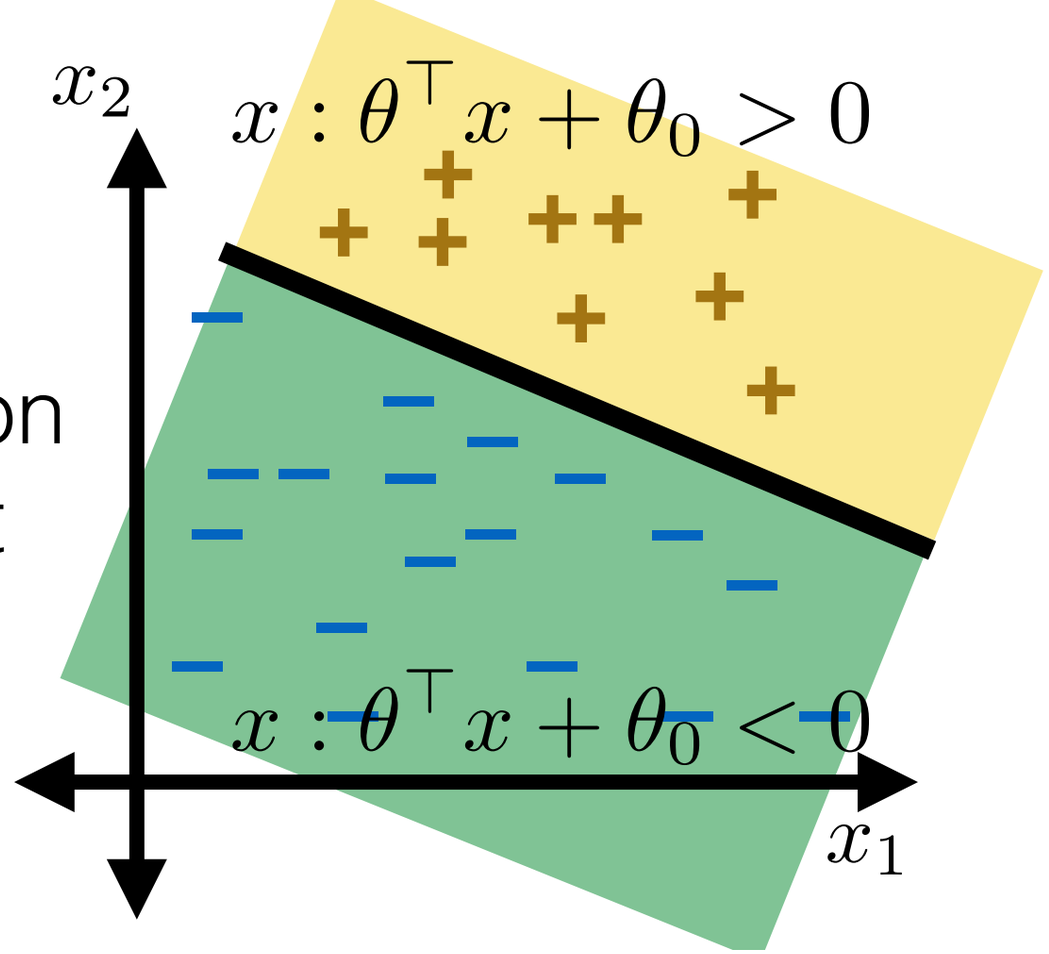


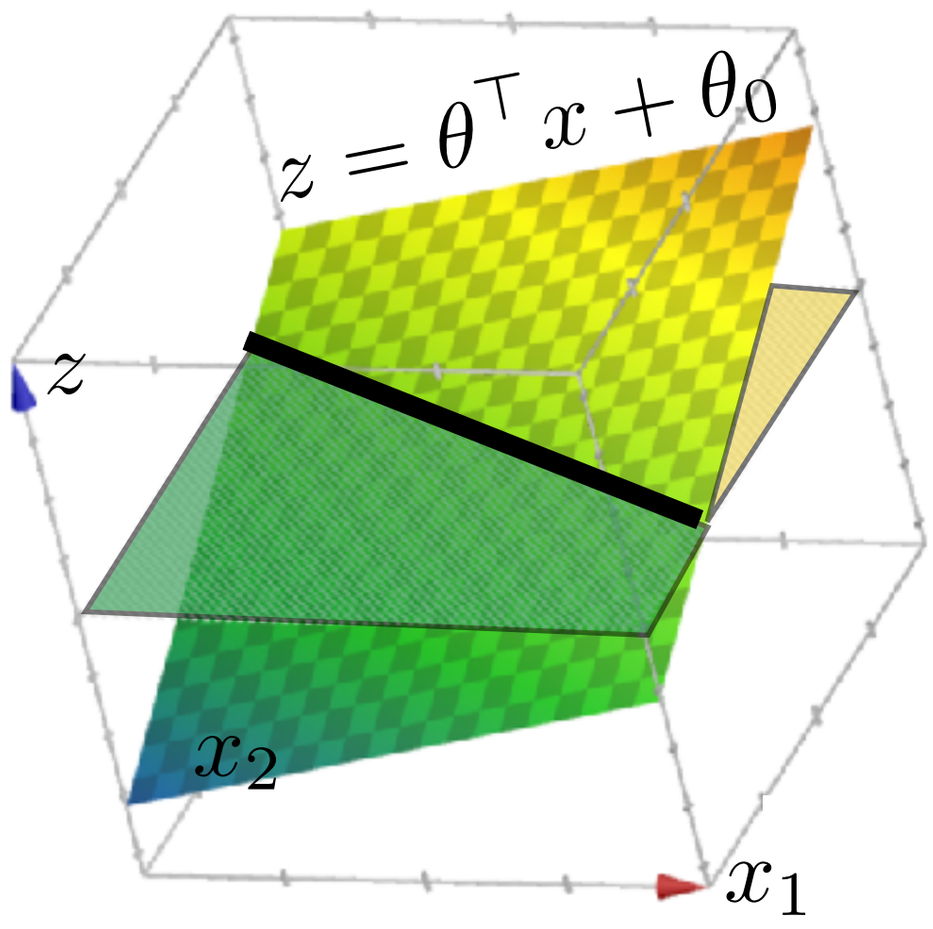


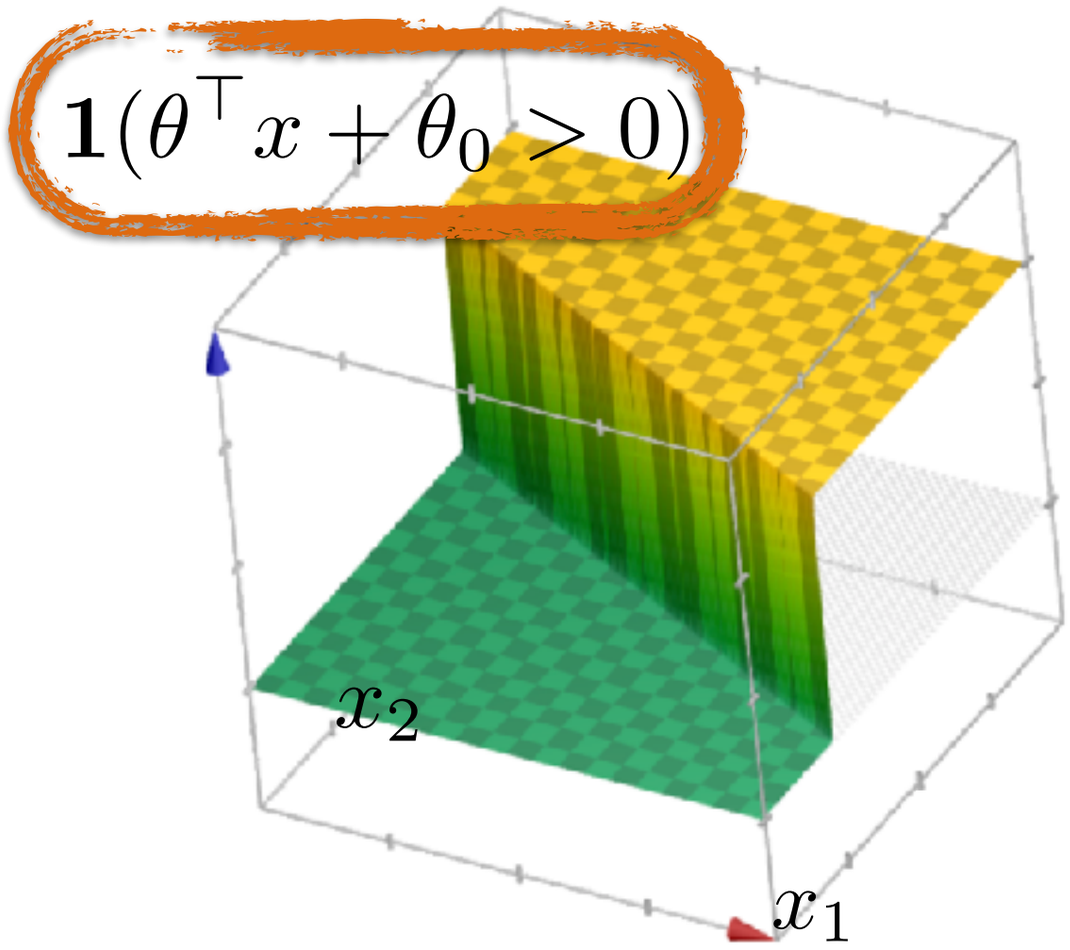

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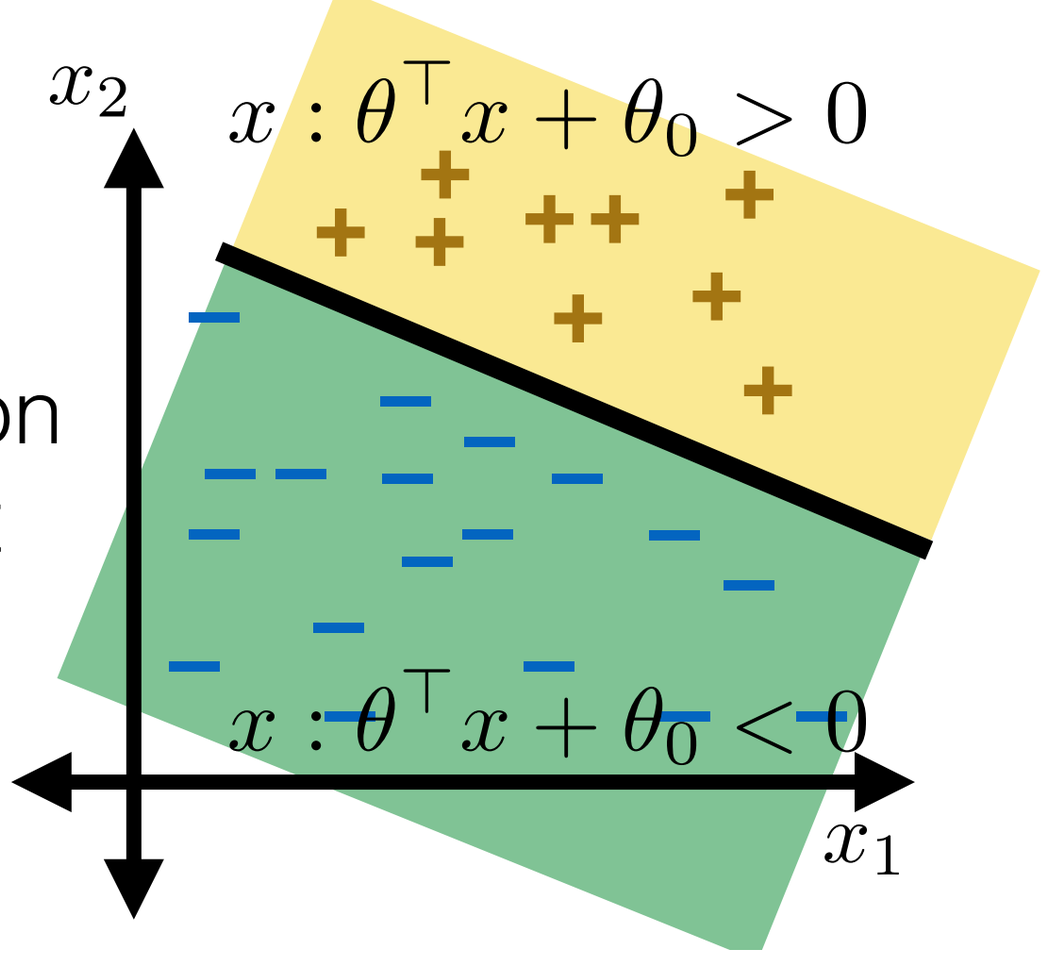


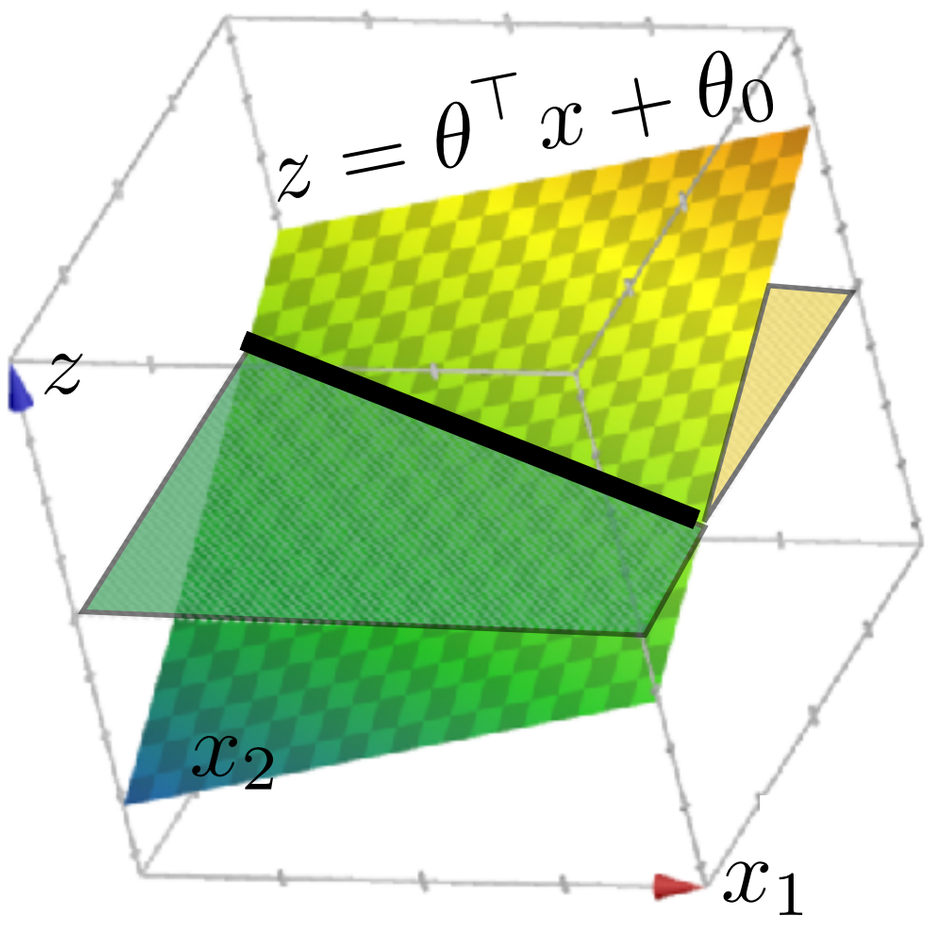


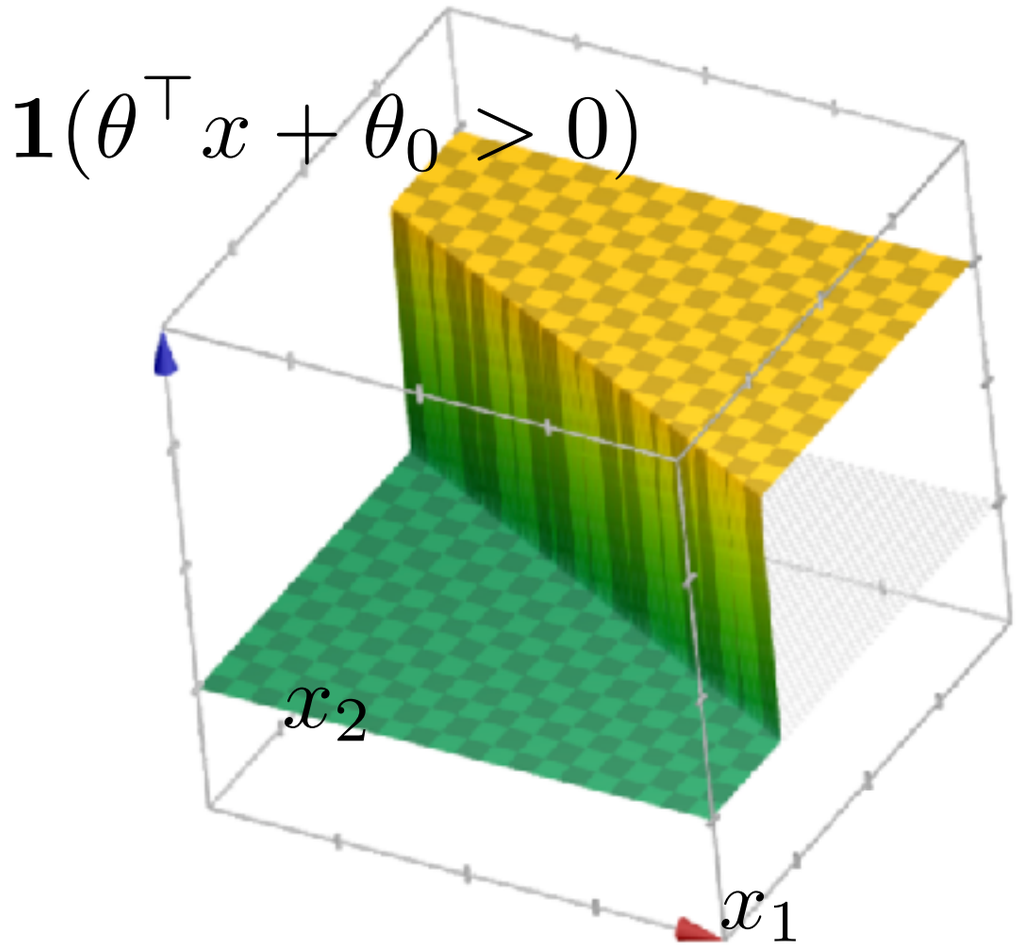


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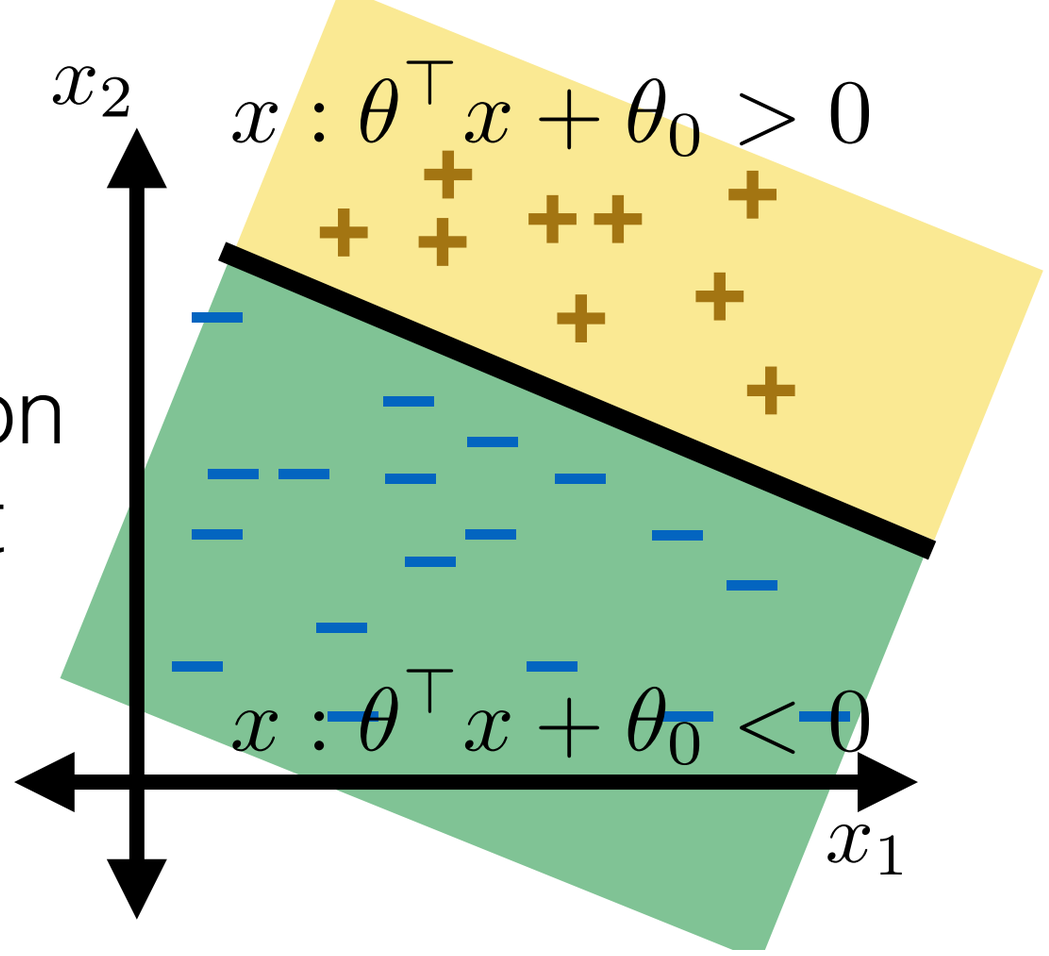


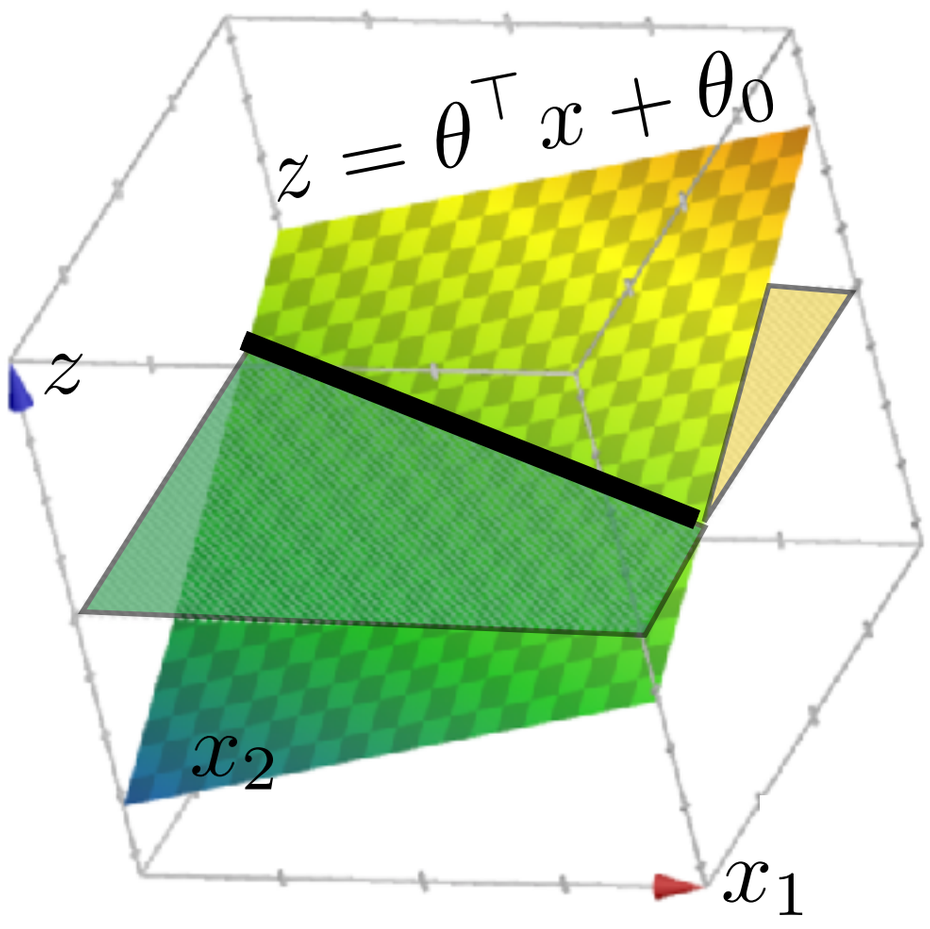


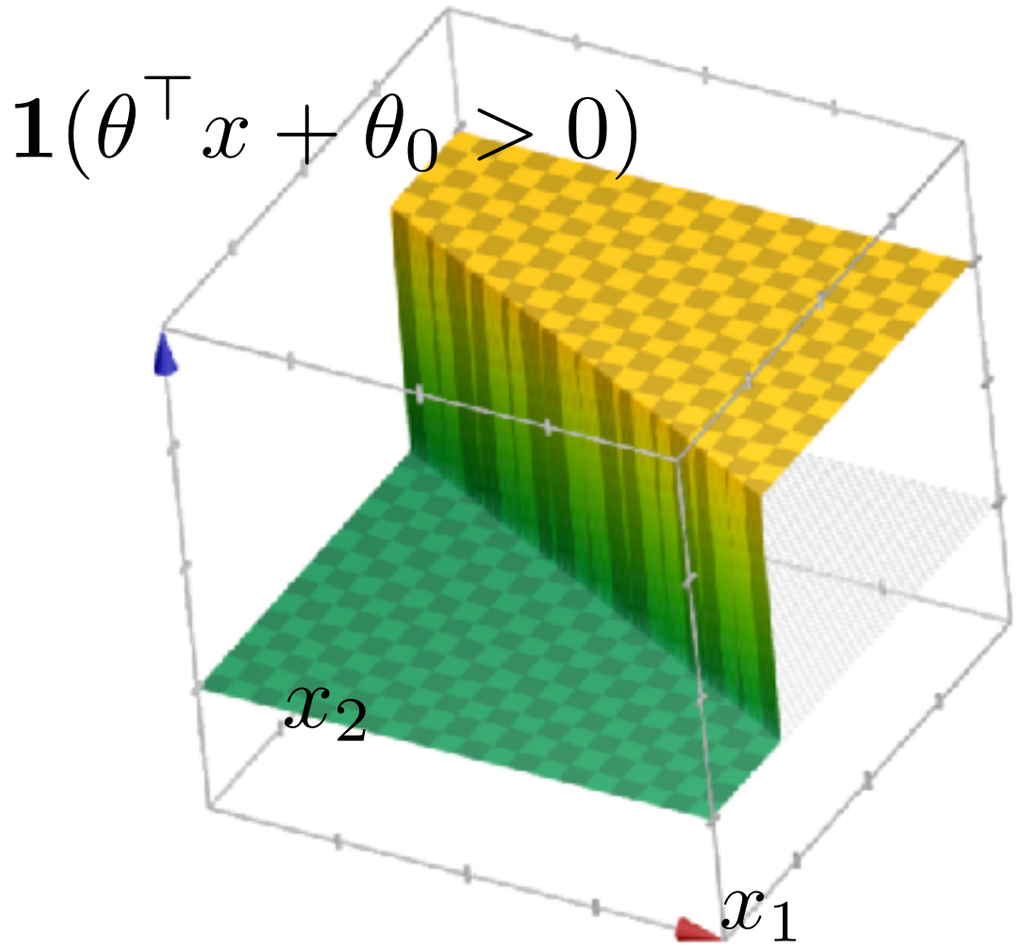


- We're used to using step functions to classify
- New idea today: we'll use step functions as *features*, with their own parameters

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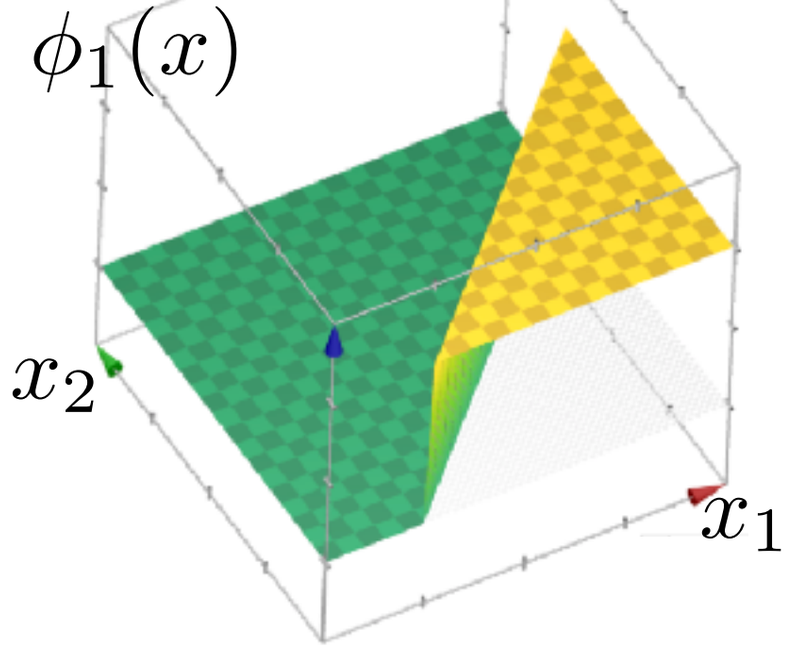

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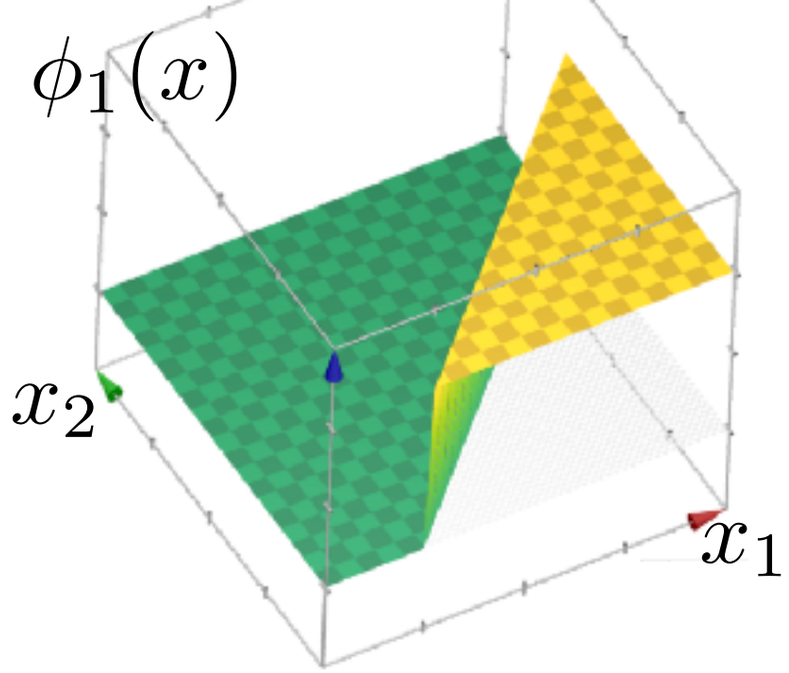

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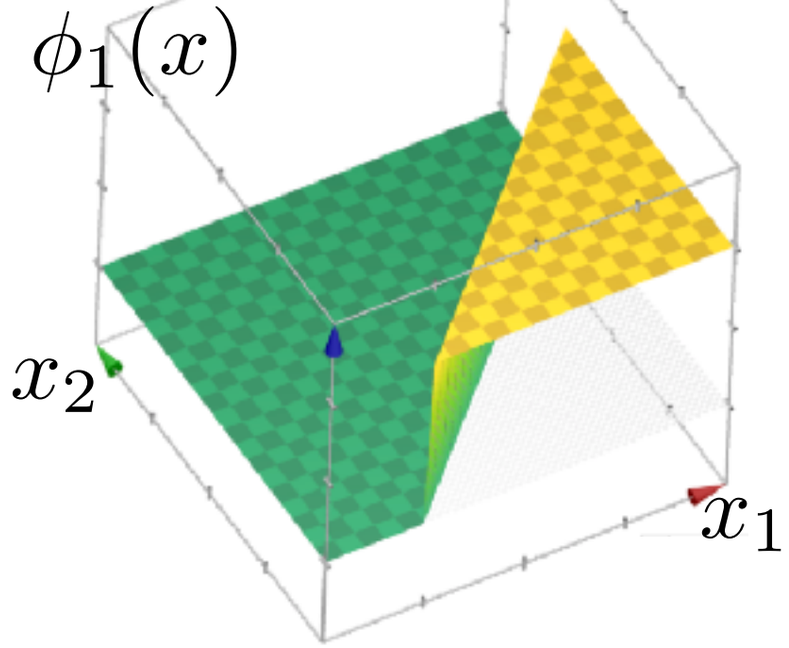


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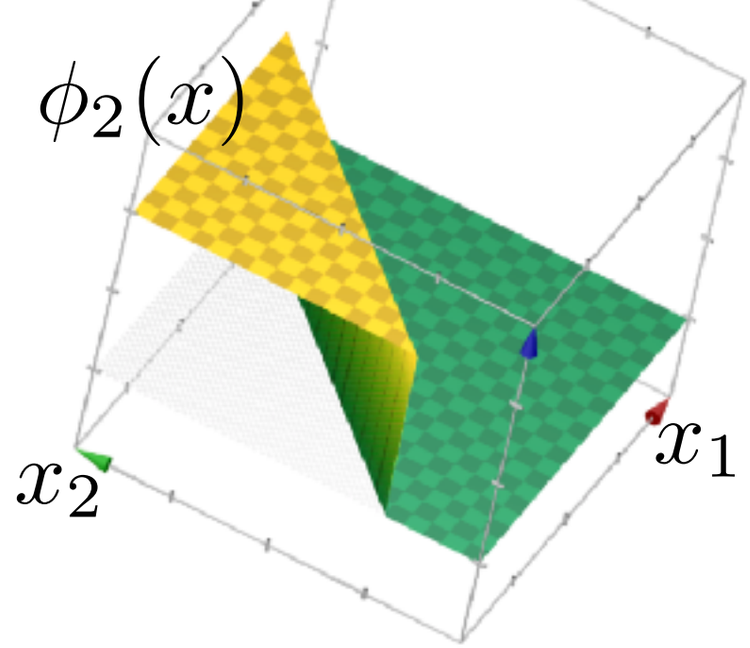

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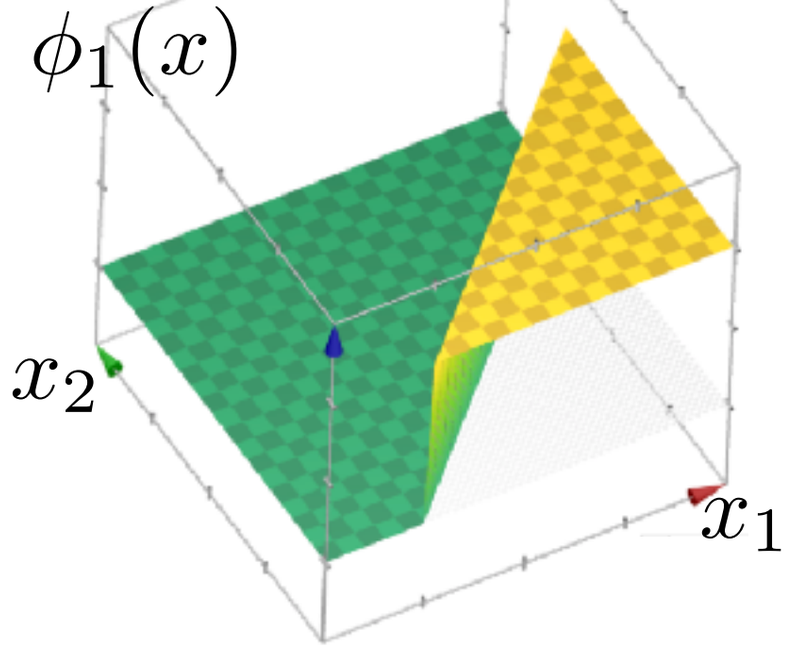


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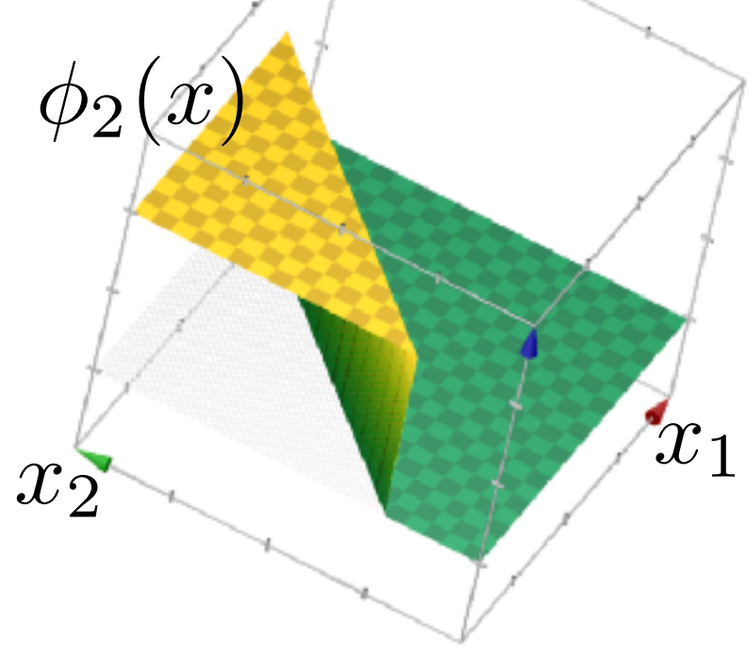


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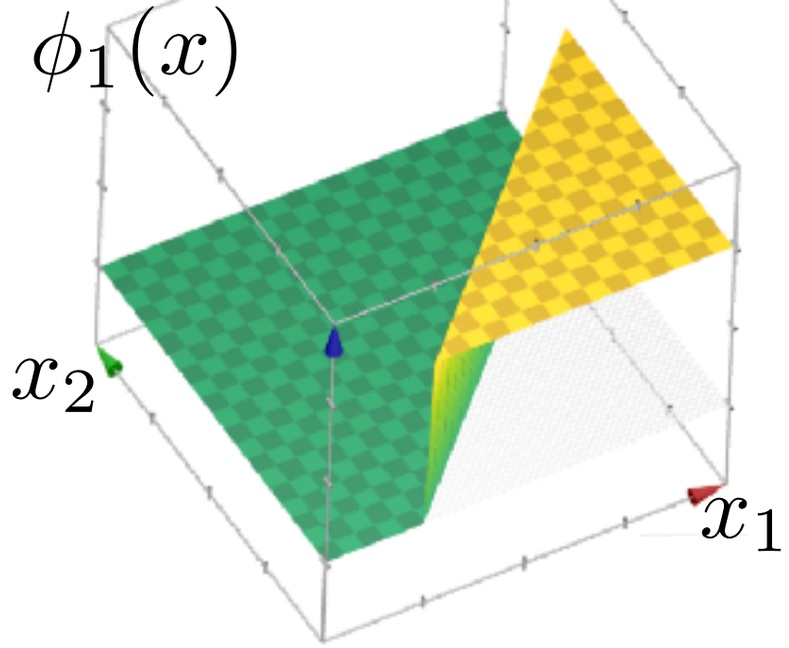
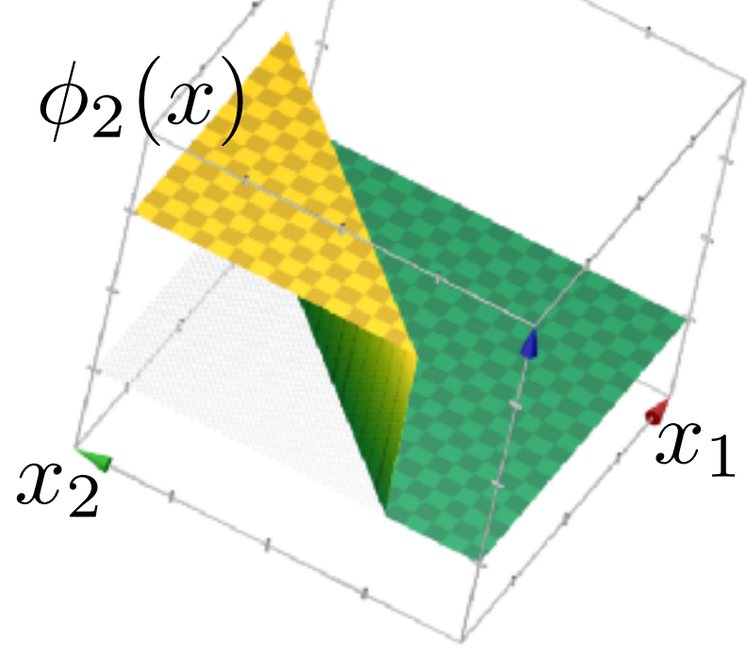



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$




$$z = \theta^\top \phi(x) + \theta_0$$
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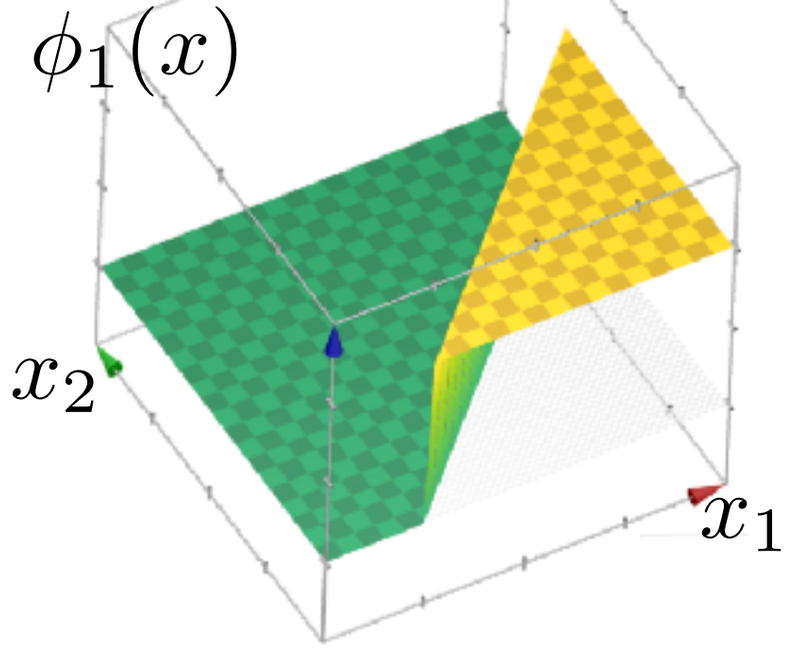


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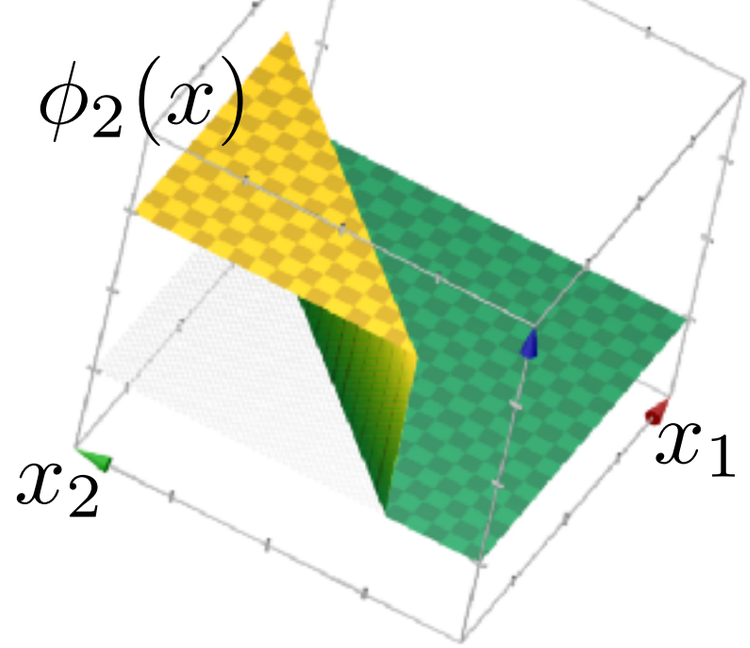


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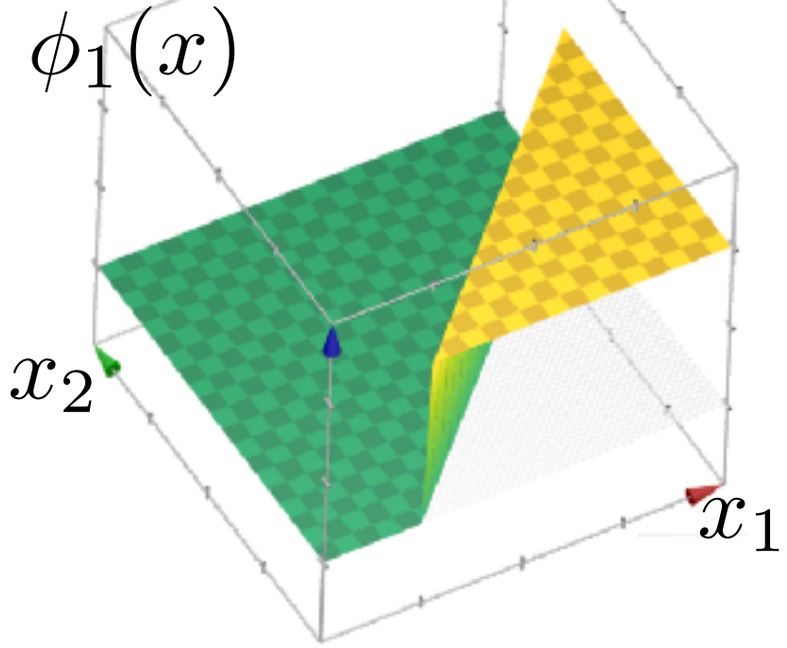


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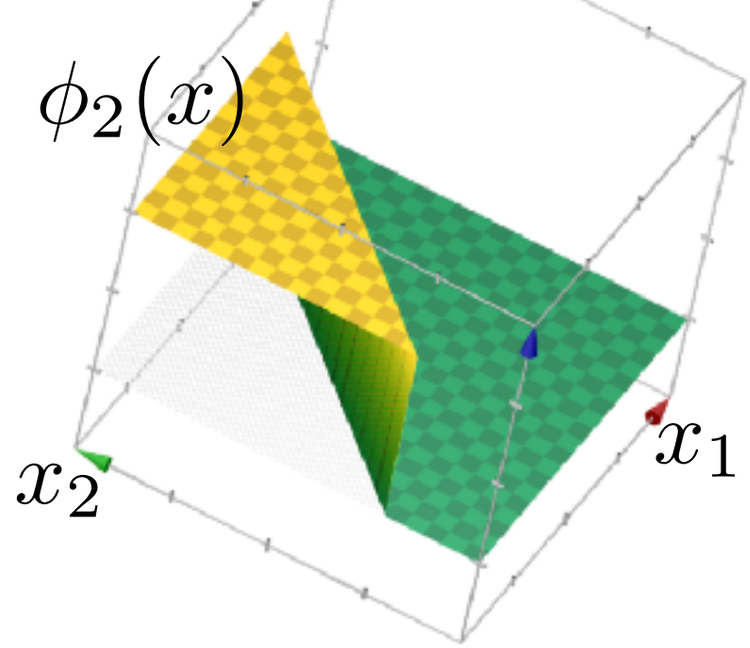


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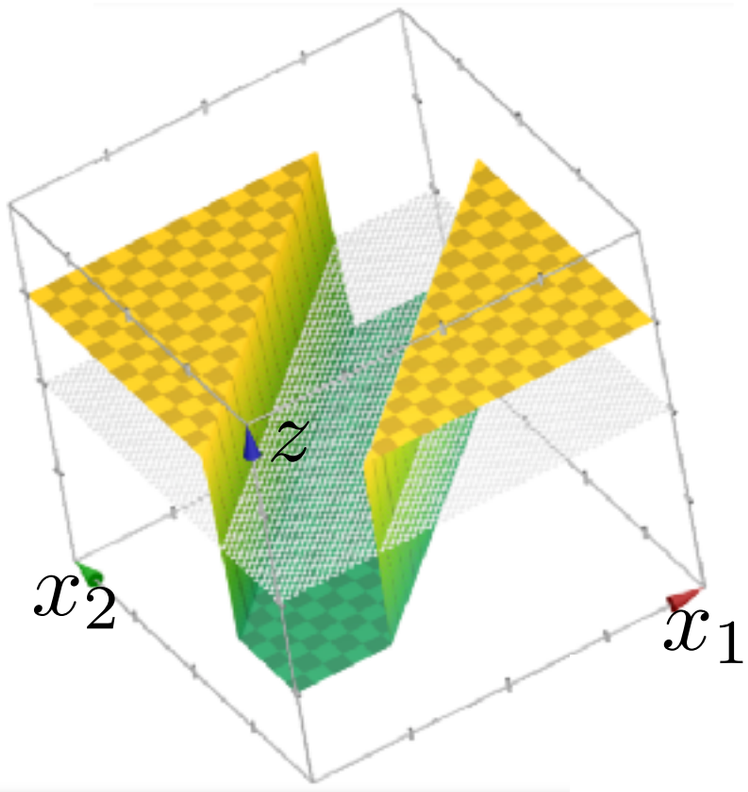

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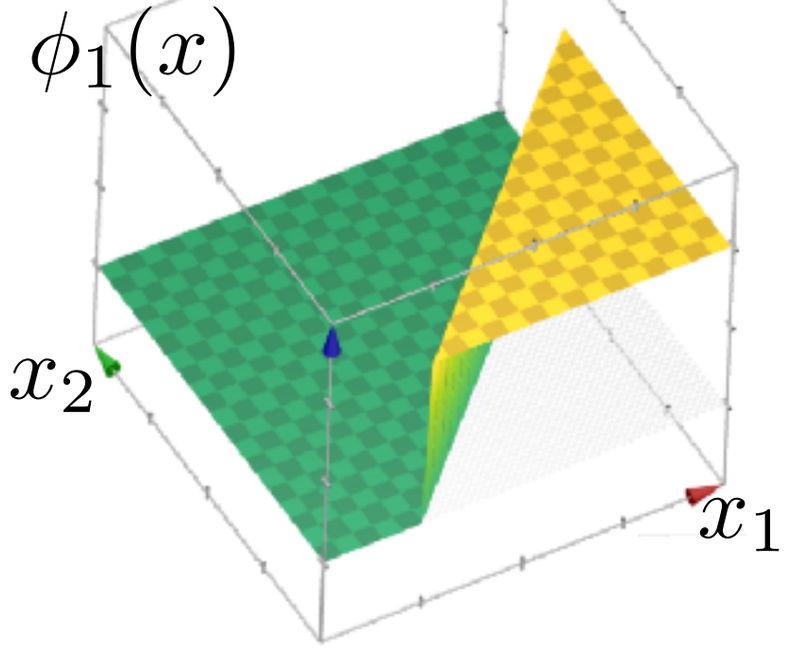


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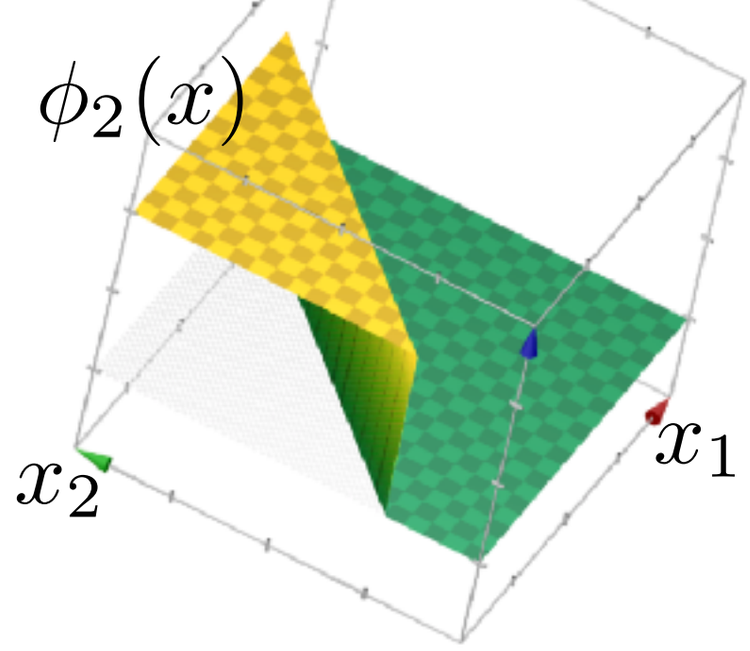


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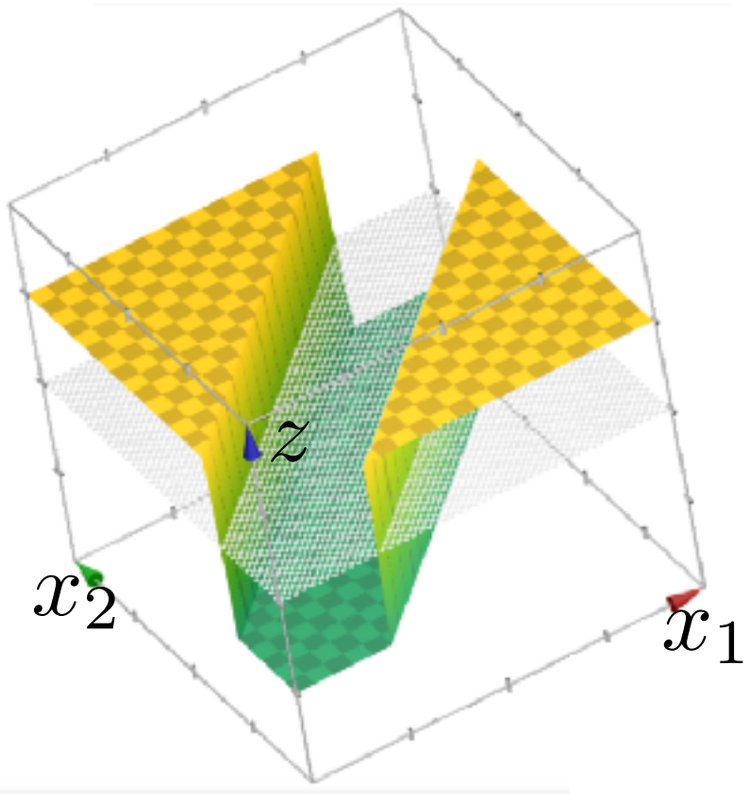


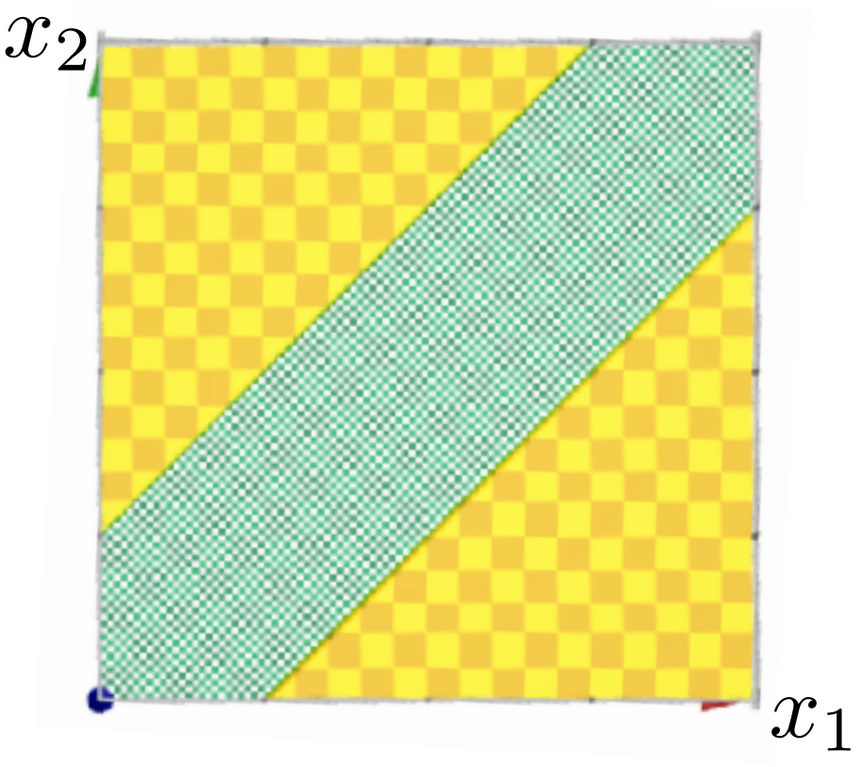

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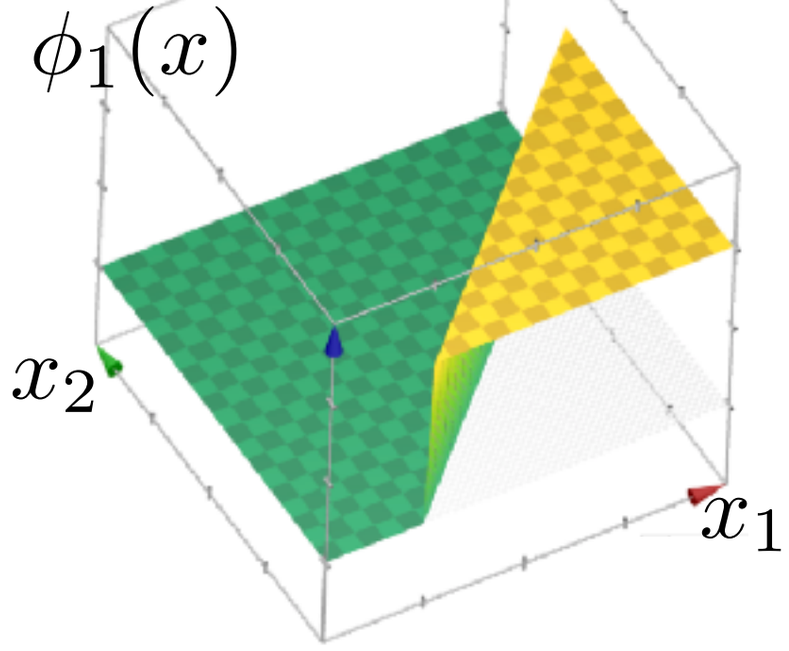


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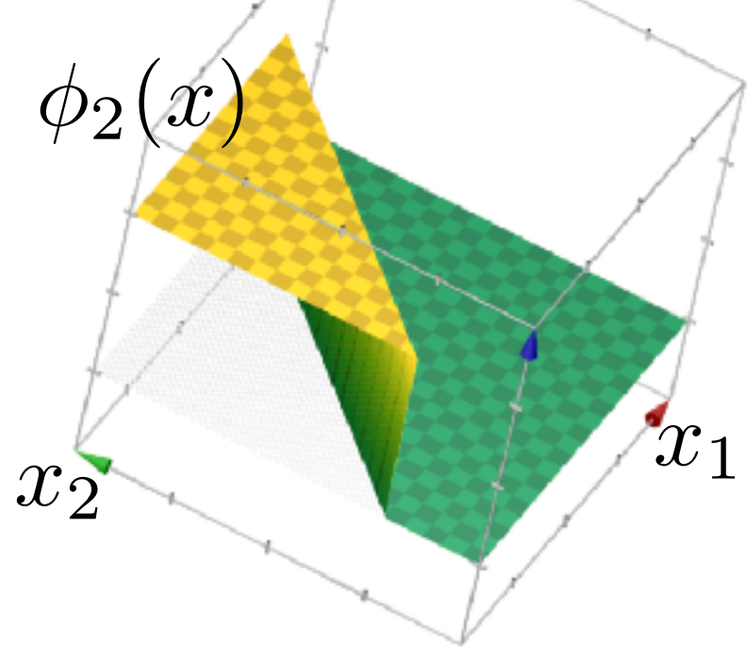


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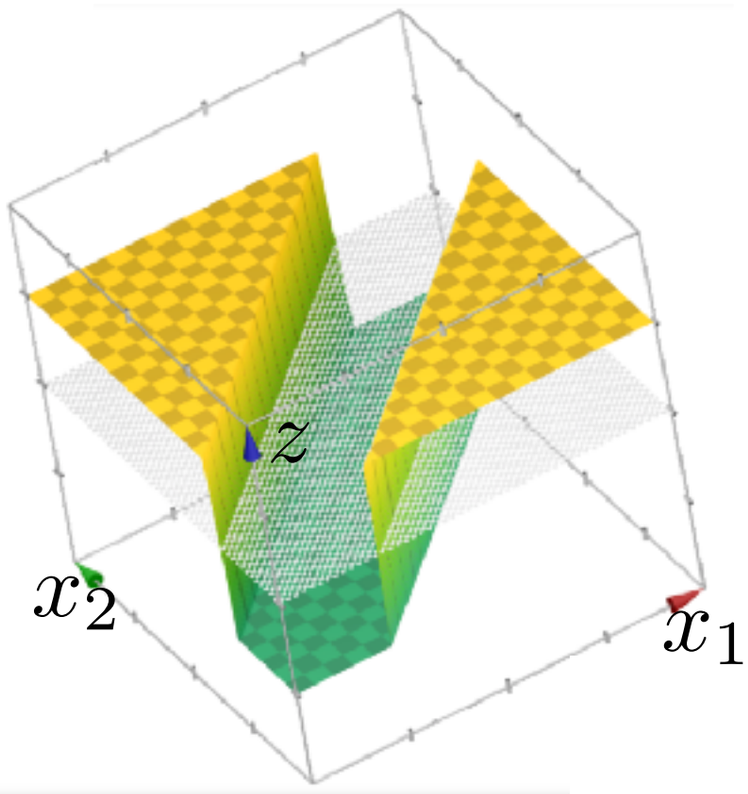


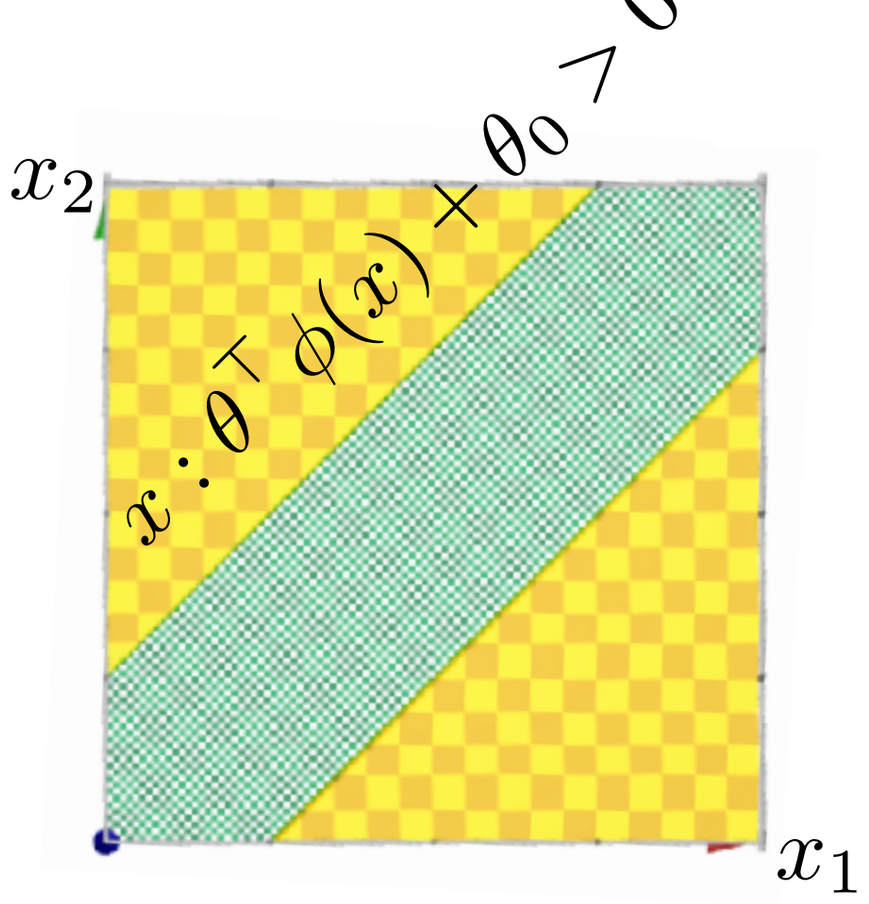

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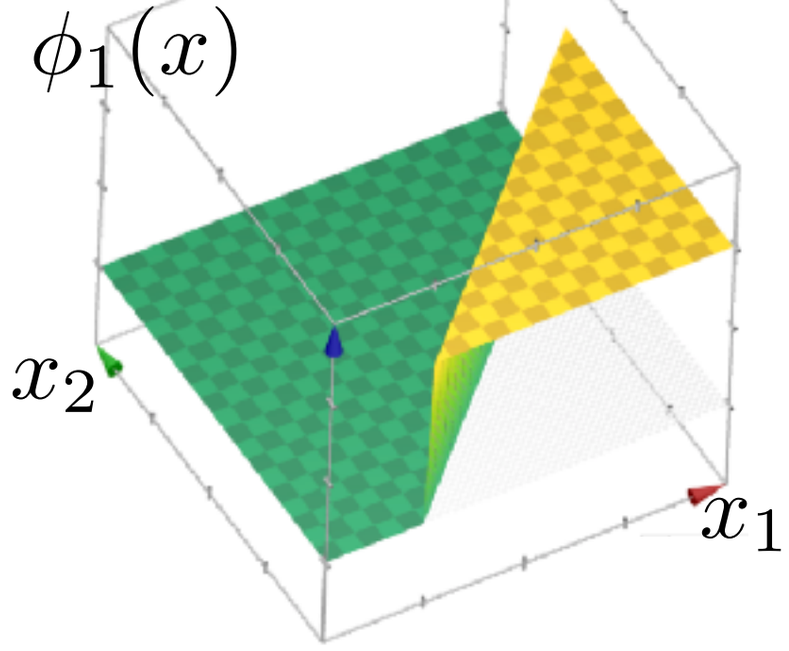


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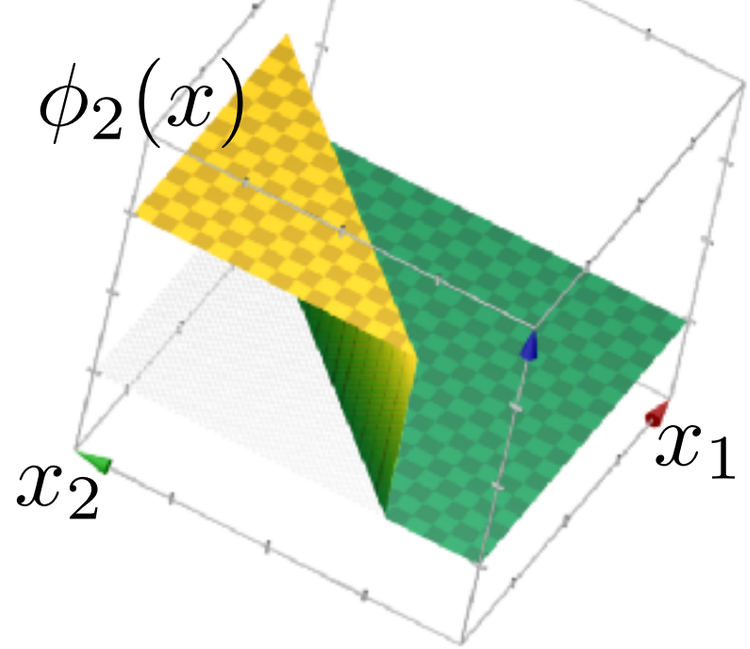


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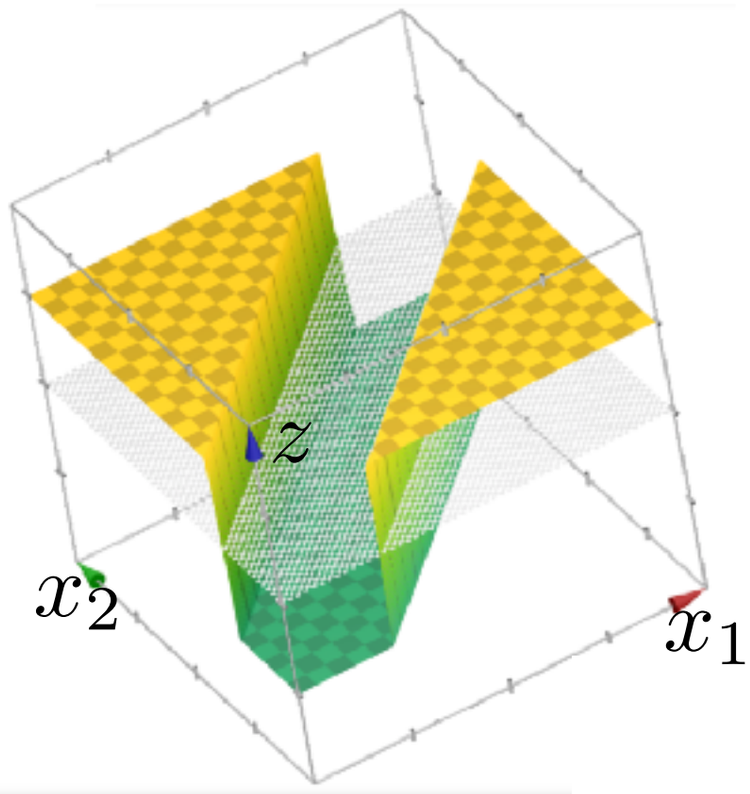


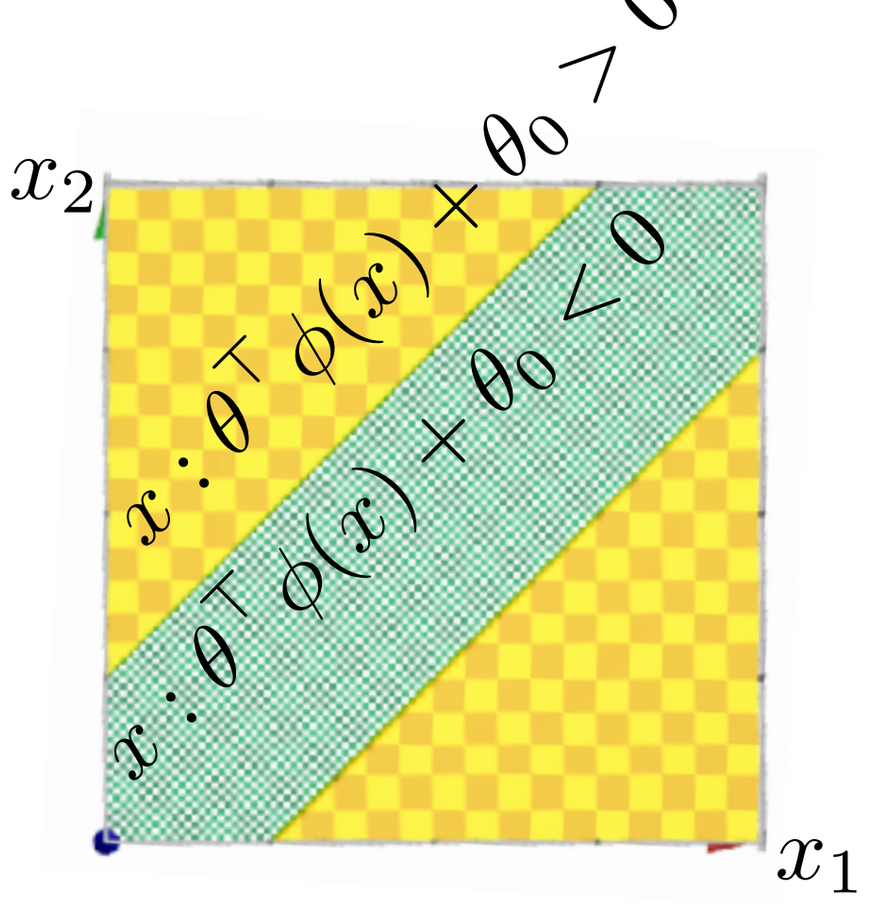

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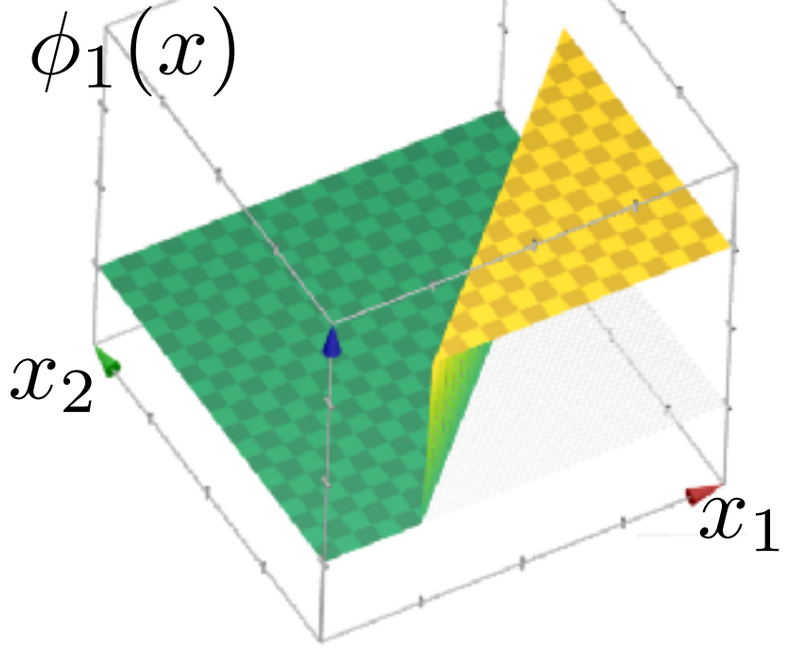


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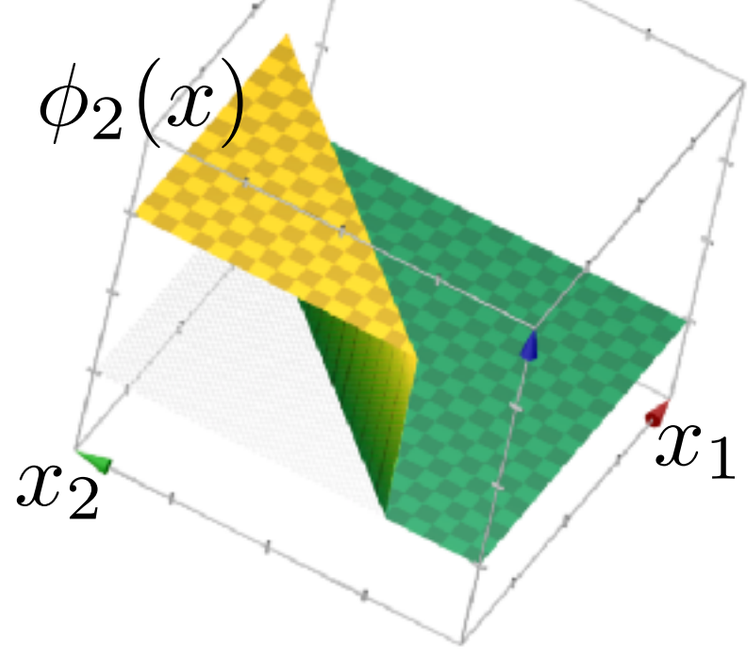


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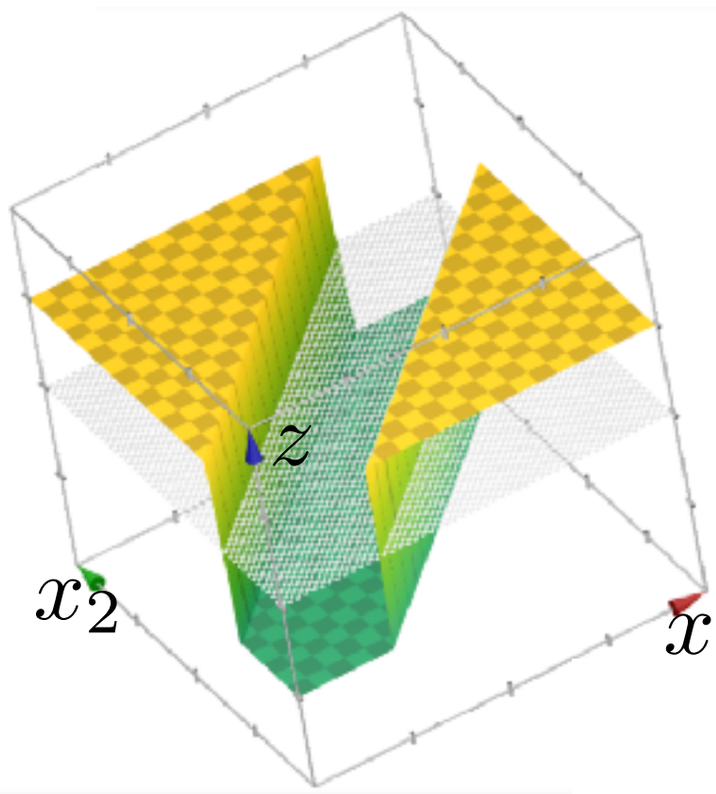
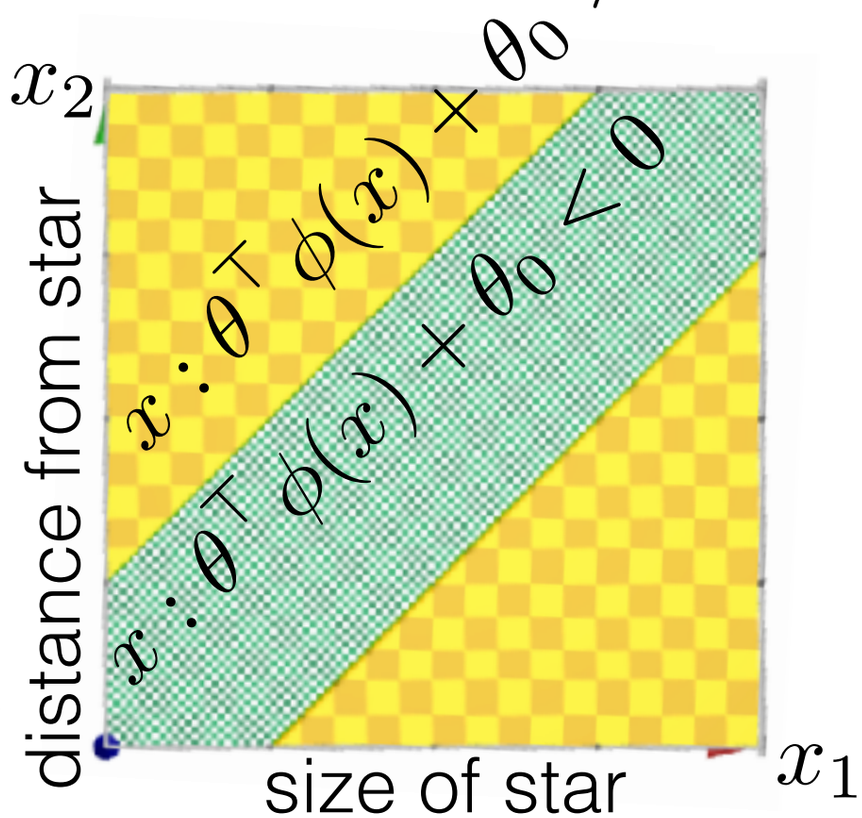



Is an exoplanet outside the habitable zone?



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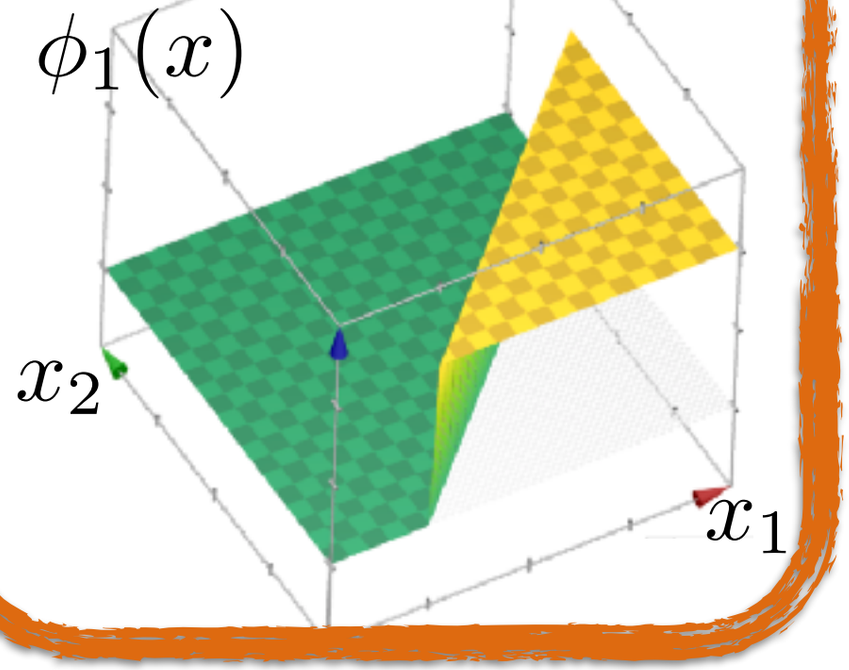


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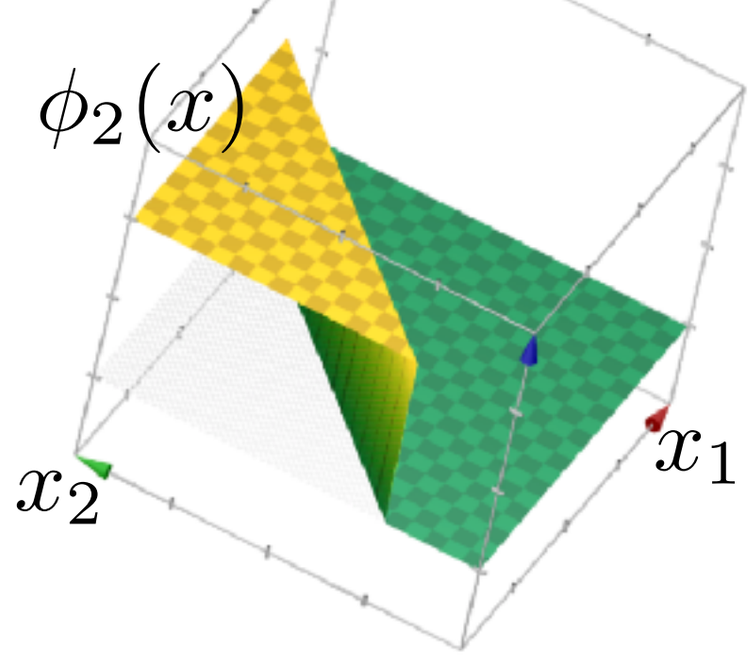


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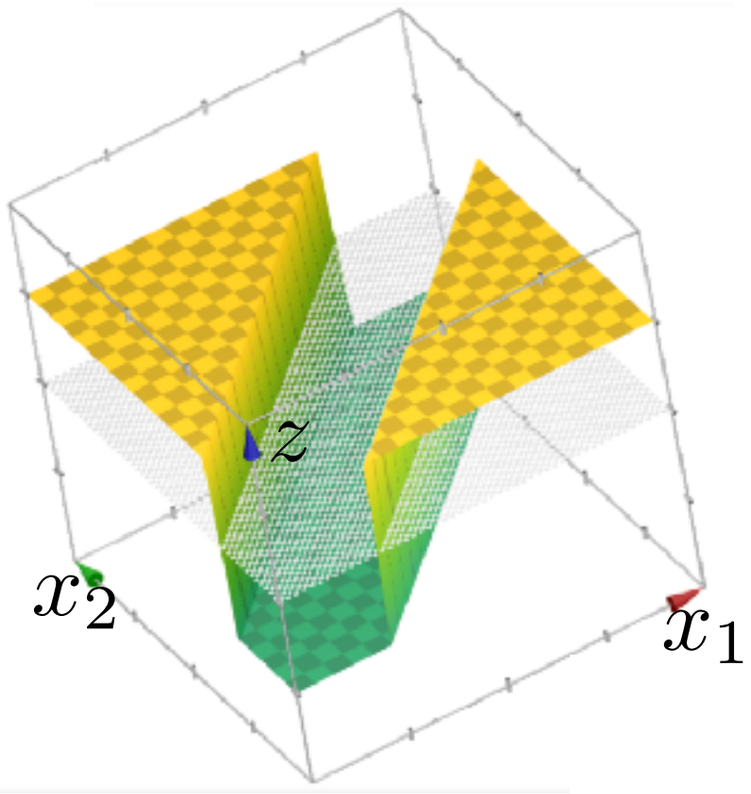


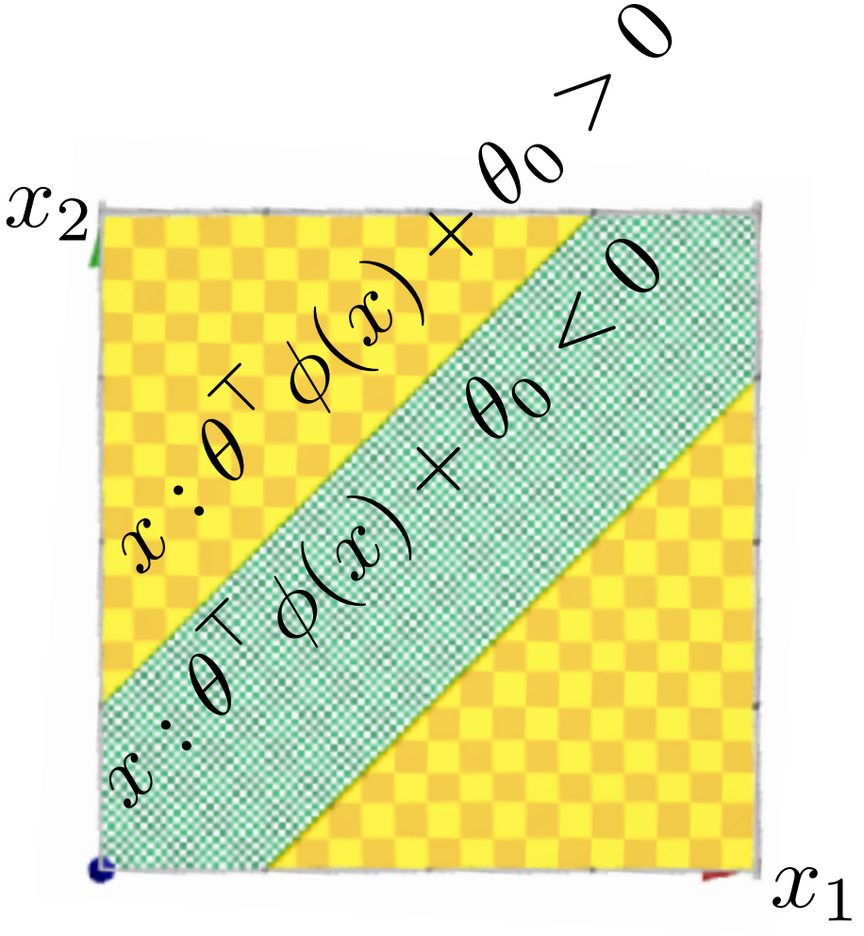

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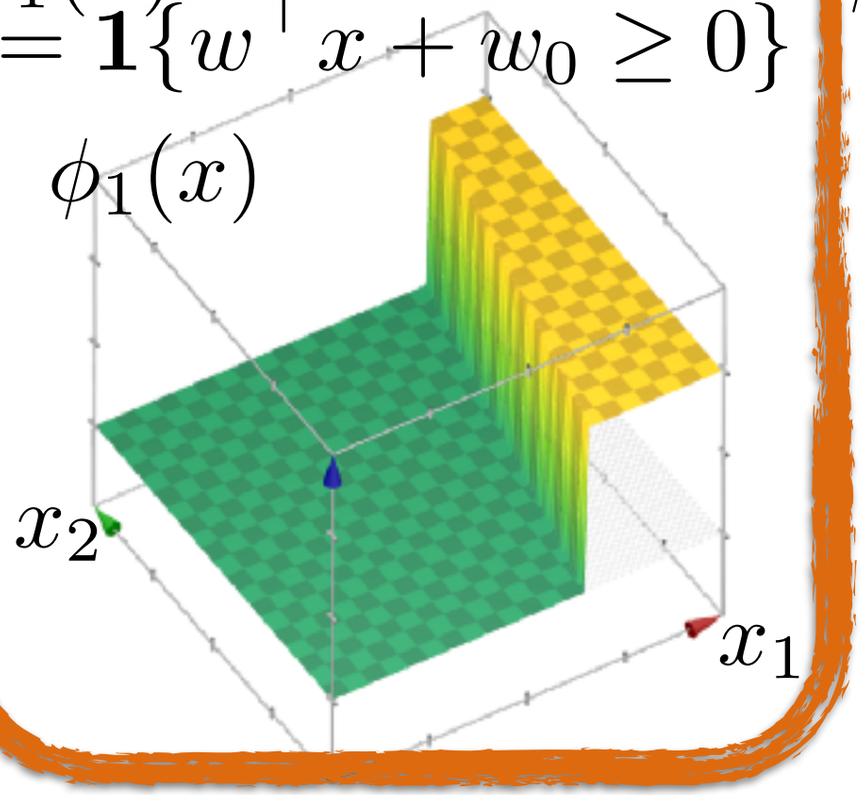


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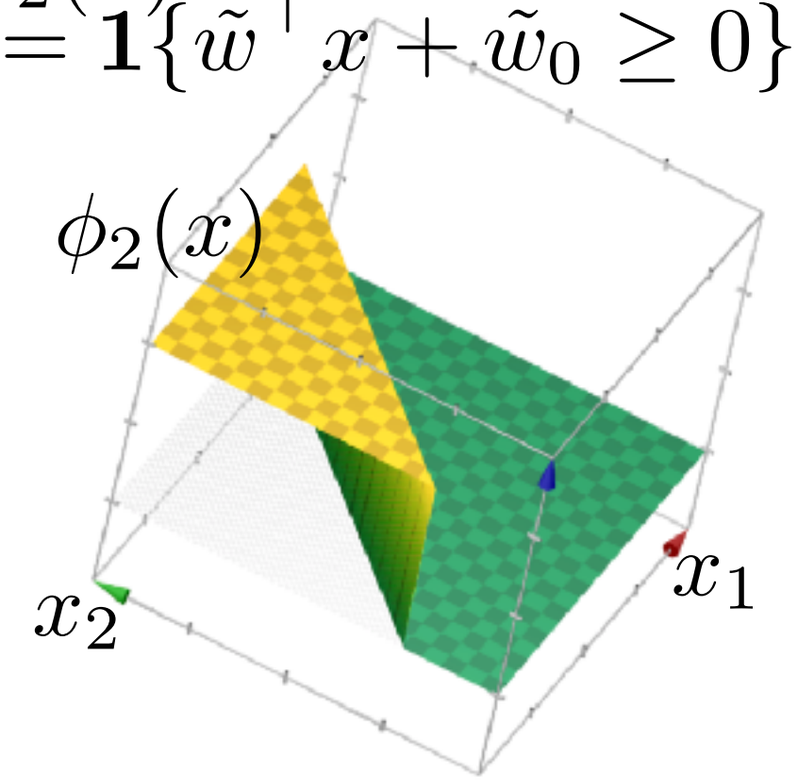


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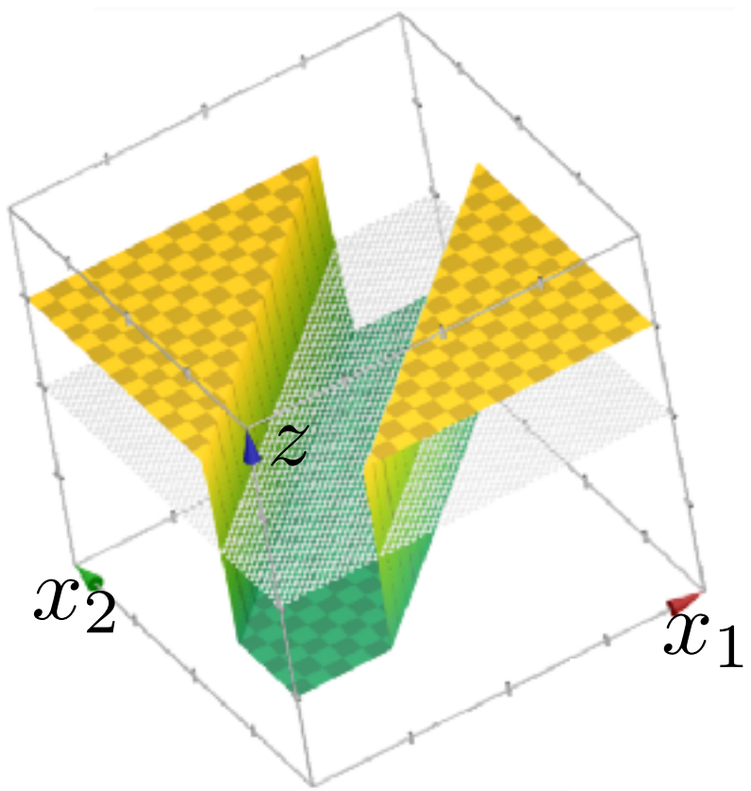


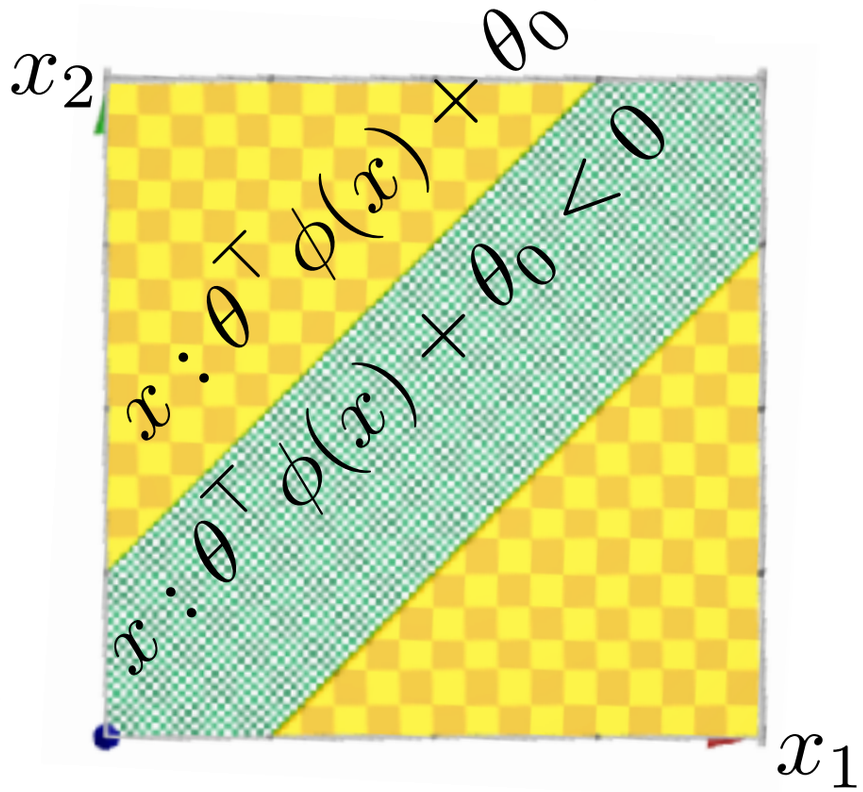

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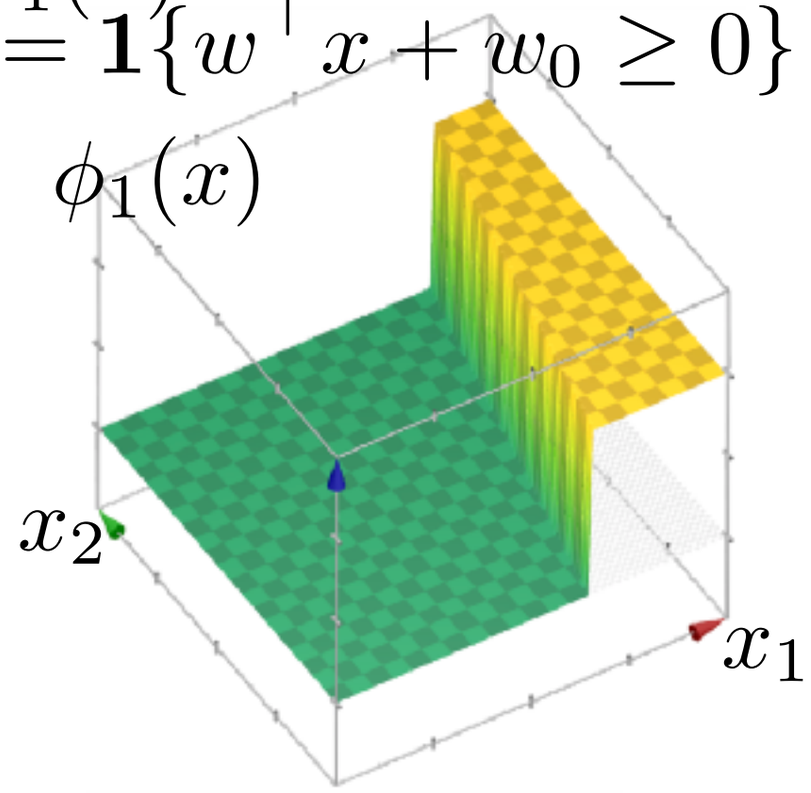


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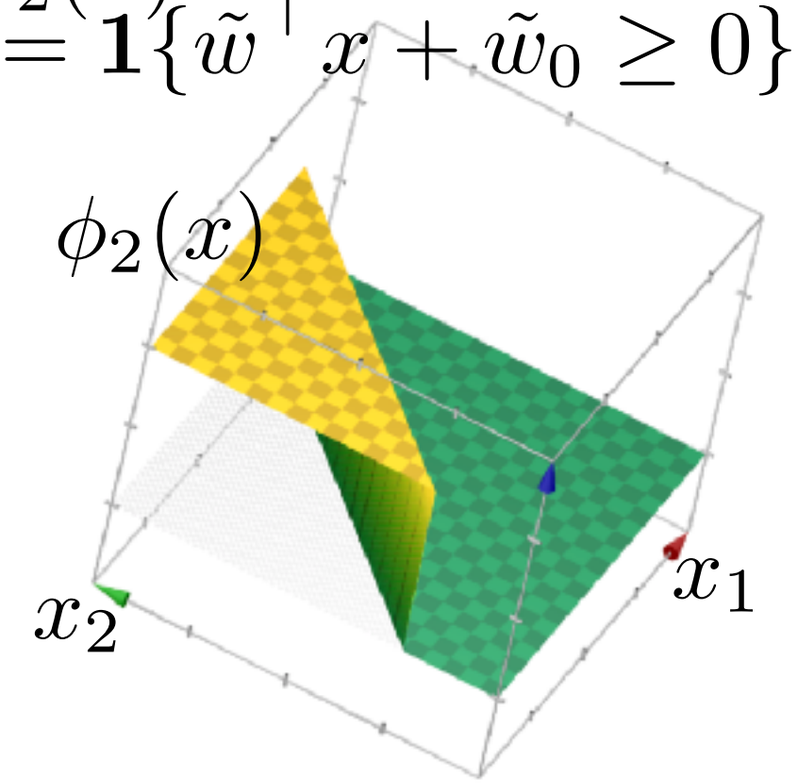


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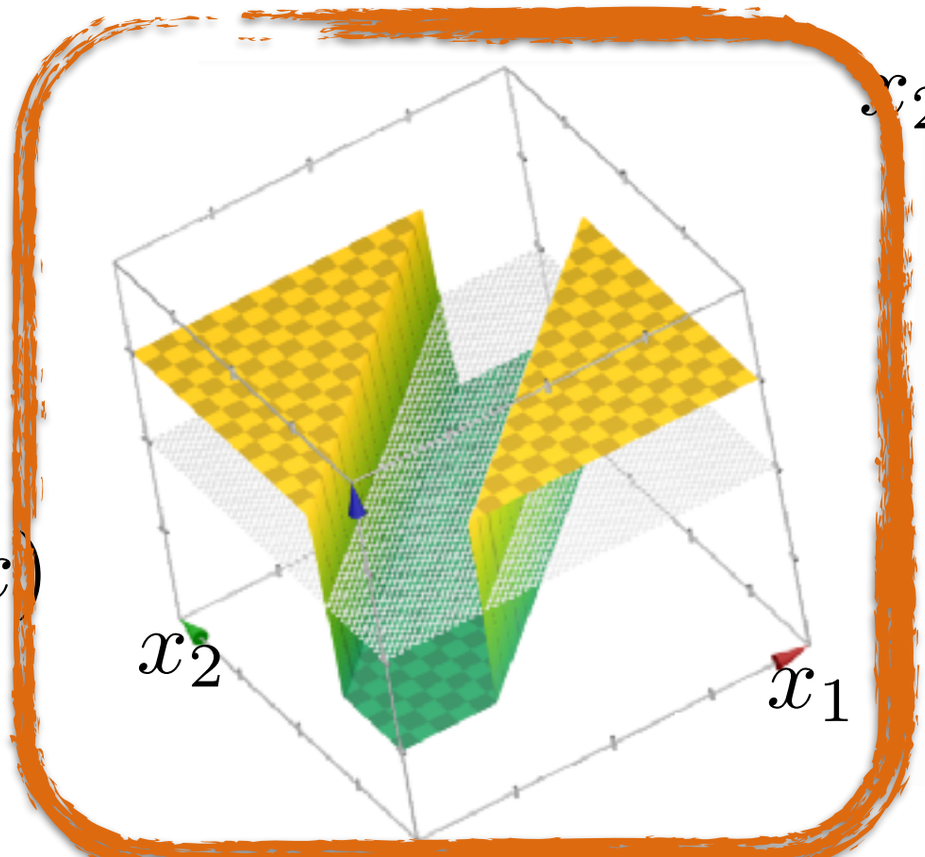


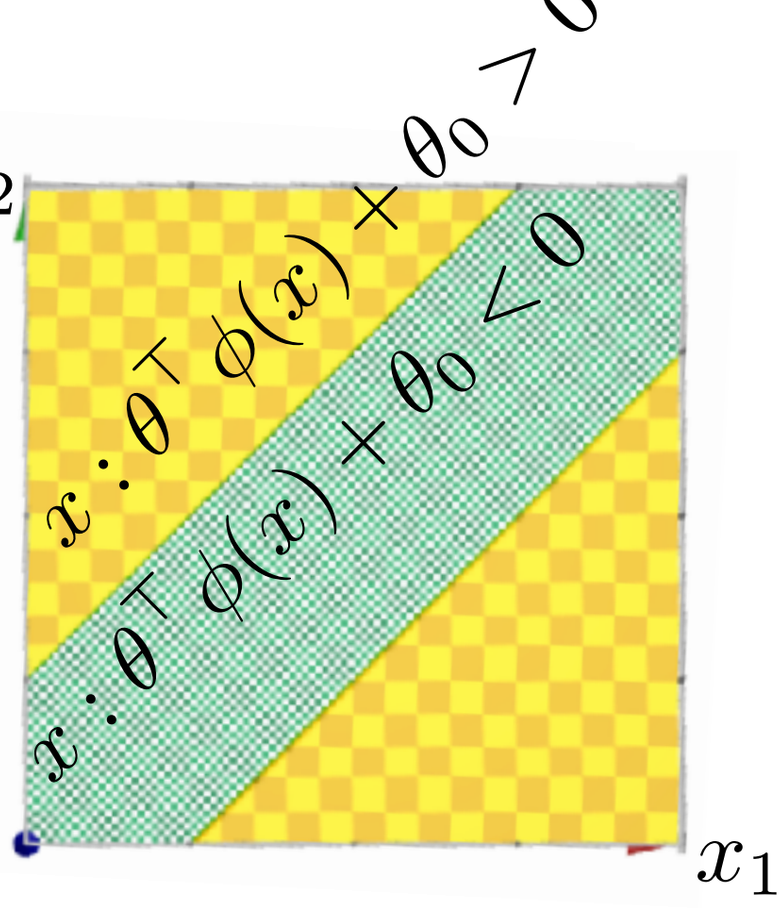

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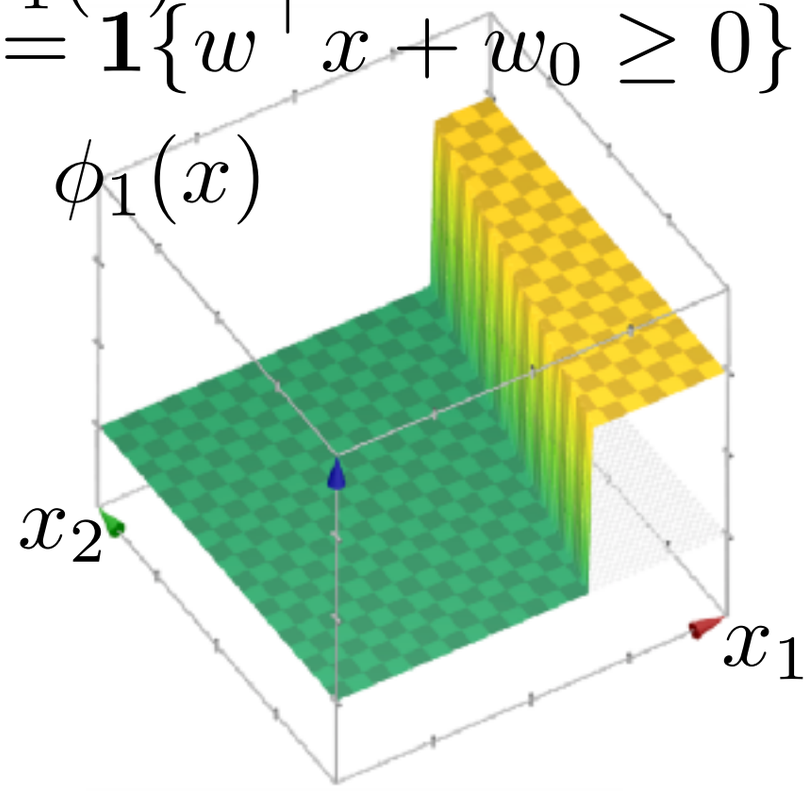


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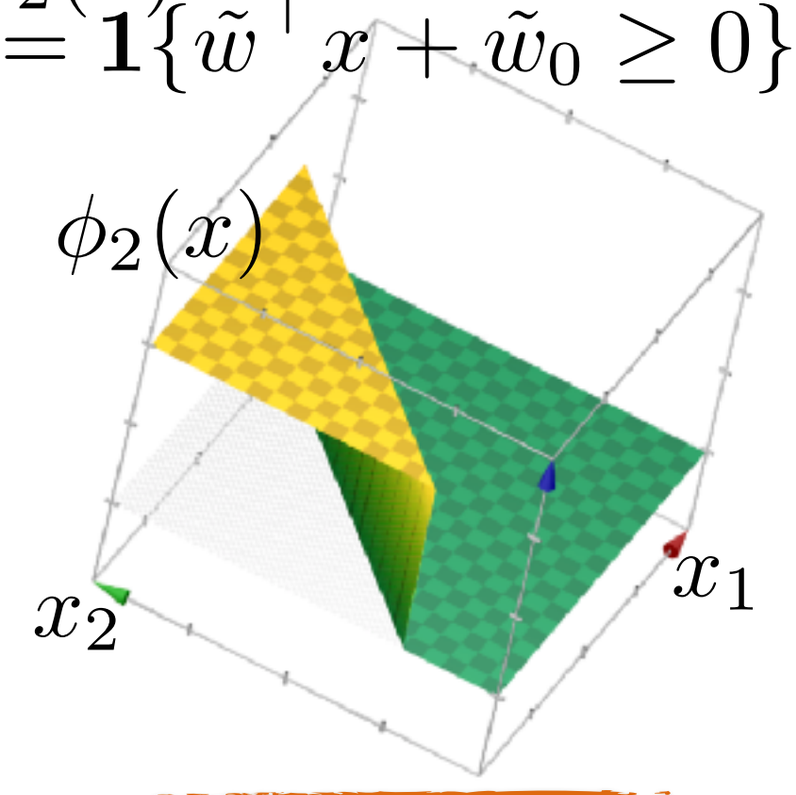


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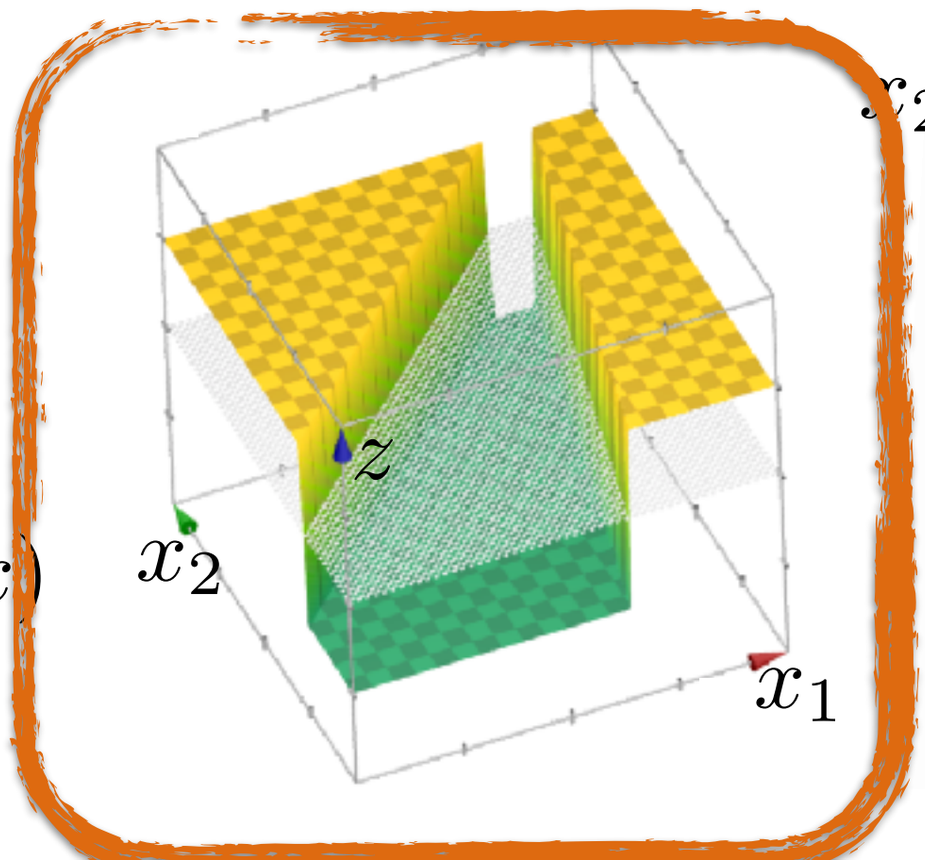


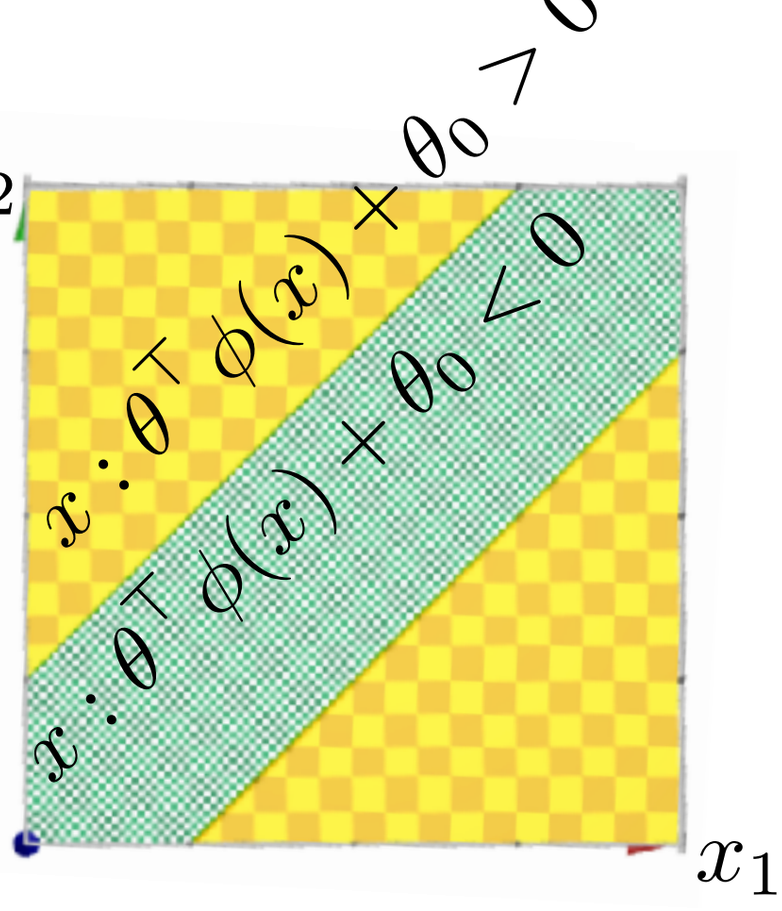

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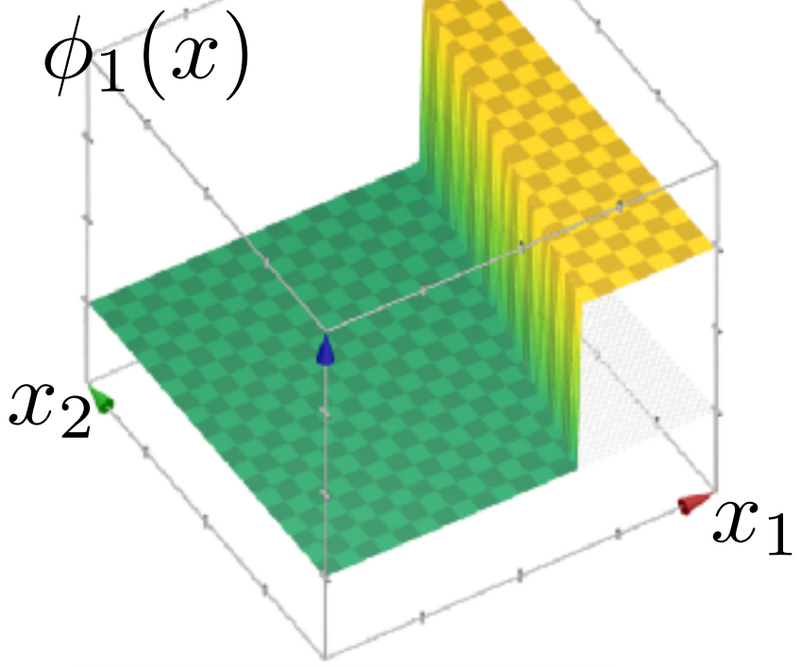


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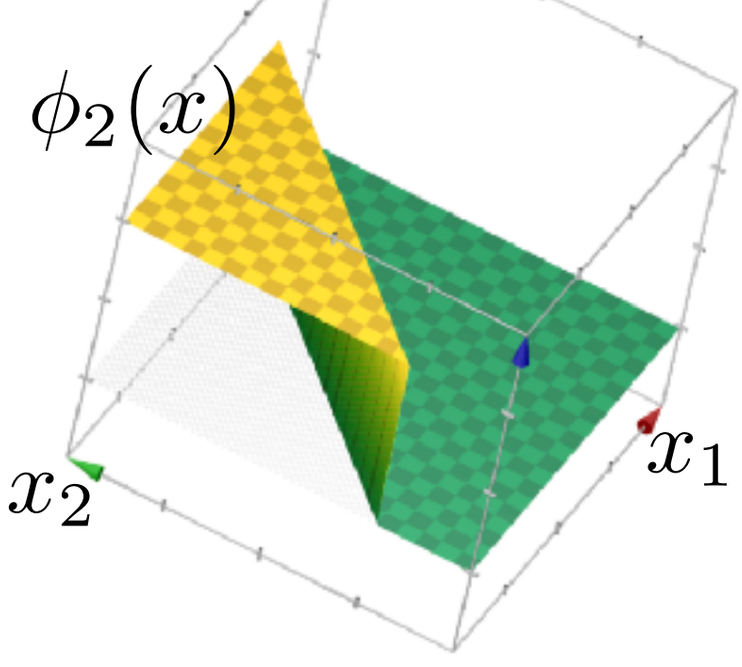


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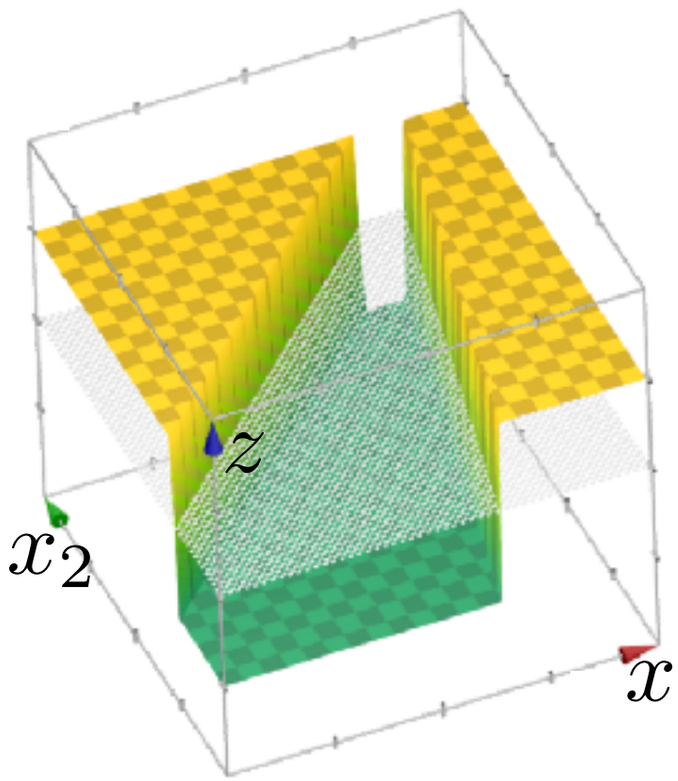


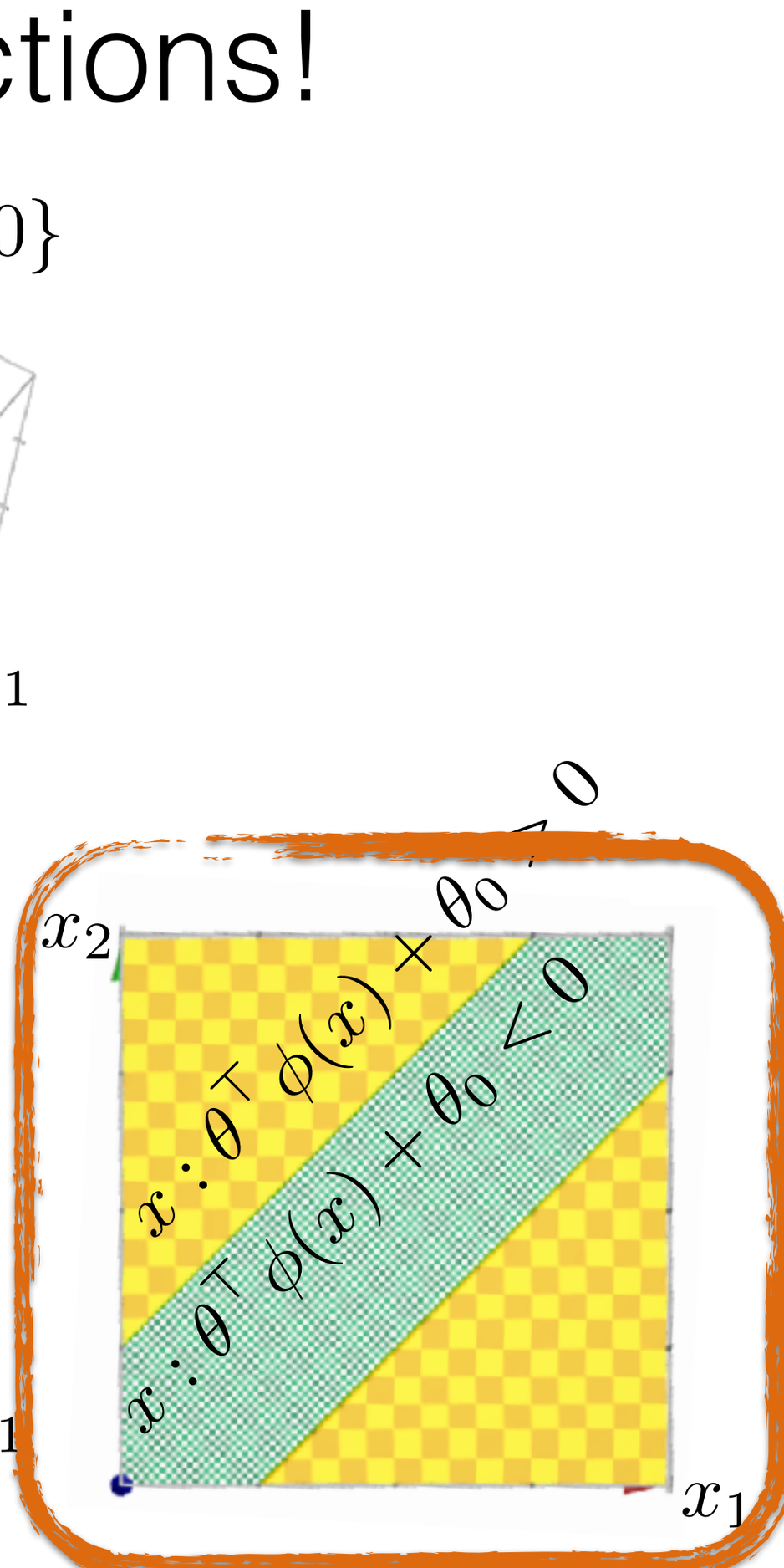

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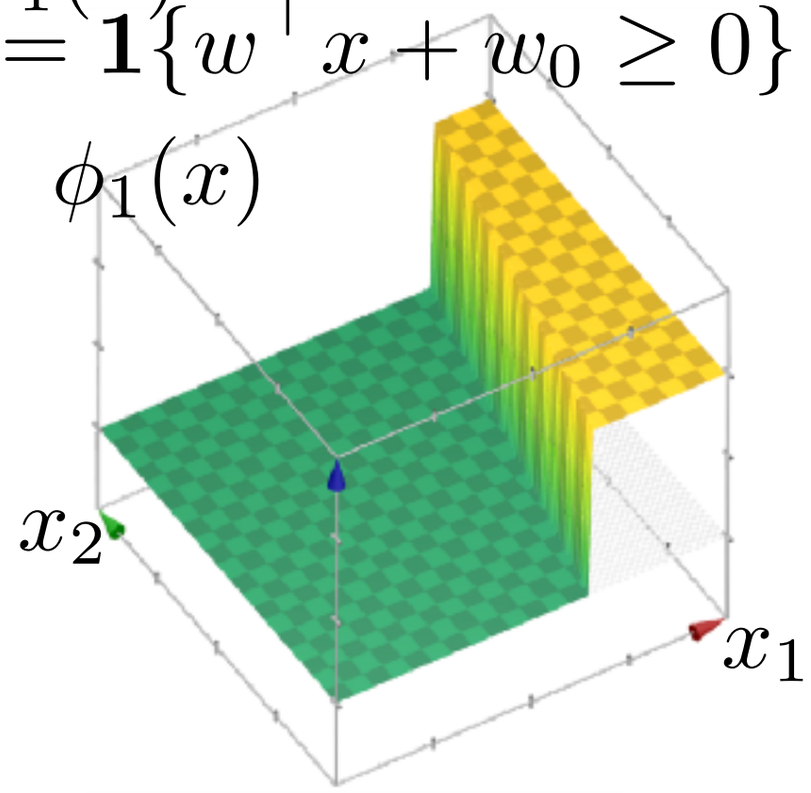


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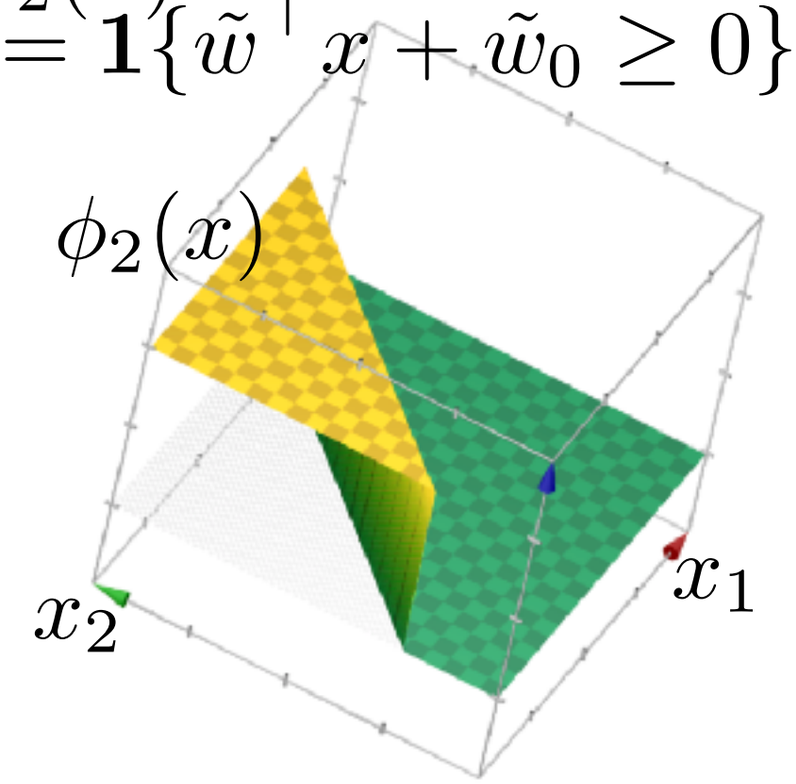


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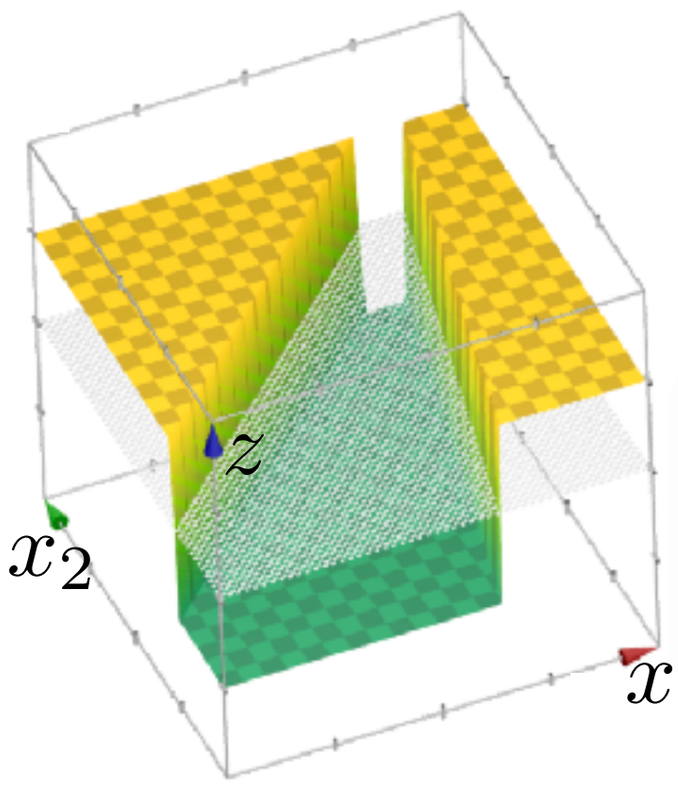
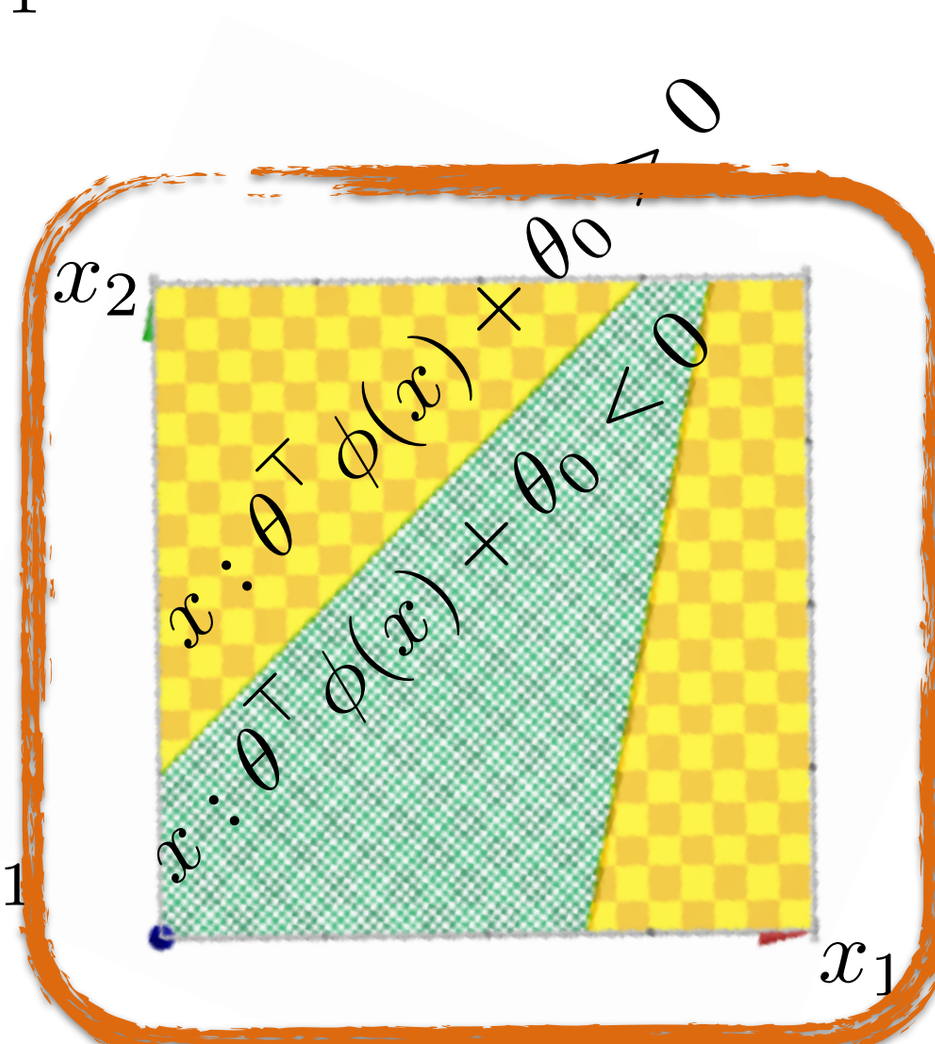
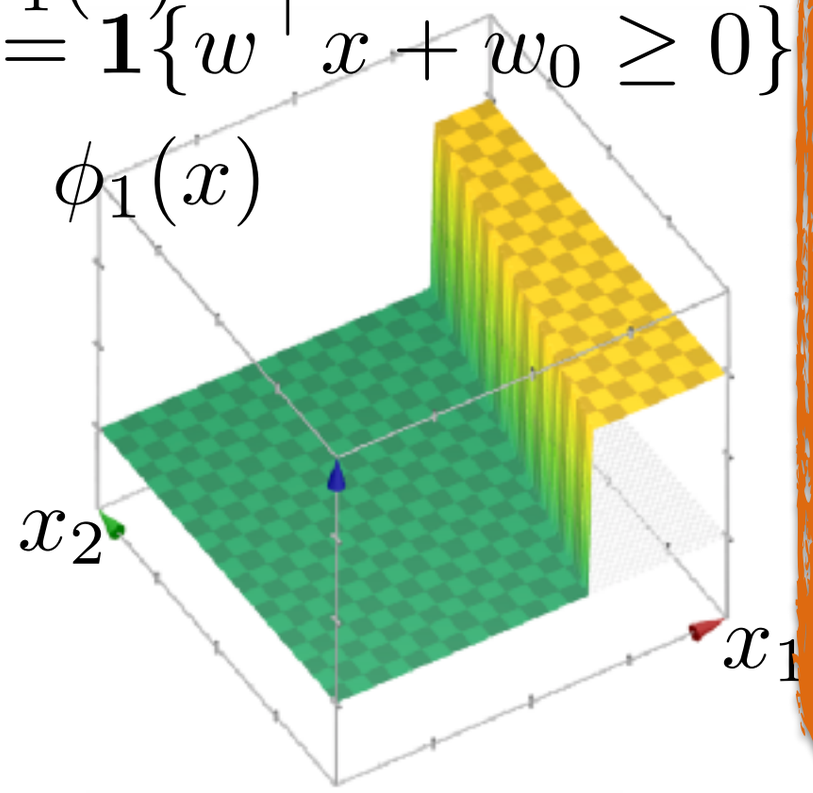


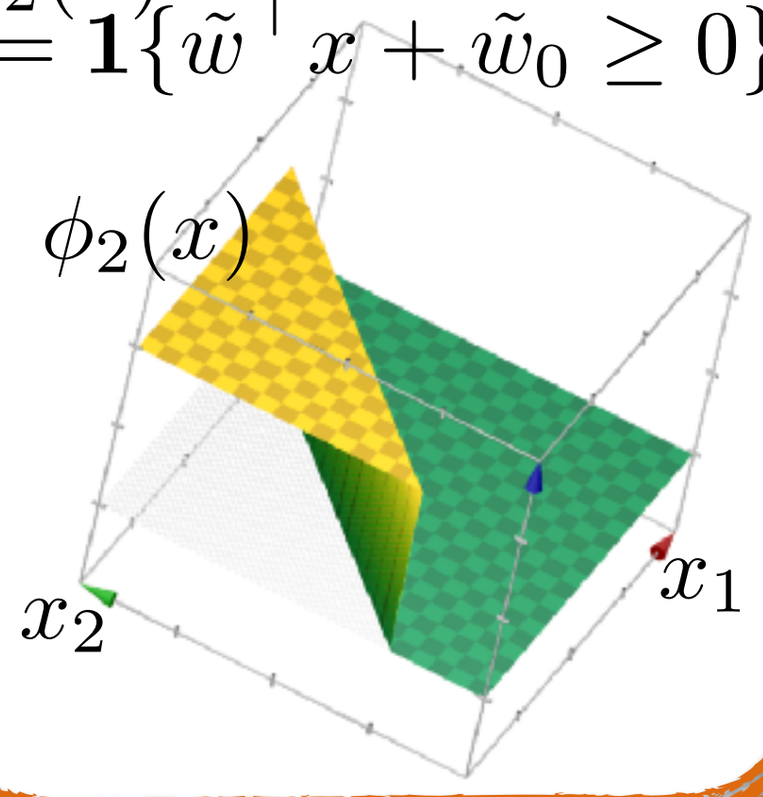



# New features: step functions!

$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$




$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$




$$z = \theta^\top \phi(x) + \theta_0$$
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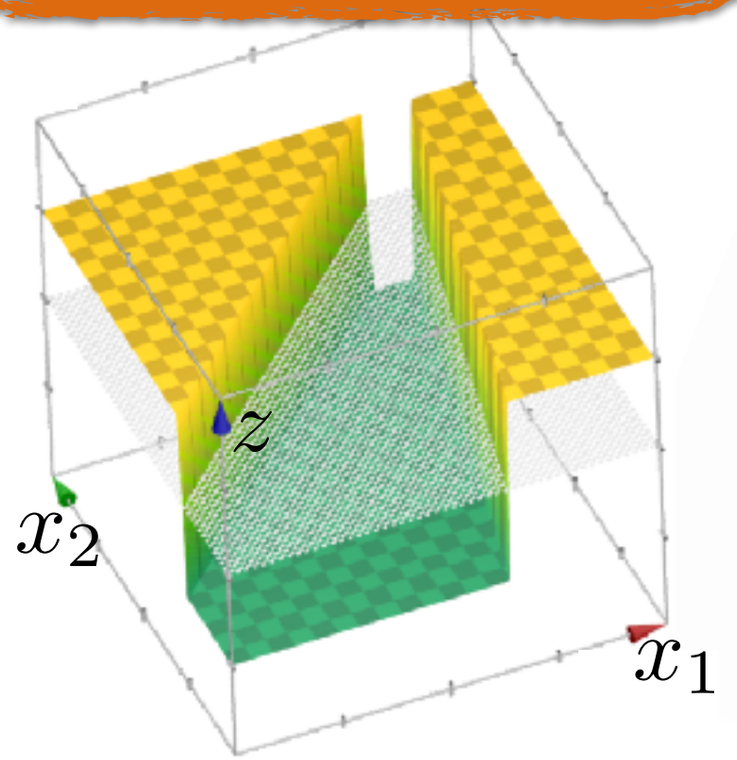


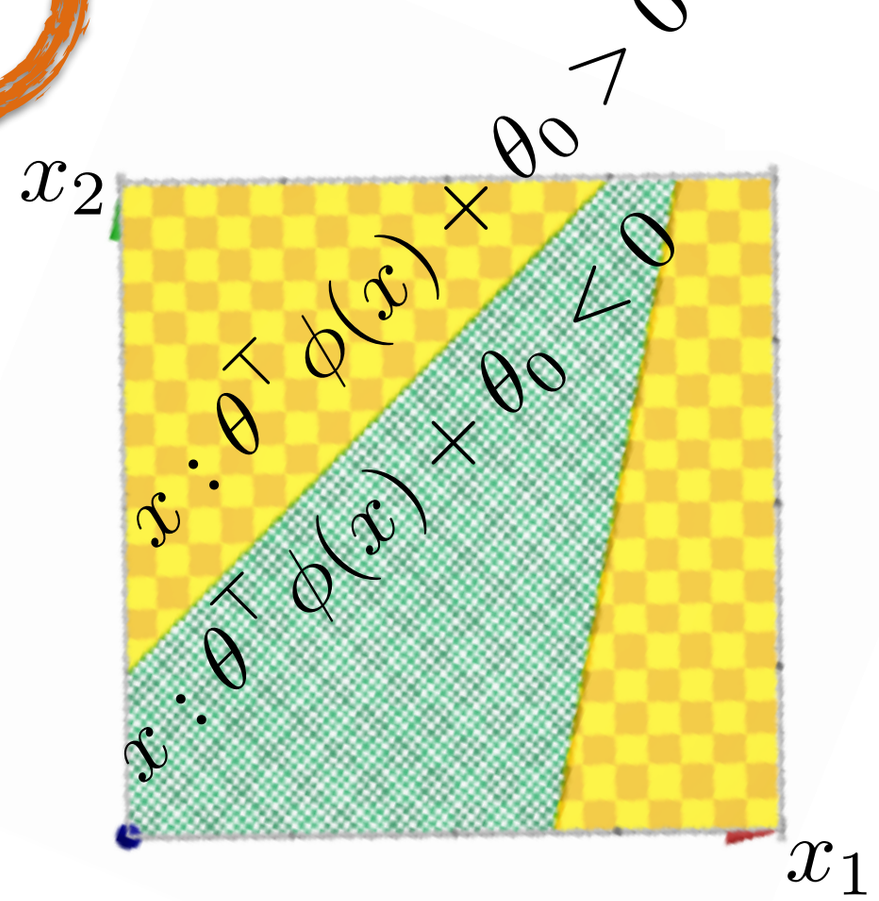

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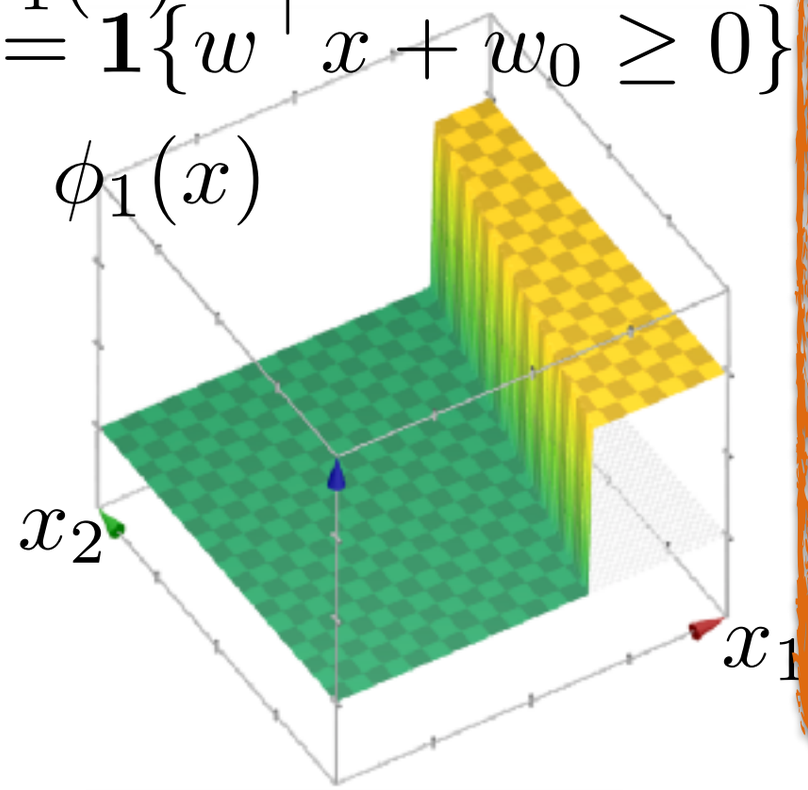
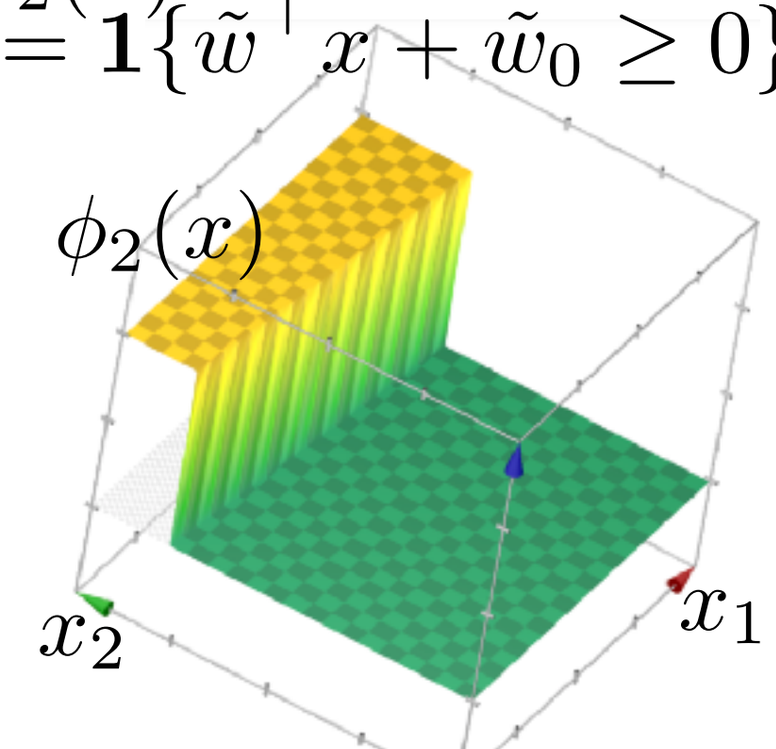



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

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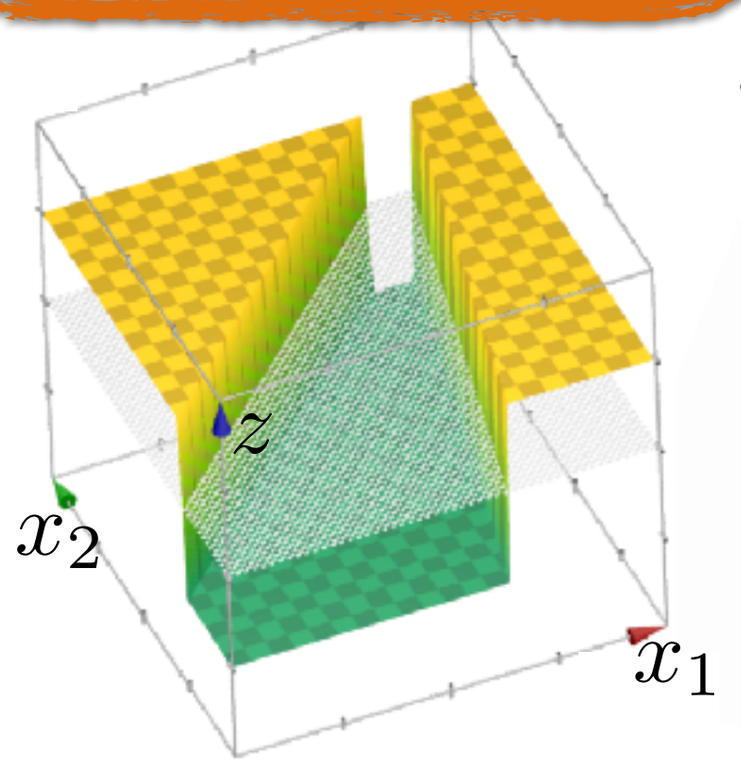


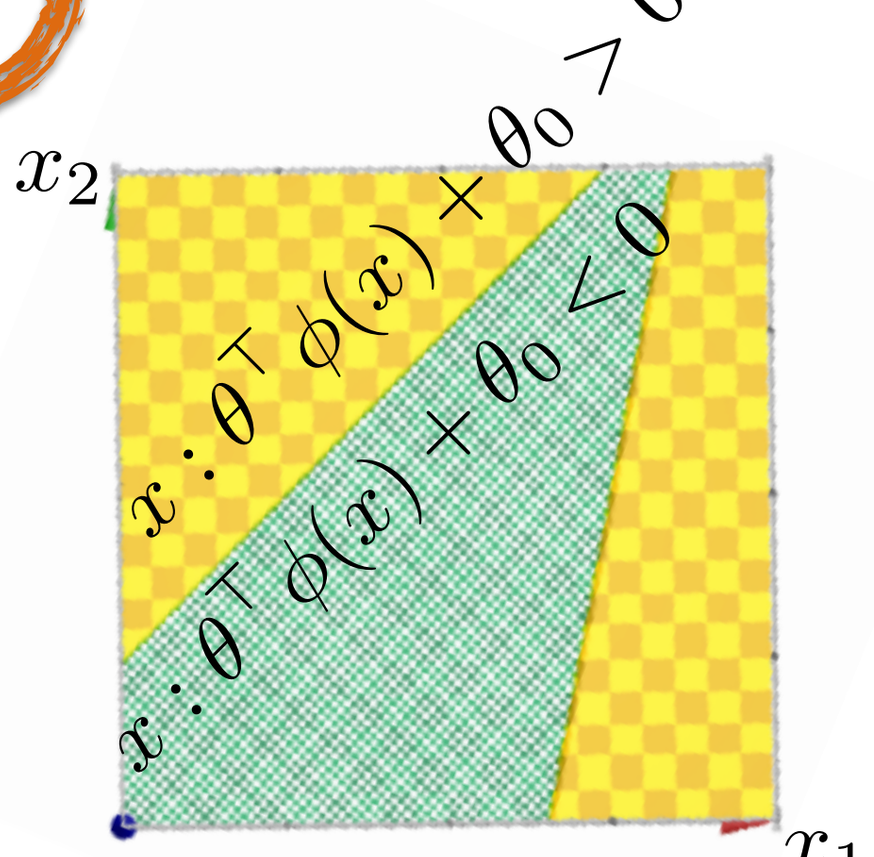

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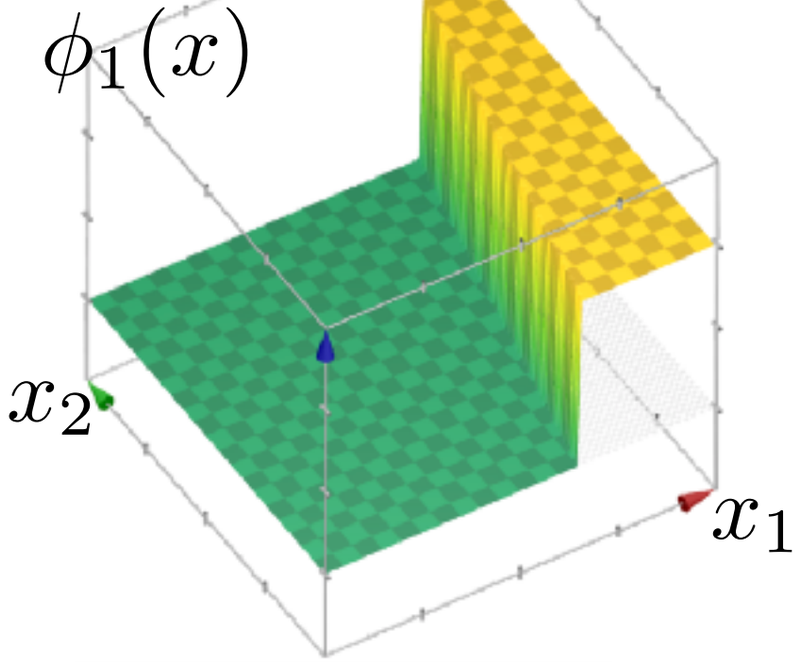


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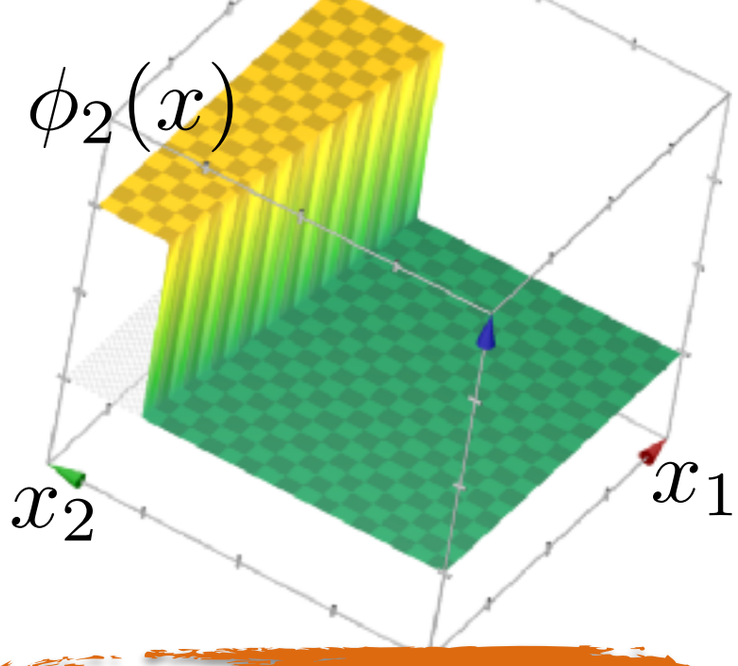


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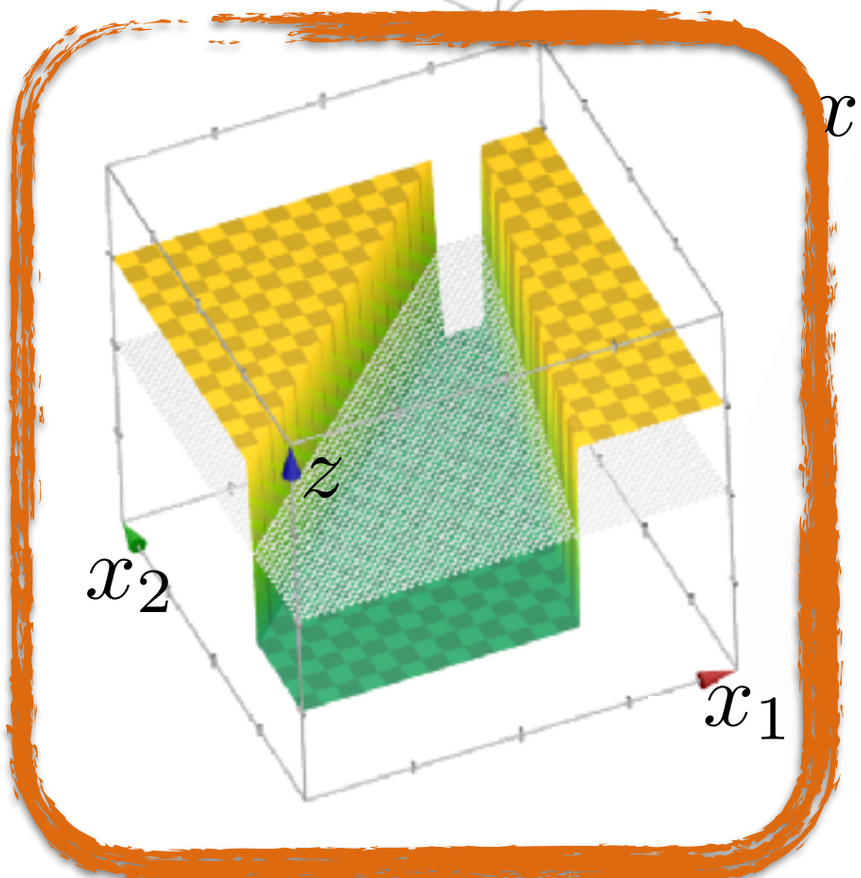


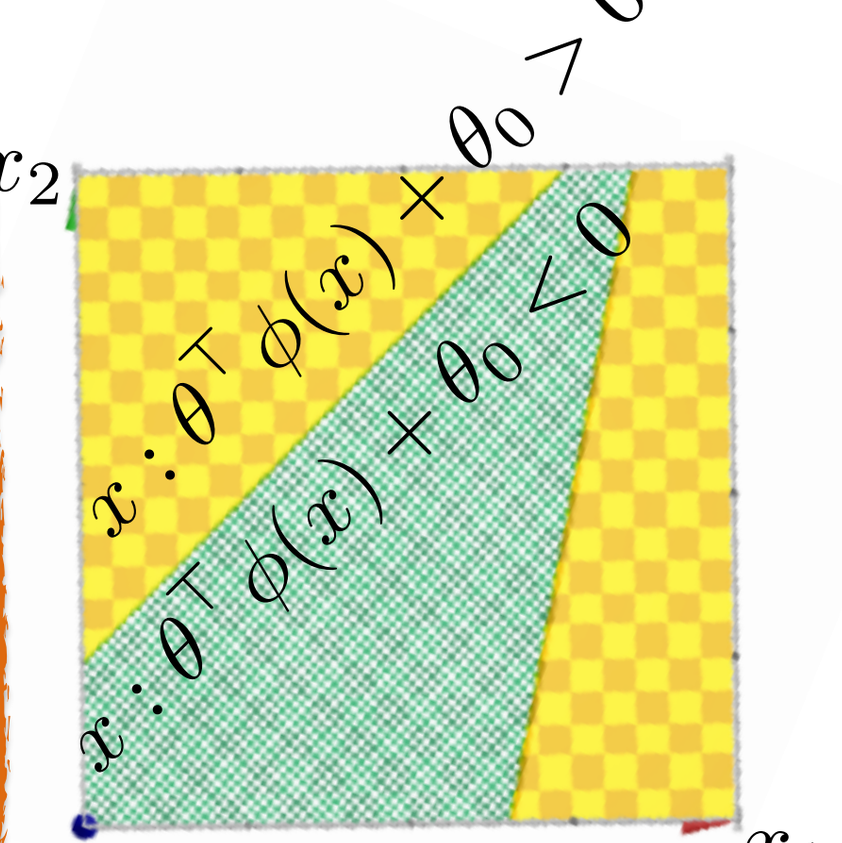

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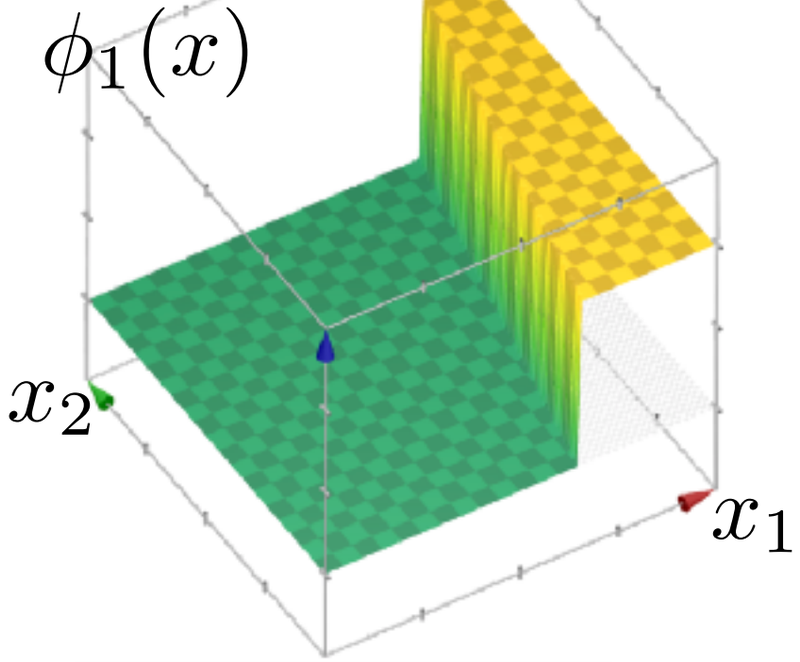


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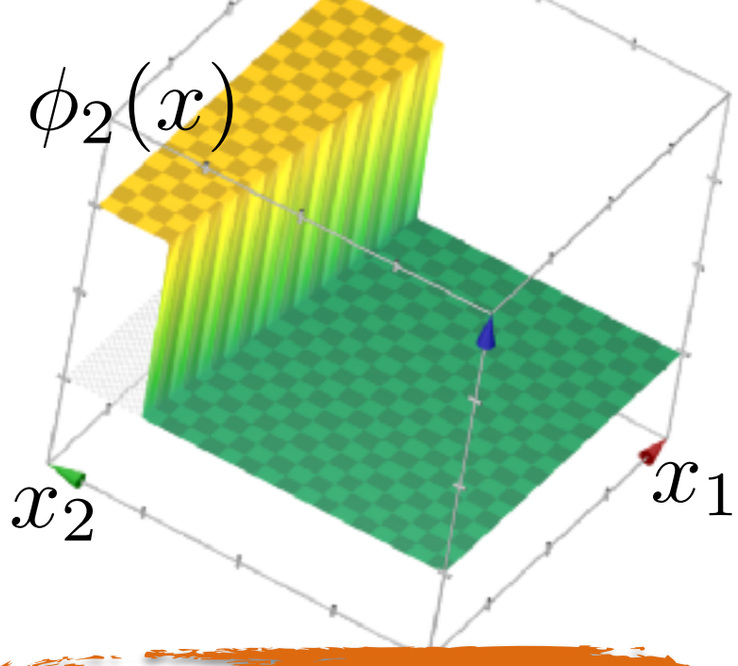


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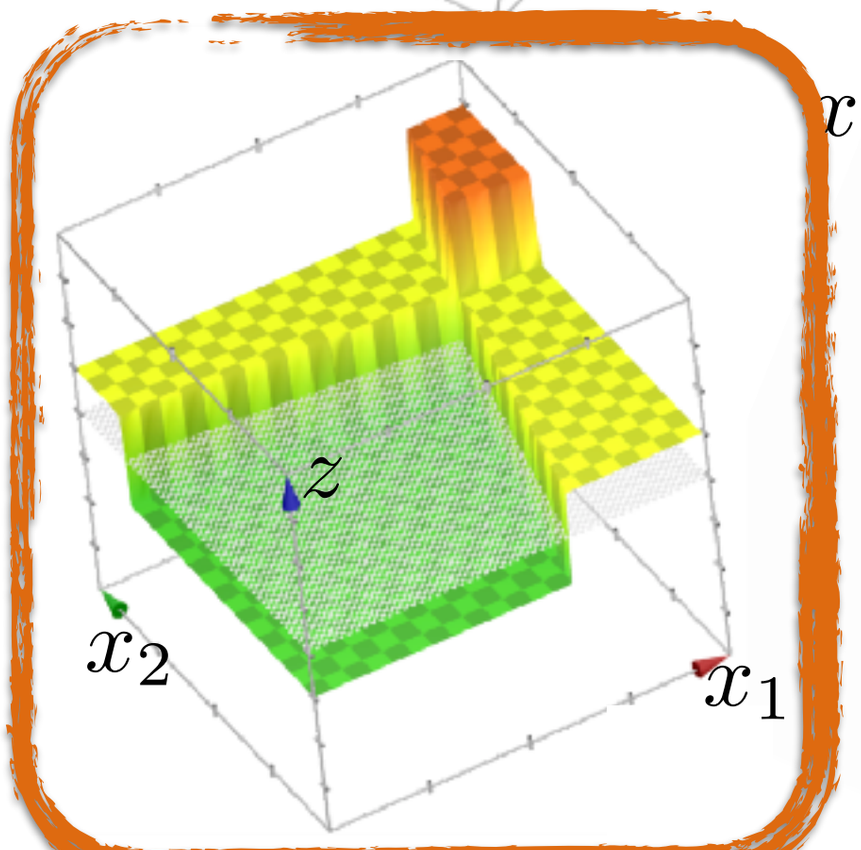


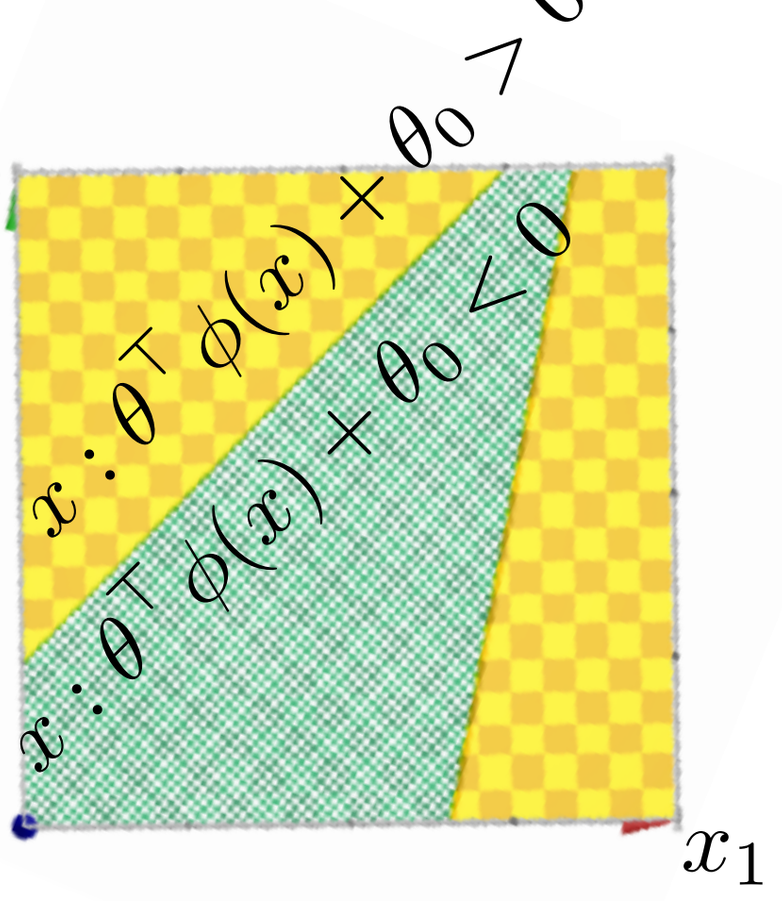

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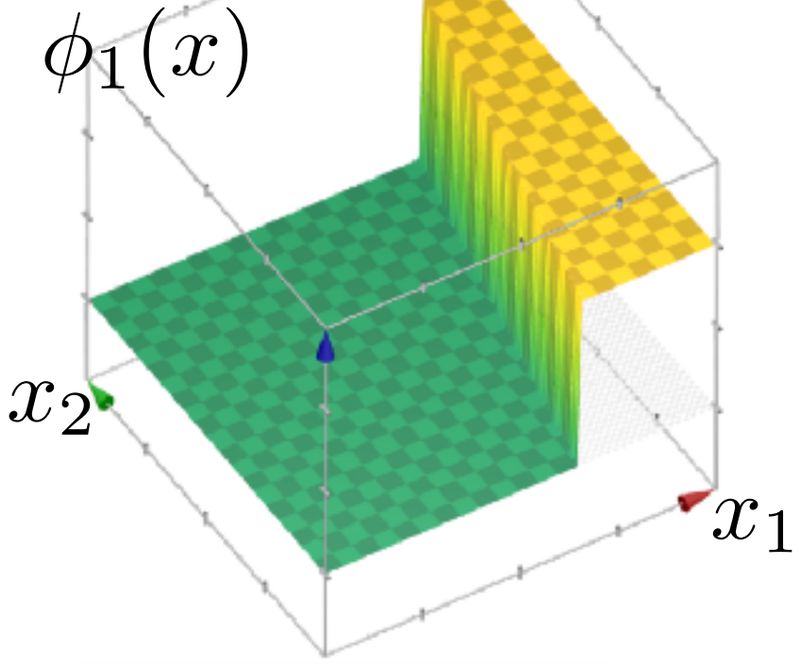


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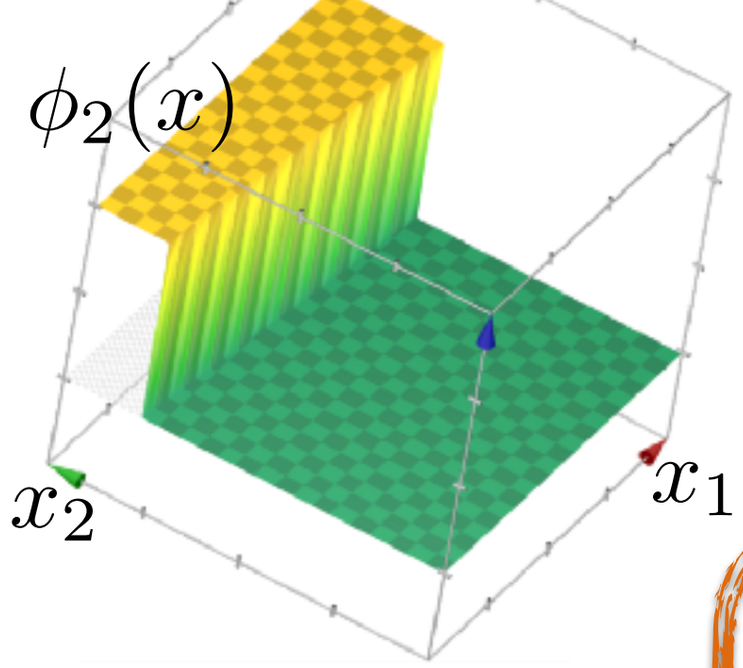


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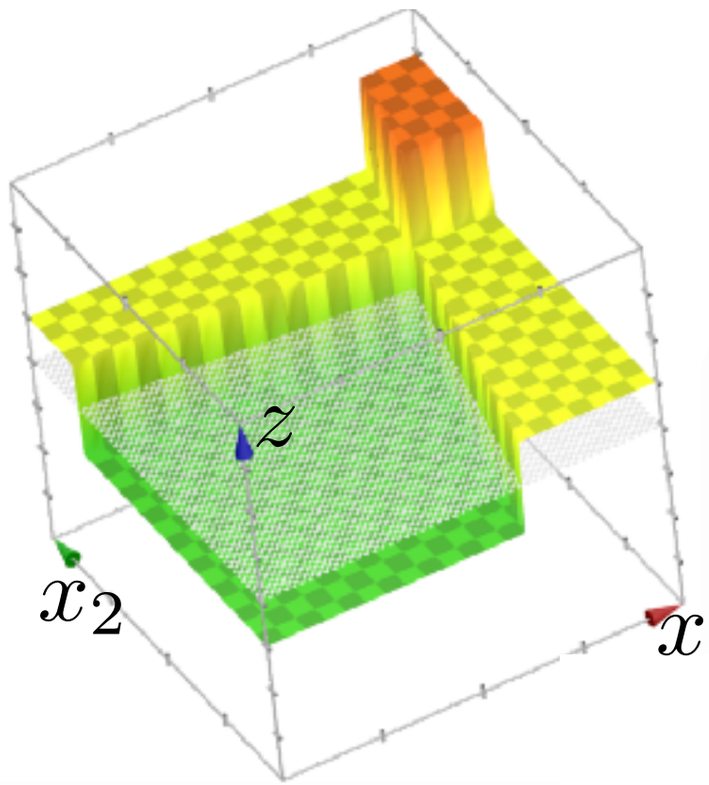


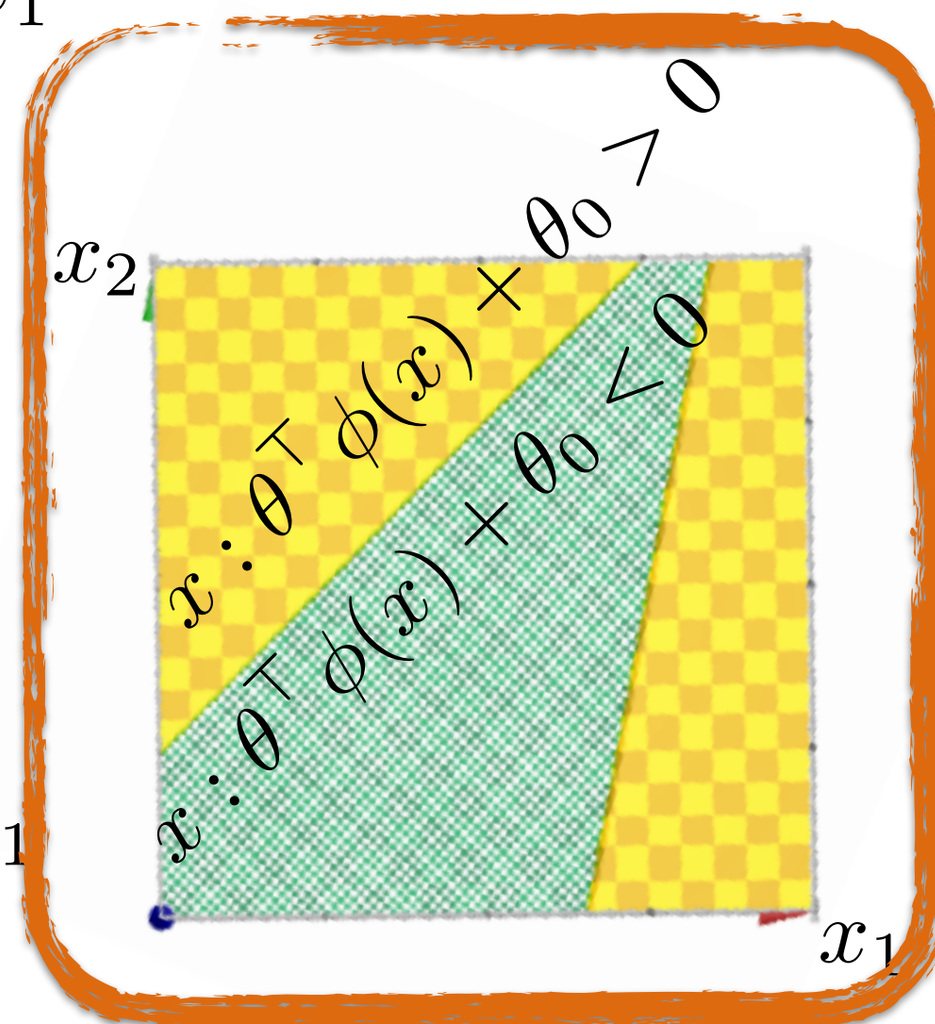

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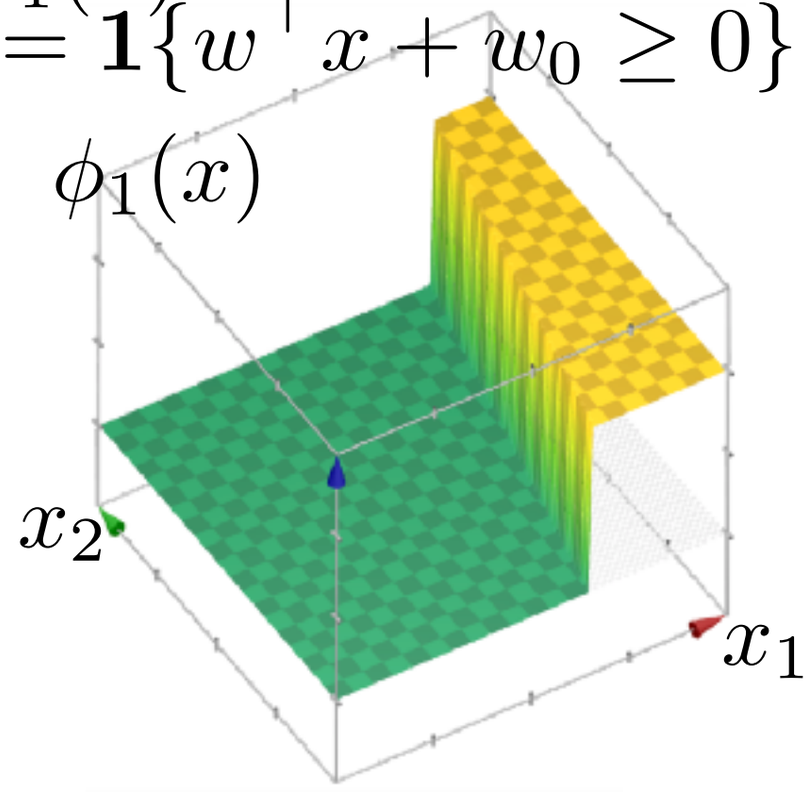


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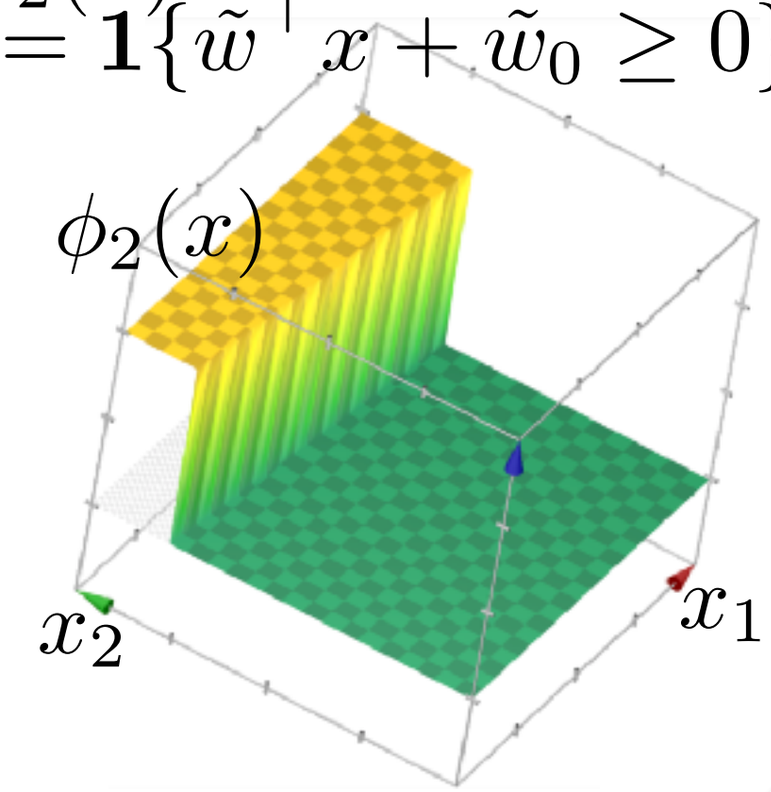


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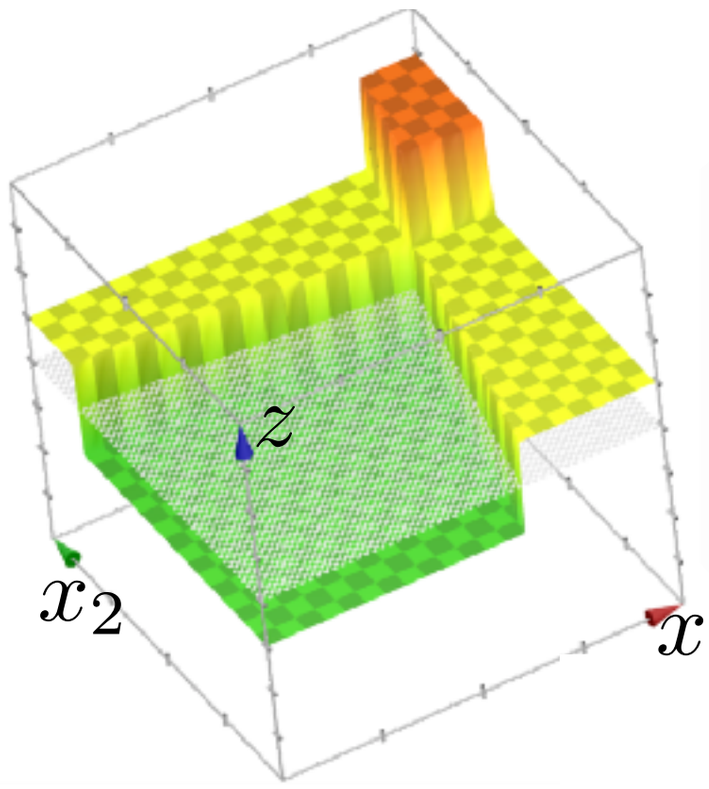


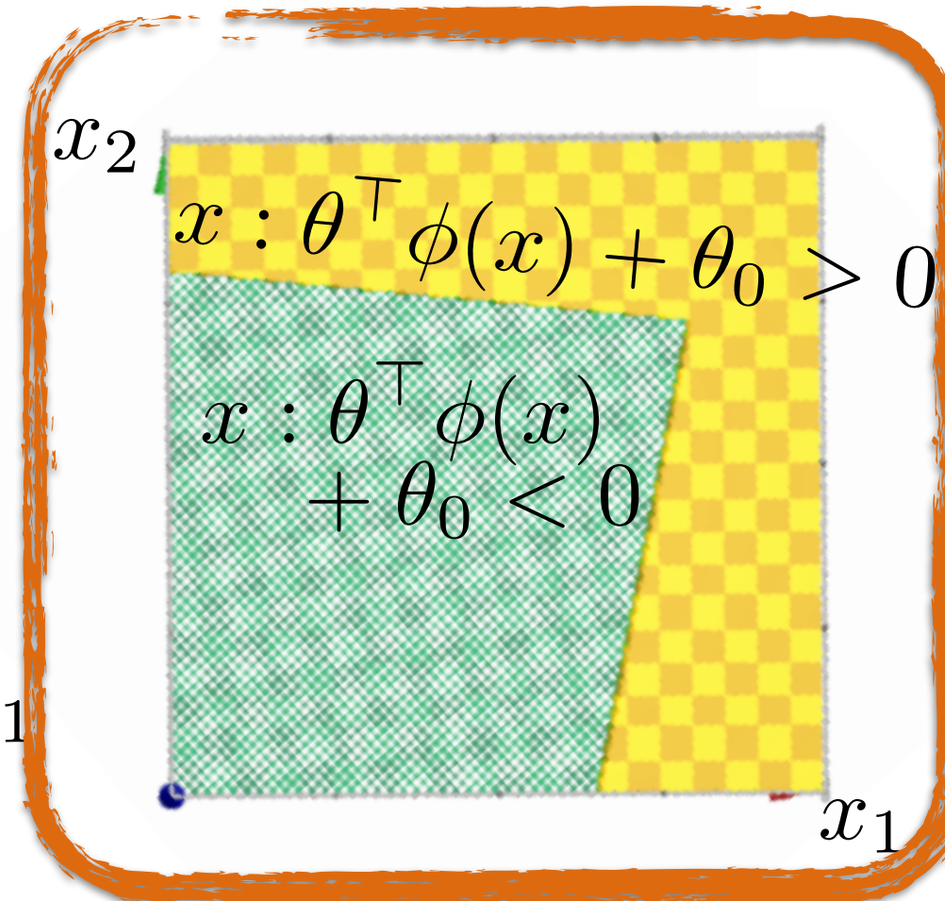

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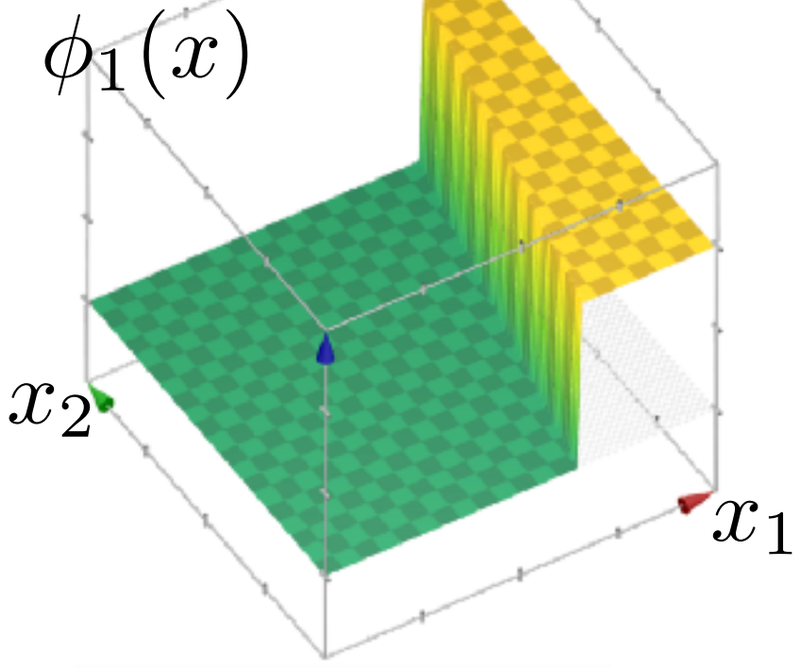


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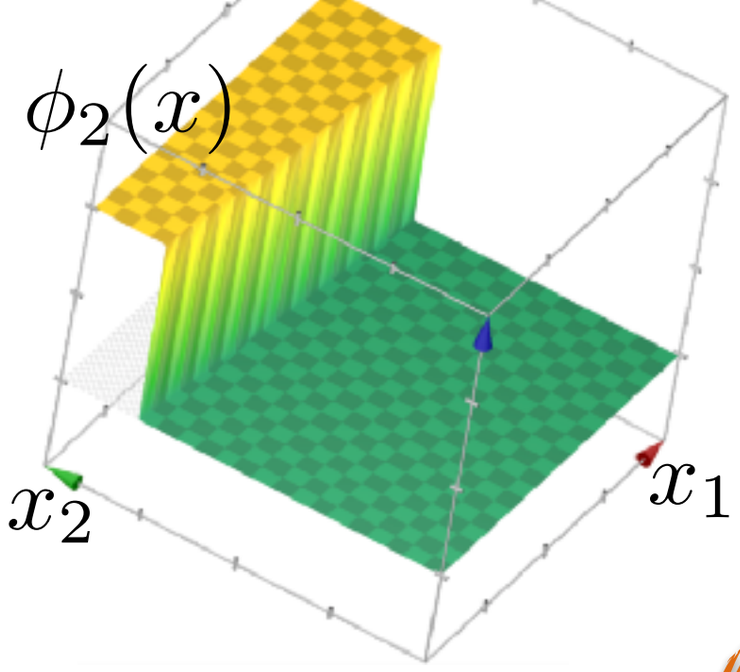


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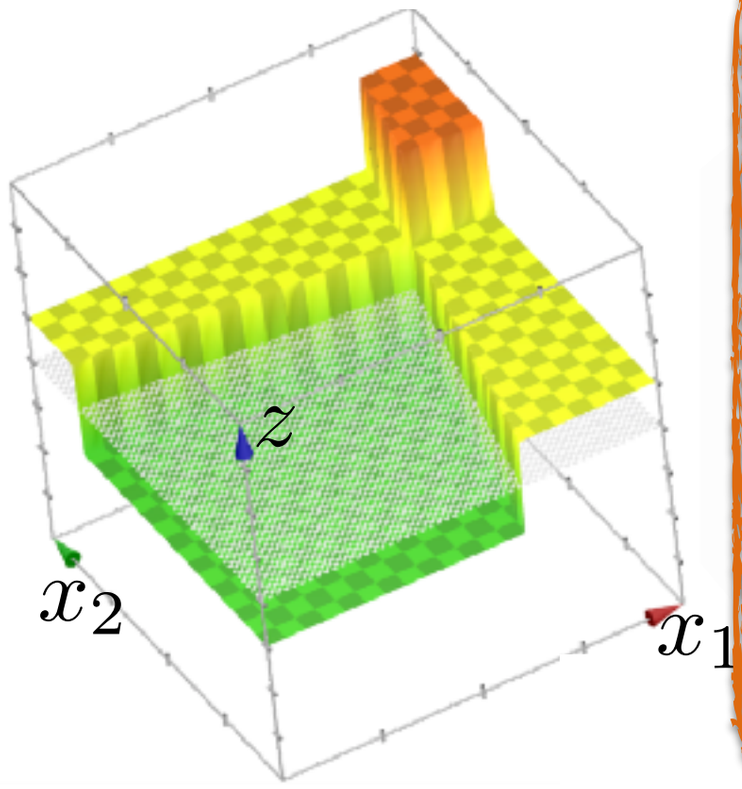


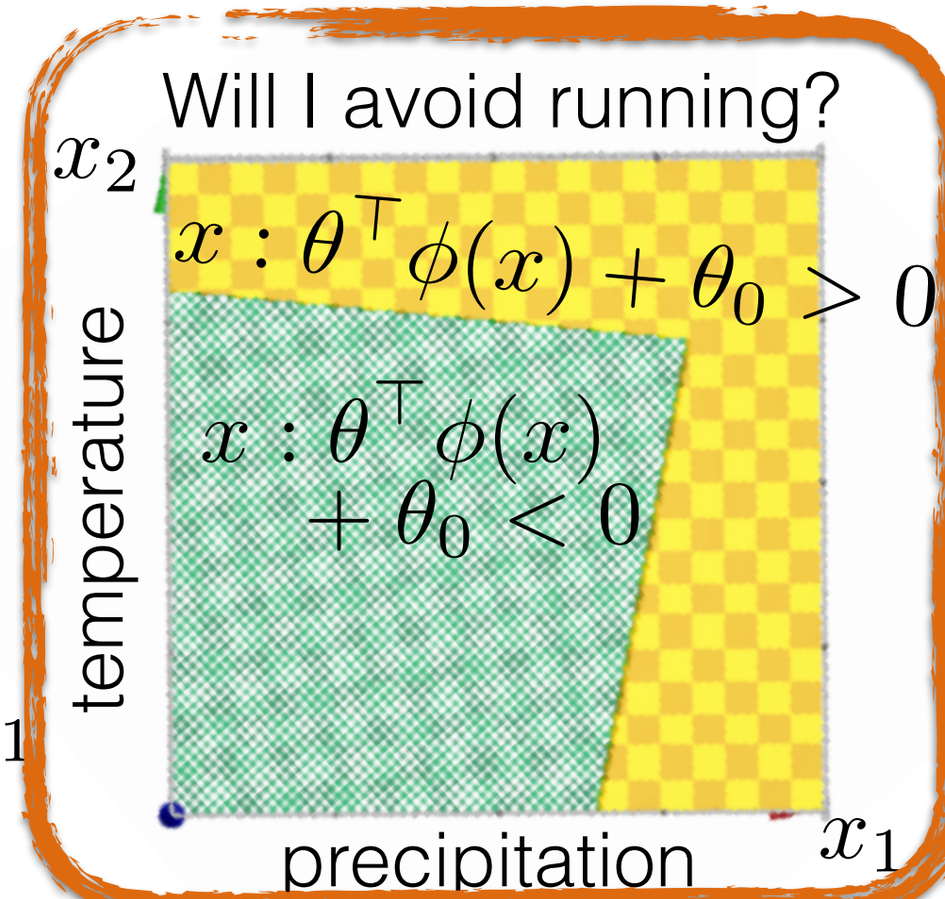

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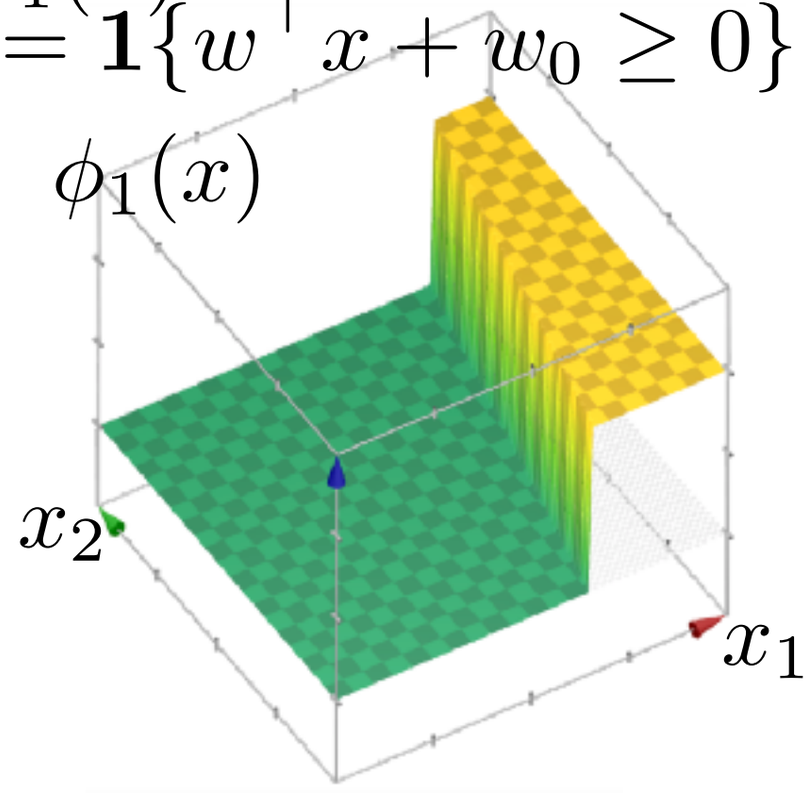


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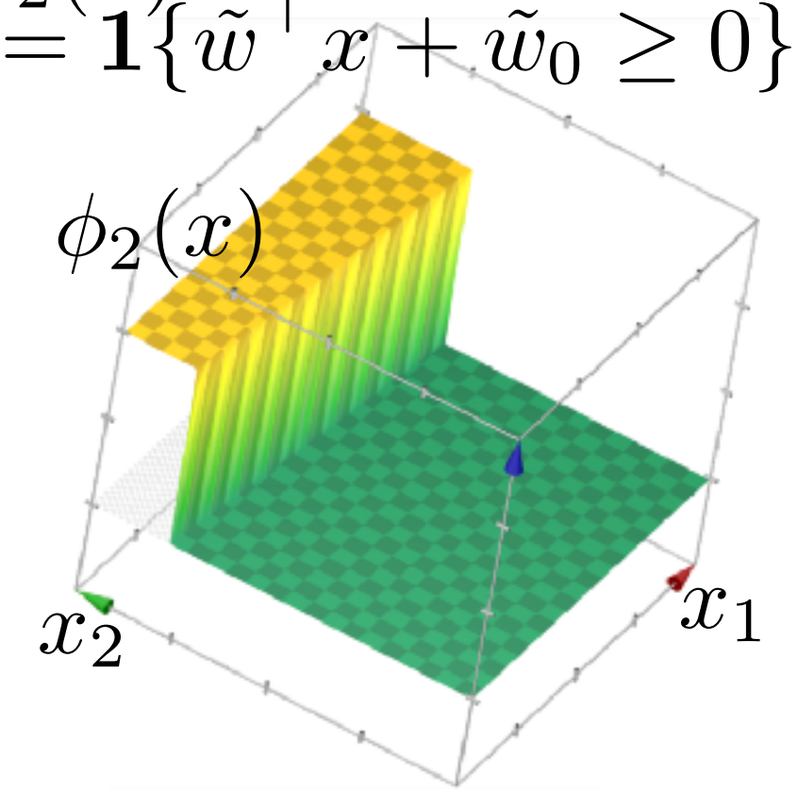


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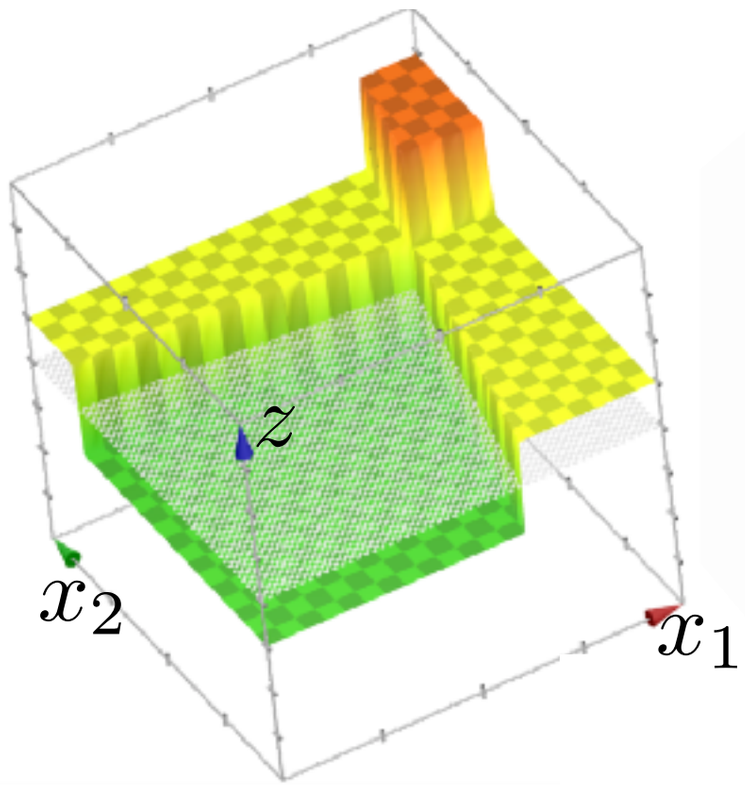


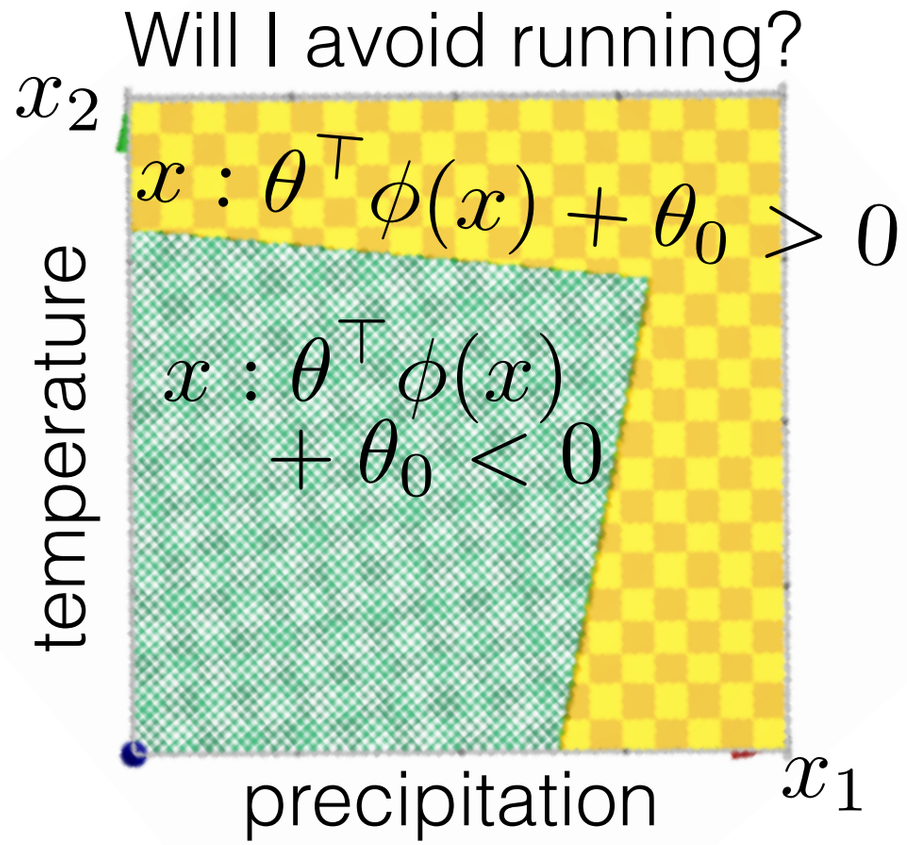

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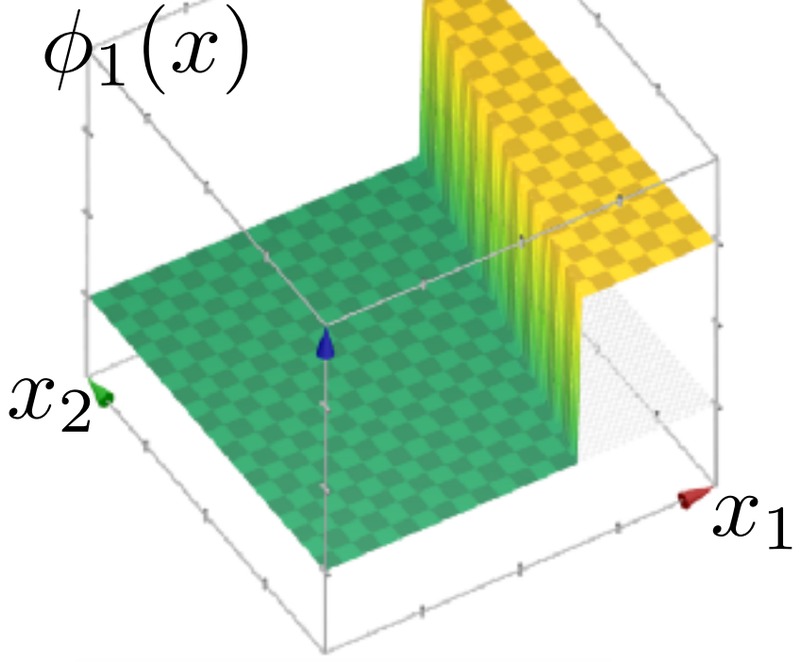


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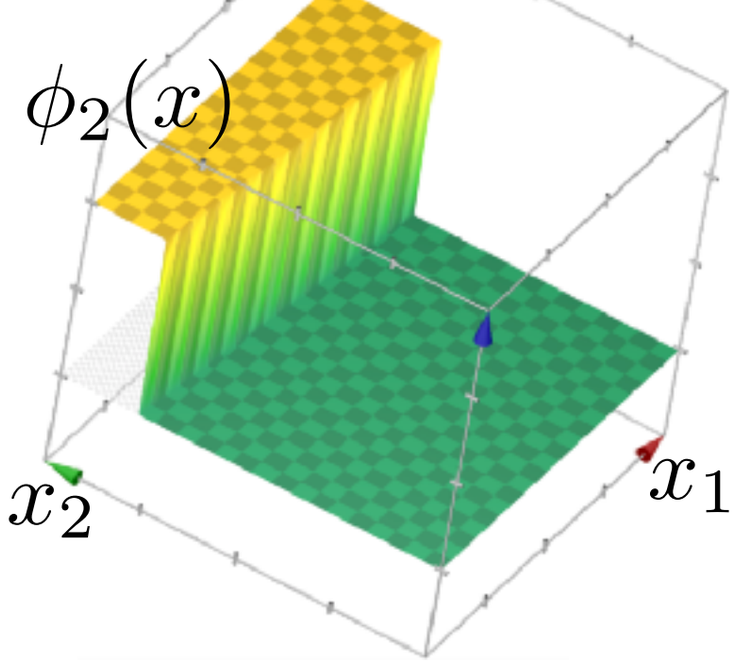
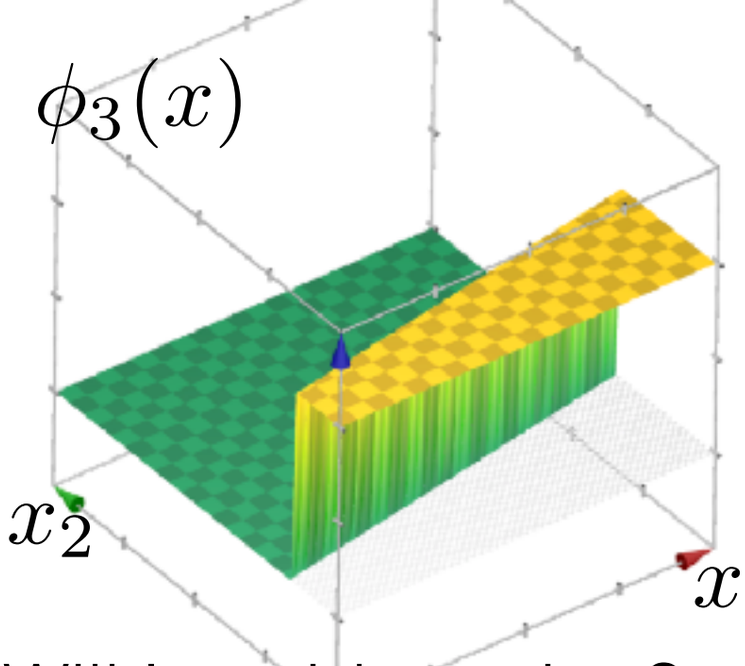



$$\phi_3(x) = \mathbf{1}\{\tilde{\tilde{w}}^\top x + \tilde{\tilde{w}}_0 \geq 0\}$$

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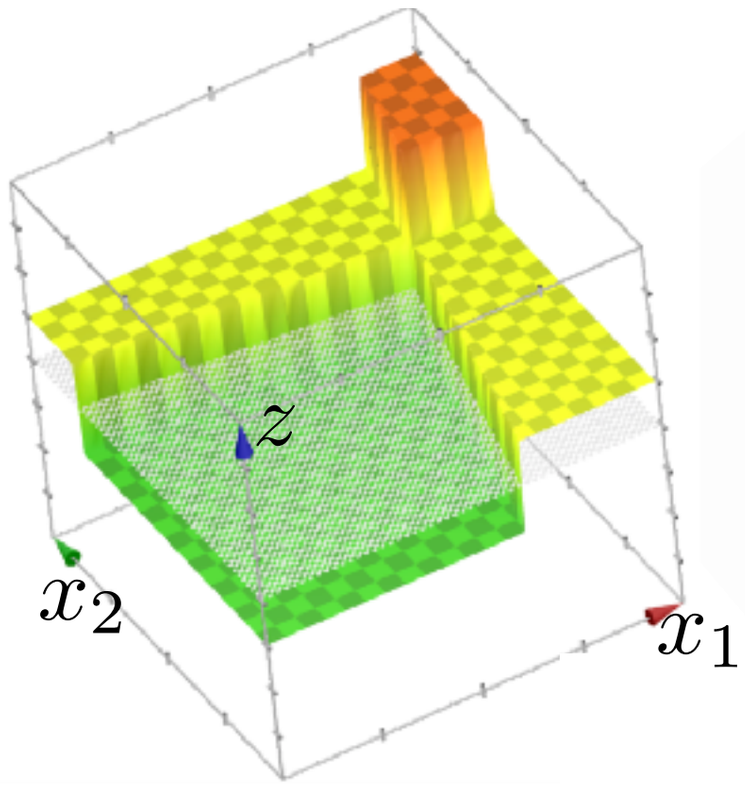
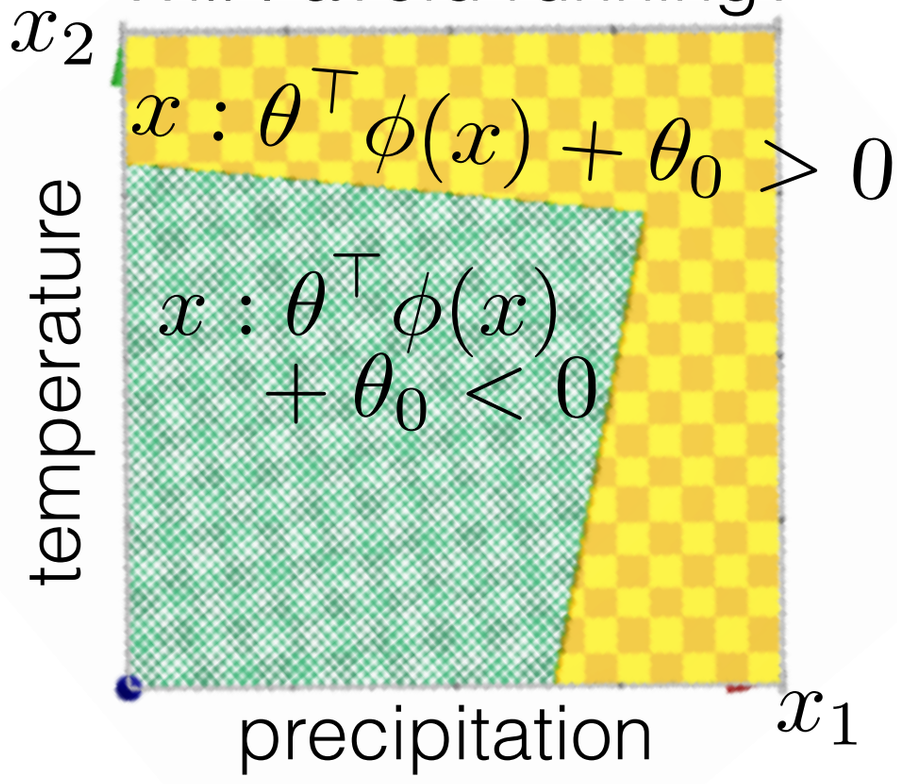



Will I avoid running?



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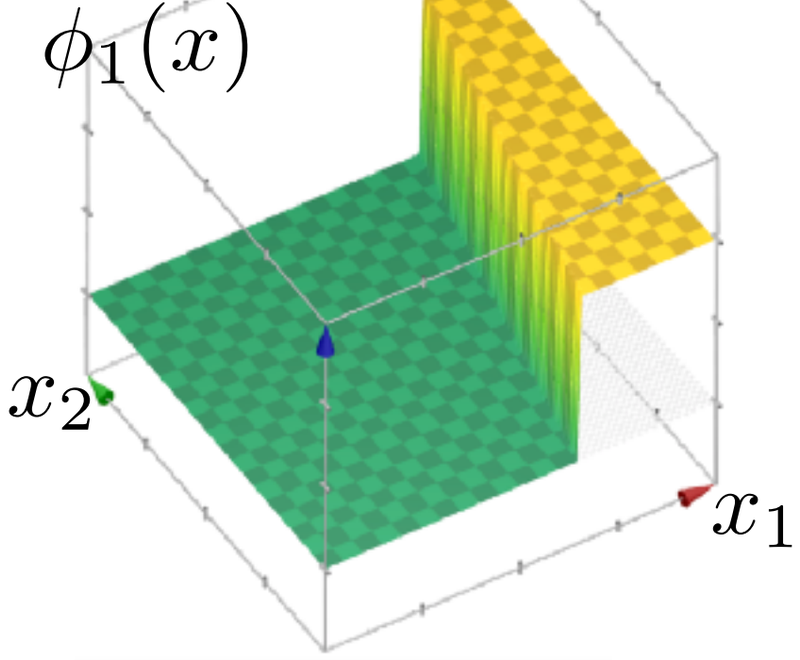


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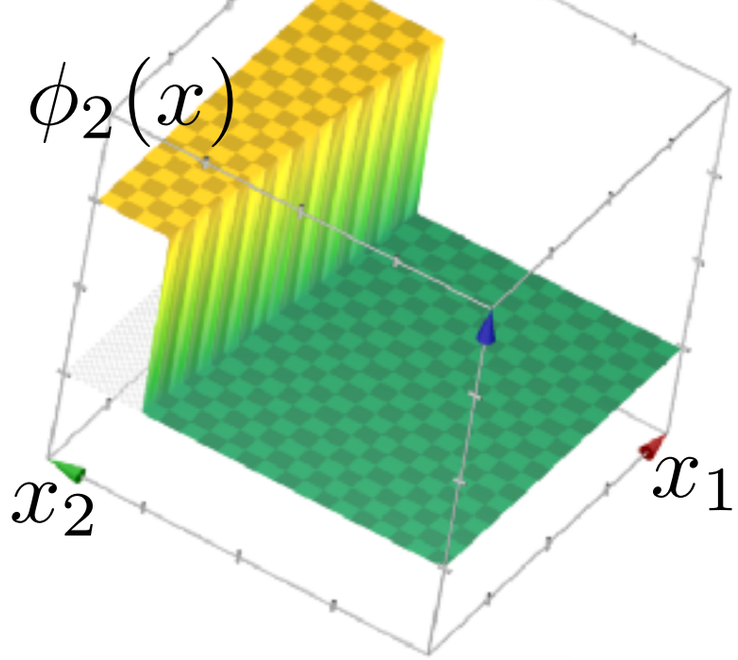


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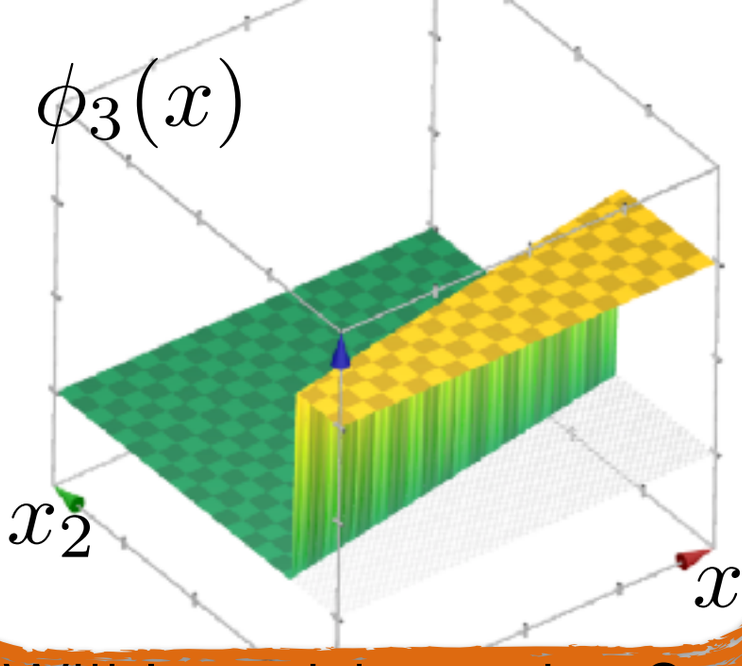


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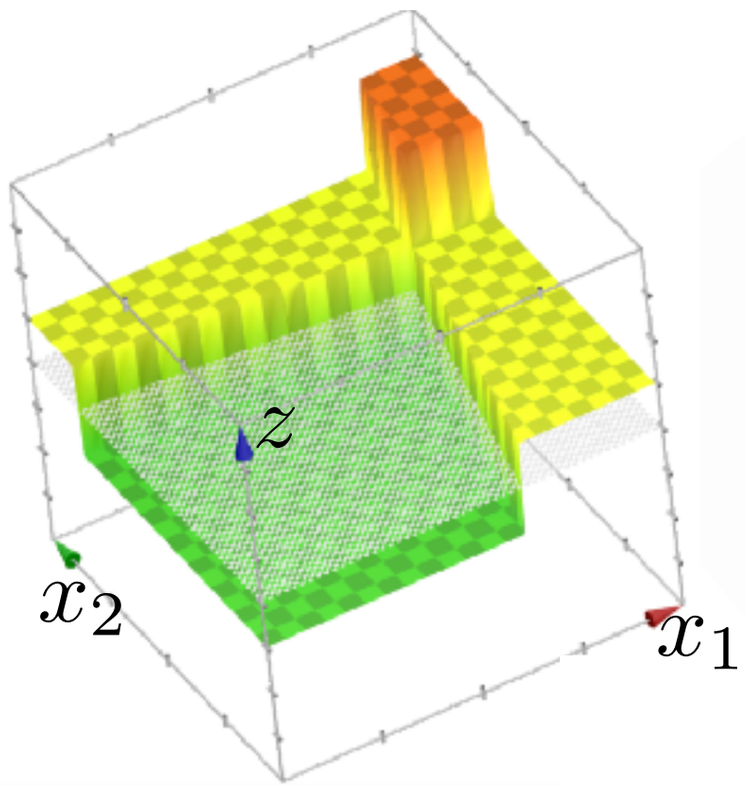


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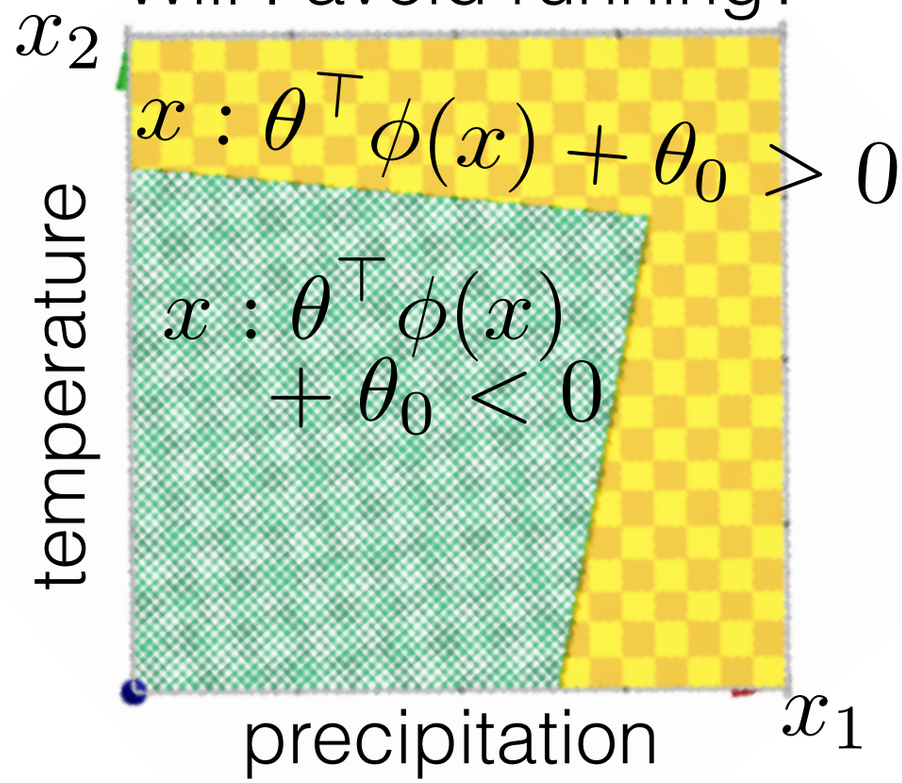

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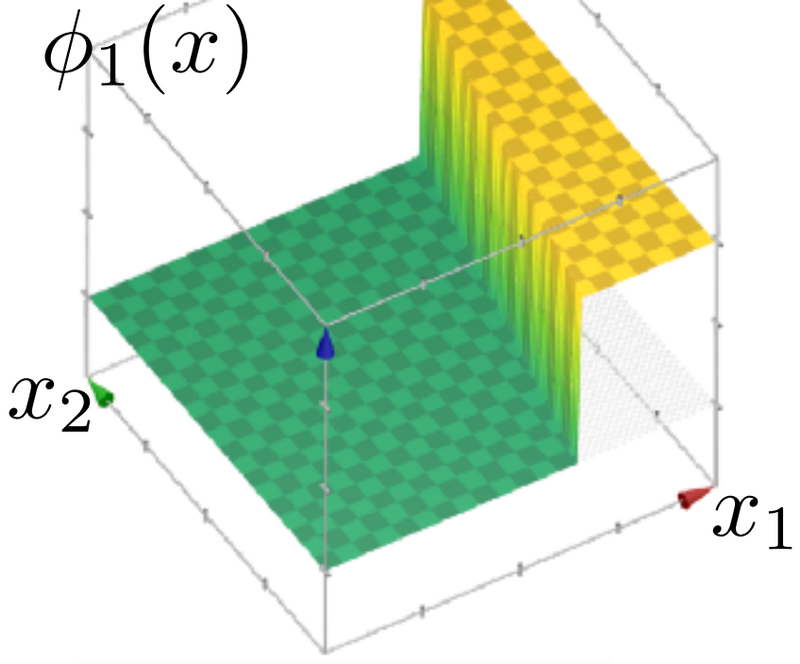


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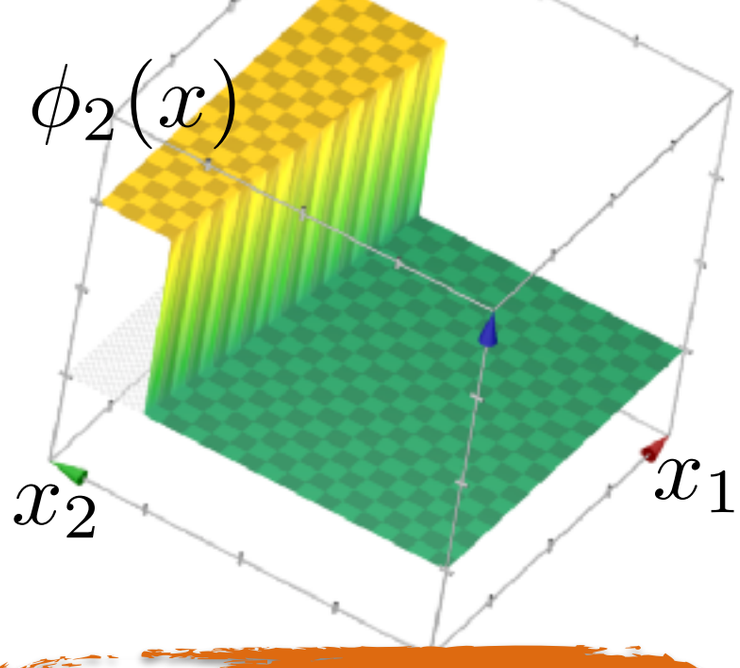


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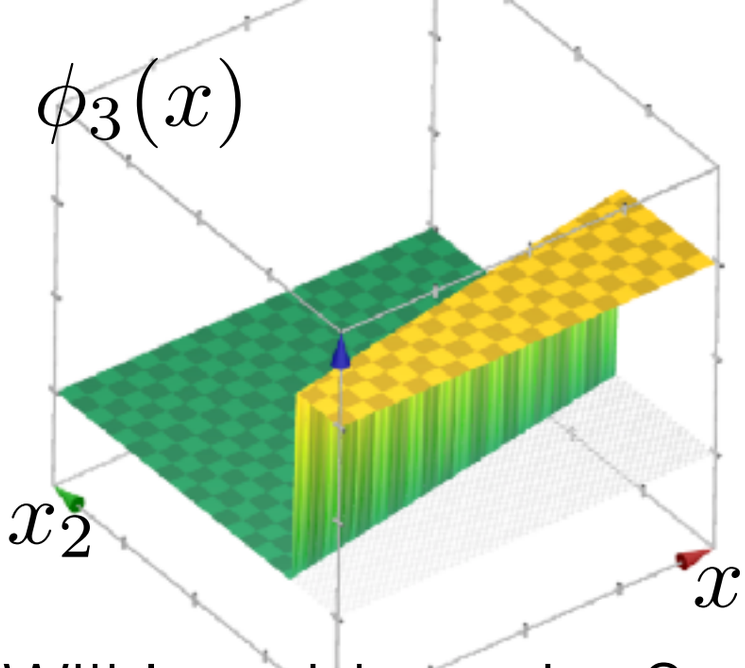


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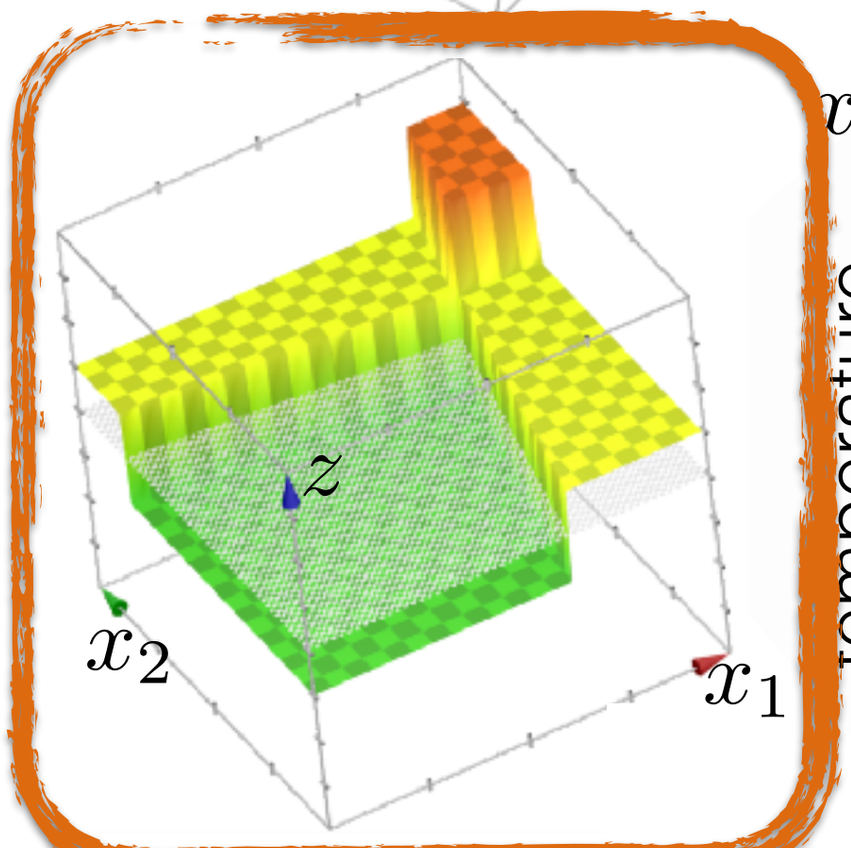


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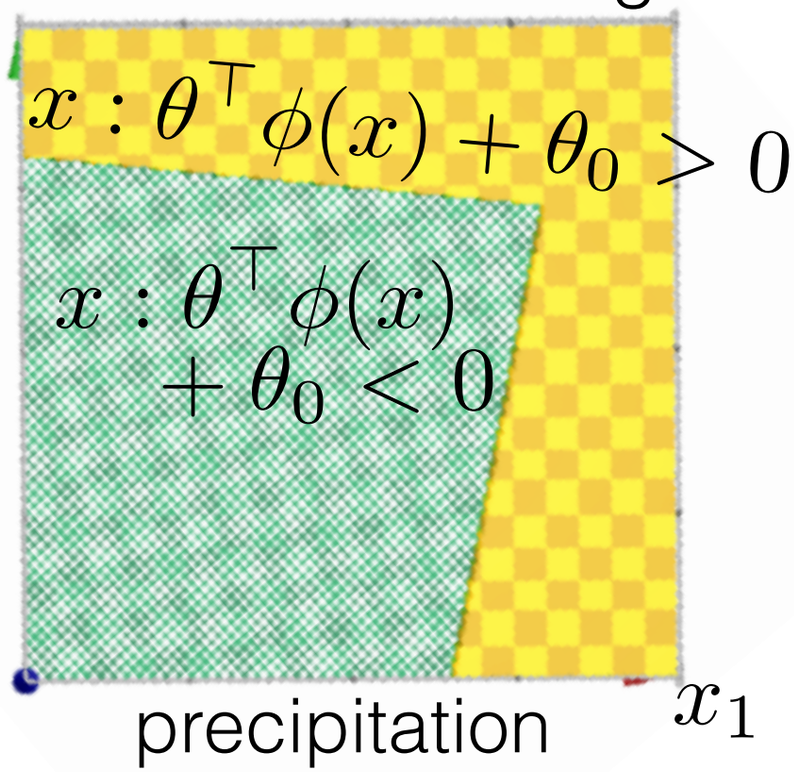

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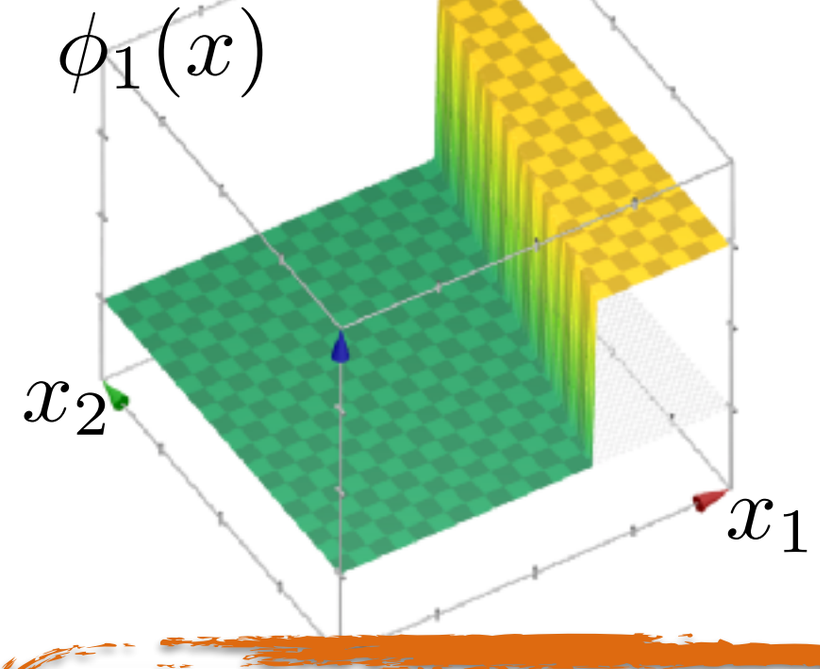


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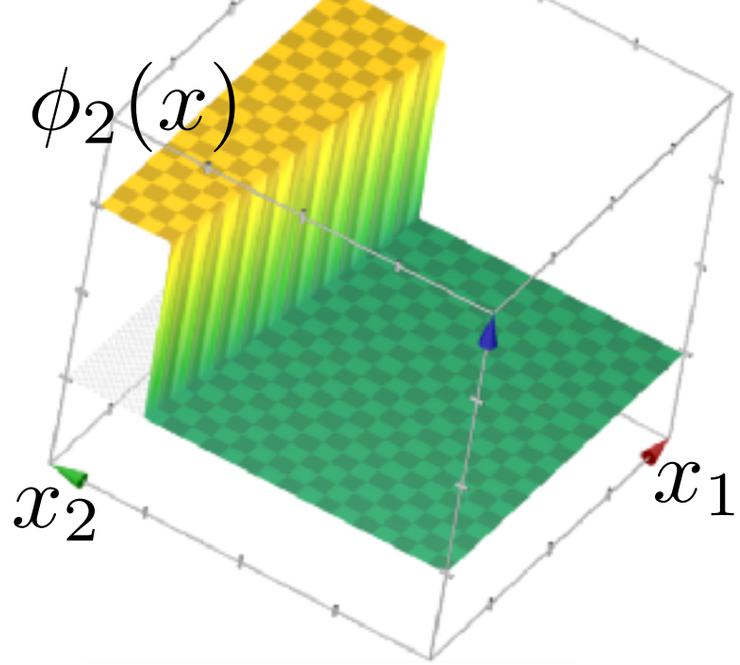


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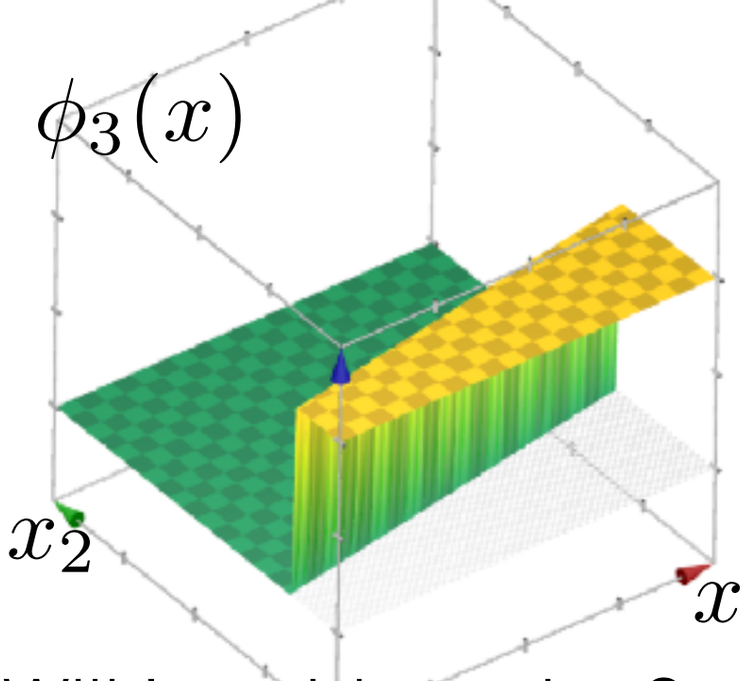


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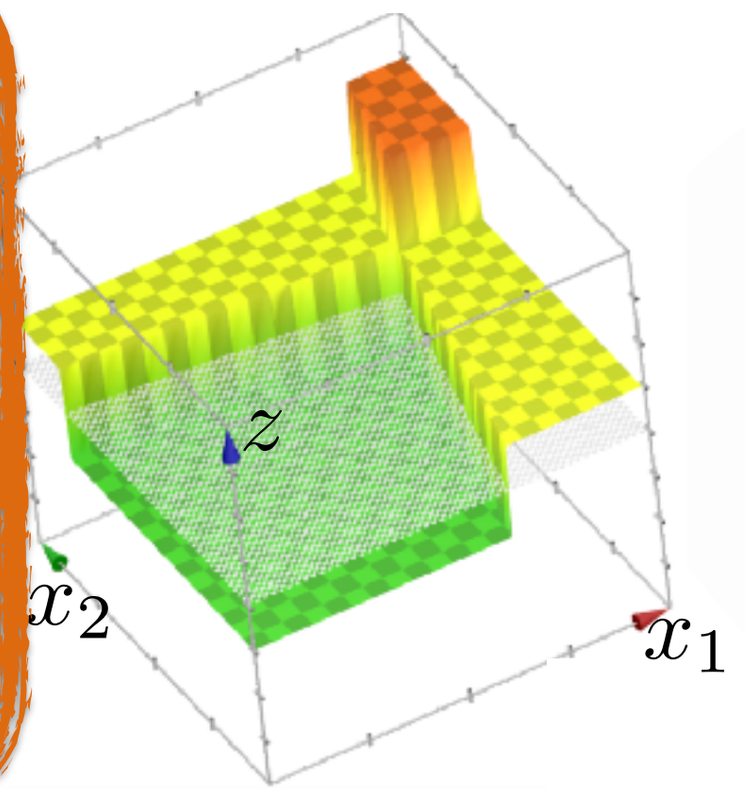


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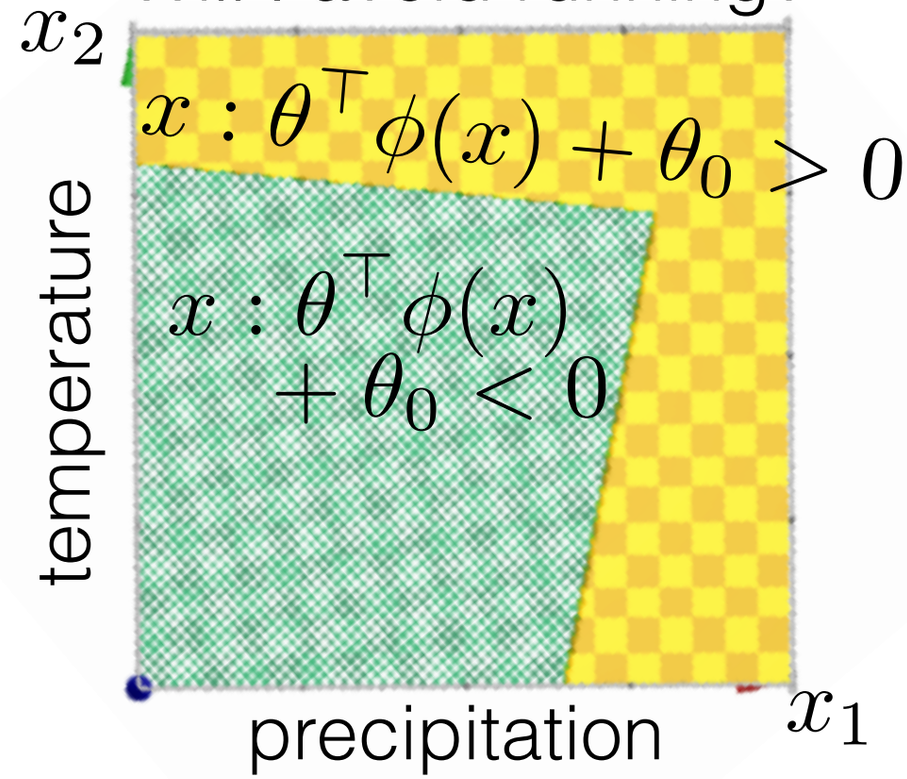

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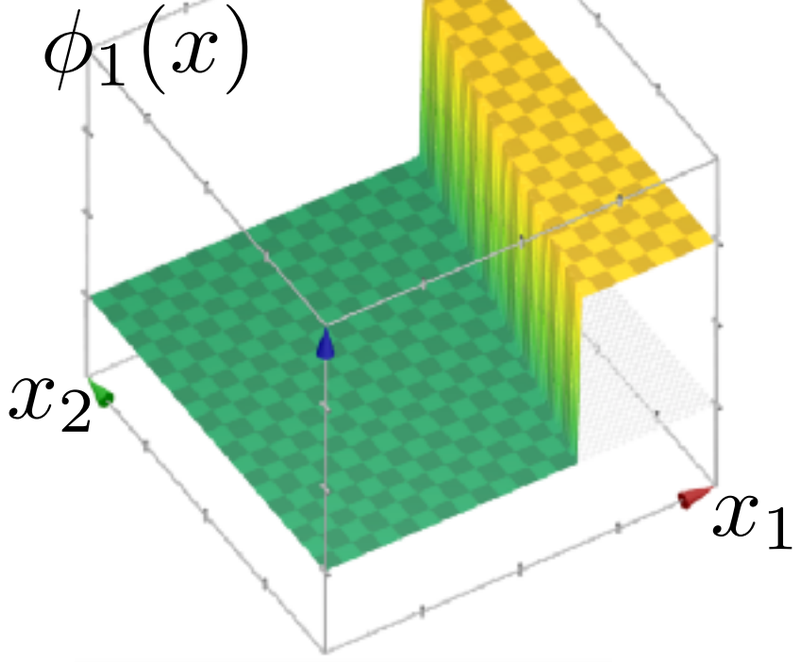


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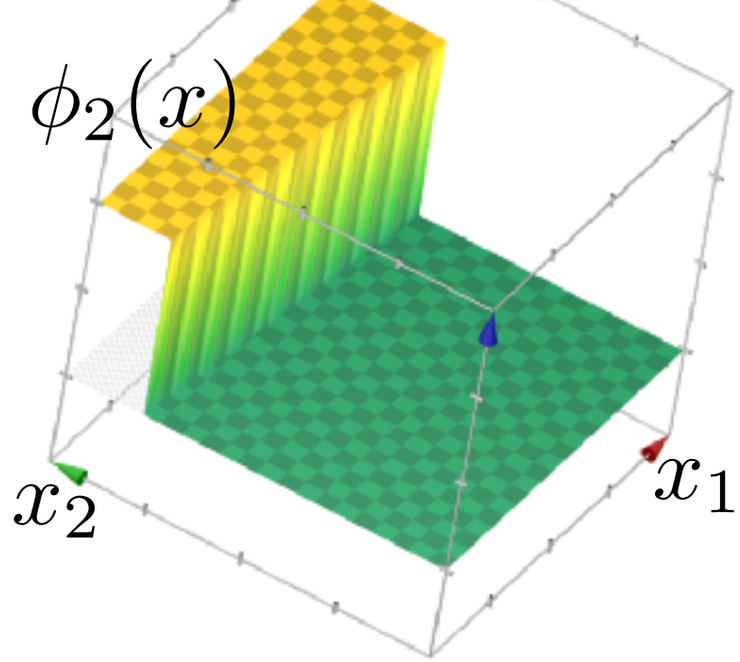


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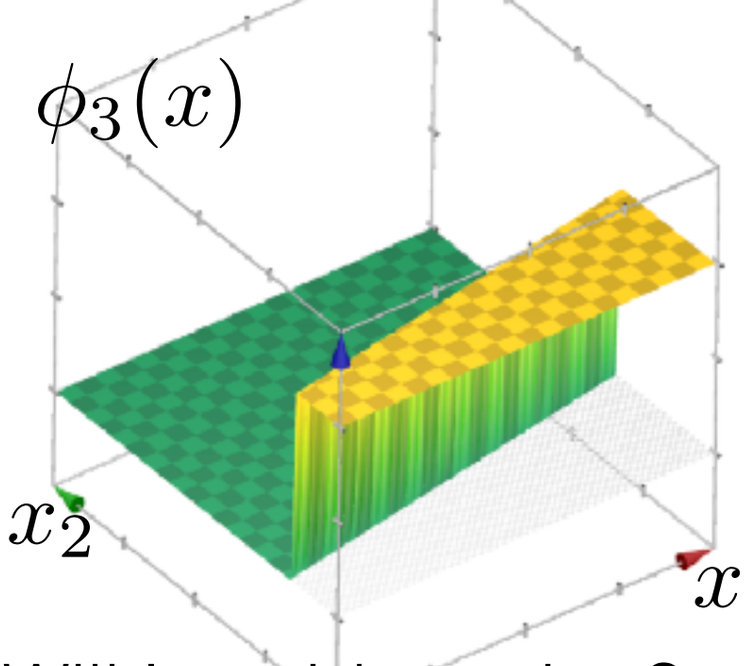


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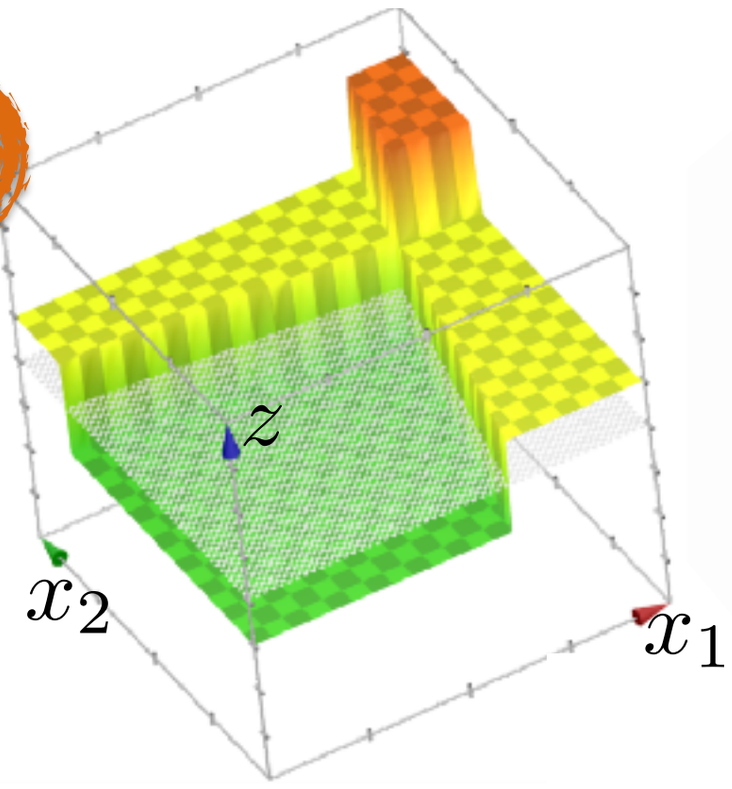


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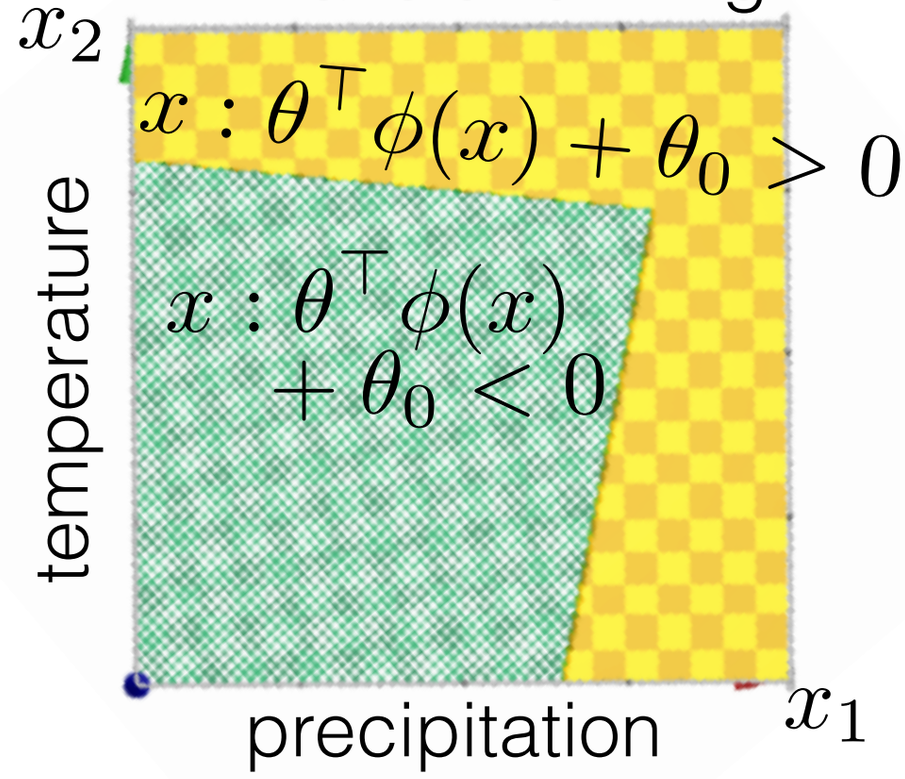

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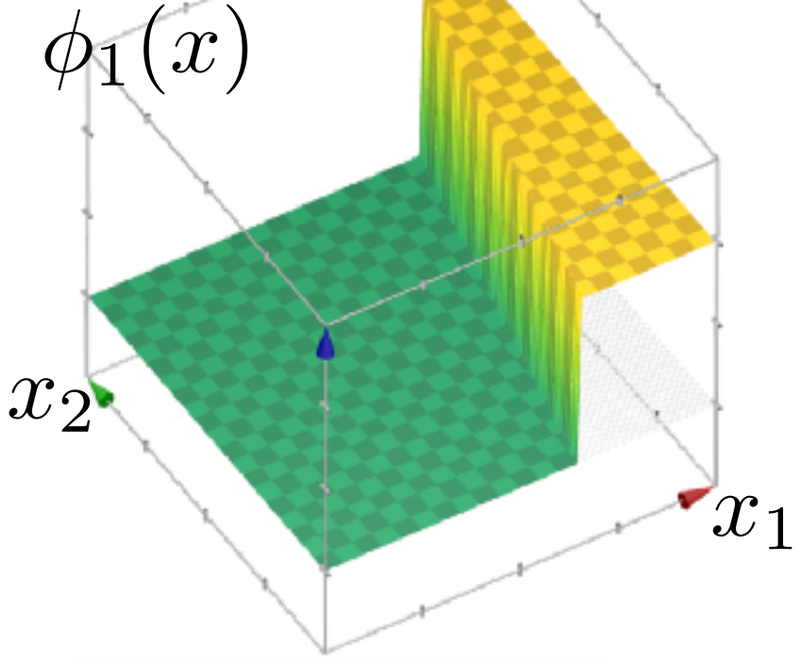


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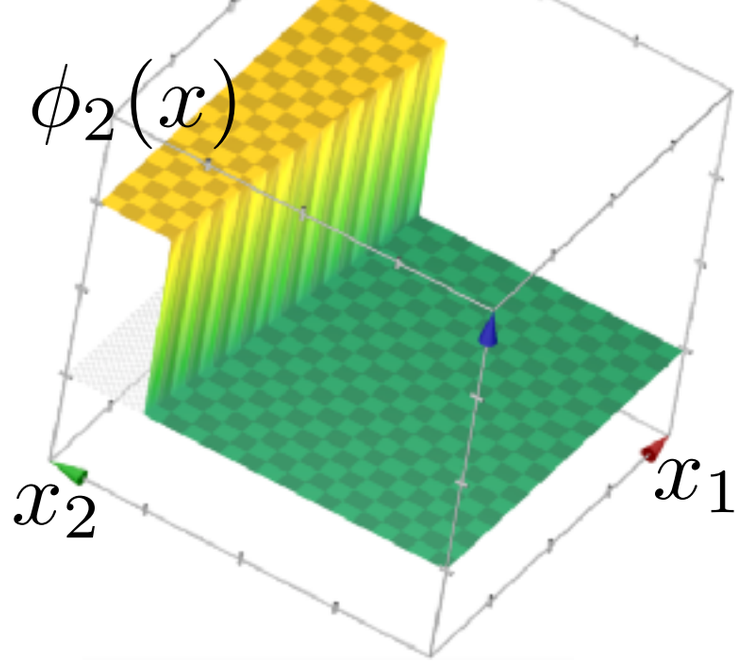


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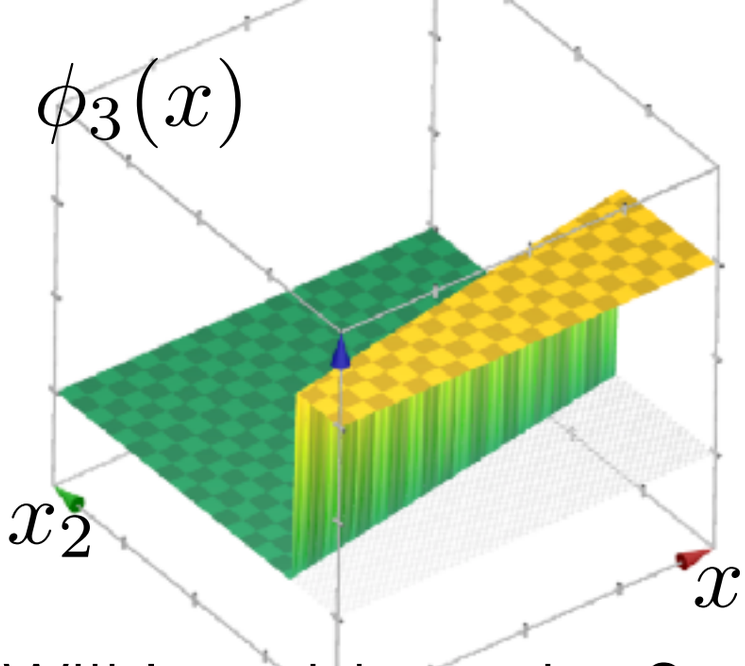


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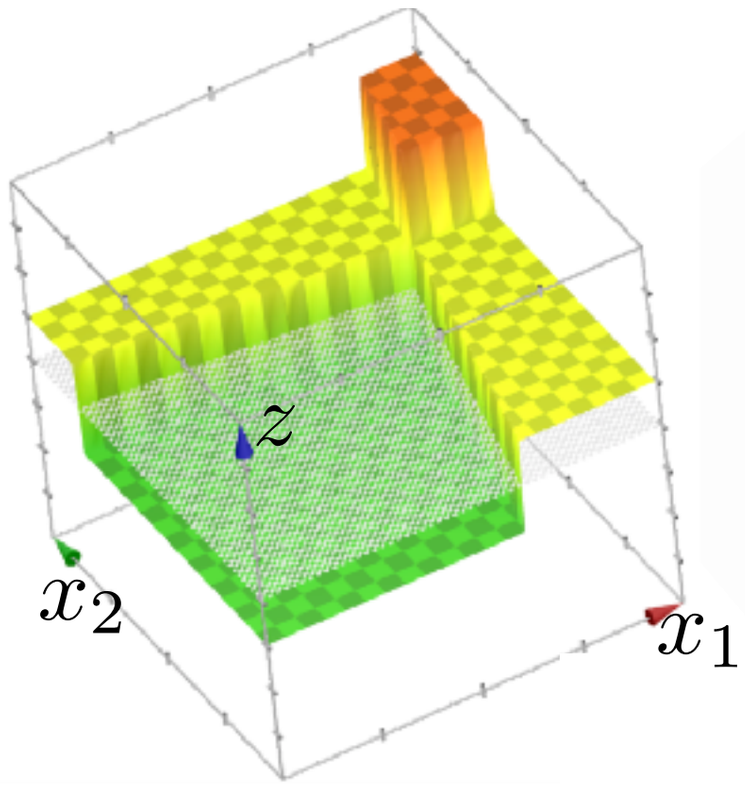


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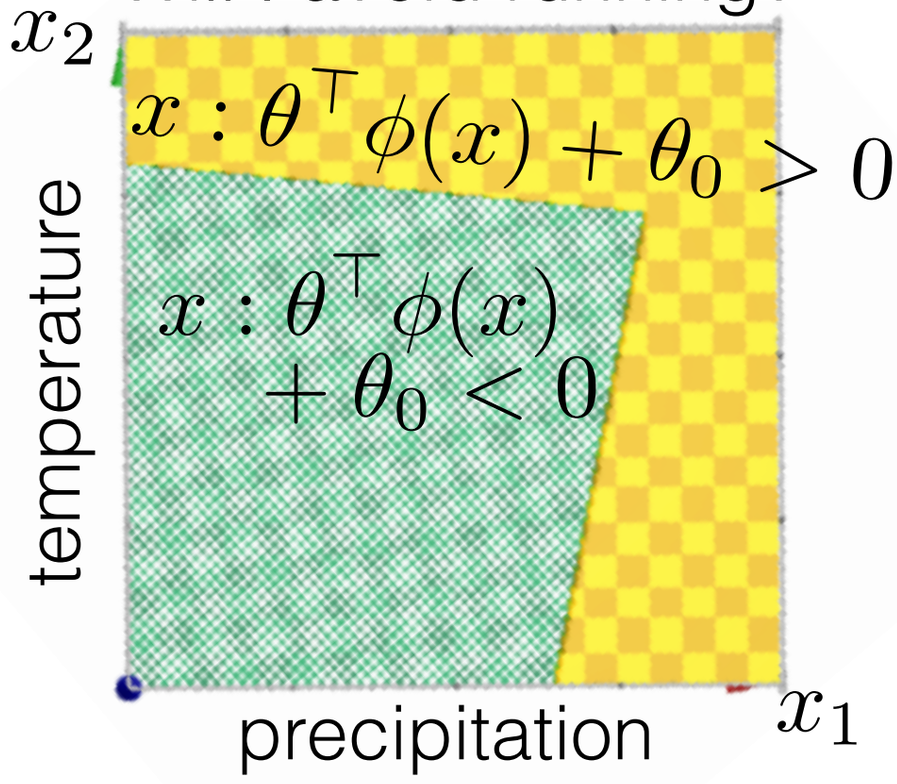

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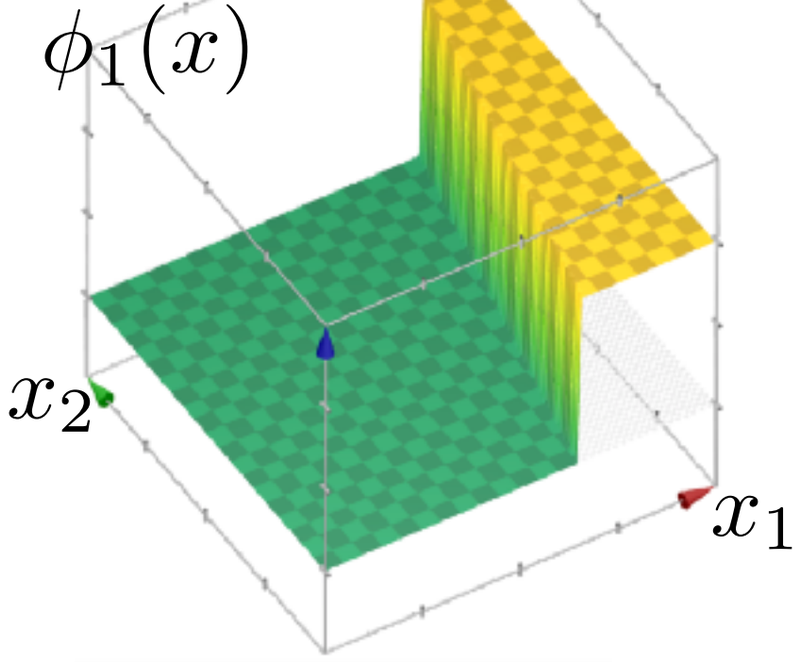


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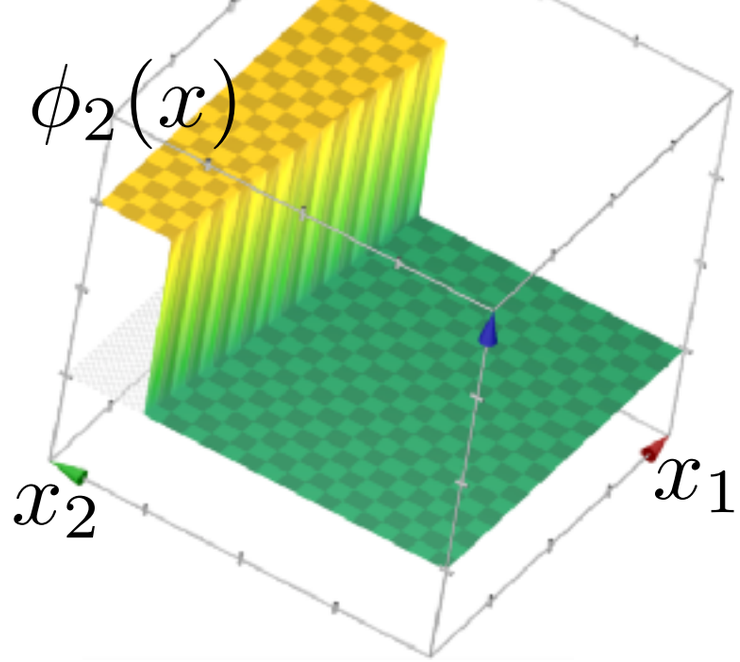


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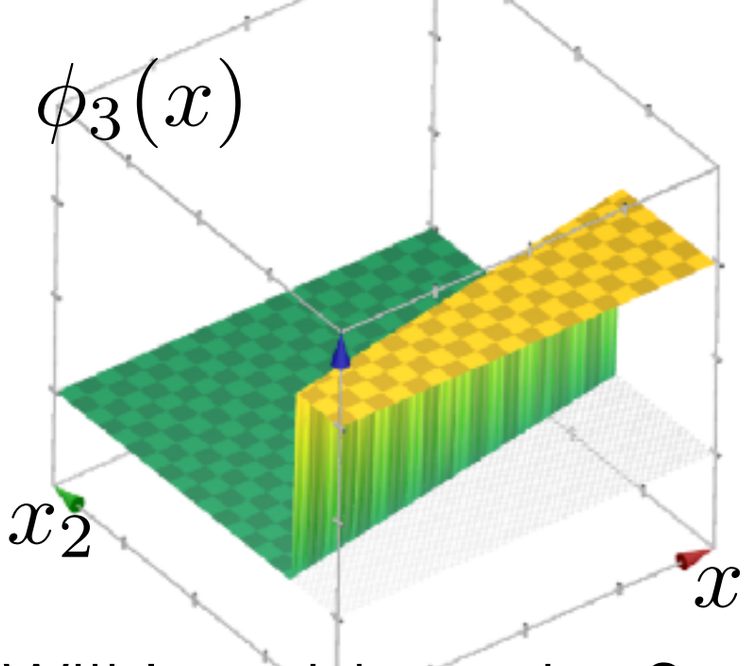


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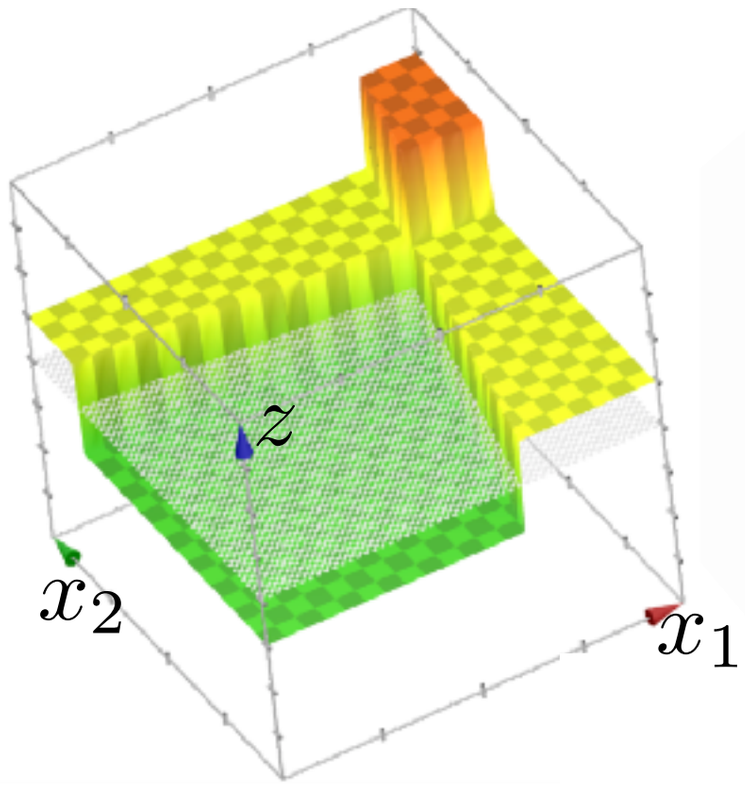


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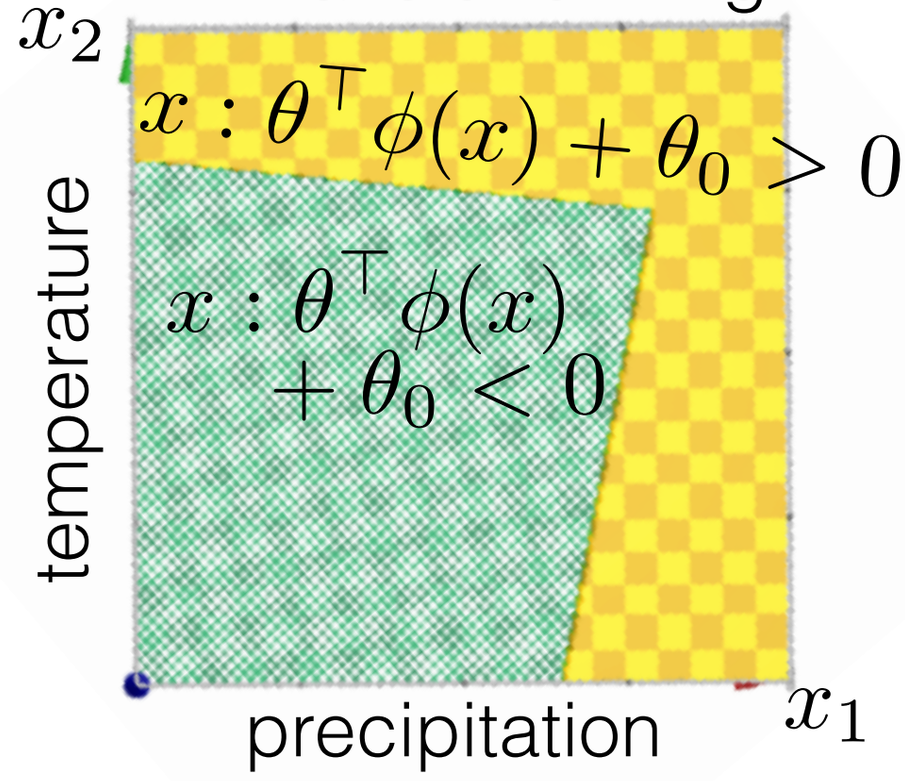

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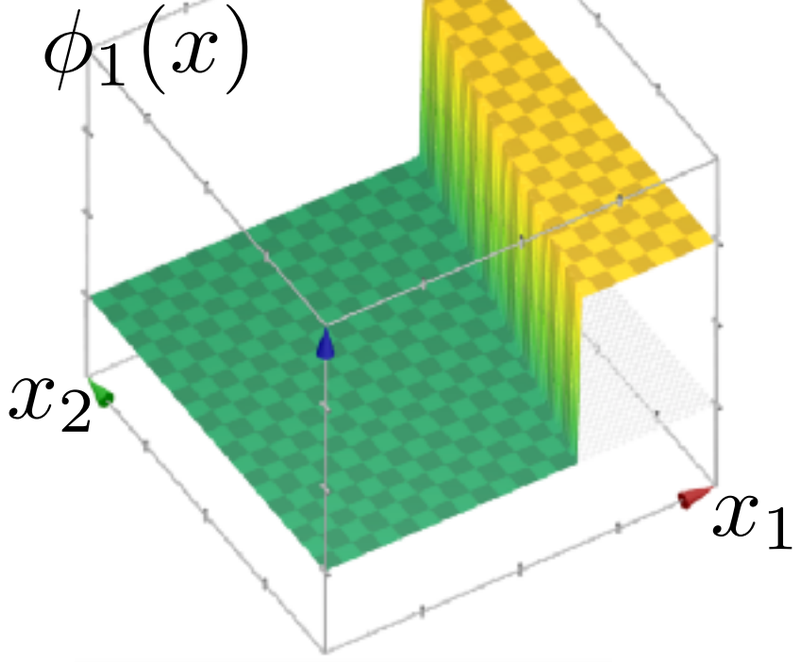


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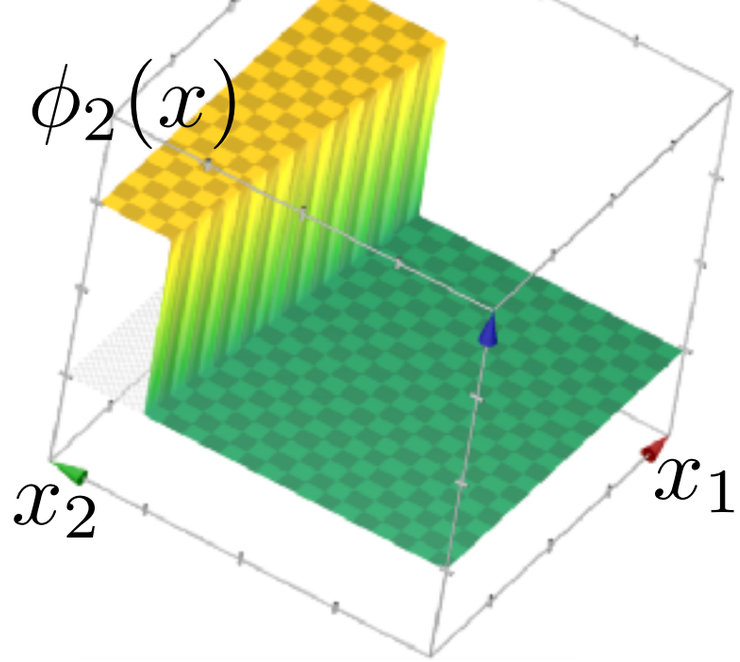


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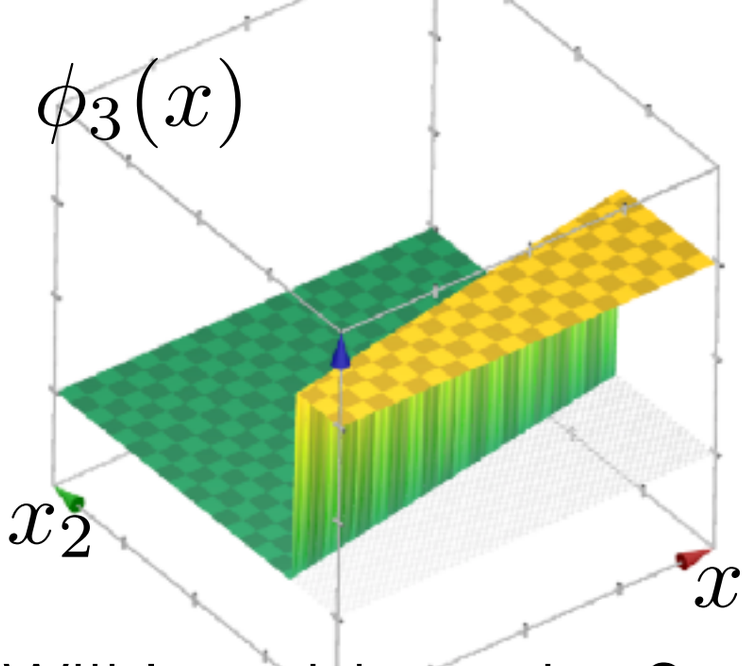


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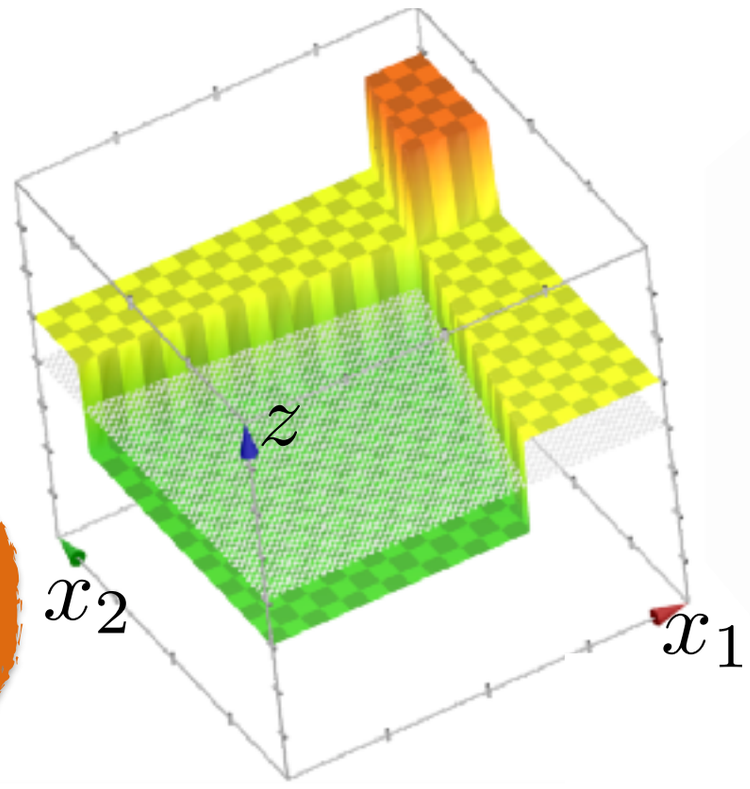


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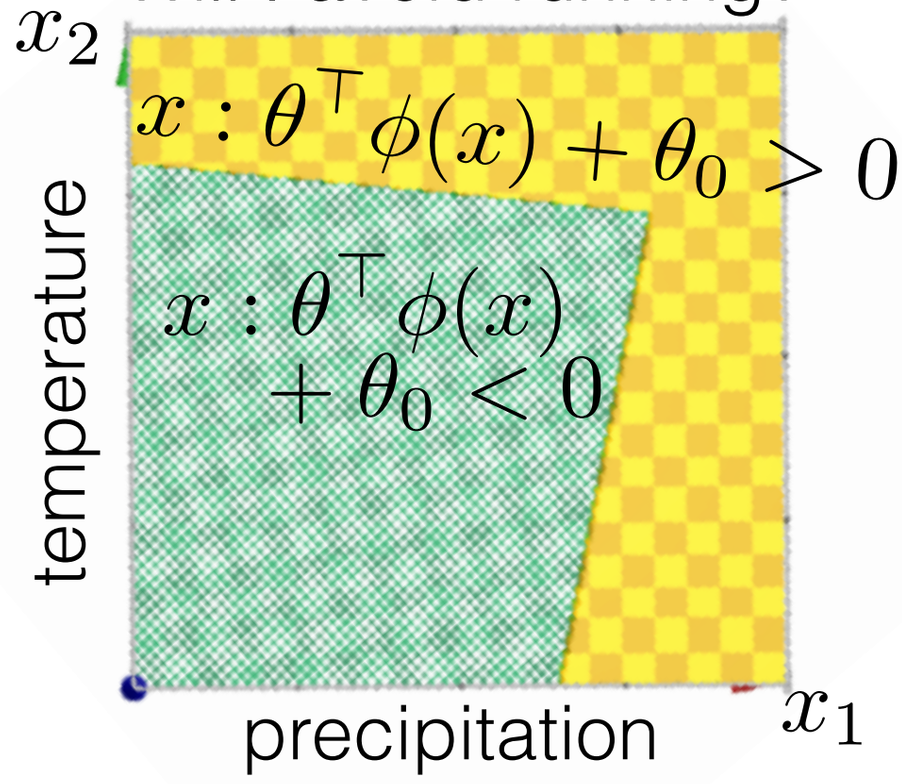

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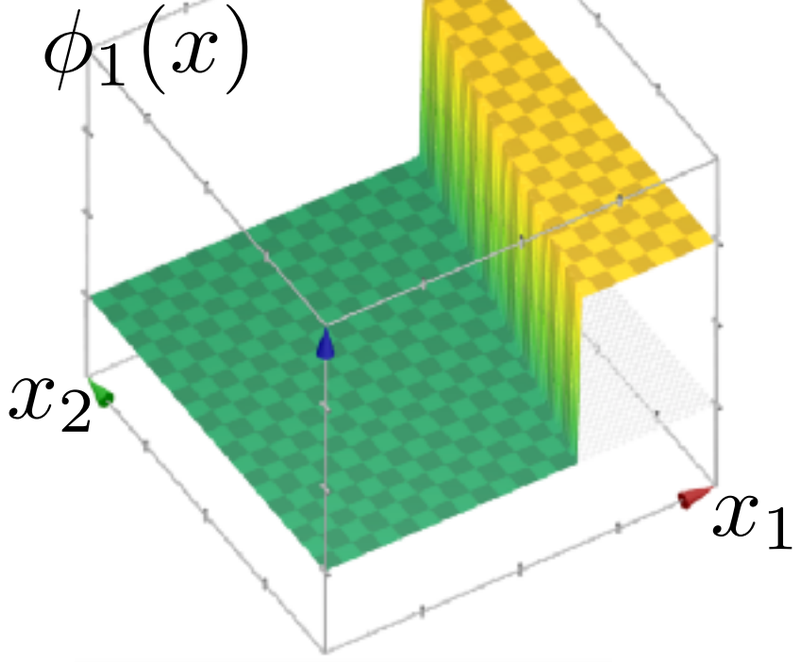


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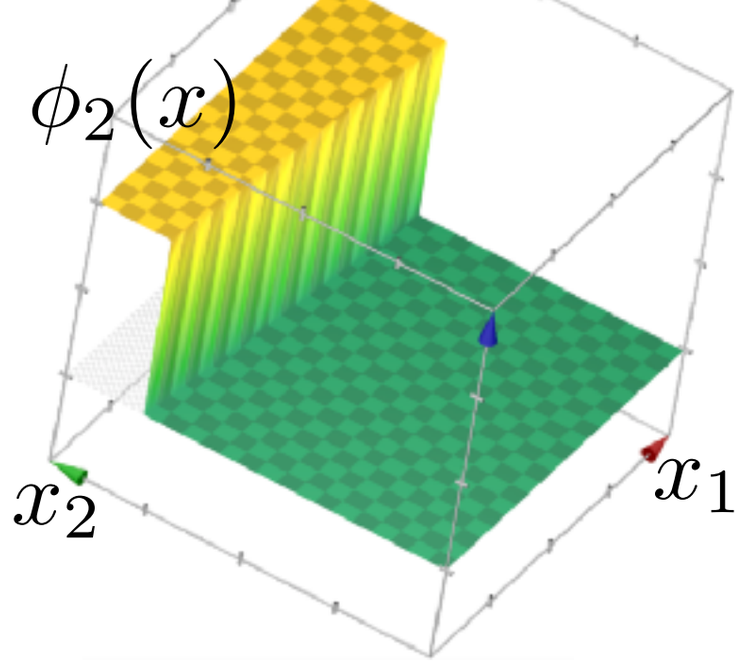


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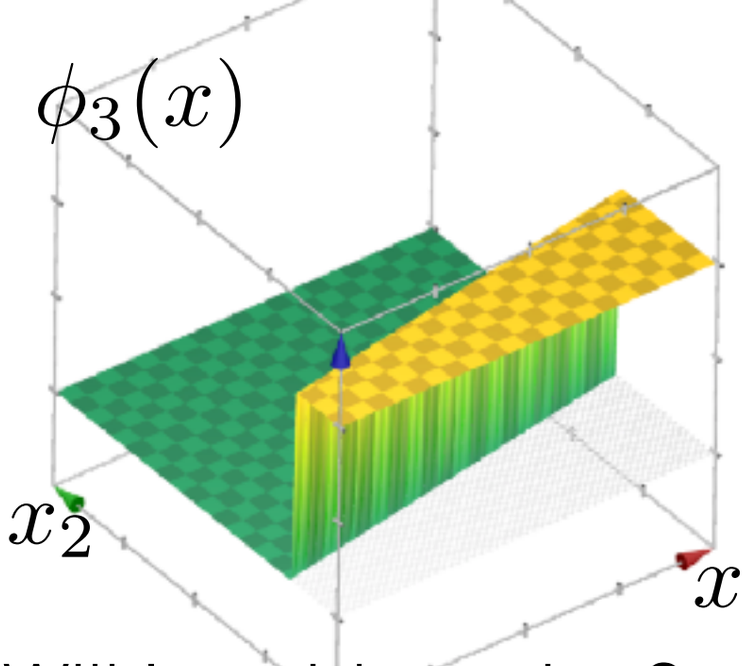


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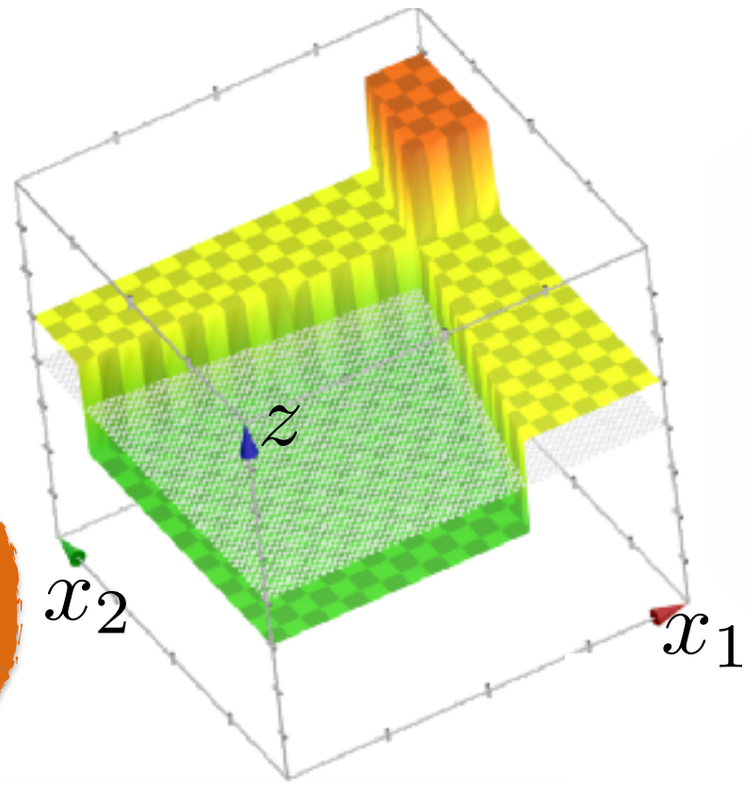


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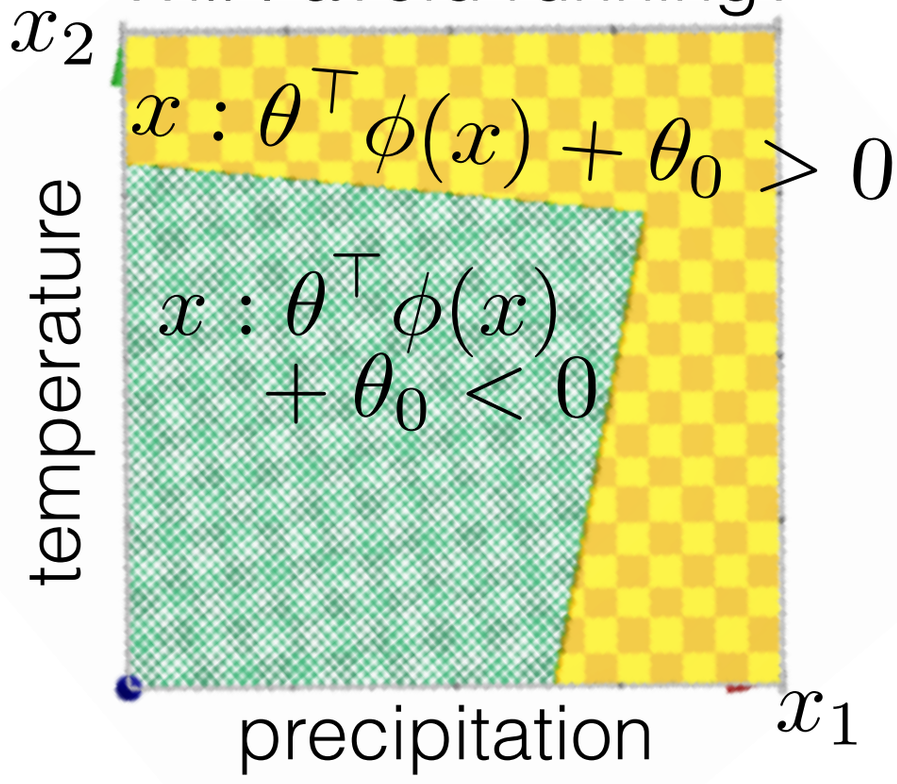

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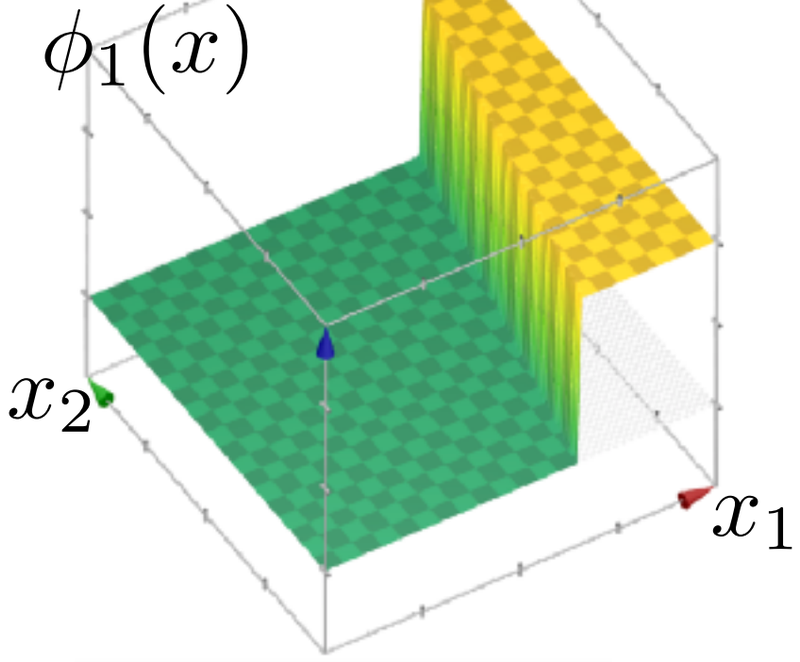


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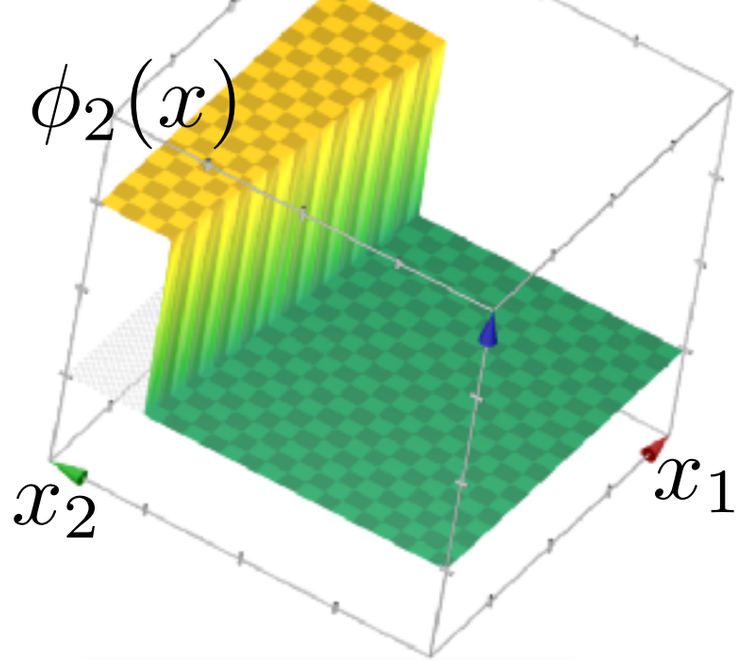


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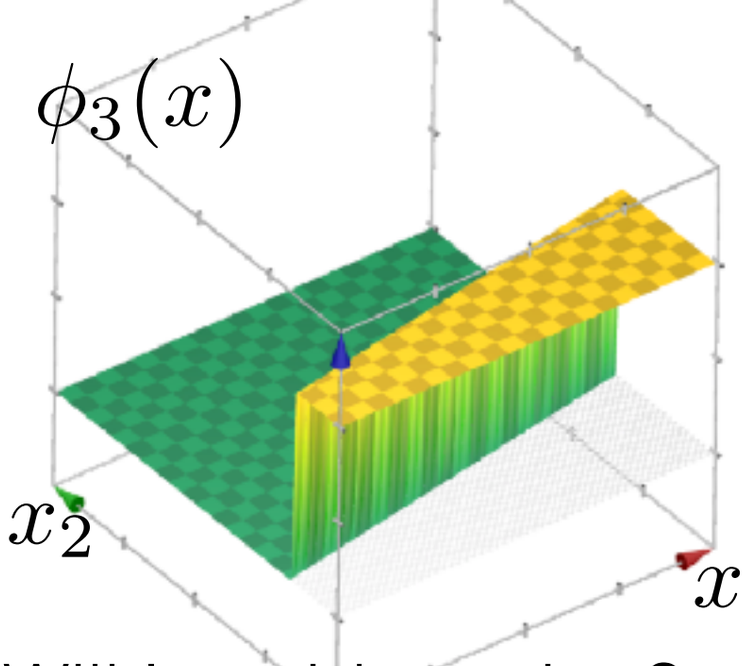


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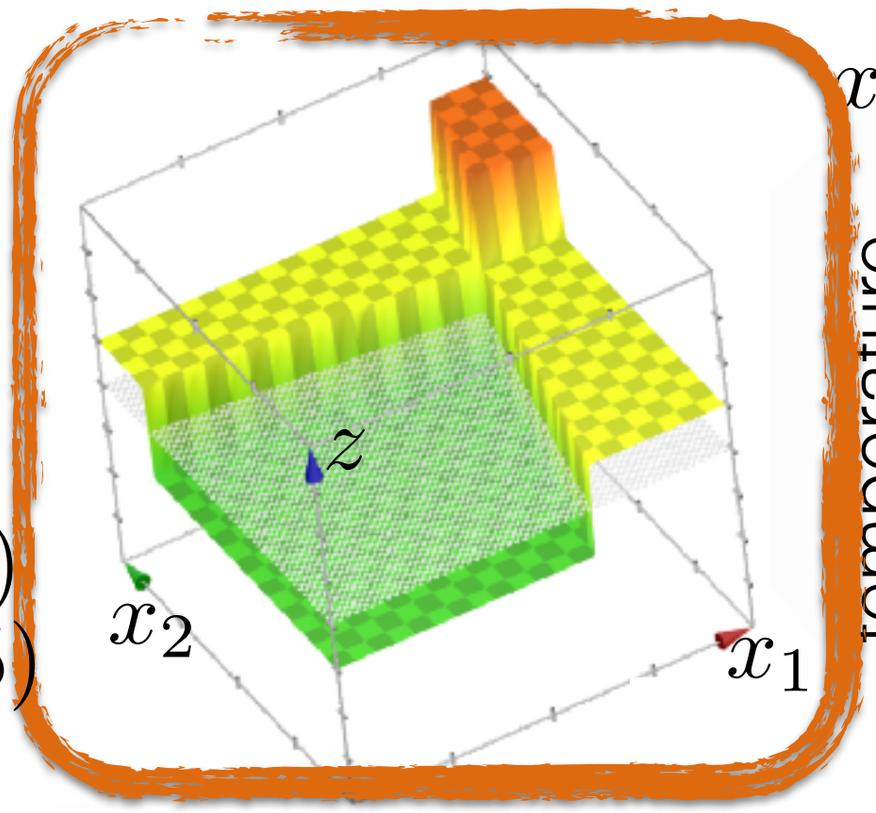


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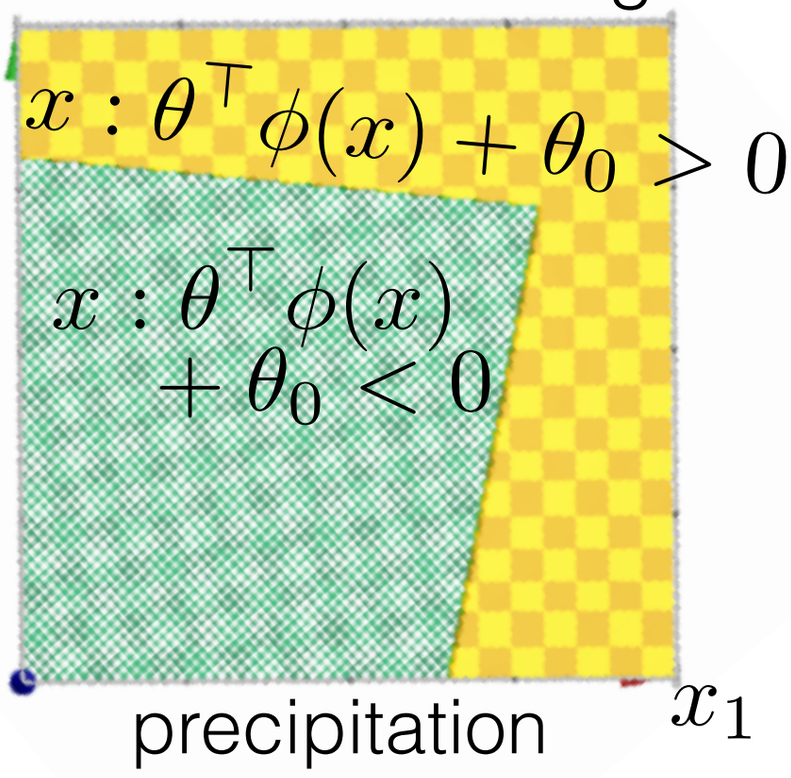

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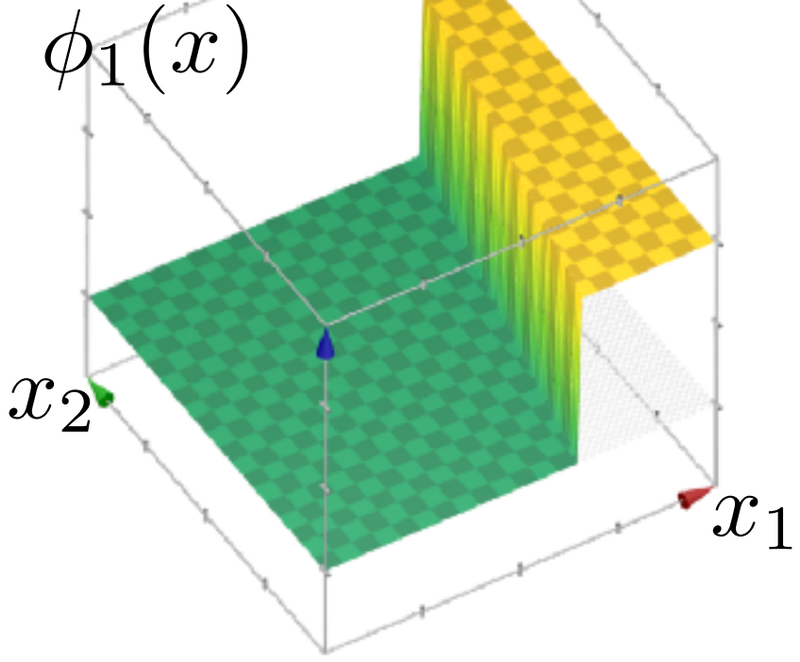


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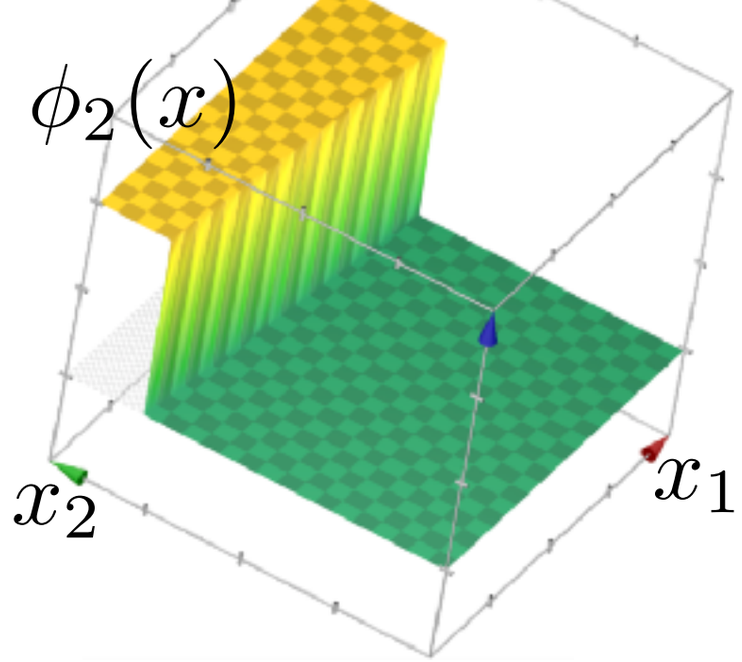


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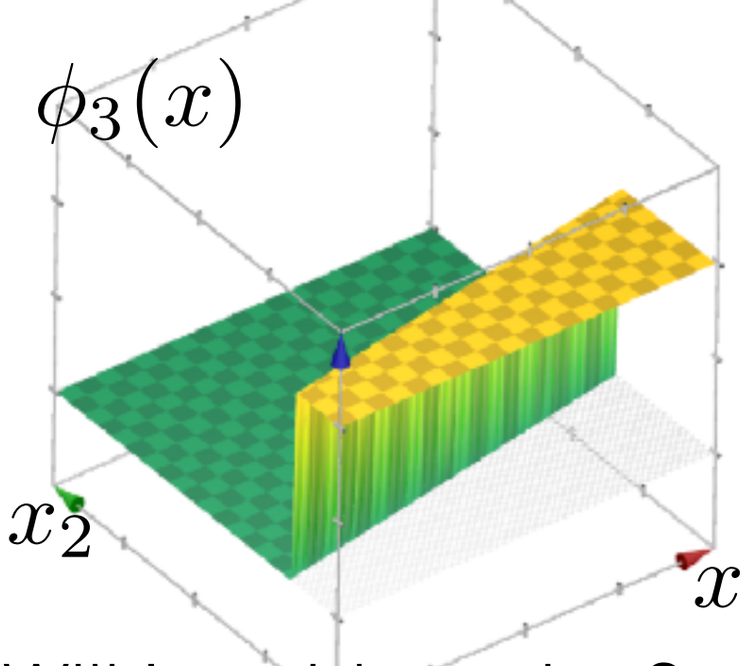


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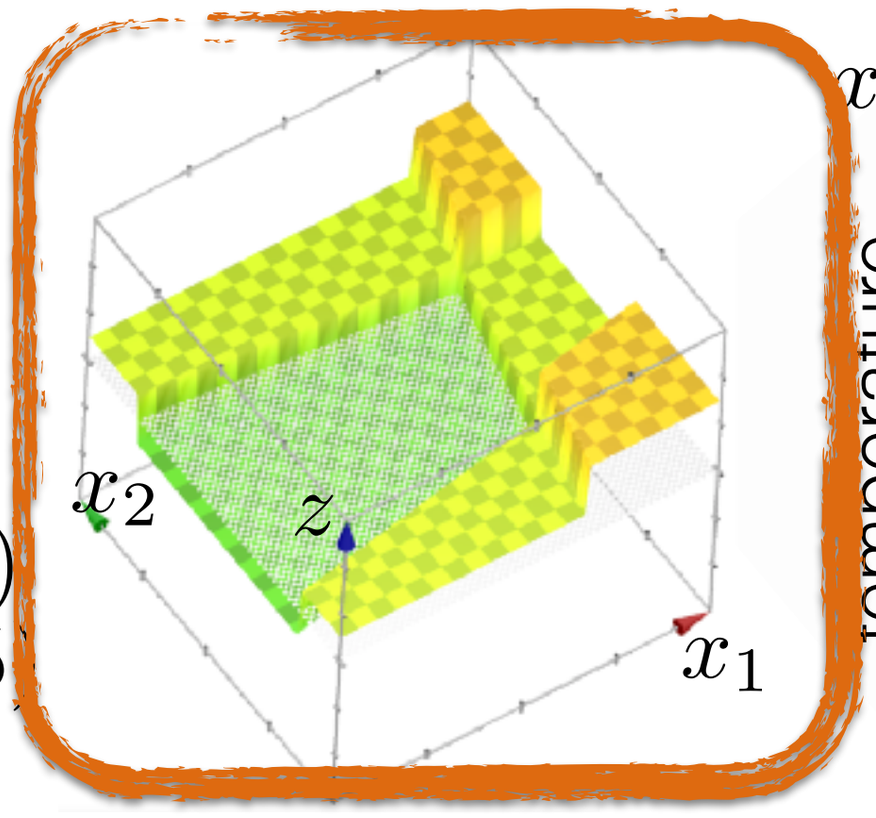


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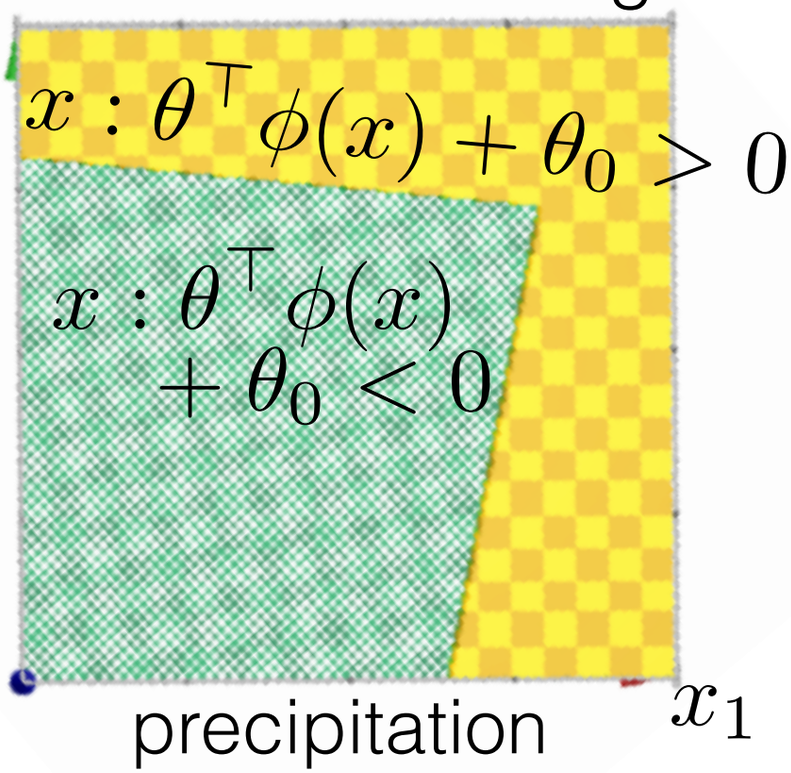

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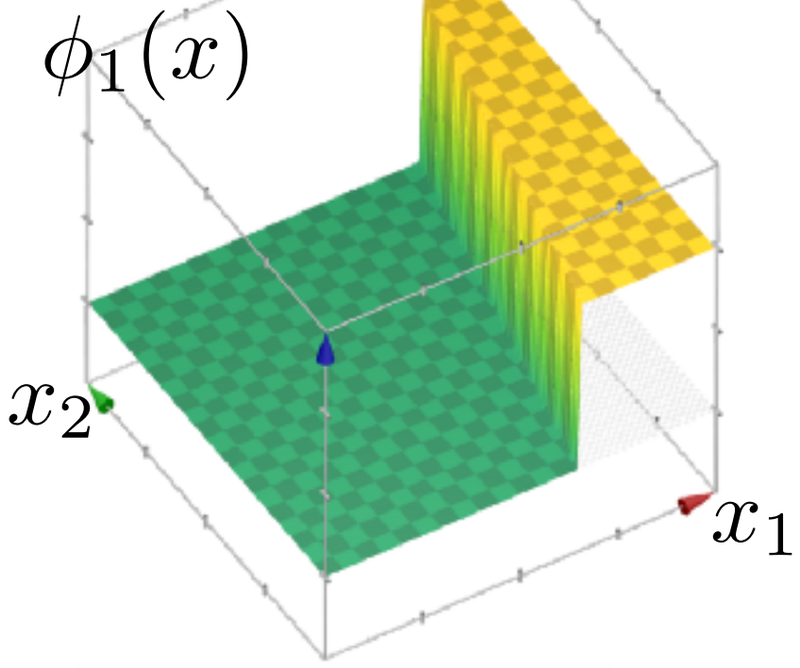


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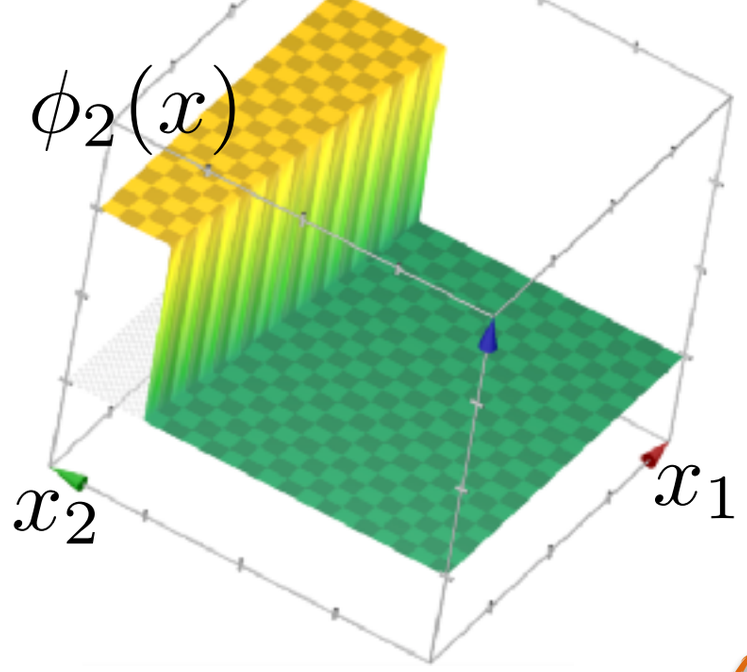


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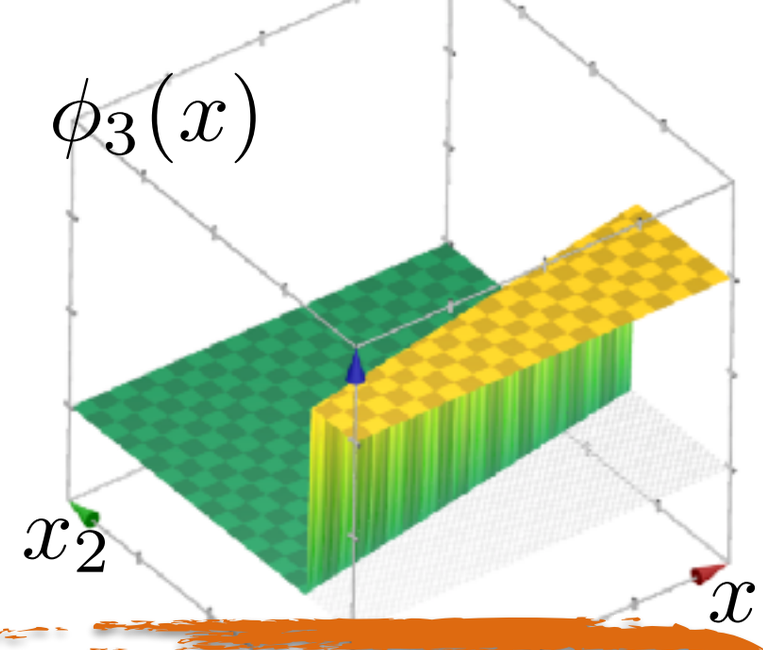


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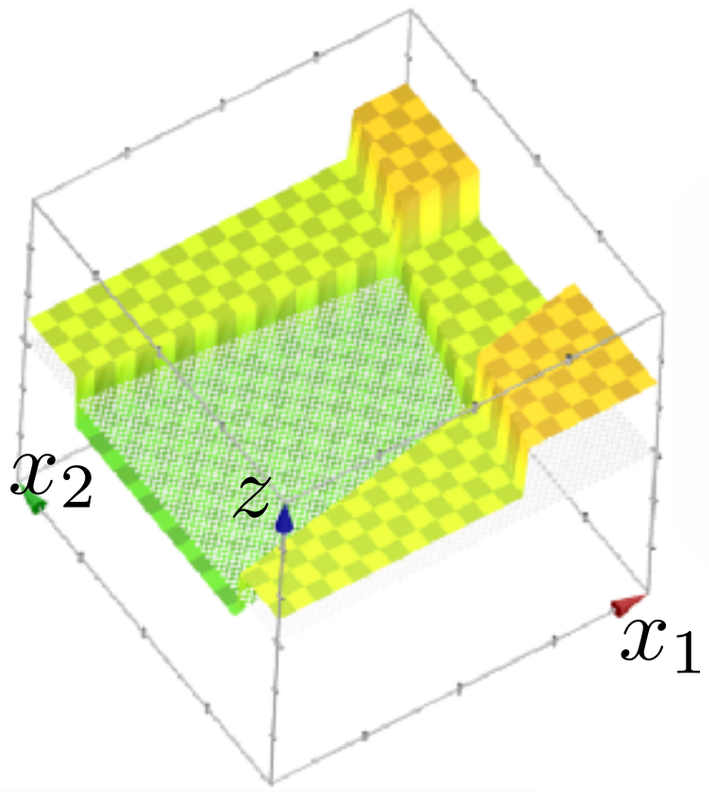


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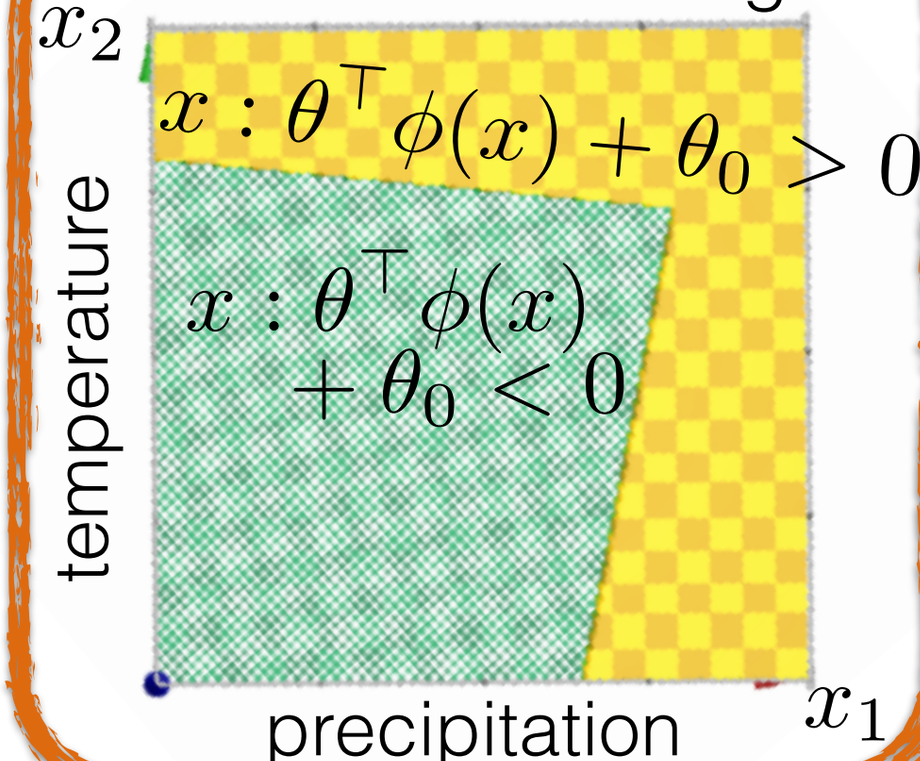

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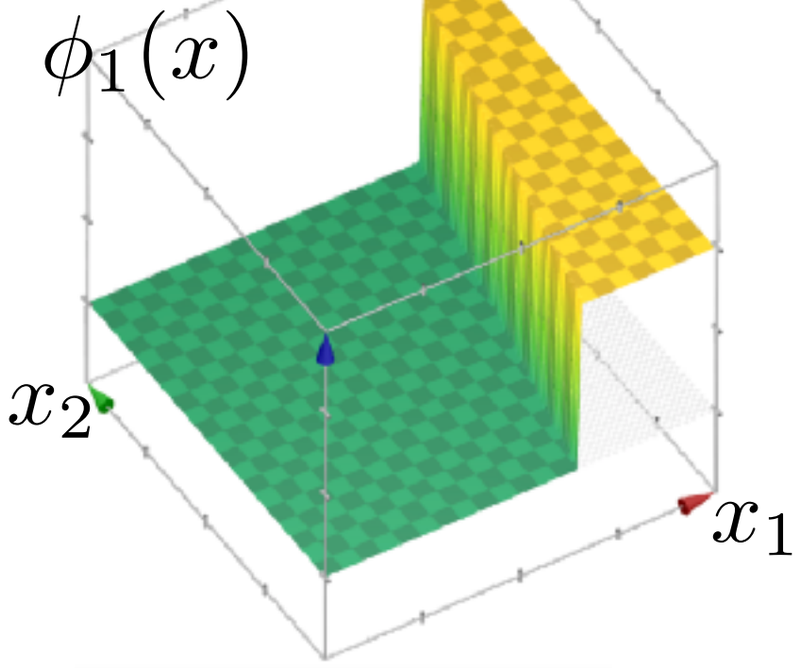
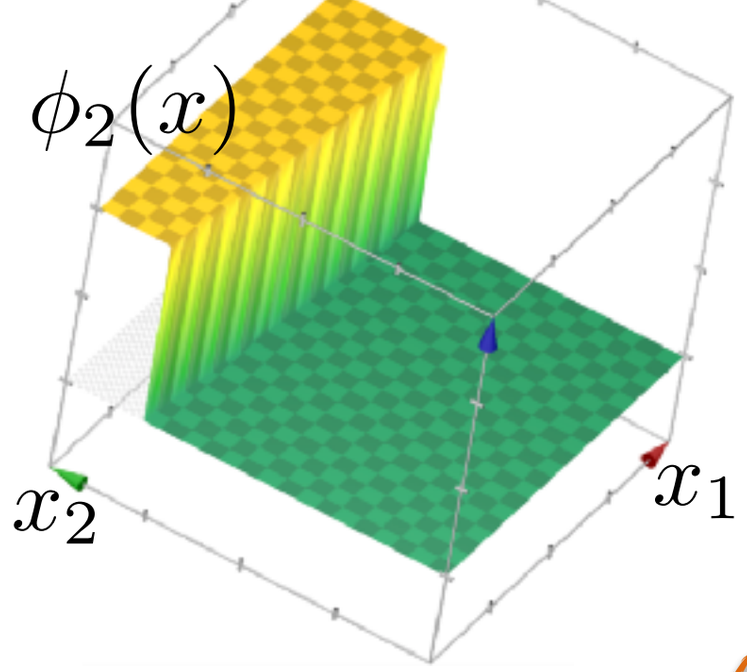
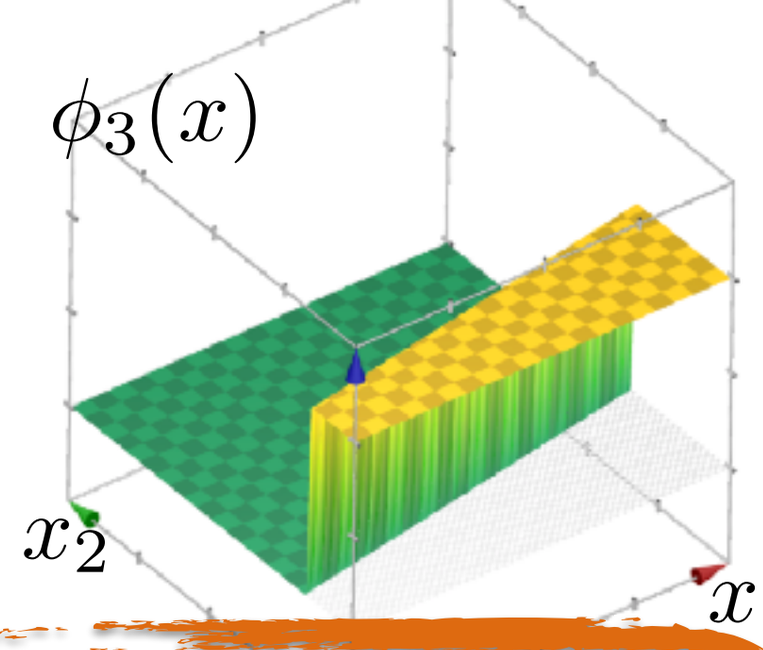


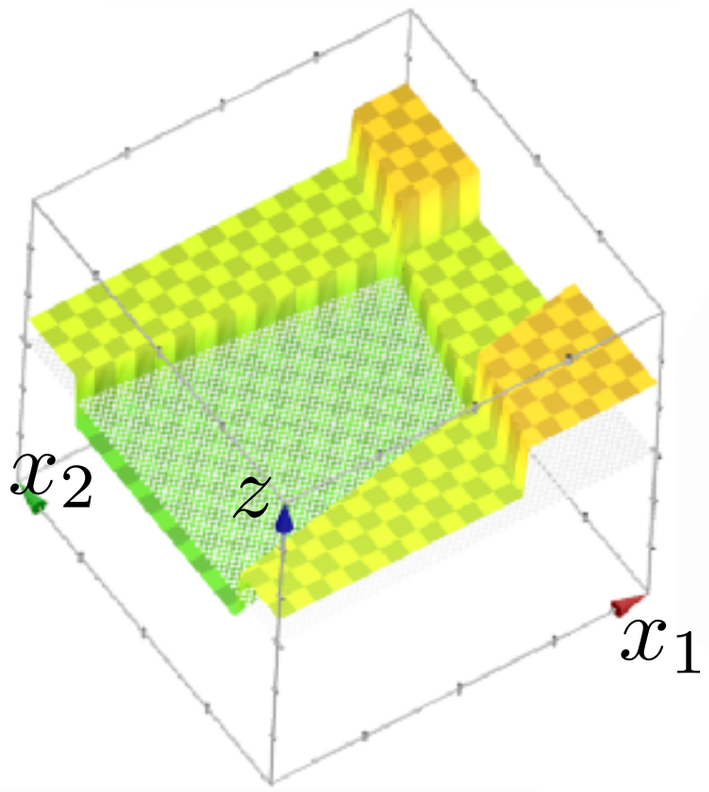

$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



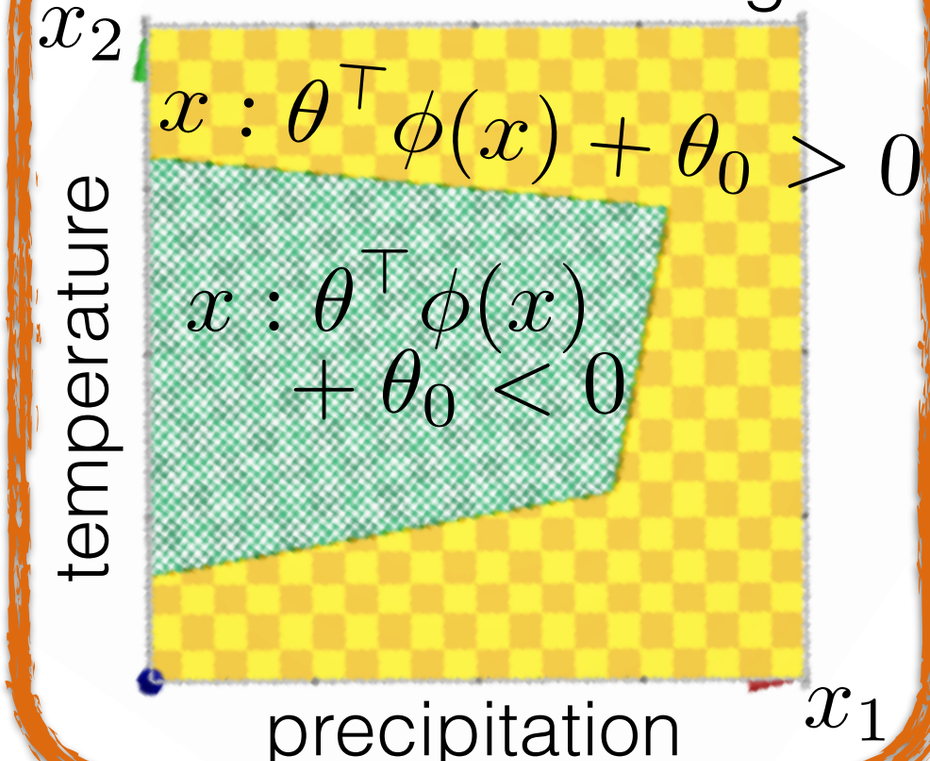

$$\phi_3(x) = \mathbf{1}\{\tilde{\tilde{w}}^\top x + \tilde{\tilde{w}}_0 \geq 0\}$$




$$z = \theta^\top \phi(x) + \theta_0$$
$$= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) + \theta_3 \phi_3(x) + \theta_0$$
$$= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) + 1 \cdot \phi_3(x) + (-0.5)$$




Will I avoid running?



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  - Input $x$ (a data point)

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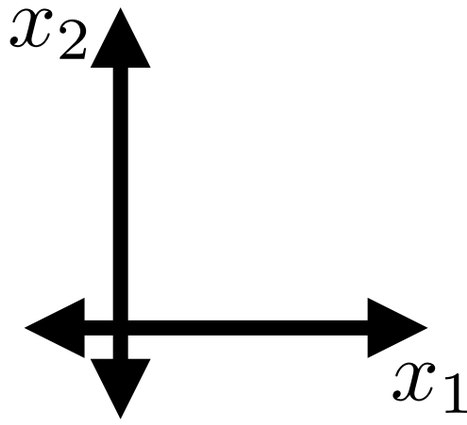

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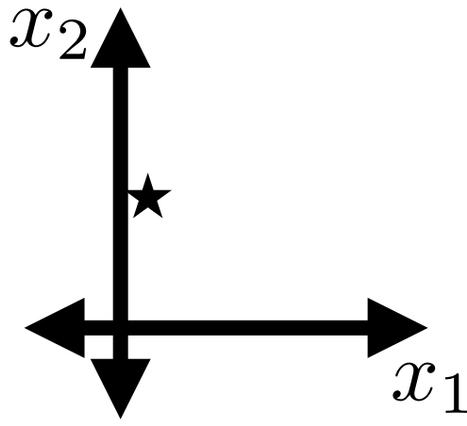

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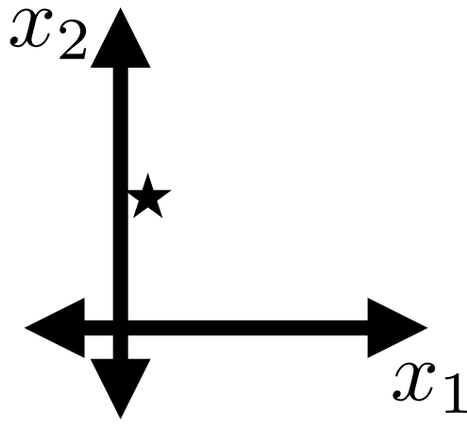
- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$



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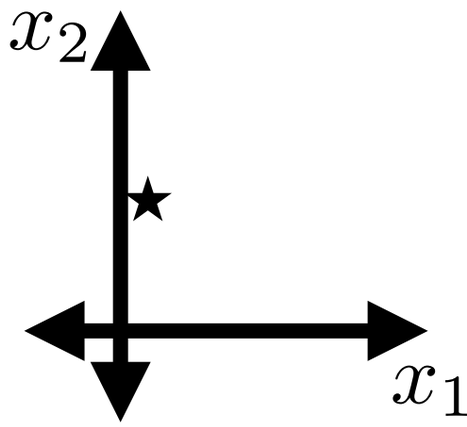
- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$

$m^{(1)} = \mathrm{d}$



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  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of feature values)

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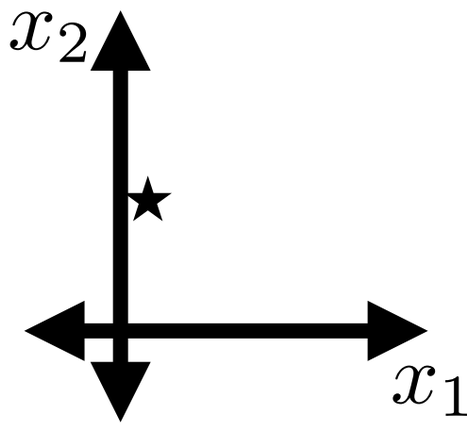

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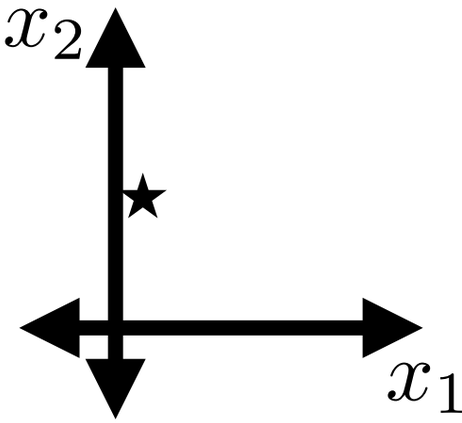


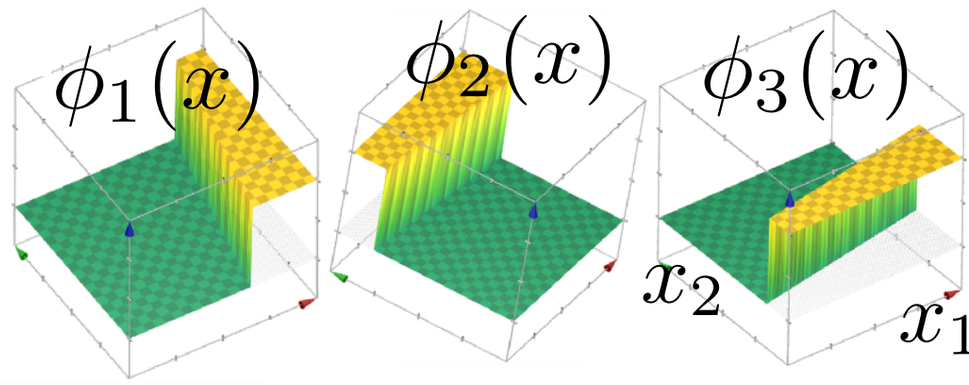

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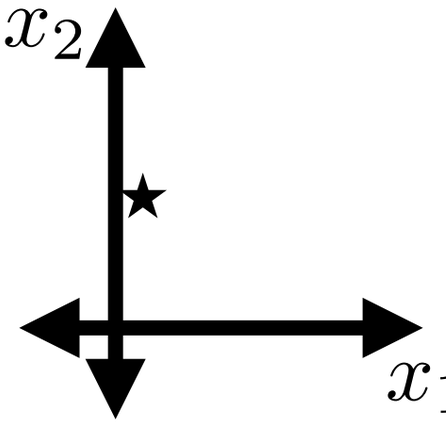


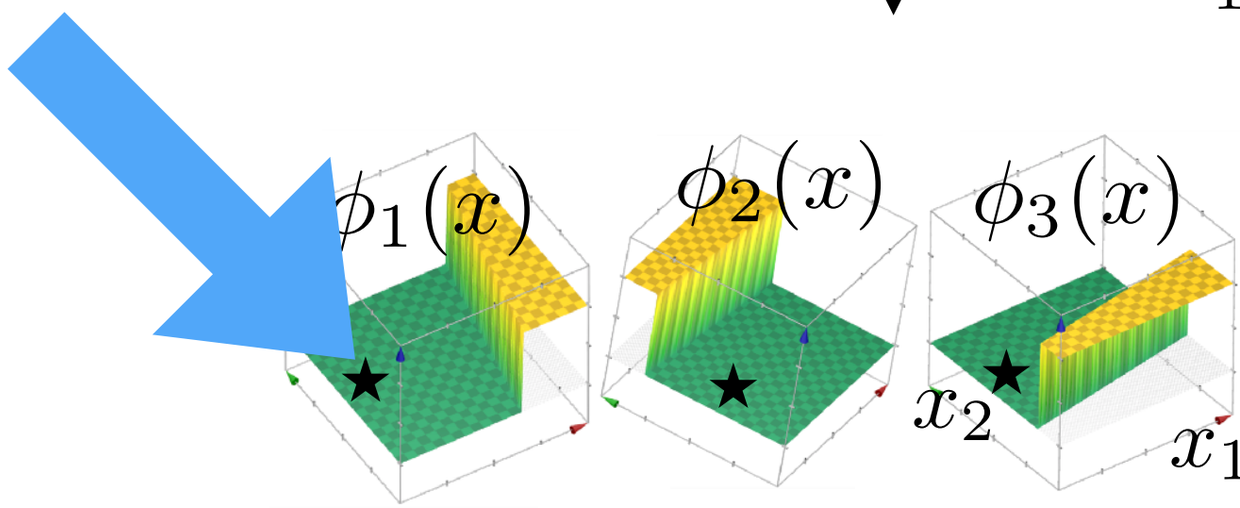

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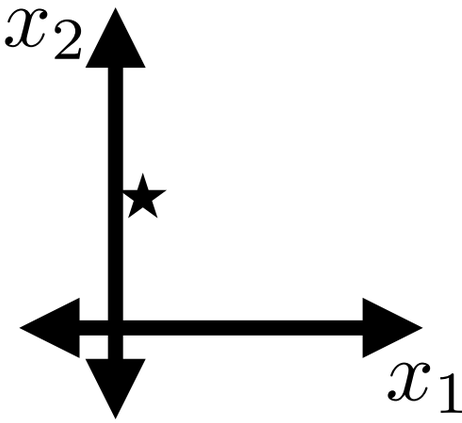


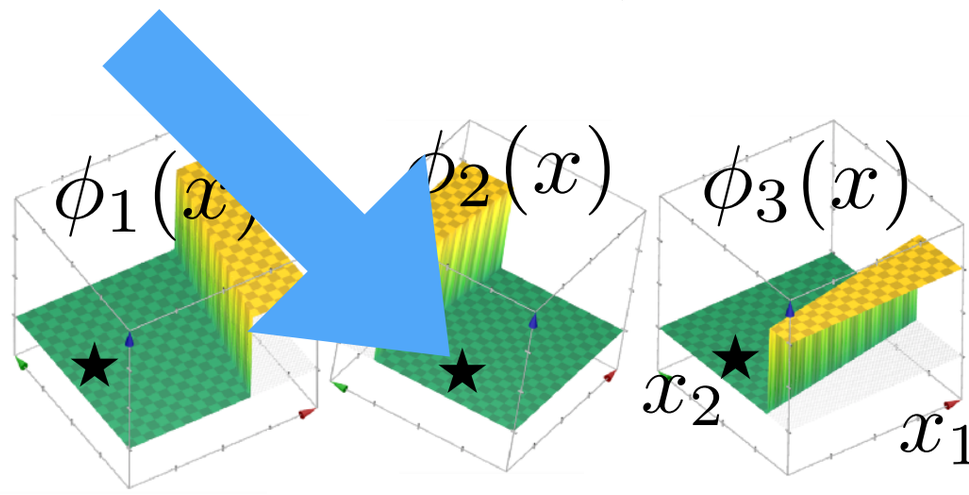

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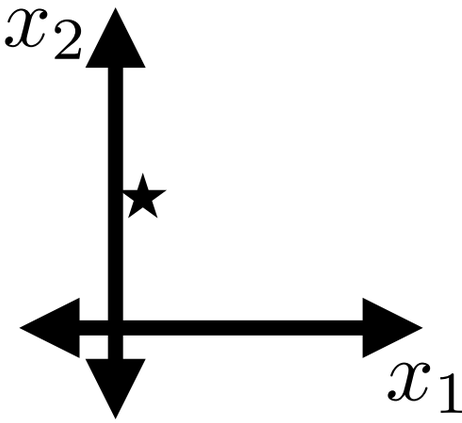


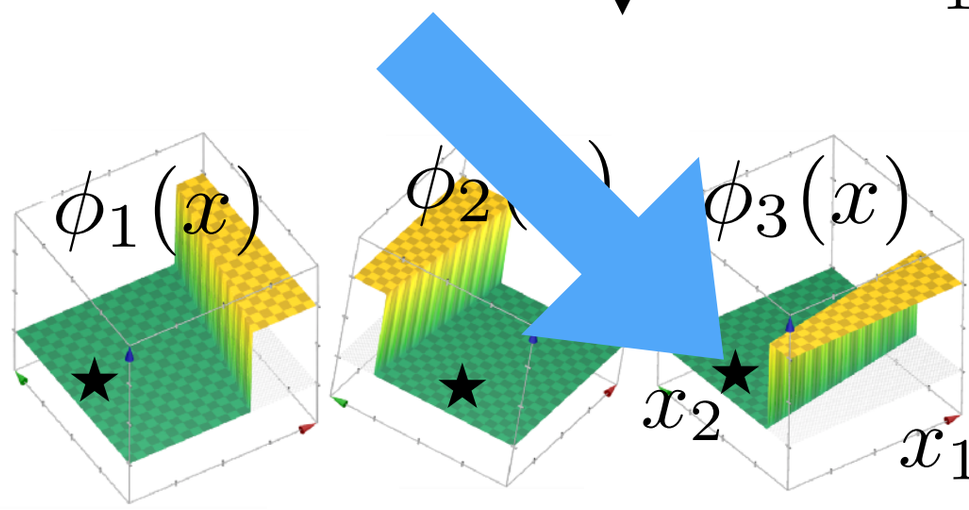

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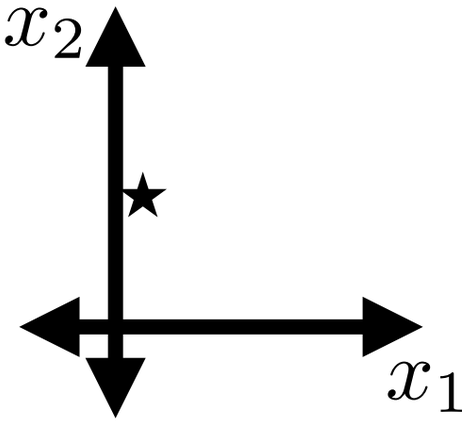


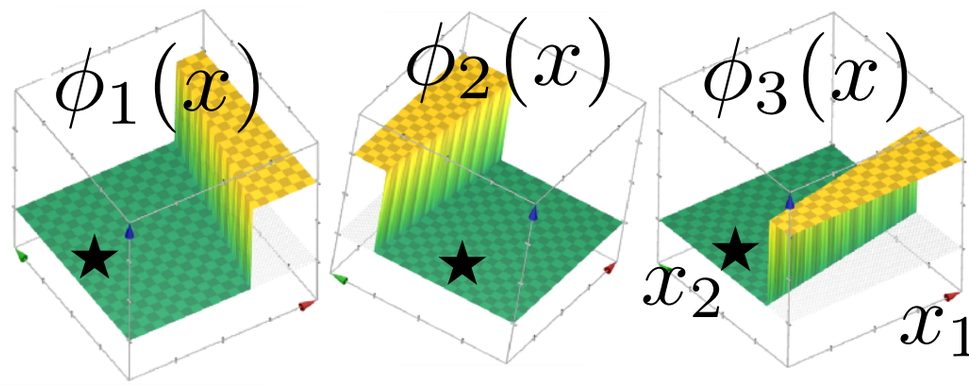

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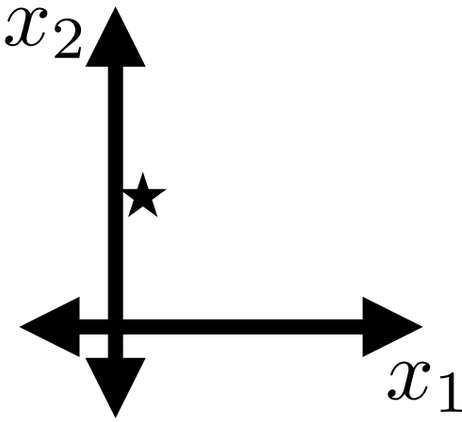


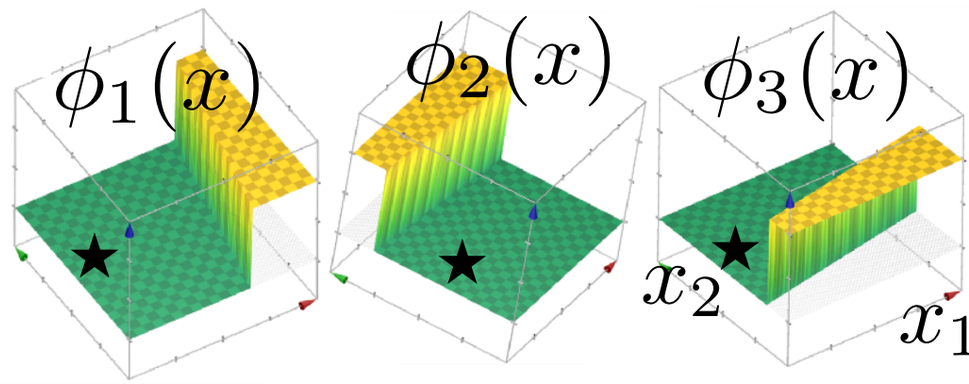

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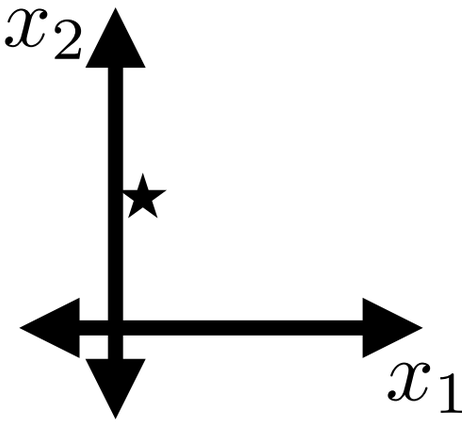


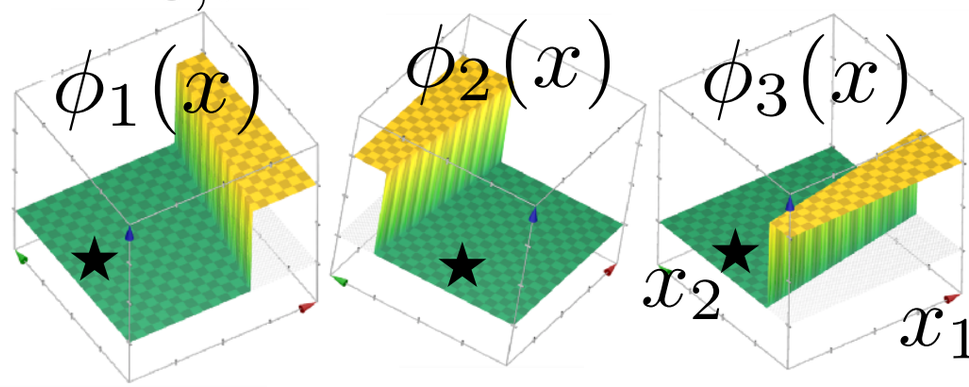

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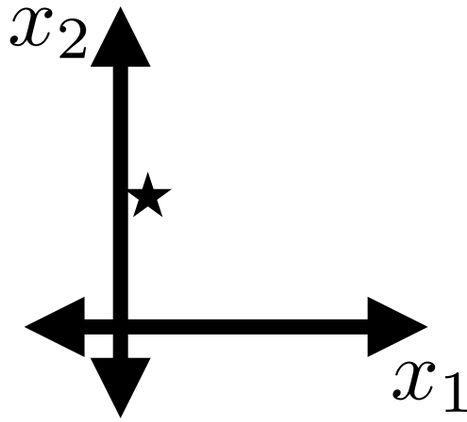


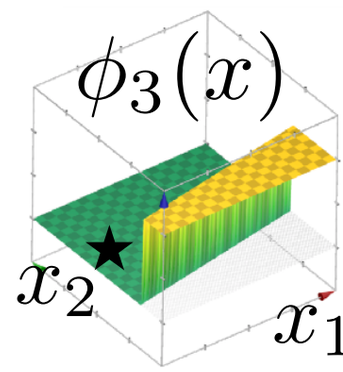

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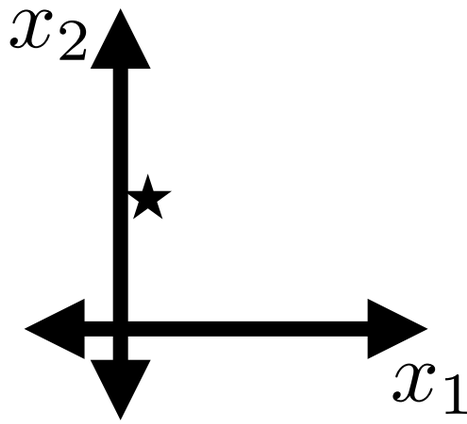


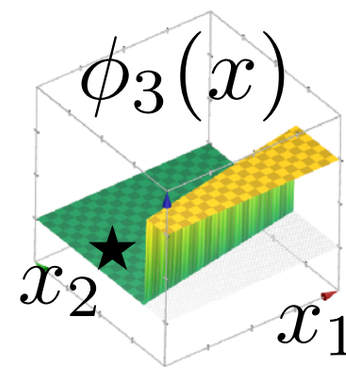

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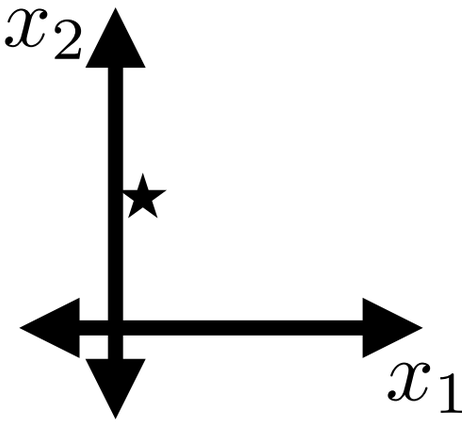


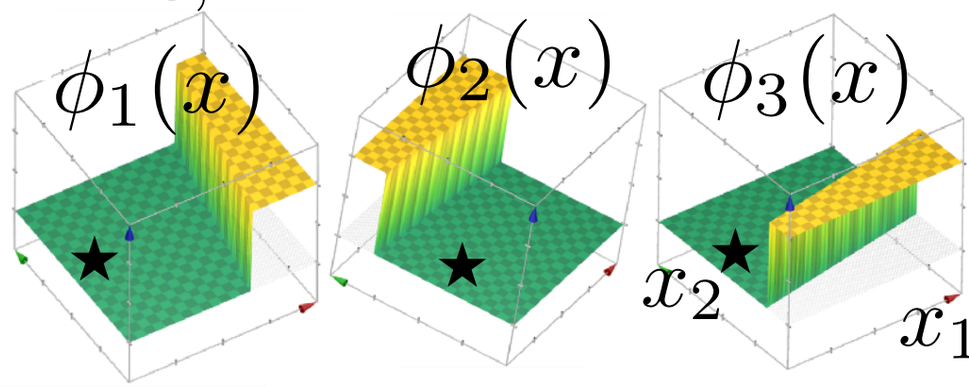

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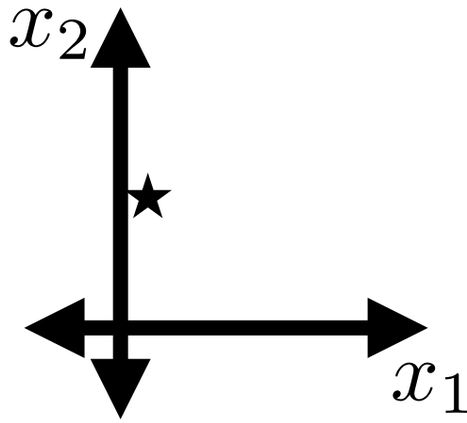
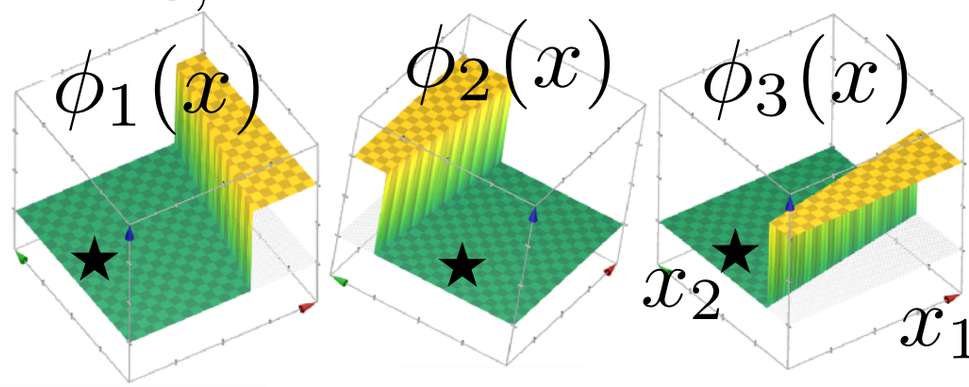
    - All the features at once:

$m^{(1)} = d$



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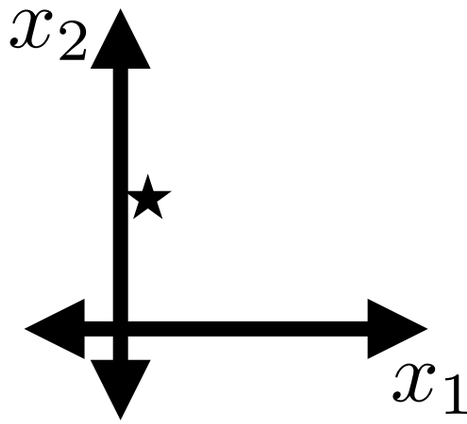


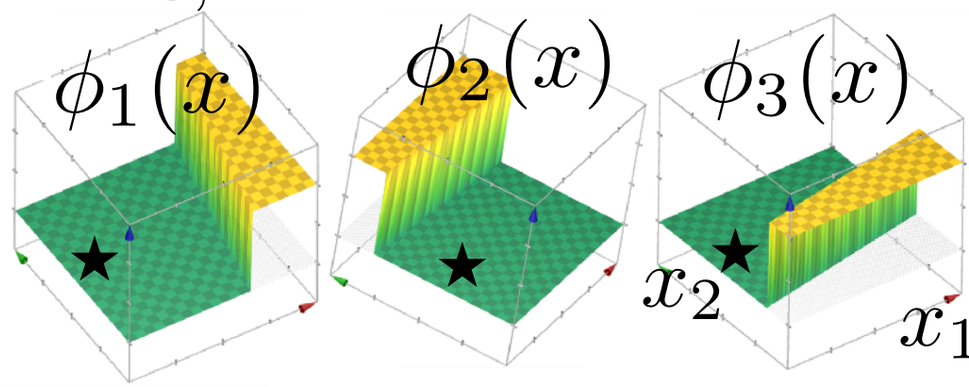

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      - $W^{(1)} : m^{(1)} \times n^{(1)}$

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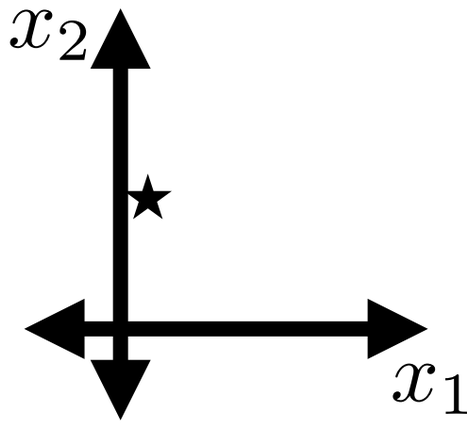


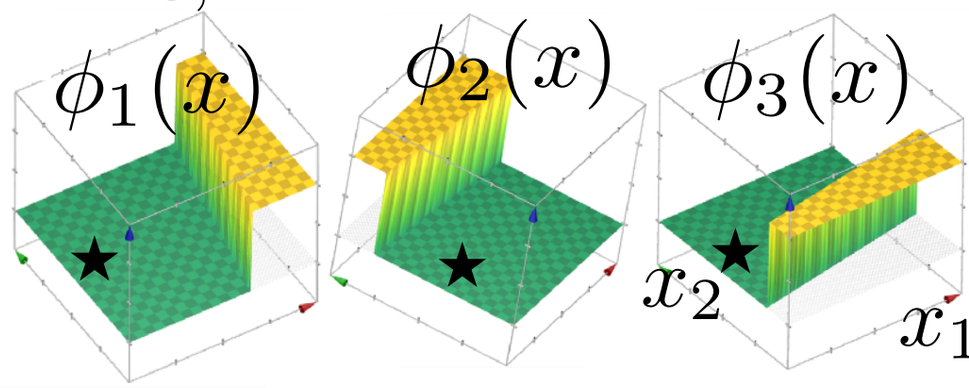

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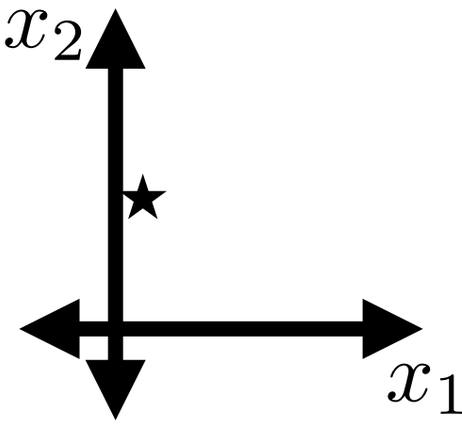


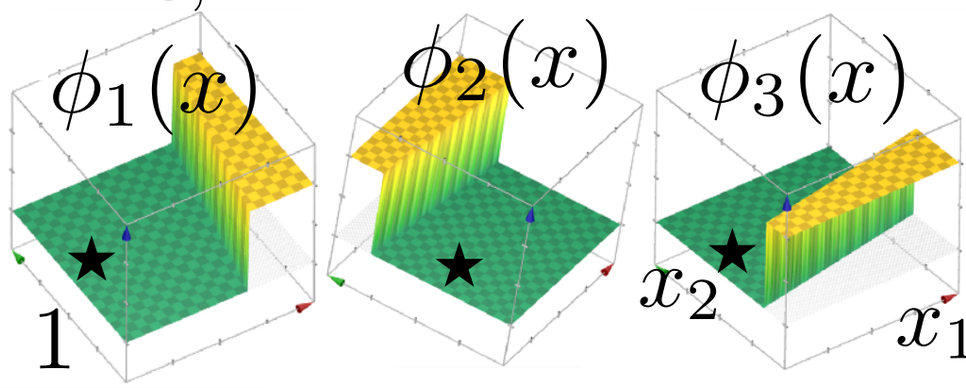

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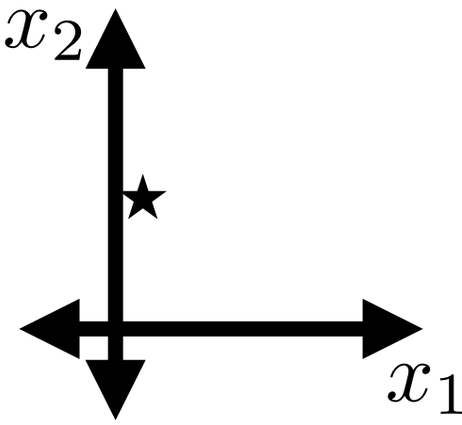


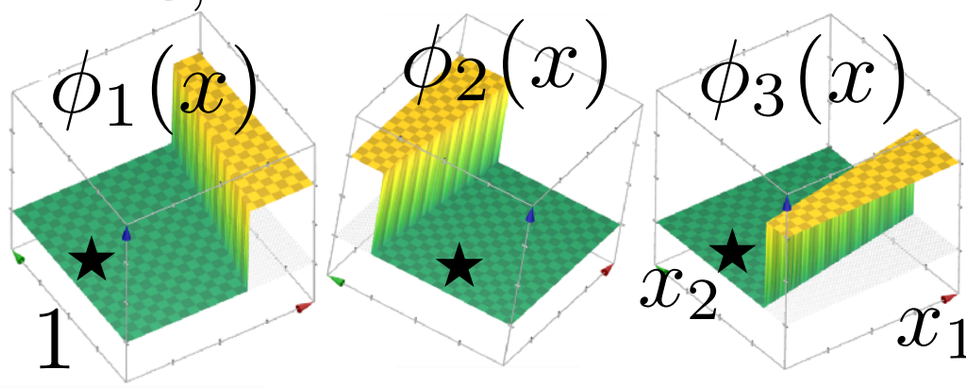

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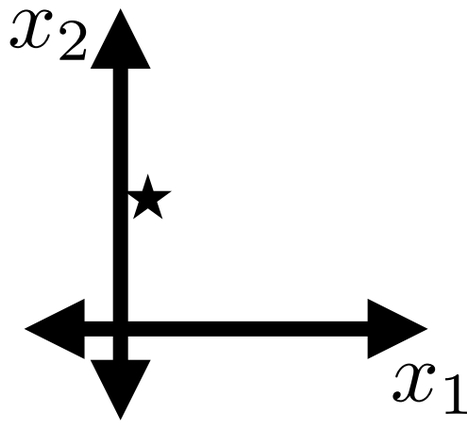


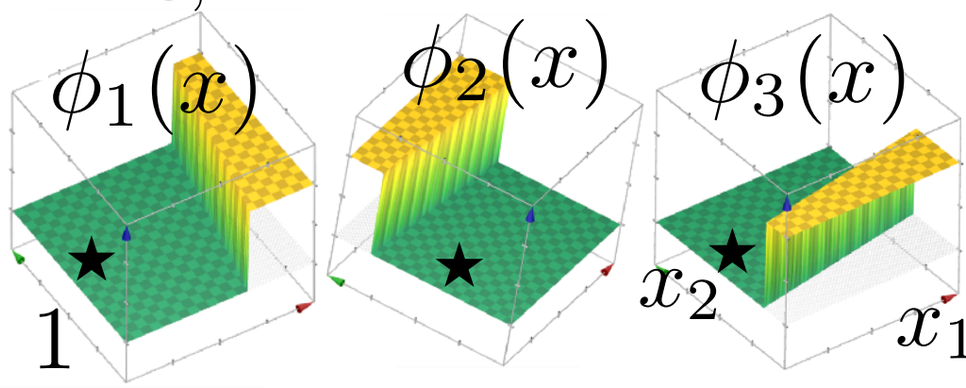

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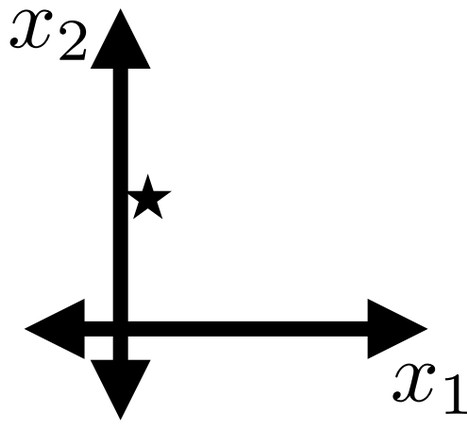
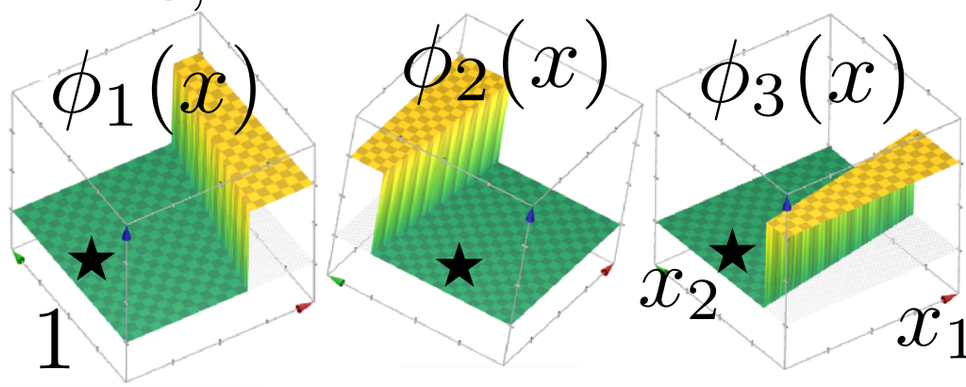
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$



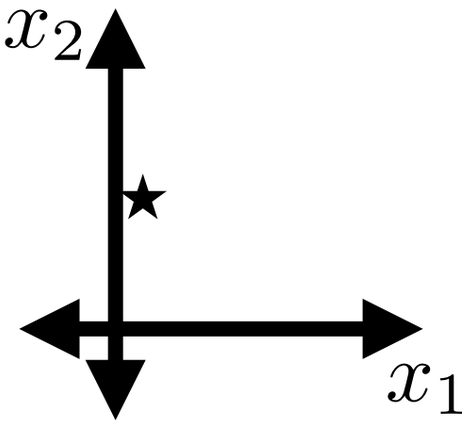



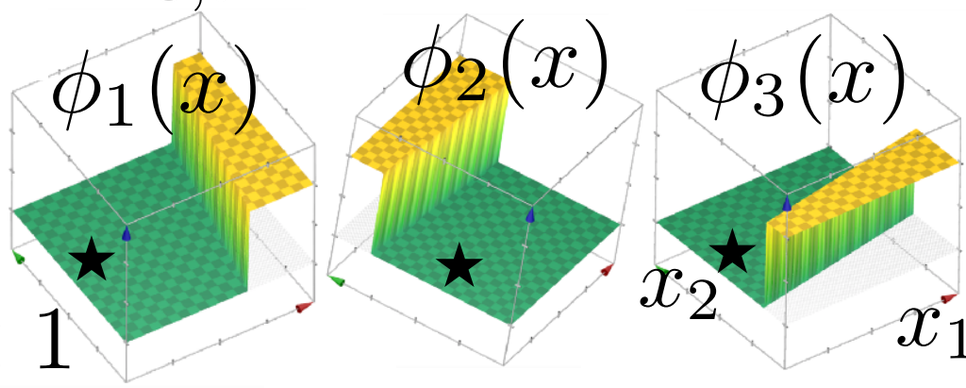

$f^{(1)}$ is applied componentwise!

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = \mathrm{d}$

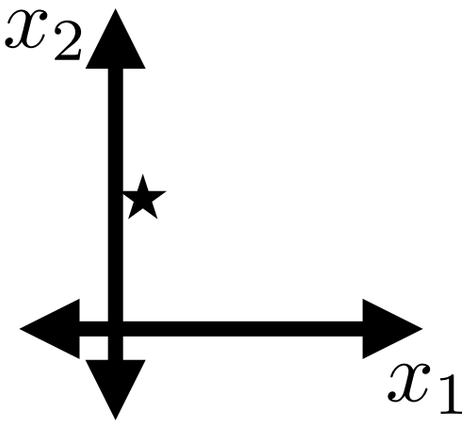


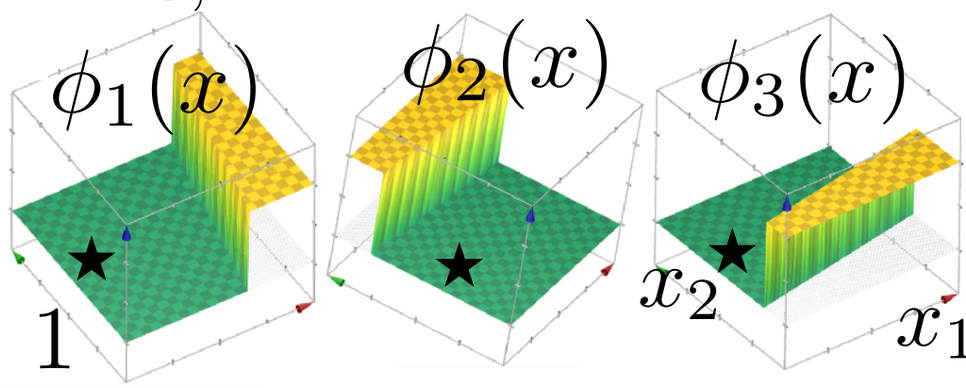



# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = \mathrm{d}$



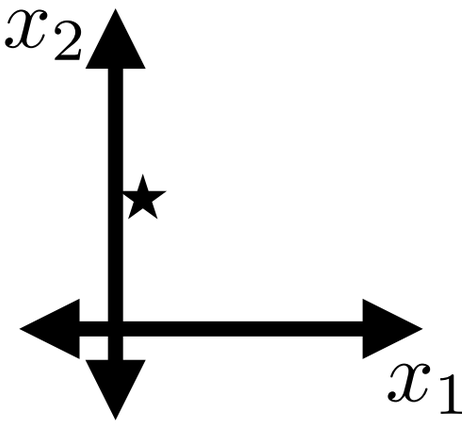



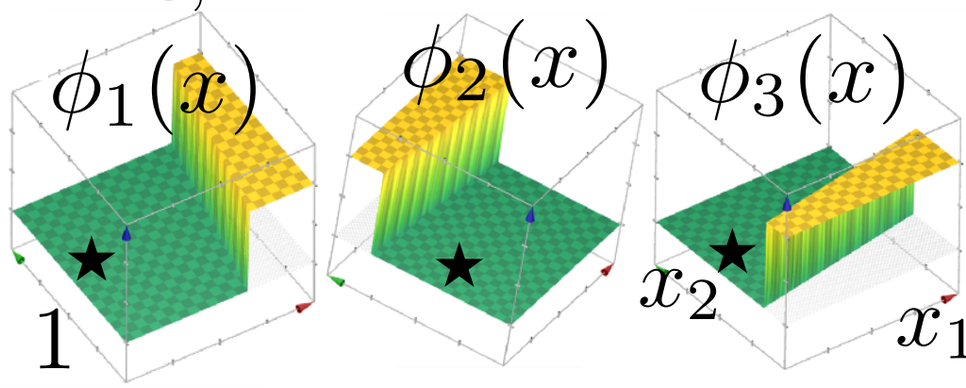

- 2nd layer, assigning a label (or labels):

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = \mathrm{d}$



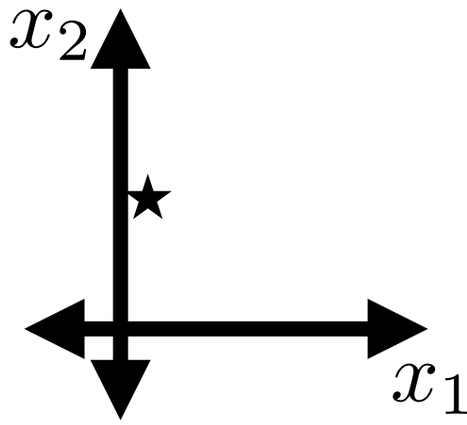



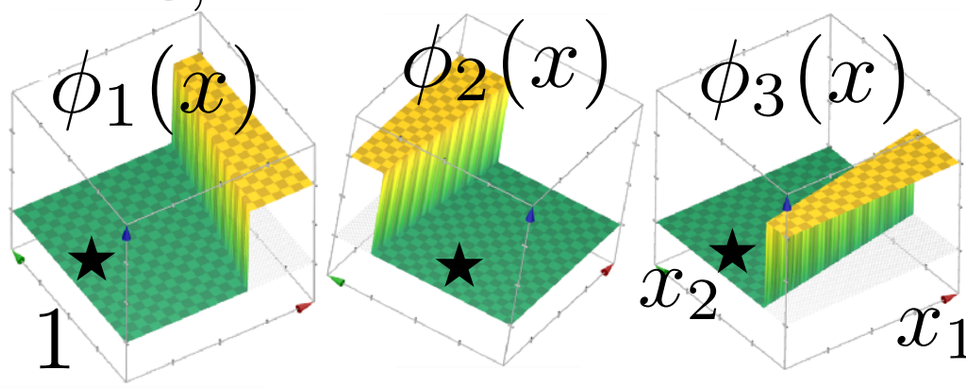

- 2nd layer, assigning a label (or labels):
  - Input (the features)

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = \mathrm{d}$

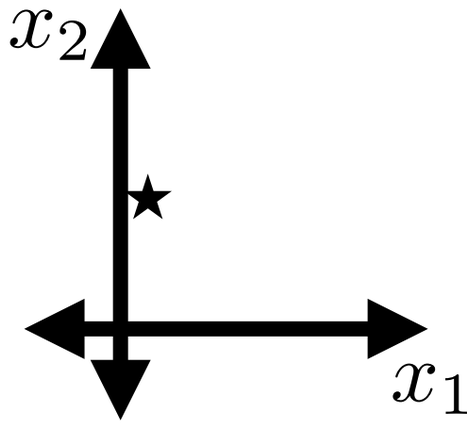


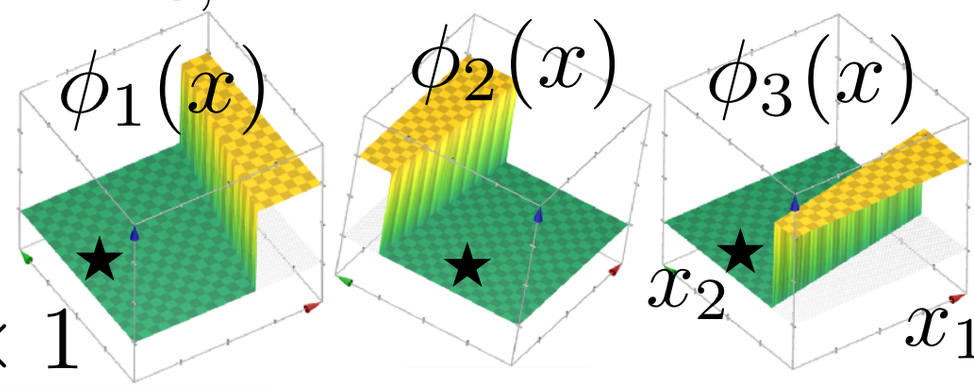




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$ $m^{(1)} = d$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



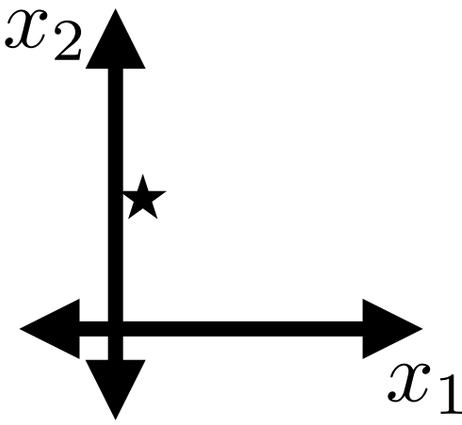



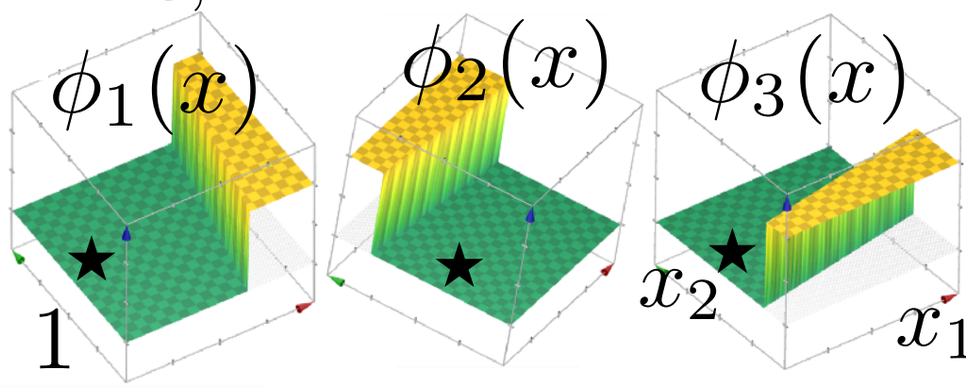

- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$ $m^{(2)} = n^{(1)}$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$



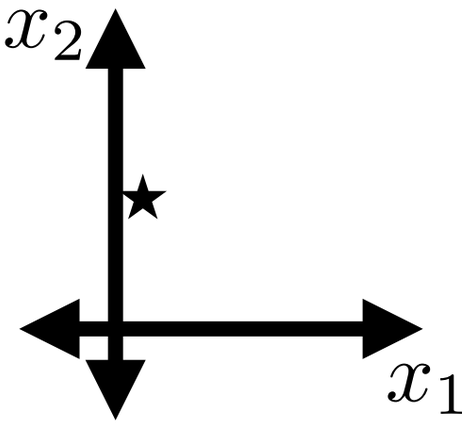

- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$

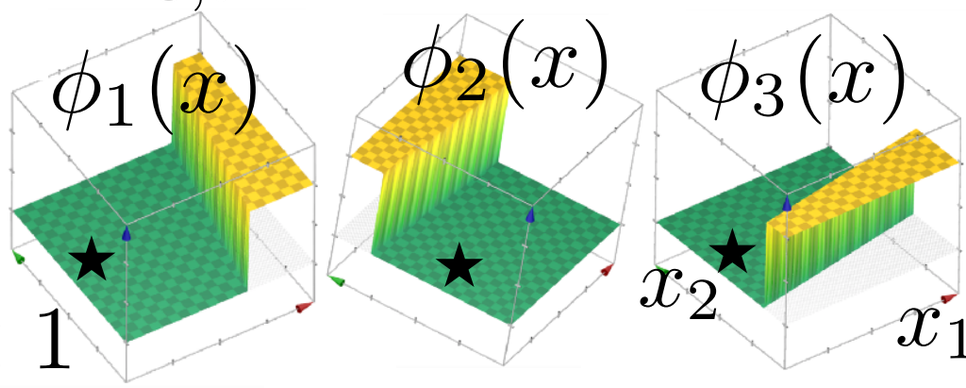
$m^{(2)} = n^{(1)}$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$

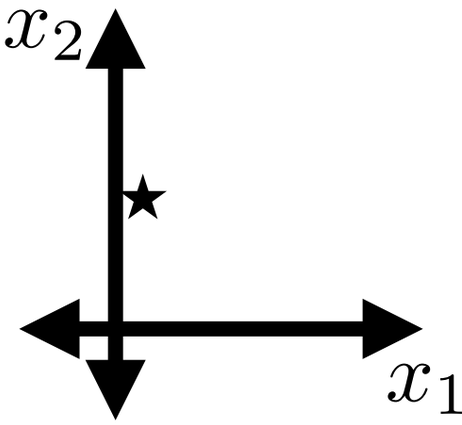
$m^{(2)} = n^{(1)}$

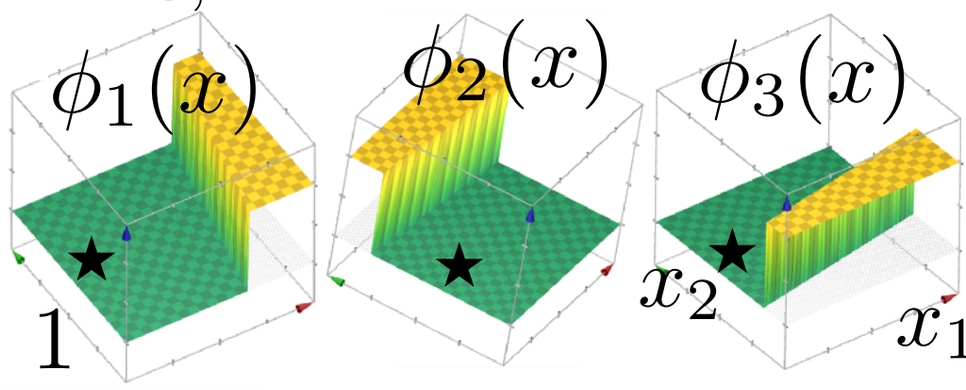
Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$

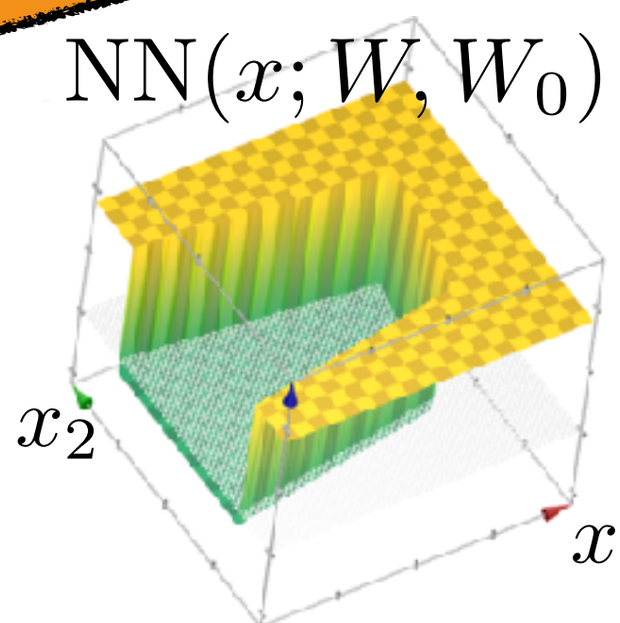



- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$

$m^{(2)} = n^{(1)}$

Like $y$



# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
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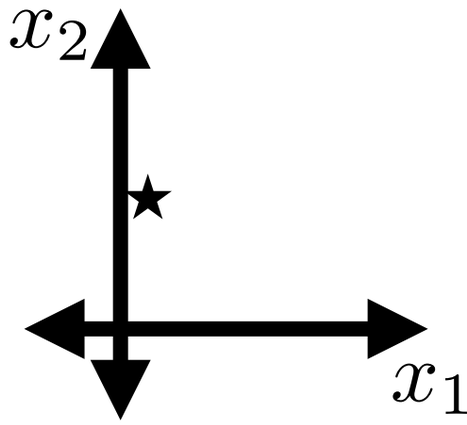
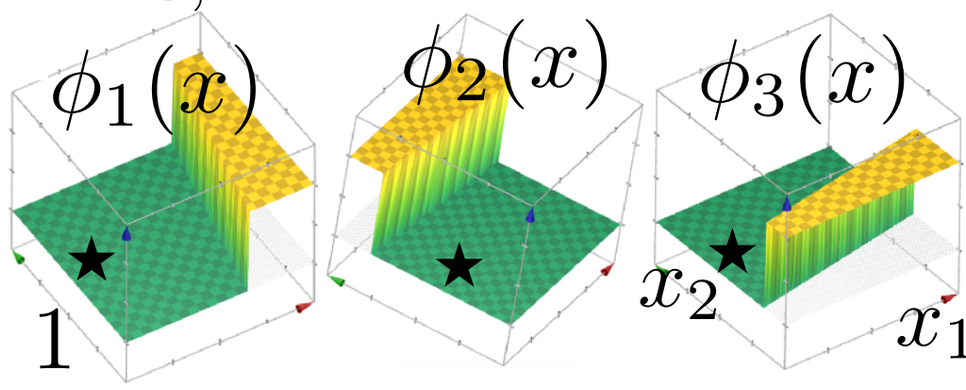
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

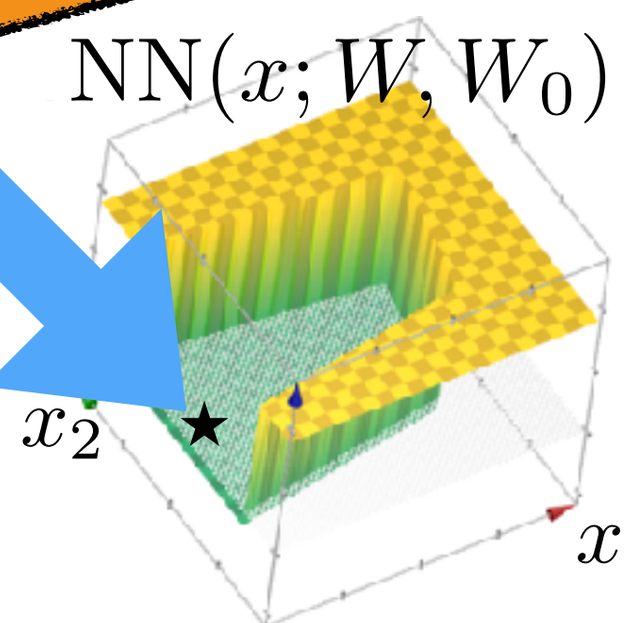
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$

$m^{(2)} = n^{(1)}$

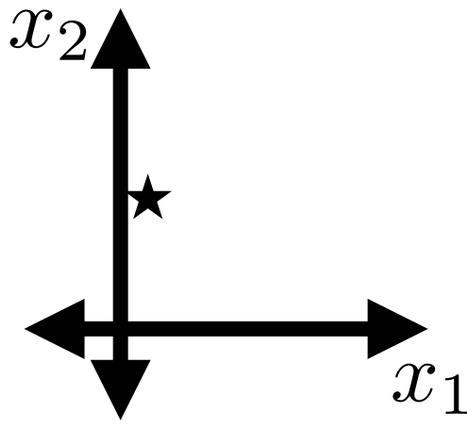


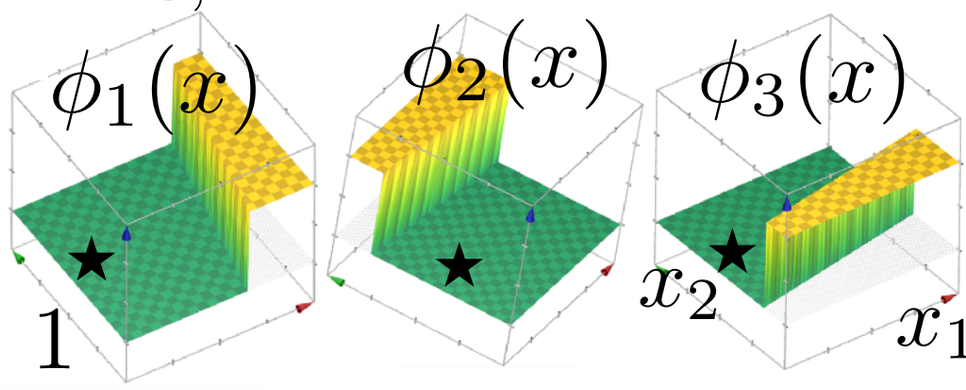

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

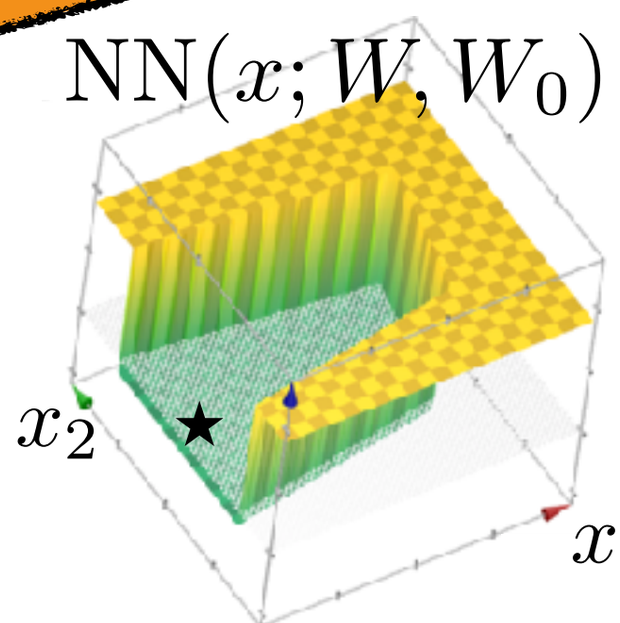
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$

$m^{(2)} = n^{(1)}$

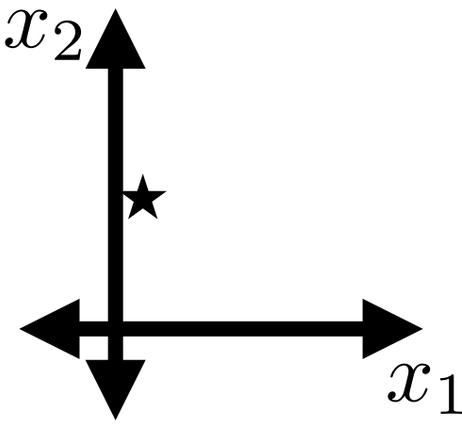


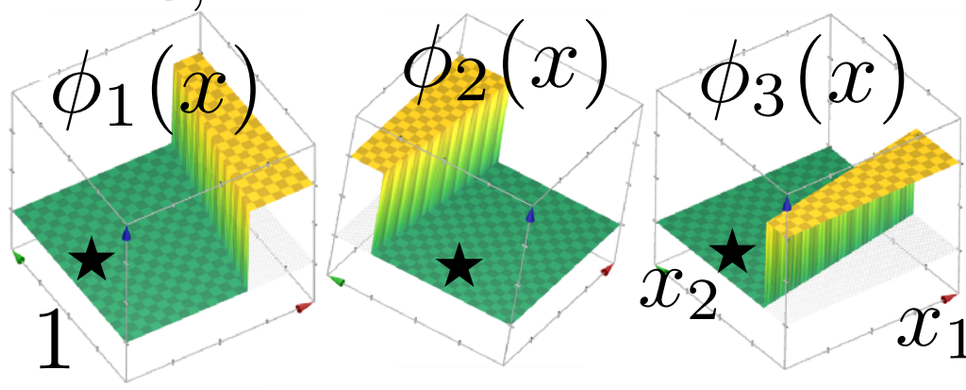

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

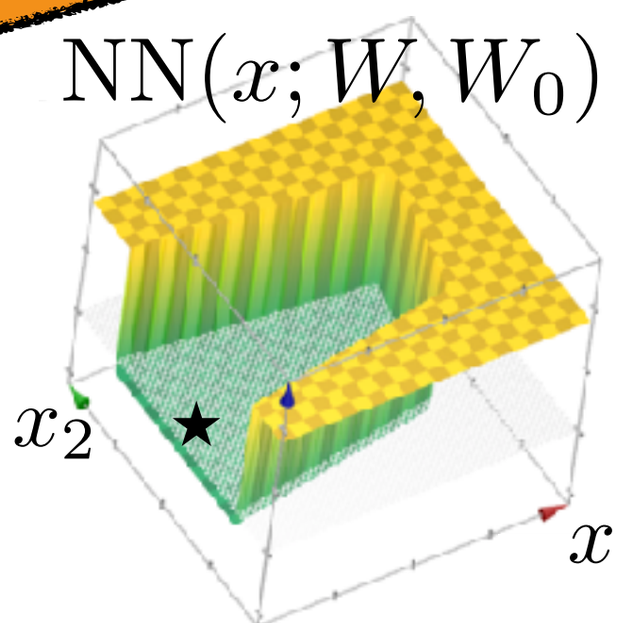
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$ (vector of labels)

$m^{(2)} = n^{(1)}$

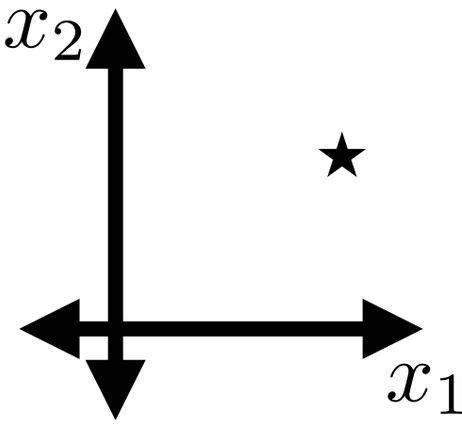


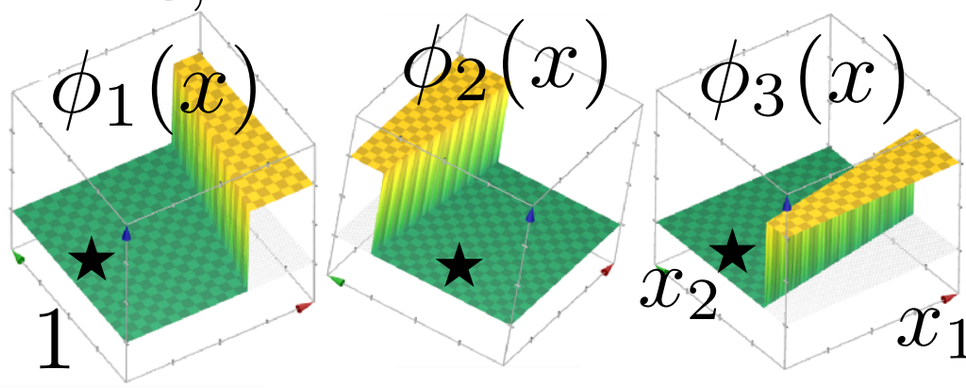

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

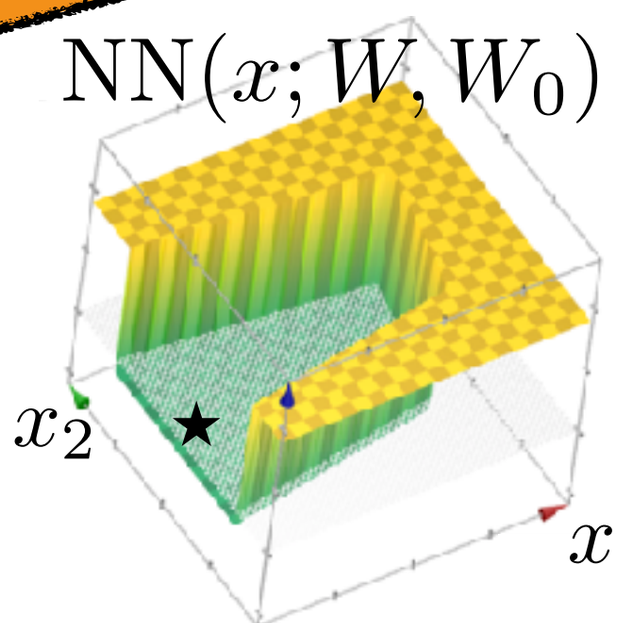
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$(vector of labels): size $n^{(2)} \times 1$

$m^{(2)} = n^{(1)}$

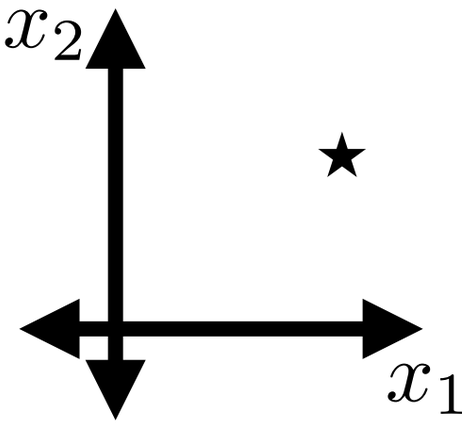


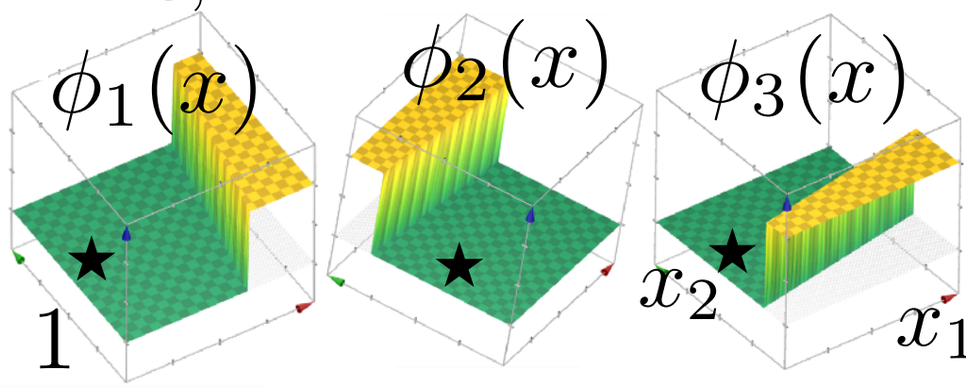

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

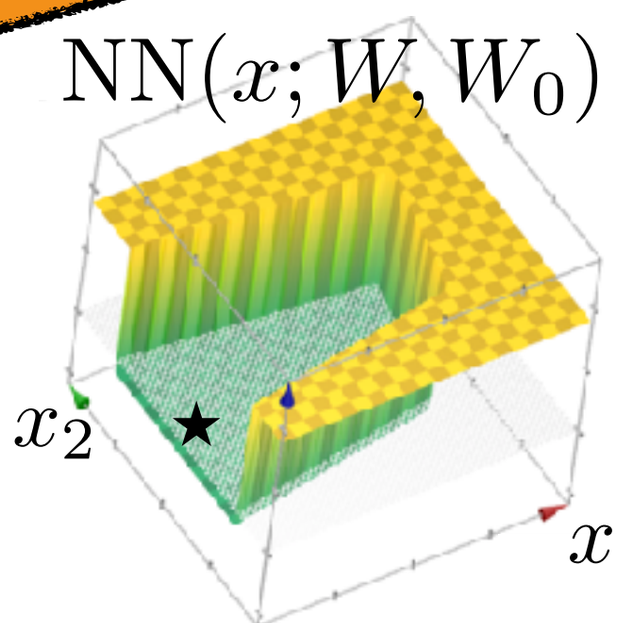
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$(vector of labels): size $n^{(2)} \times 1$
  - The $i$th label:

$m^{(2)} = n^{(1)}$

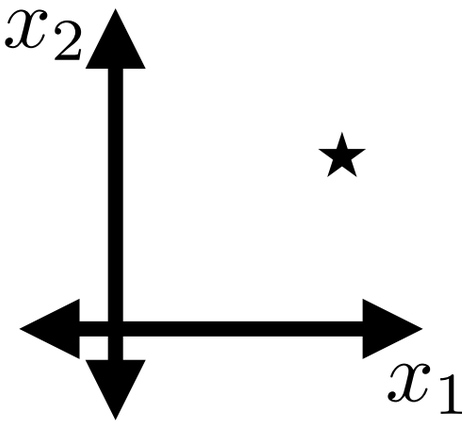


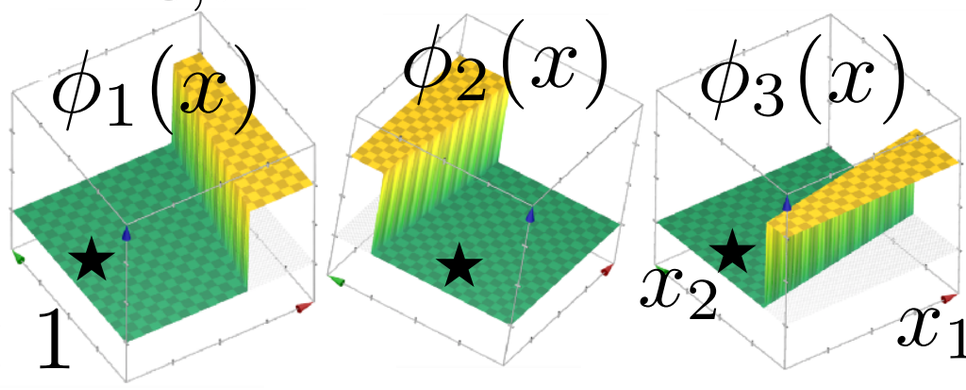

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

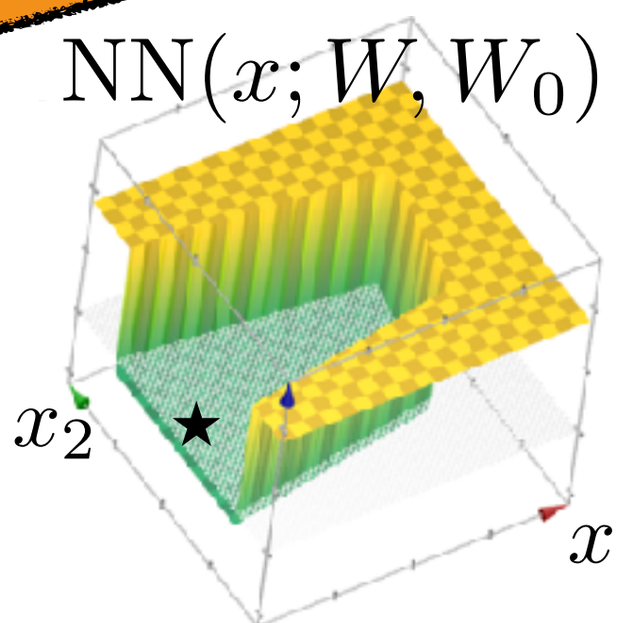
$m^{(1)} = d$




- 2nd layer, assigning a label (or labels):
  - Input (the features): size $m^{(2)} \times 1$
  - Output $A^{(2)}$(vector of labels): size $n^{(2)} \times 1$
  - The $i$th label: $A_i^{(2)} = f^{(2)}(w_i^{(2)\top} A^{(1)} + w_{0,i}^{(2)})$

$m^{(2)} = n^{(1)}$

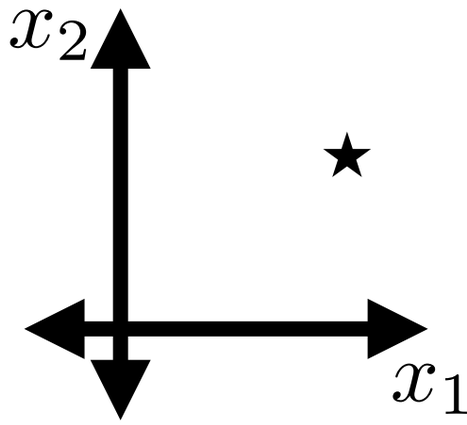


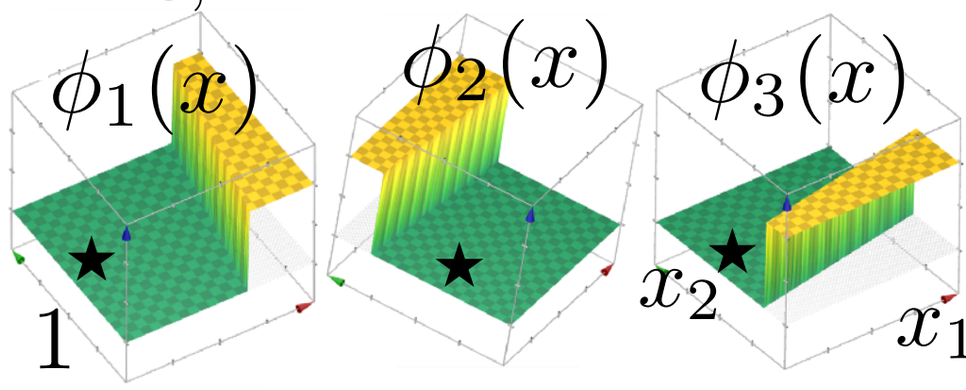

Like $y$

# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
  - The $i$th feature: $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
    - All the features at once:
      - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$

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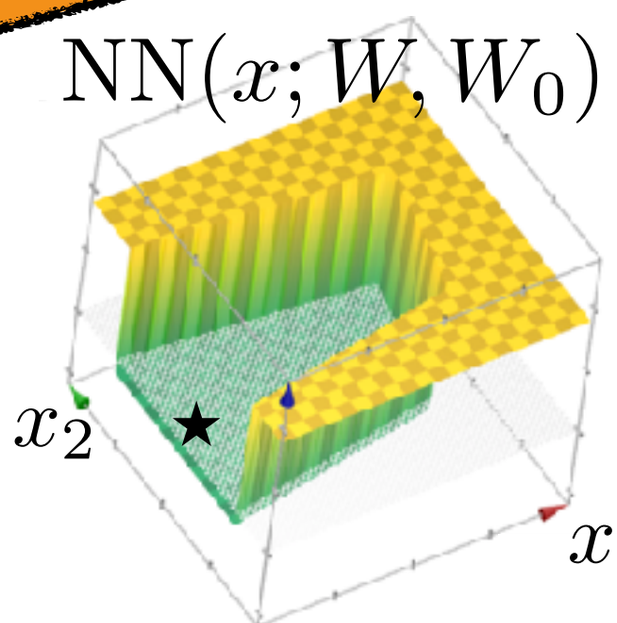
  - The $i$th label: $A_i^{(2)} = f^{(2)}(w_i^{(2)\top} A^{(1)} + w_{0,i}^{(2)})$
    - All: $y$

$m^{(2)} = n^{(1)}$

Like



# Let's get some new notation

- 1st layer, constructing the features:
  - Input $x$ (a data point): size $m^{(1)} \times 1$ $m^{(1)} = d$
  - Output $A^{(1)}$ (vector of features): size $n^{(1)} \times 1$
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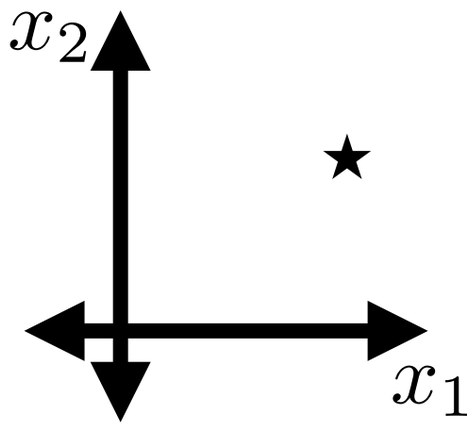


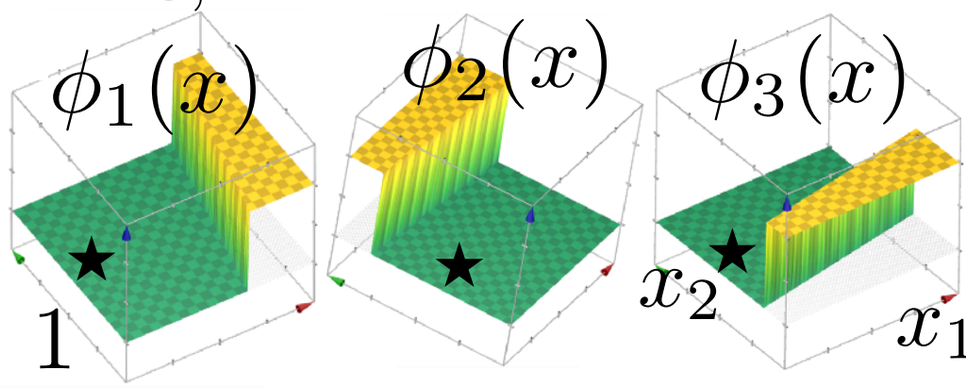


- 2nd layer, assigning a label (or labels):
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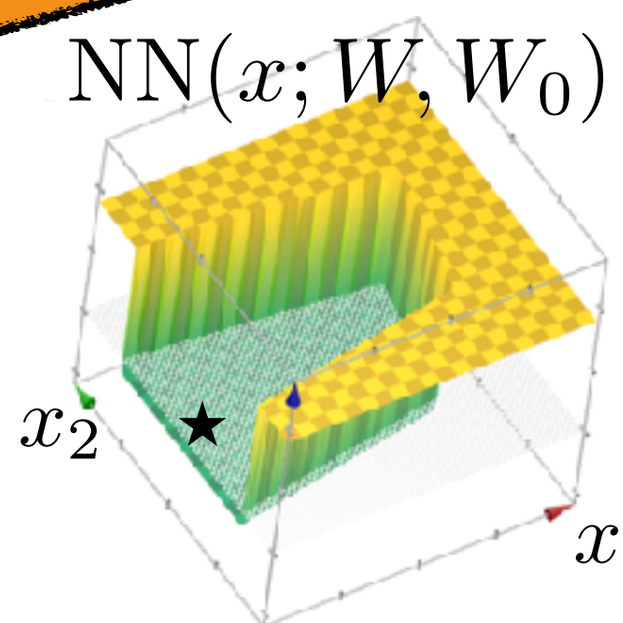
  - All: $A^{(2)} = f^{(2)}(W^{(2)\top} A^{(1)} + W_0^{(2)})$

Like $y$



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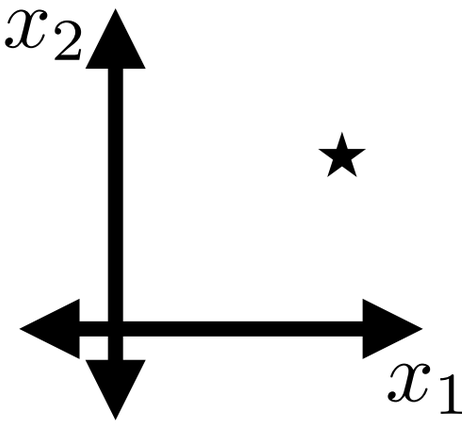
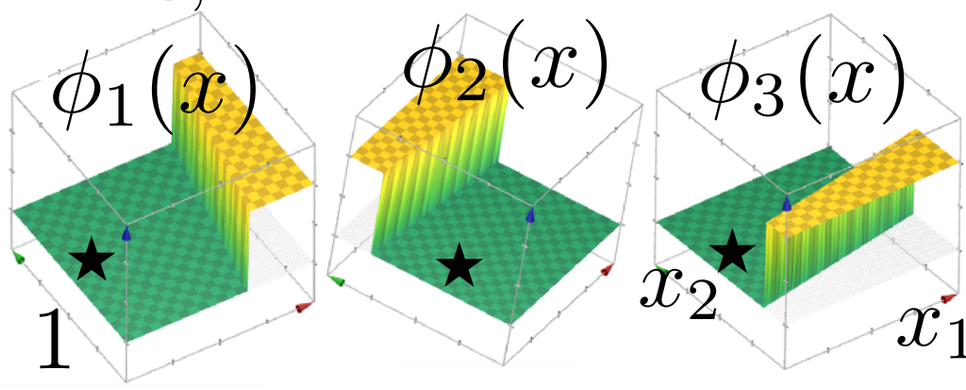
      - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

$m^{(1)} = d$

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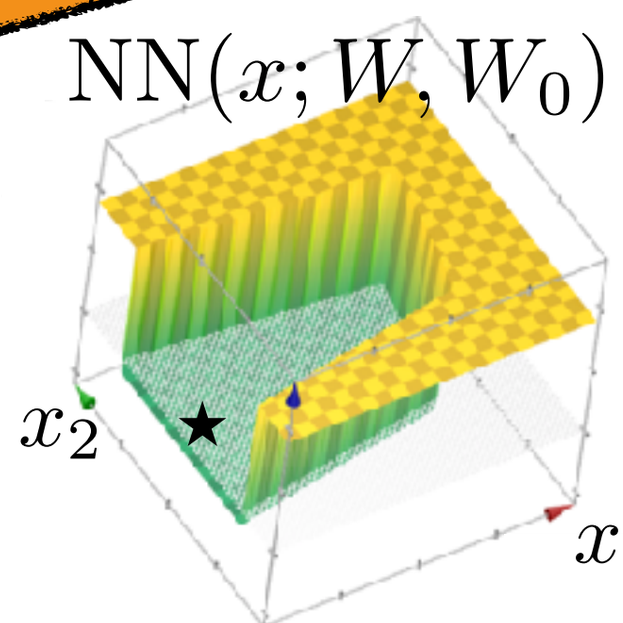
    - $W^{(2)} : m^{(2)} \times n^{(2)}$

$m^{(2)} = n^{(1)}$

Like $y$



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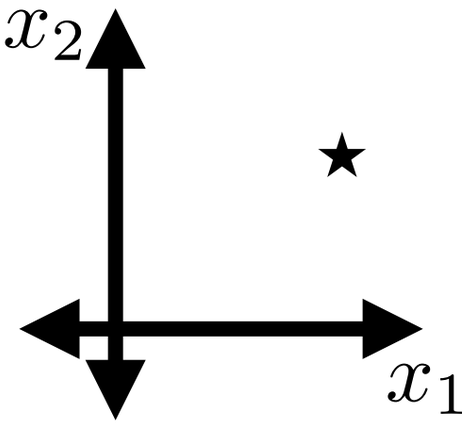


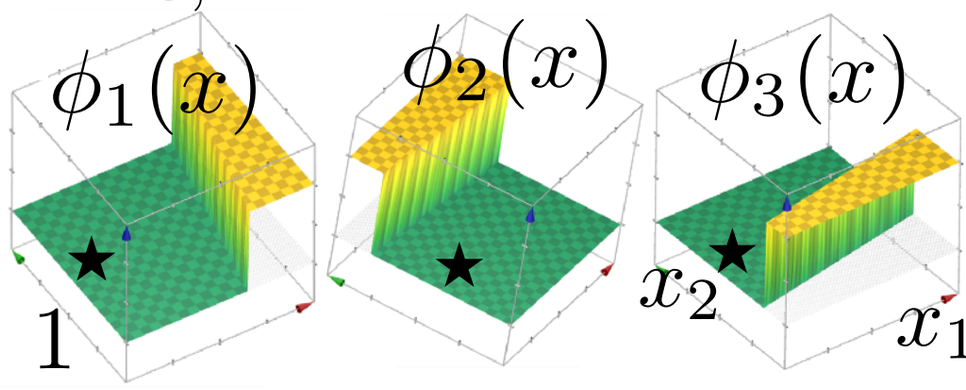


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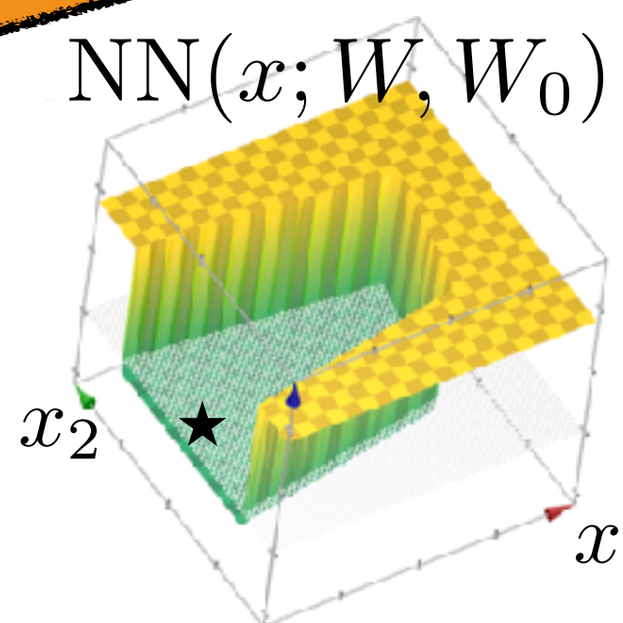
Like $y$



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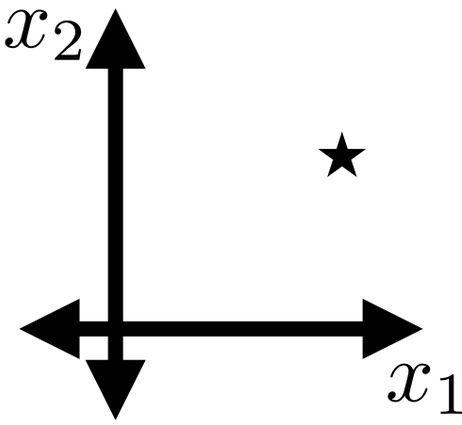


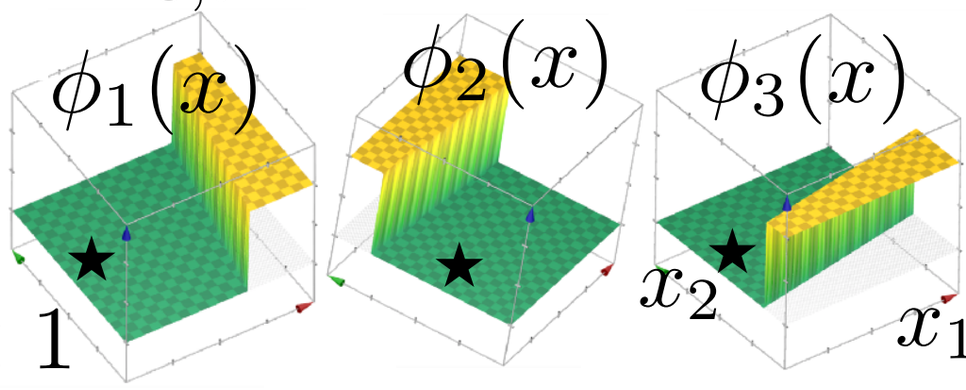


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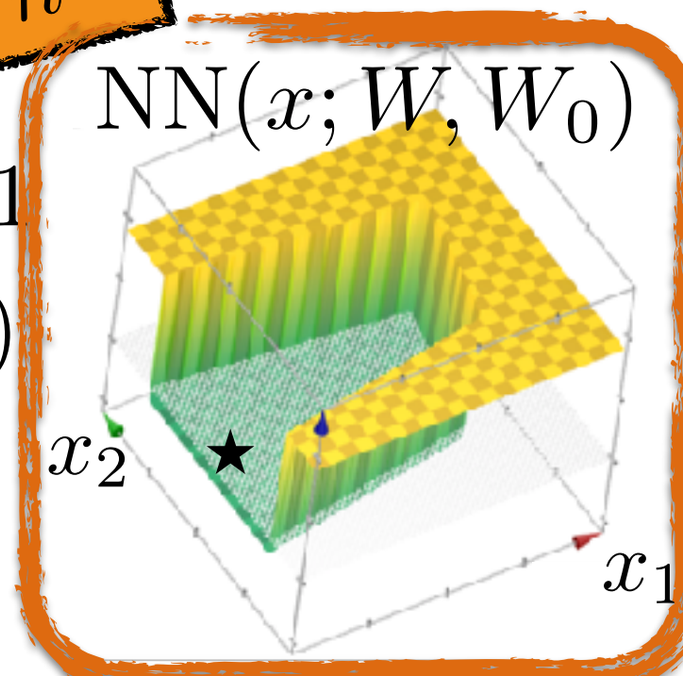
Like $y$



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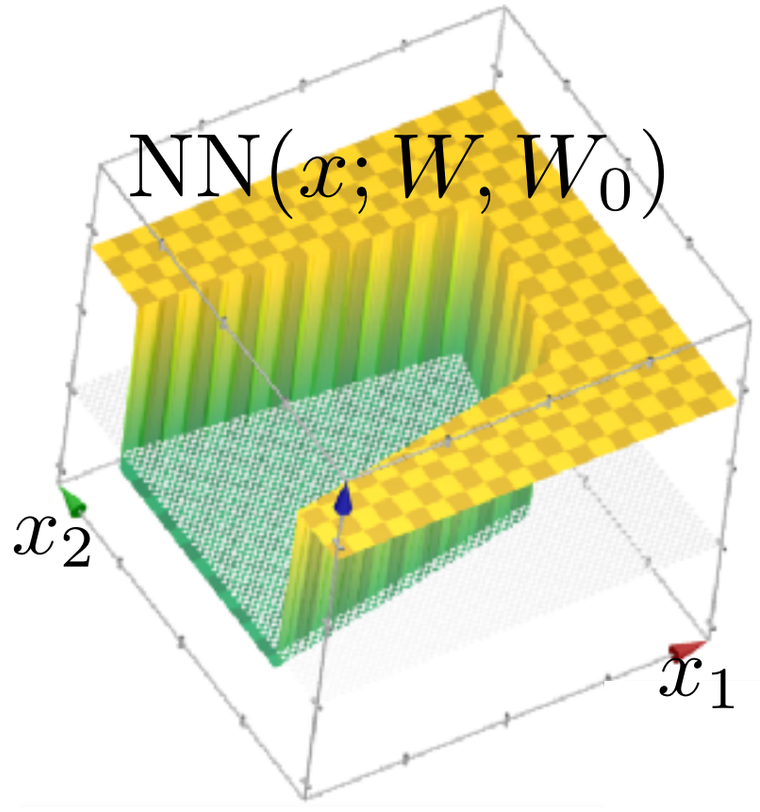

# Function graph representation

- 1st layer: $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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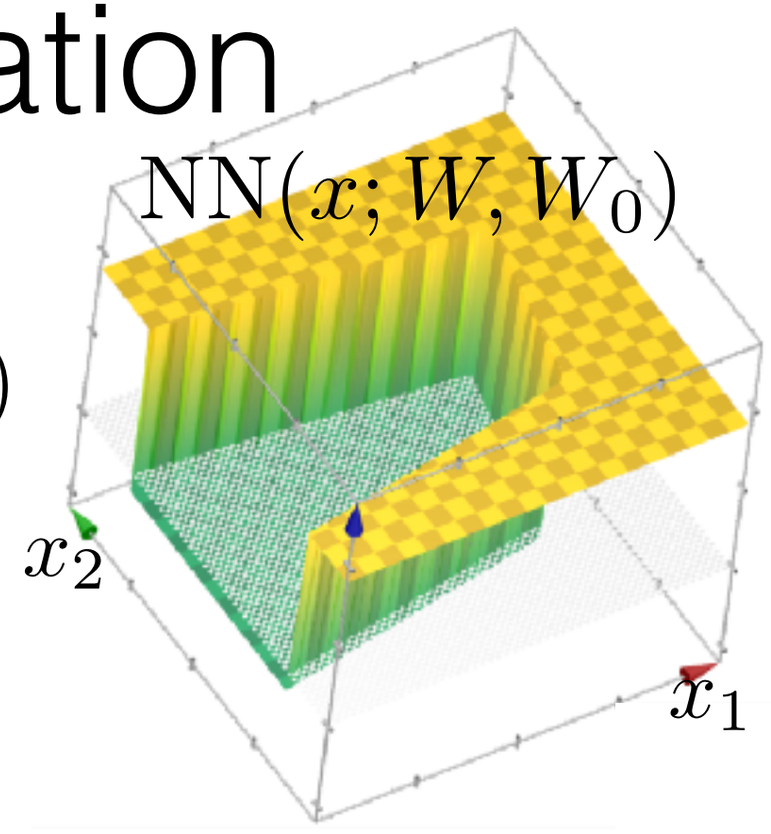

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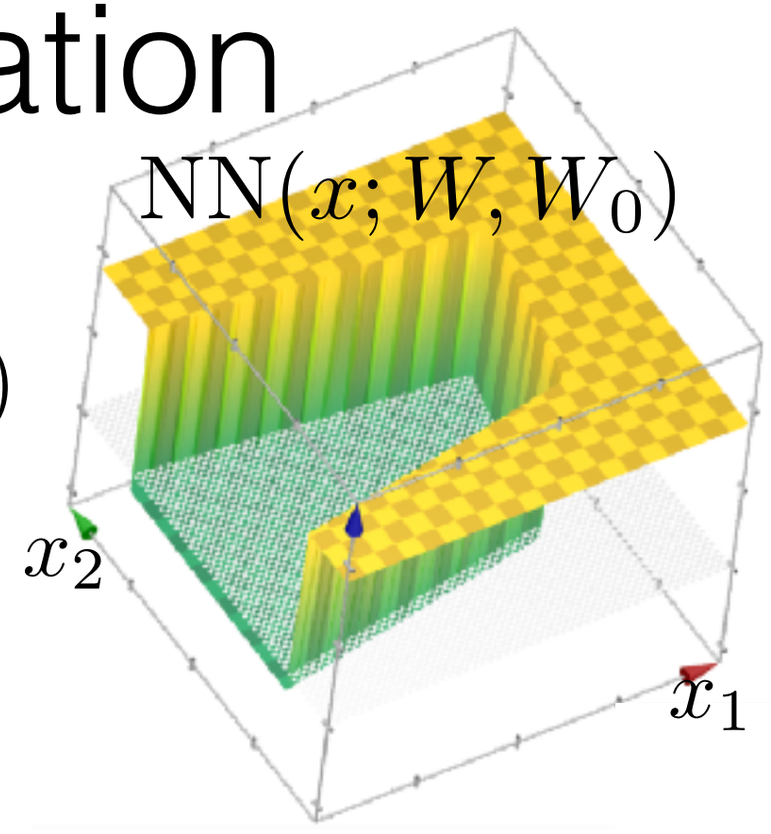

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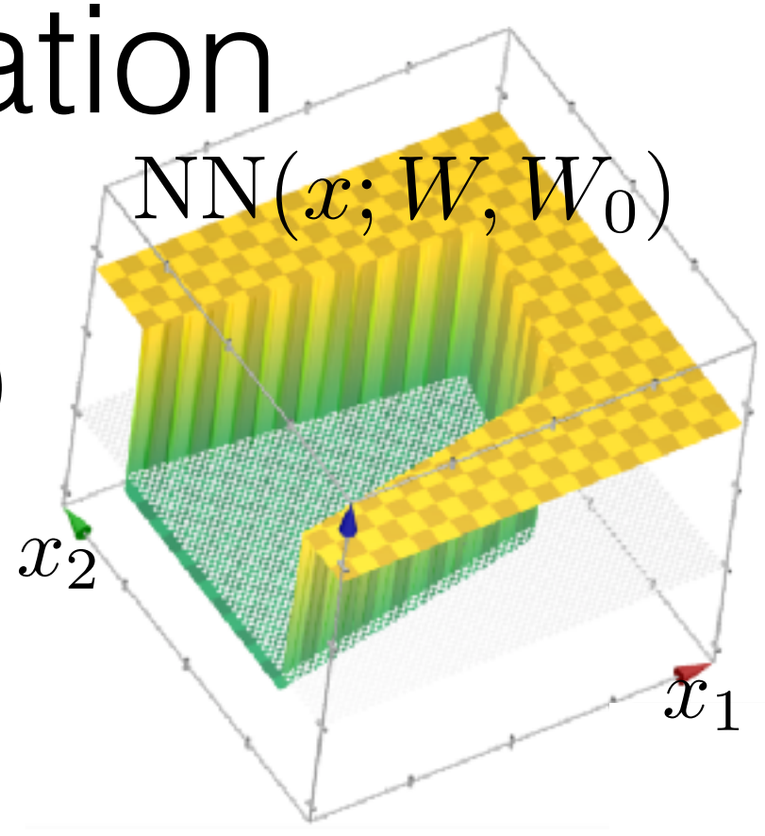


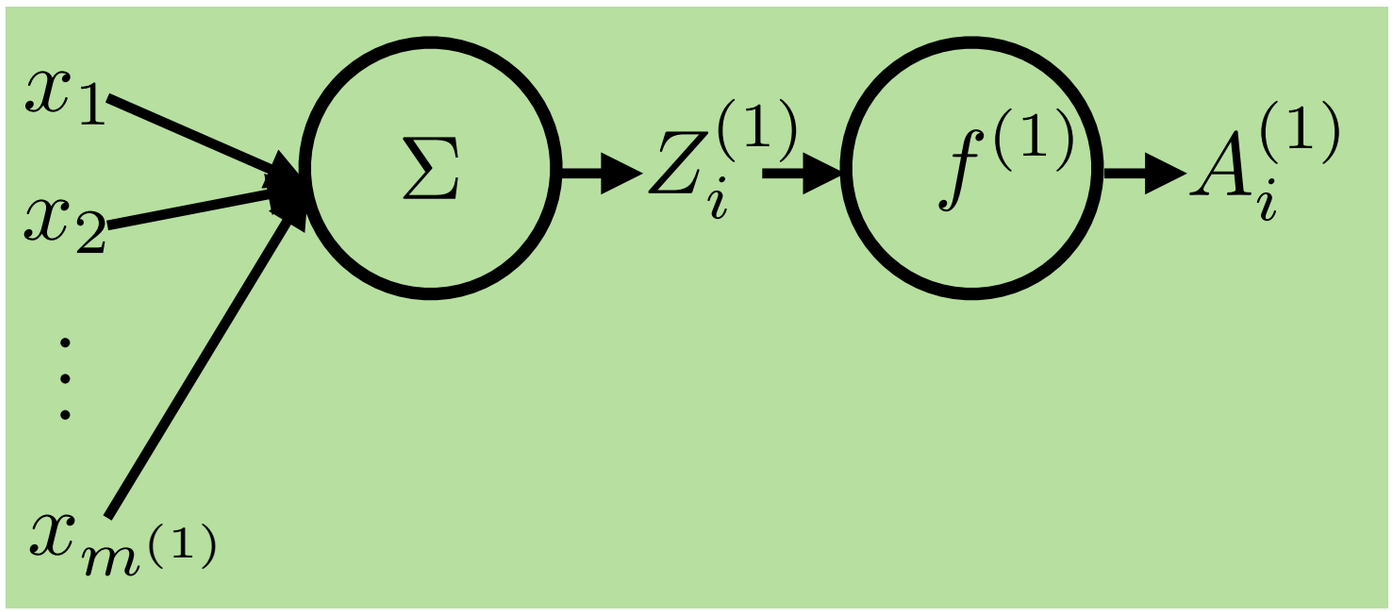

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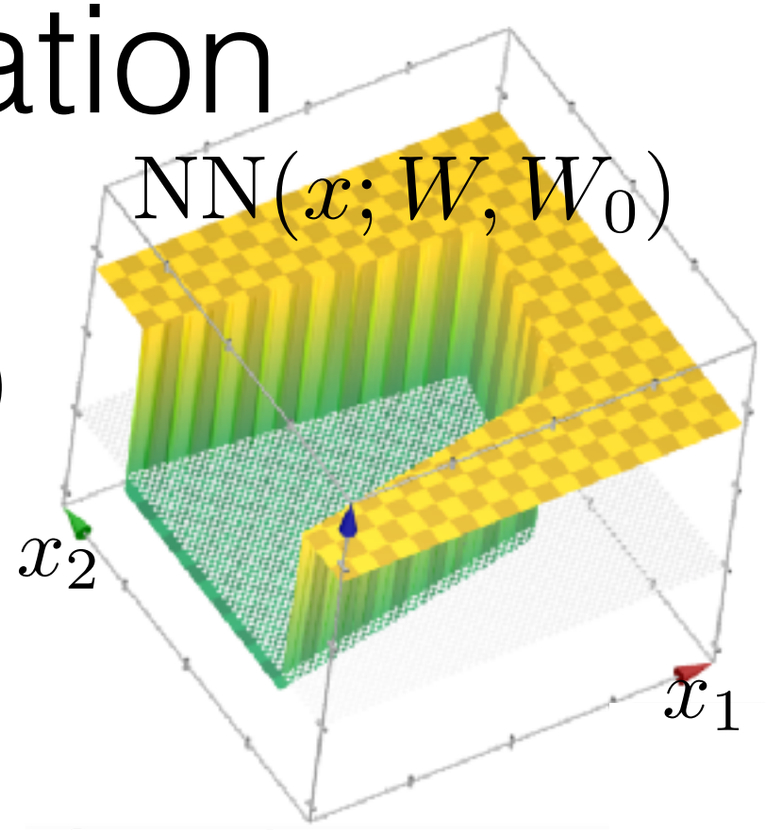
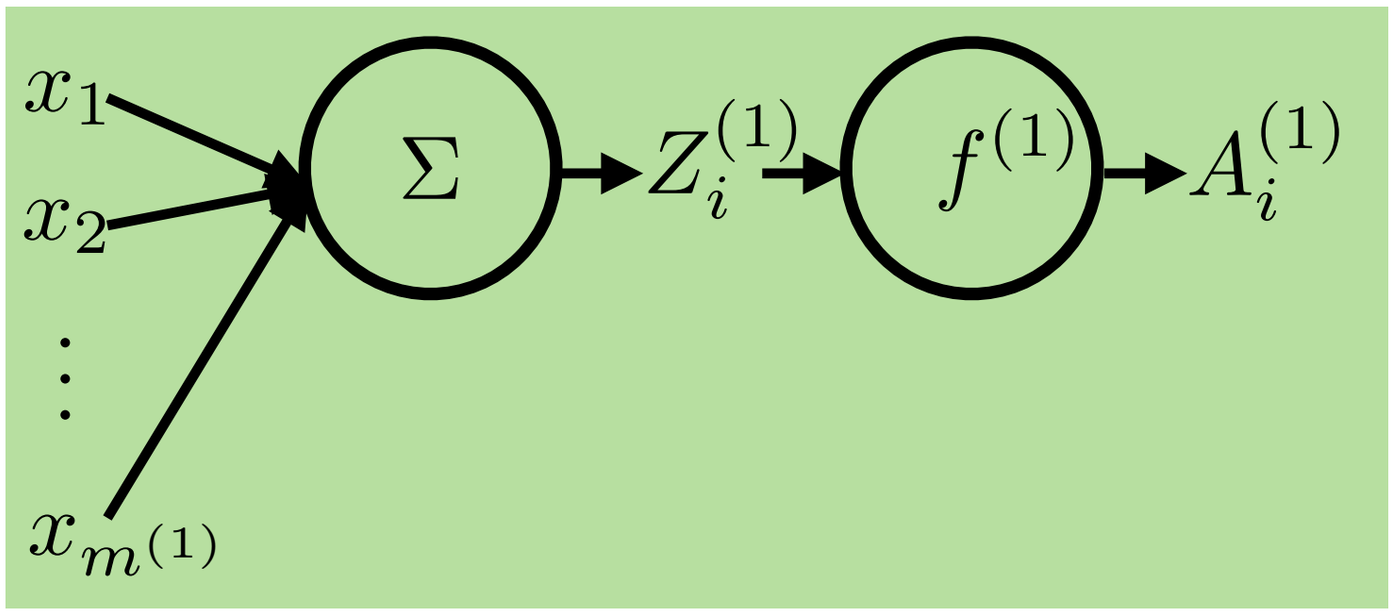



- Circle: function evaluation



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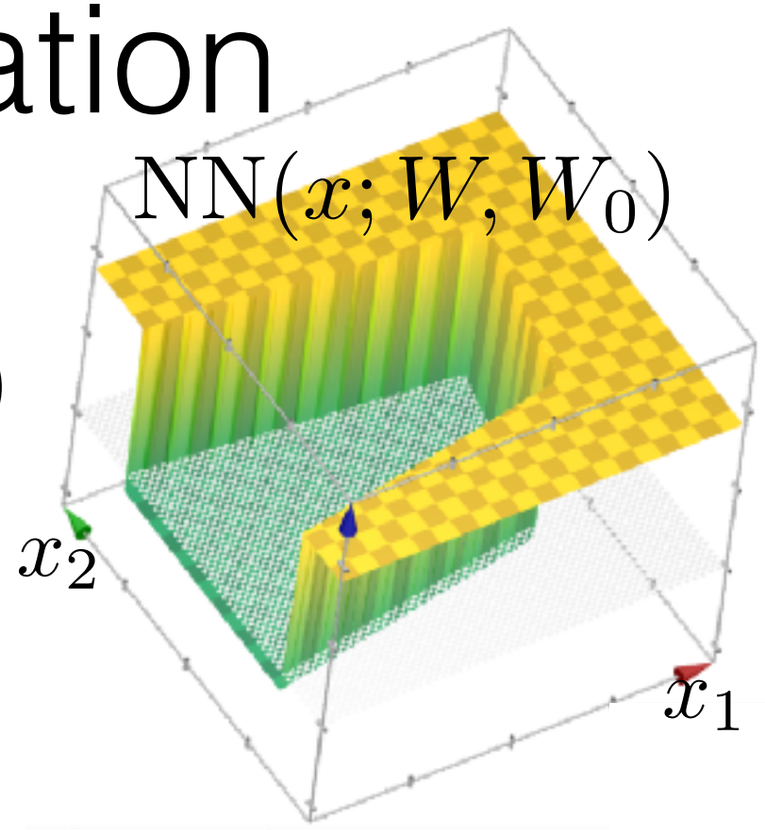


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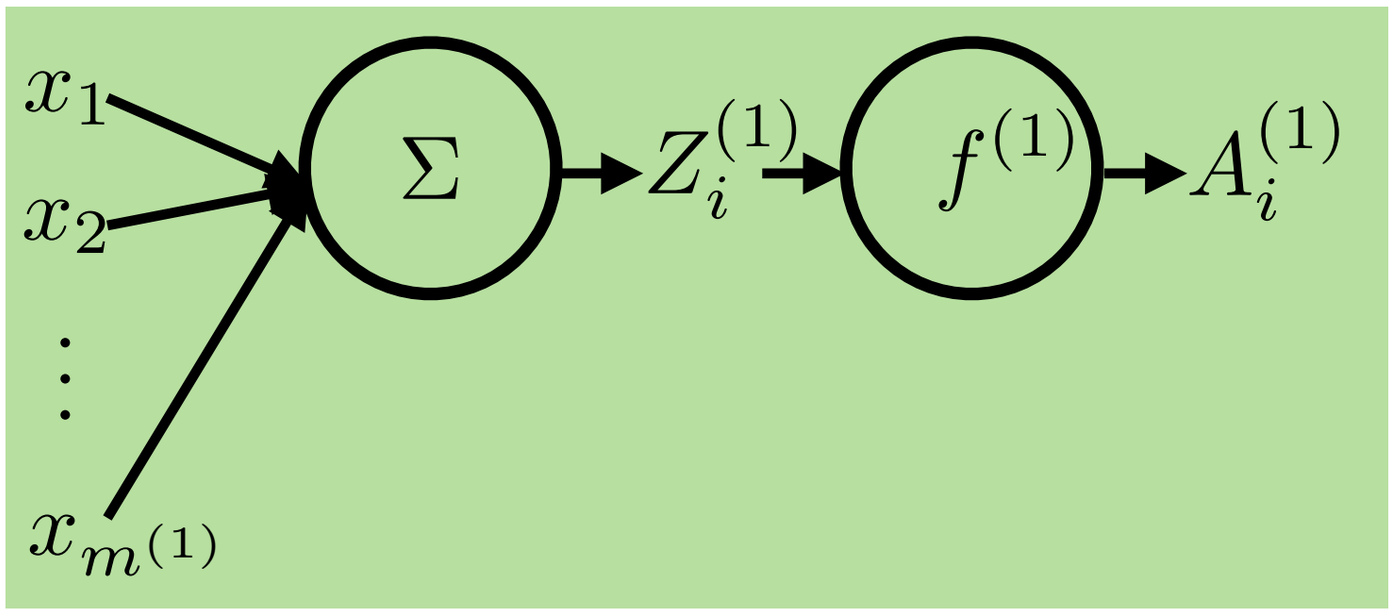

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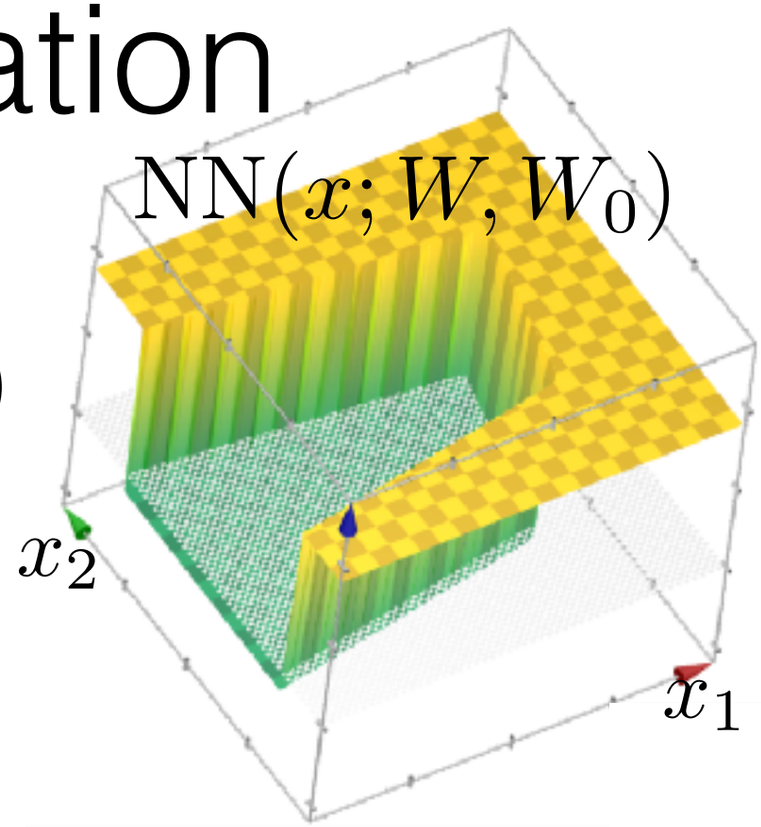


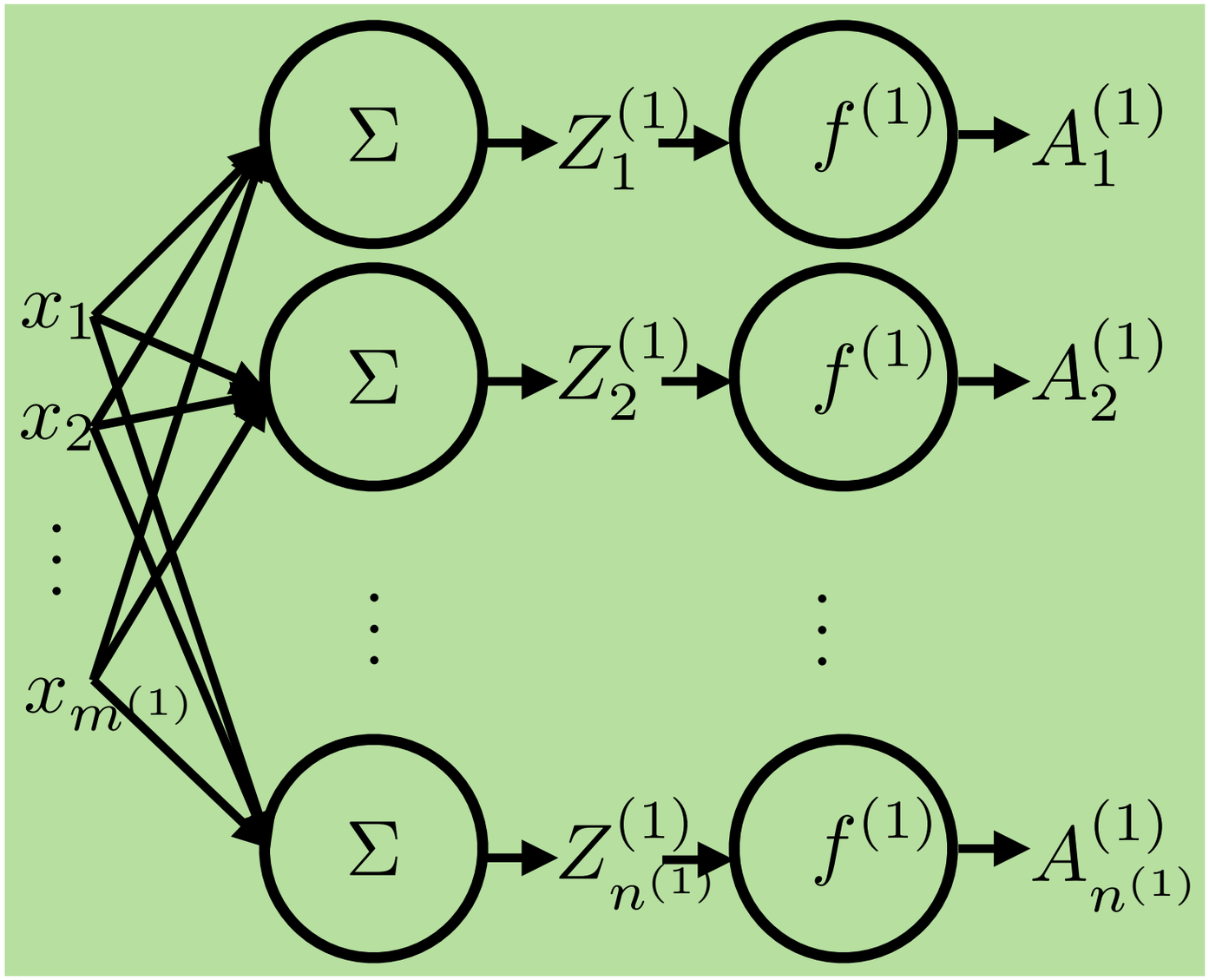


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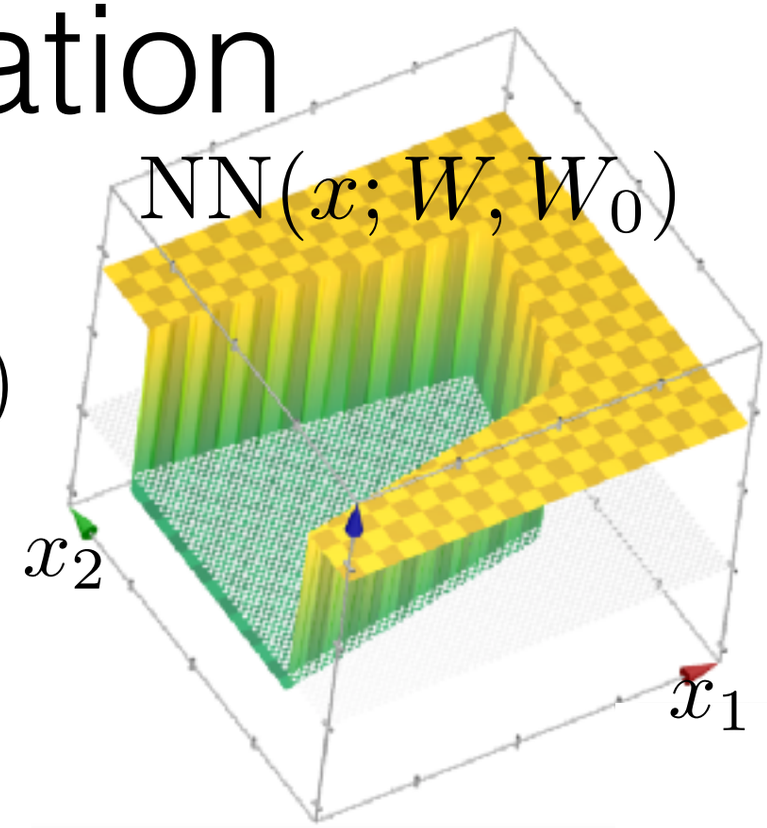


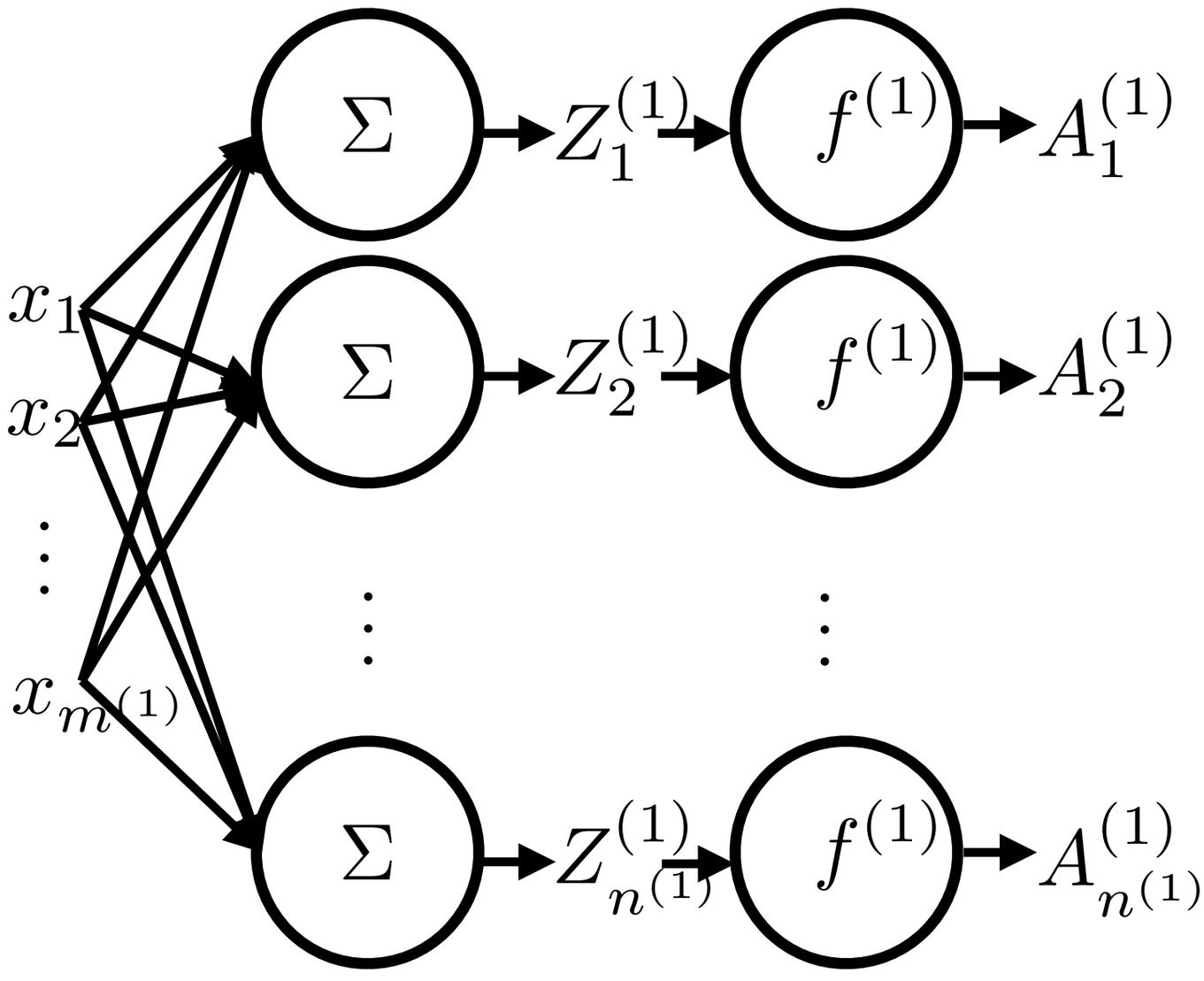


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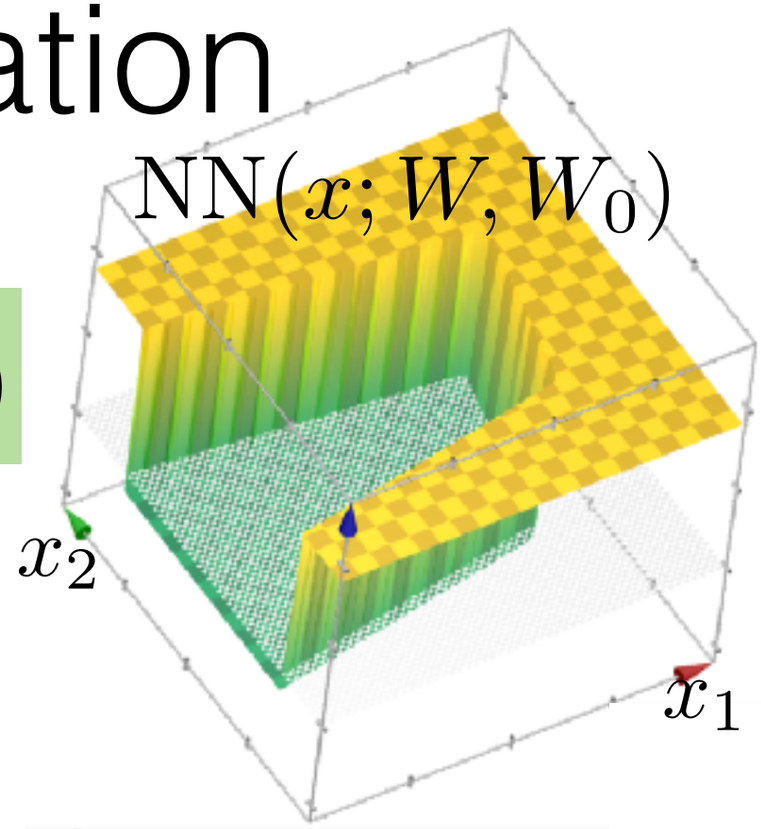


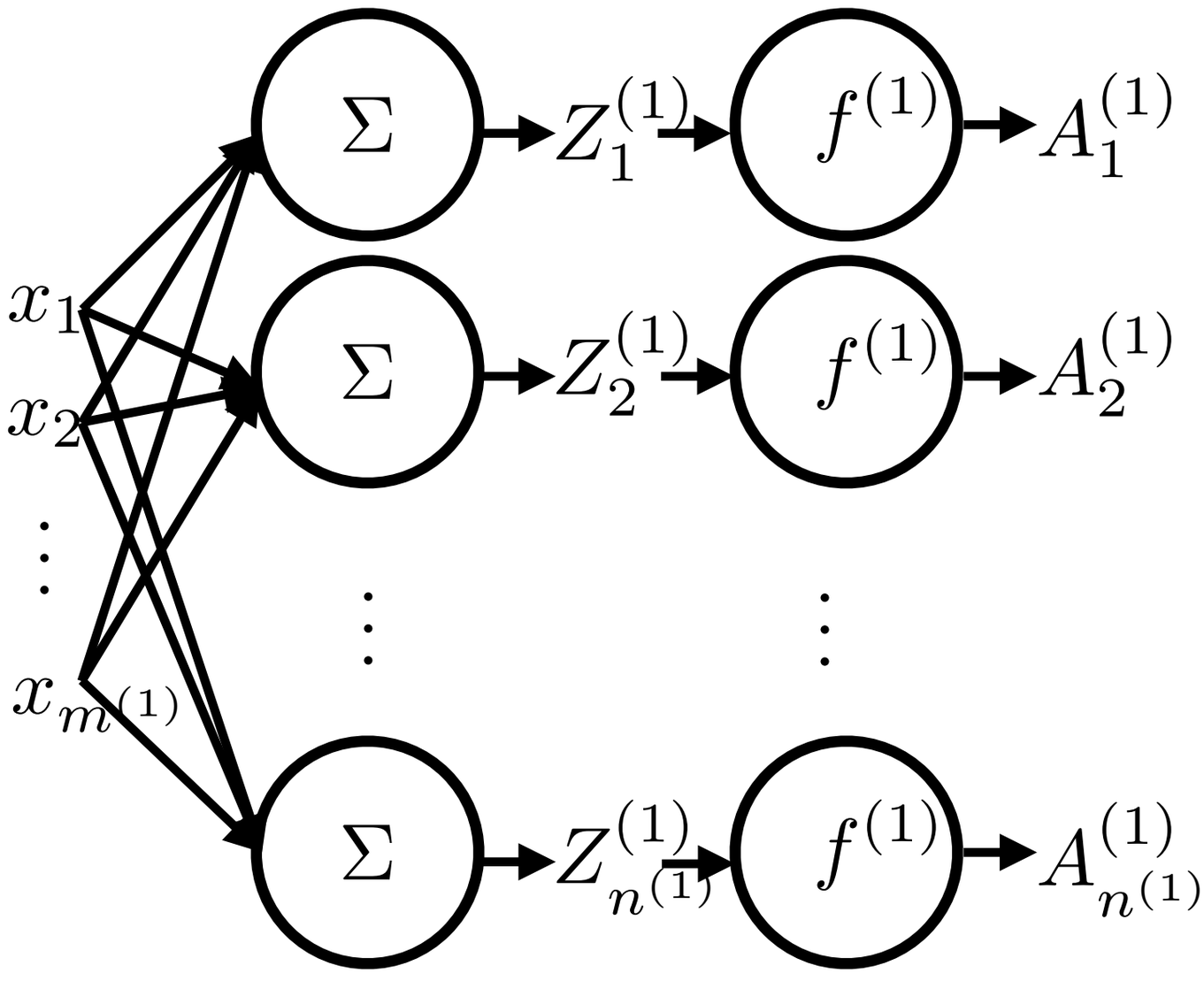


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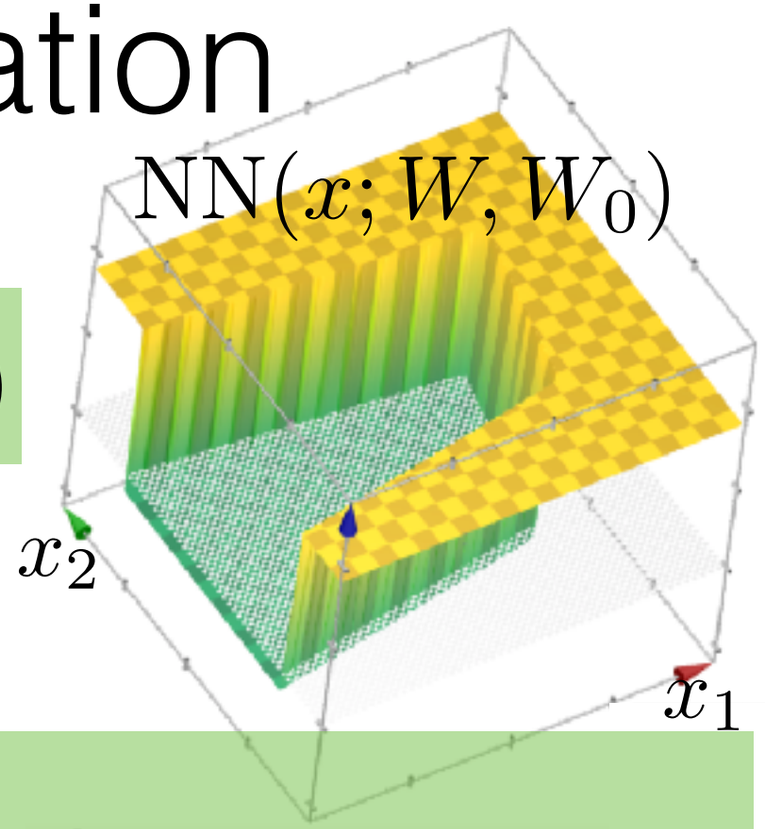


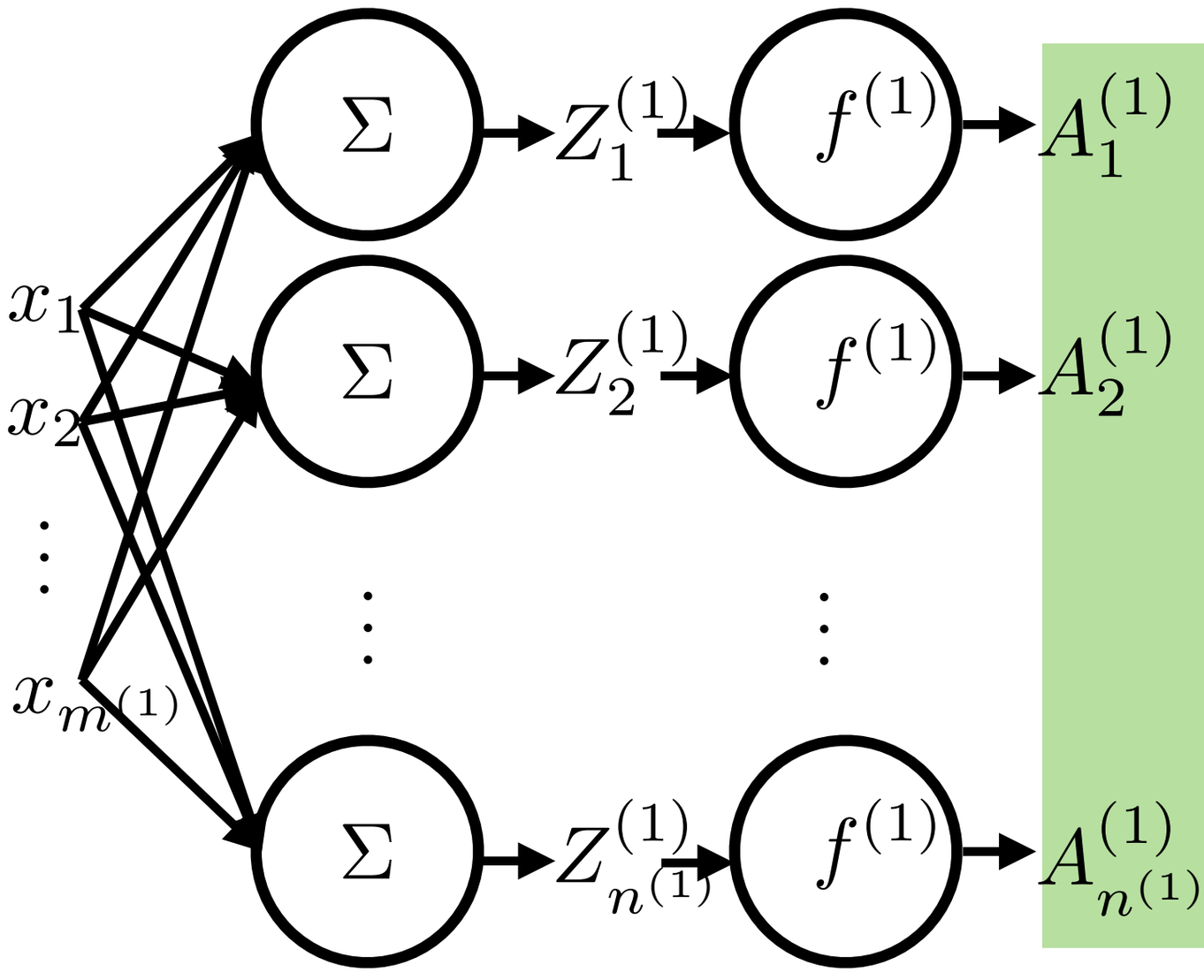


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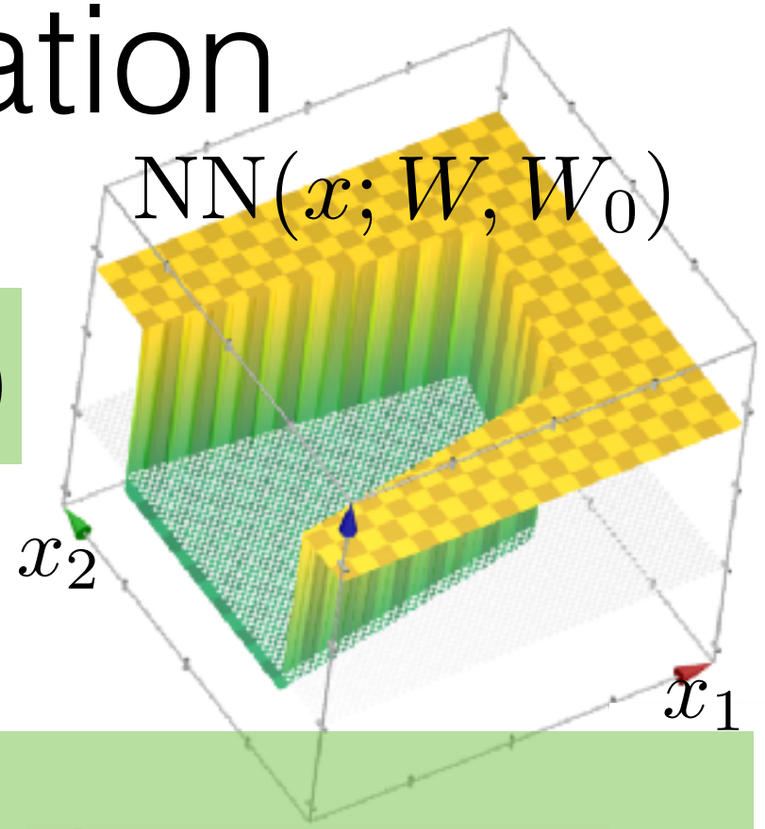


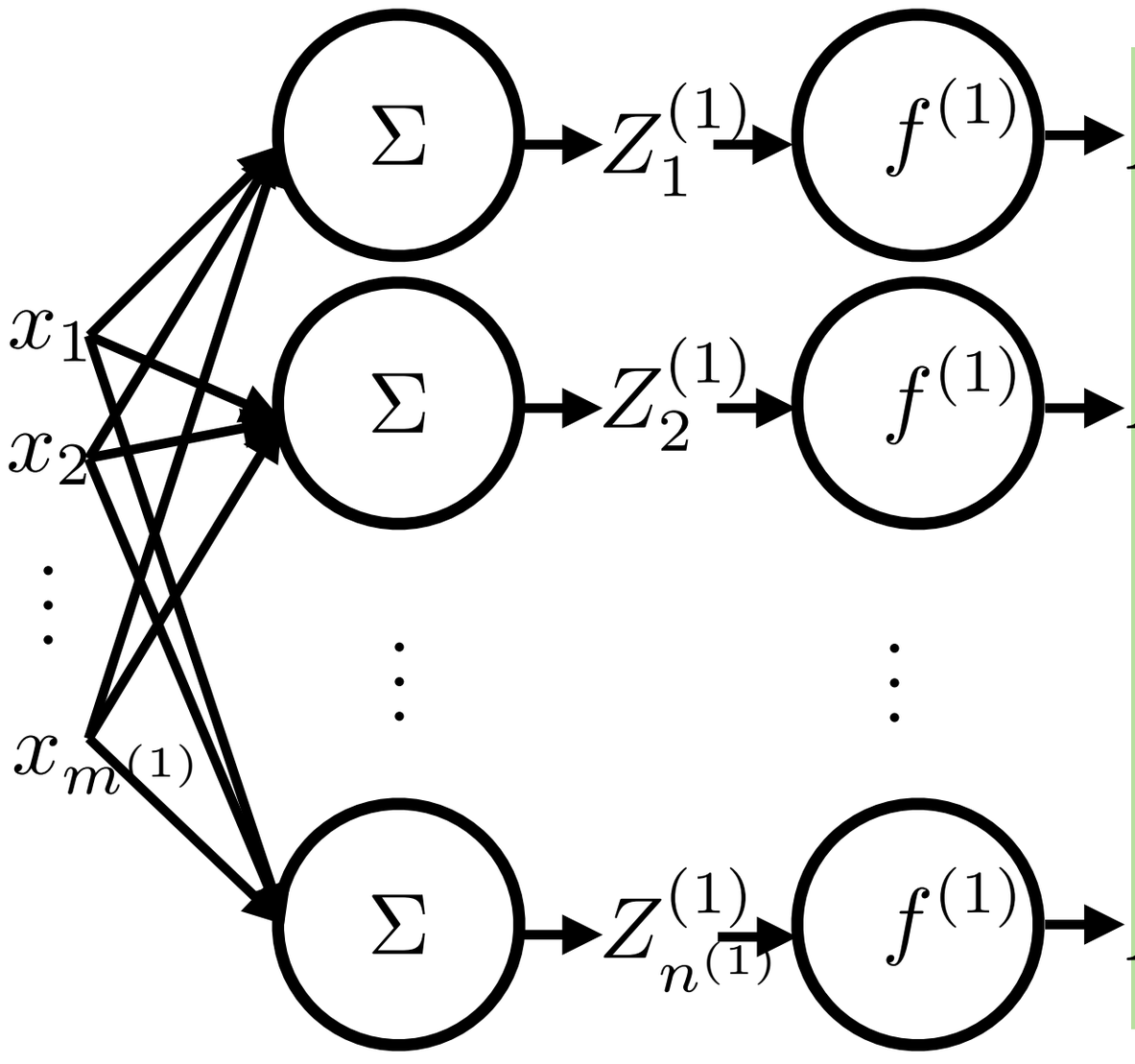


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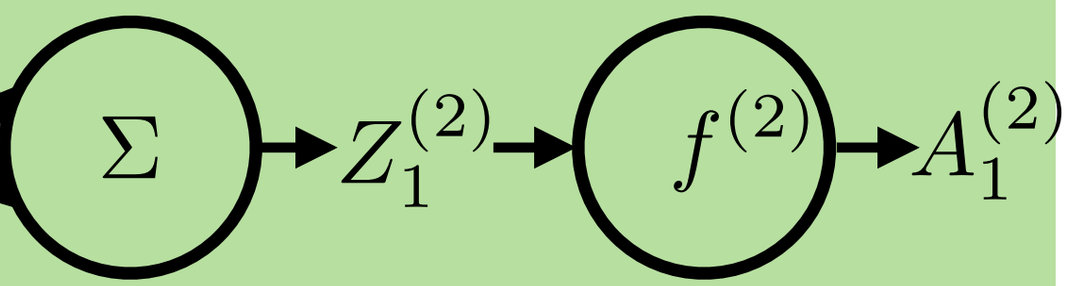

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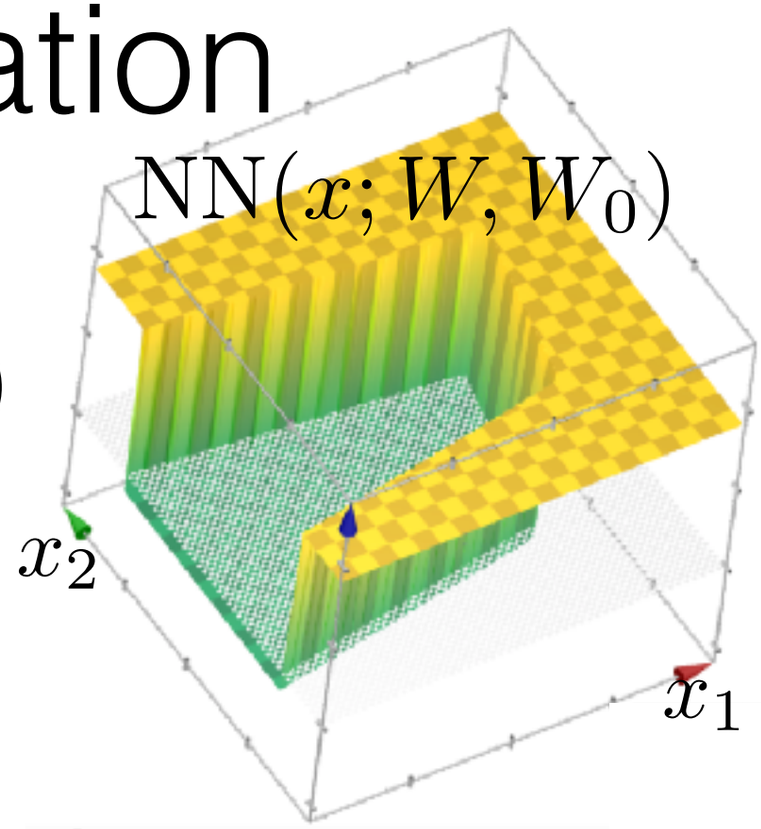


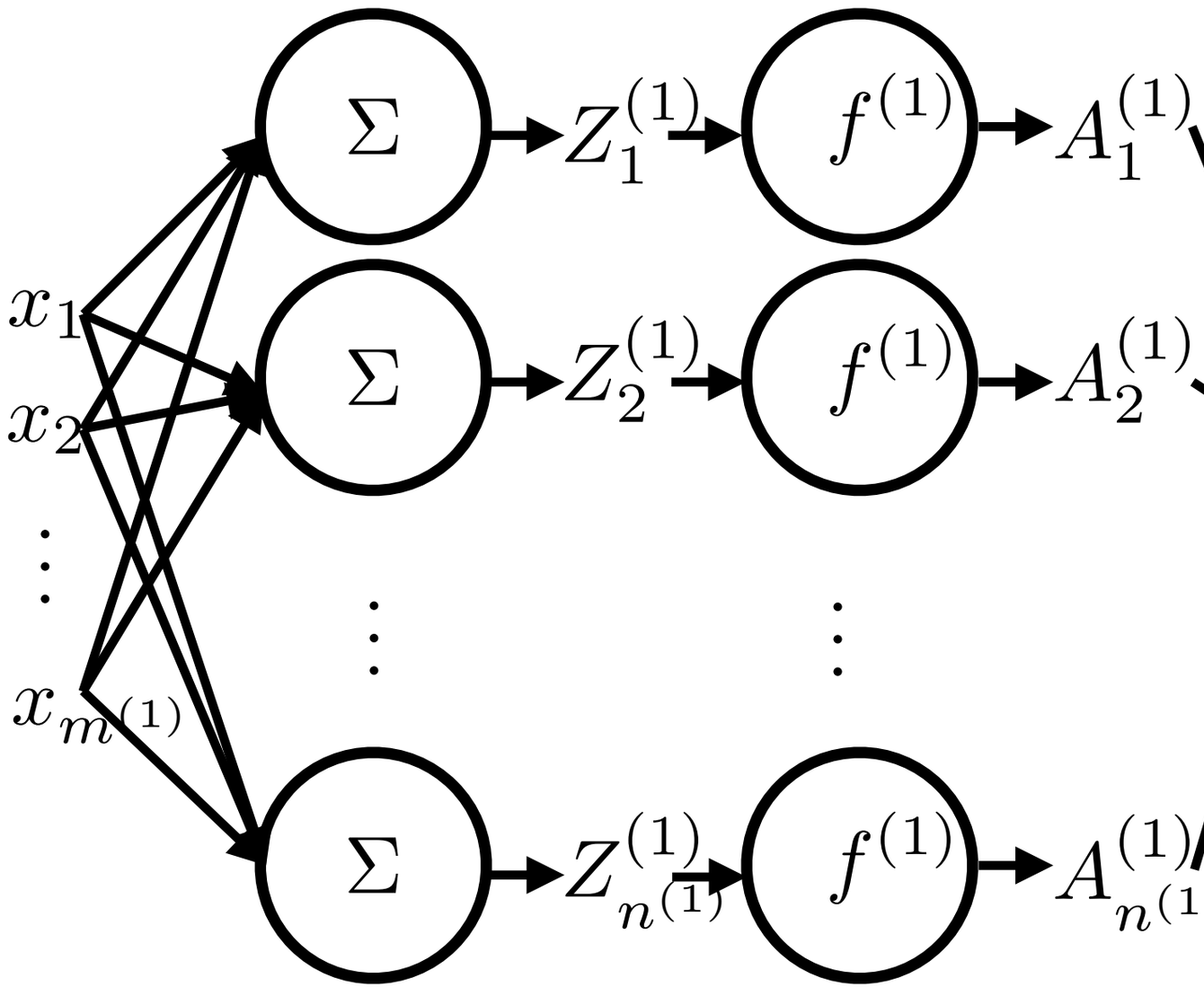


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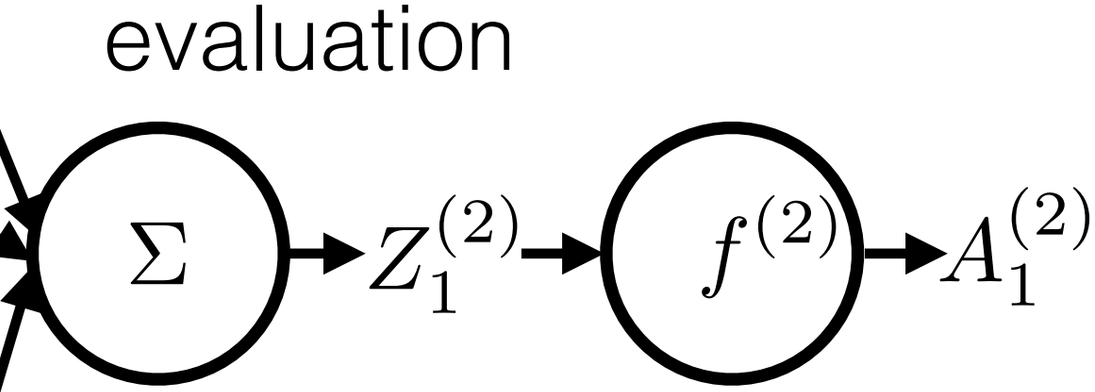

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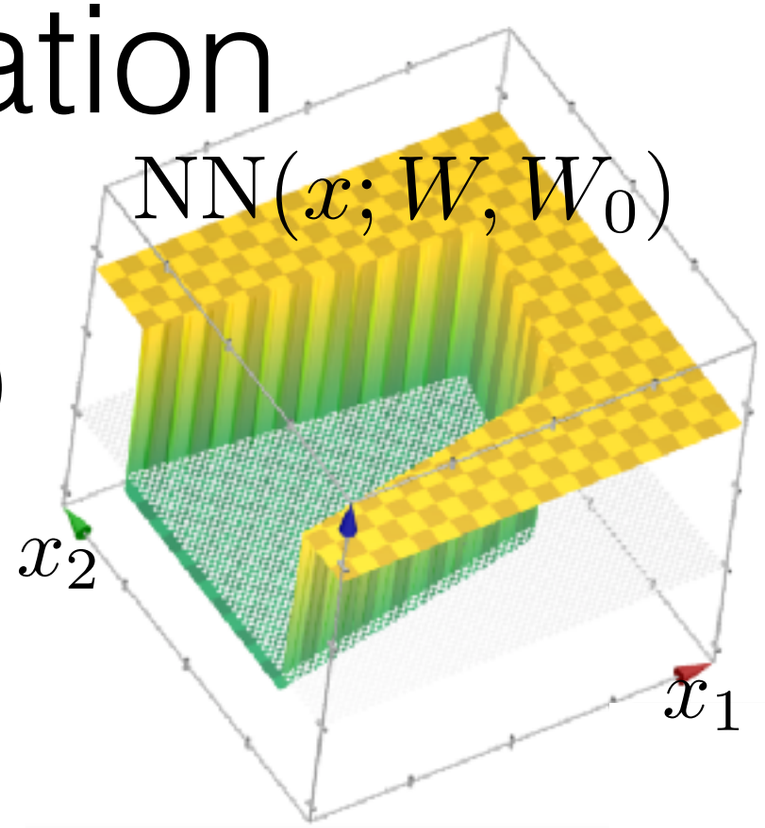


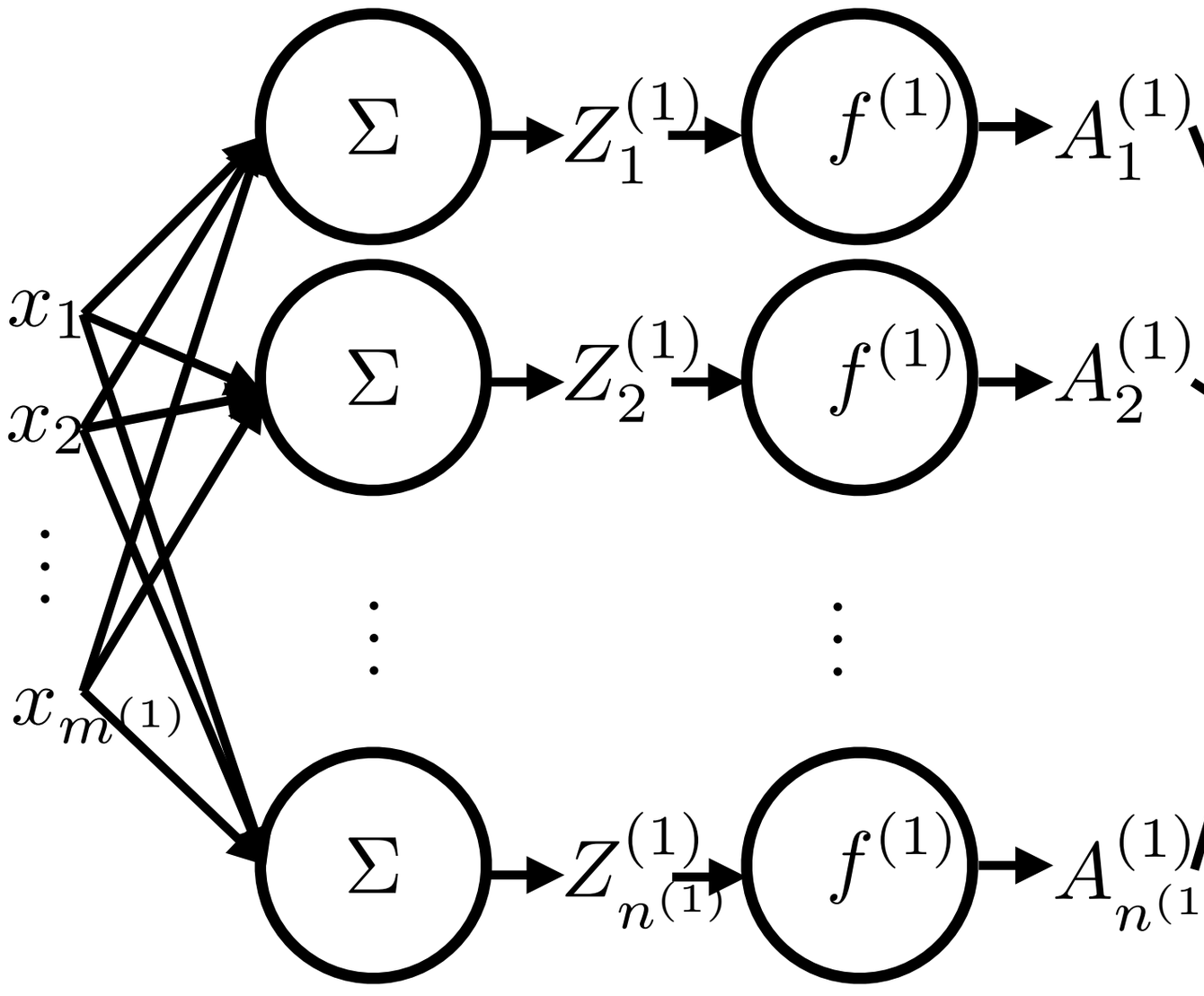


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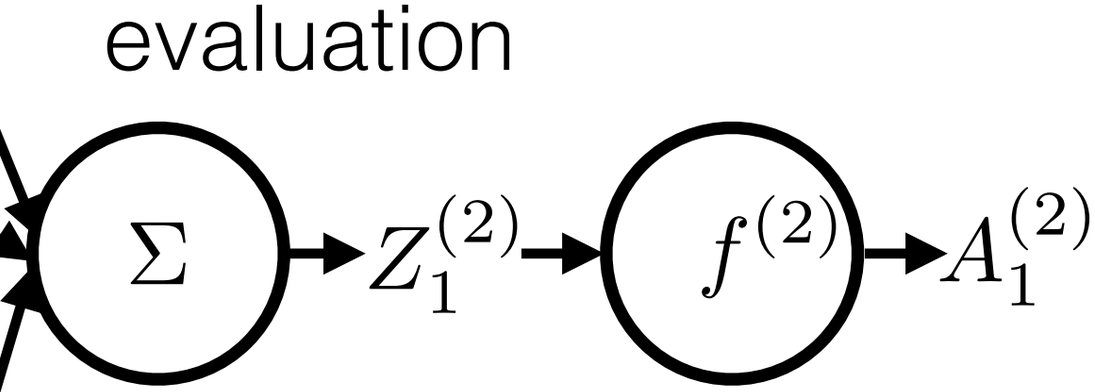

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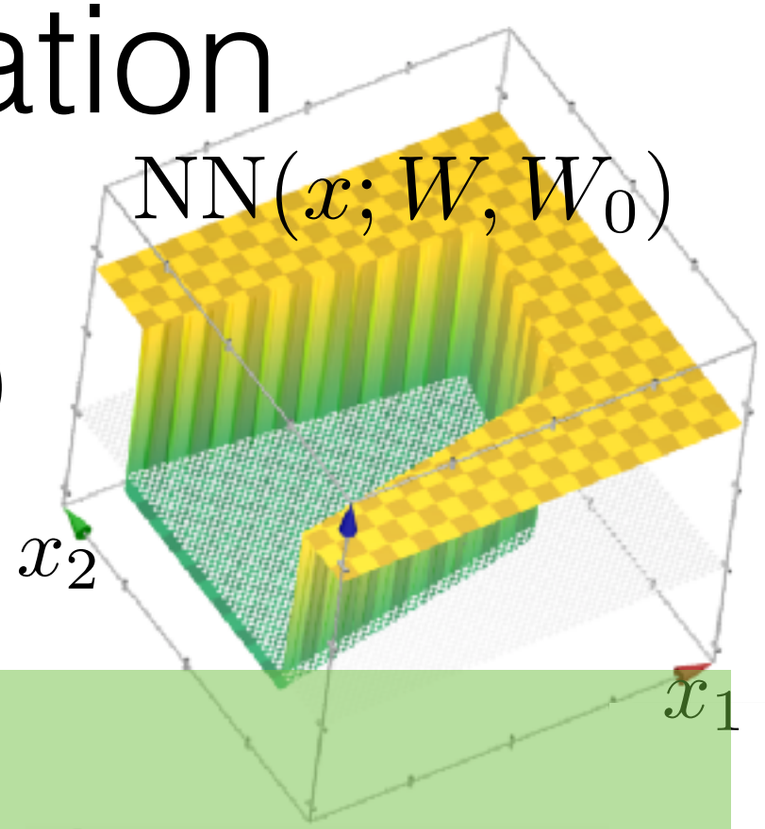


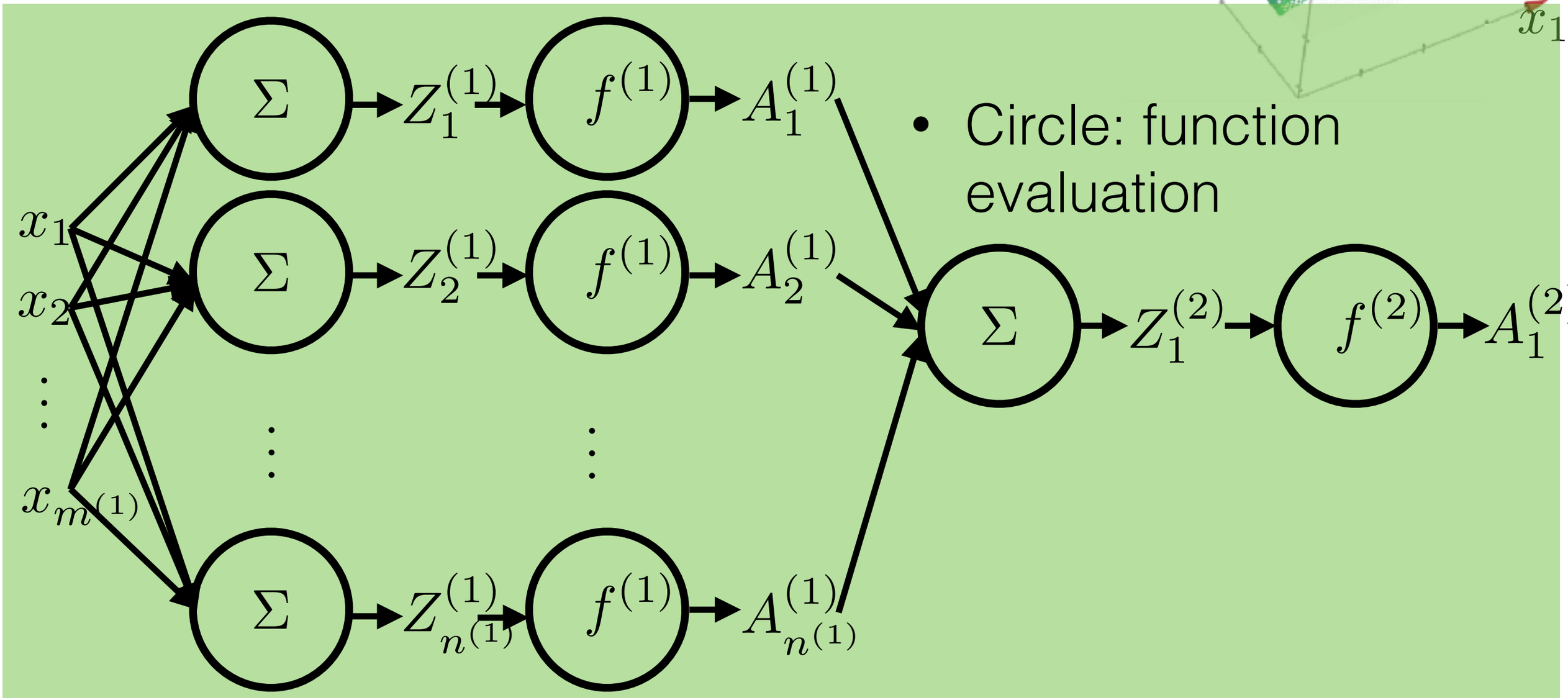


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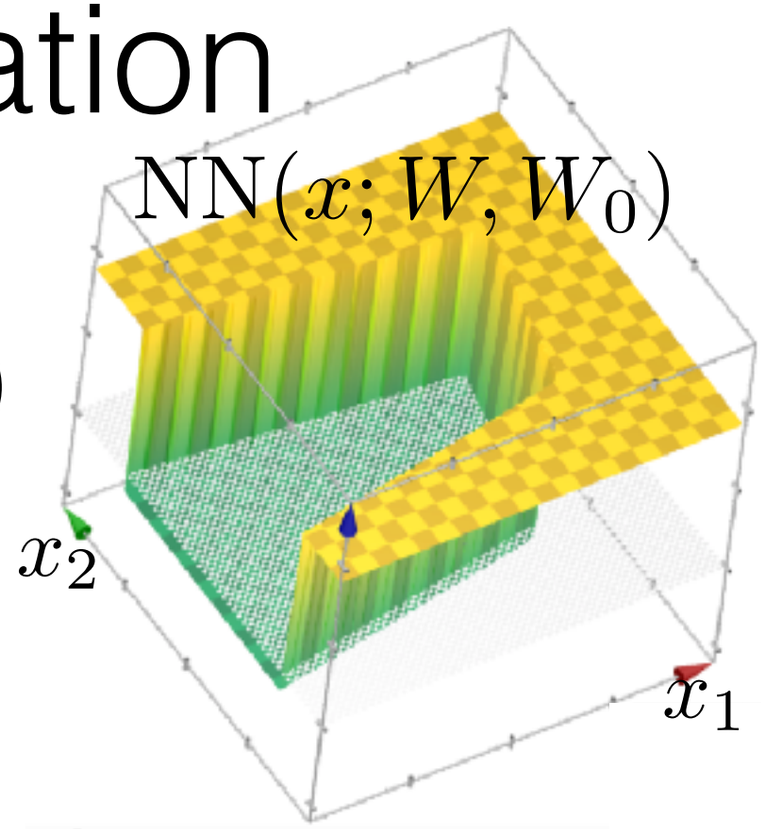


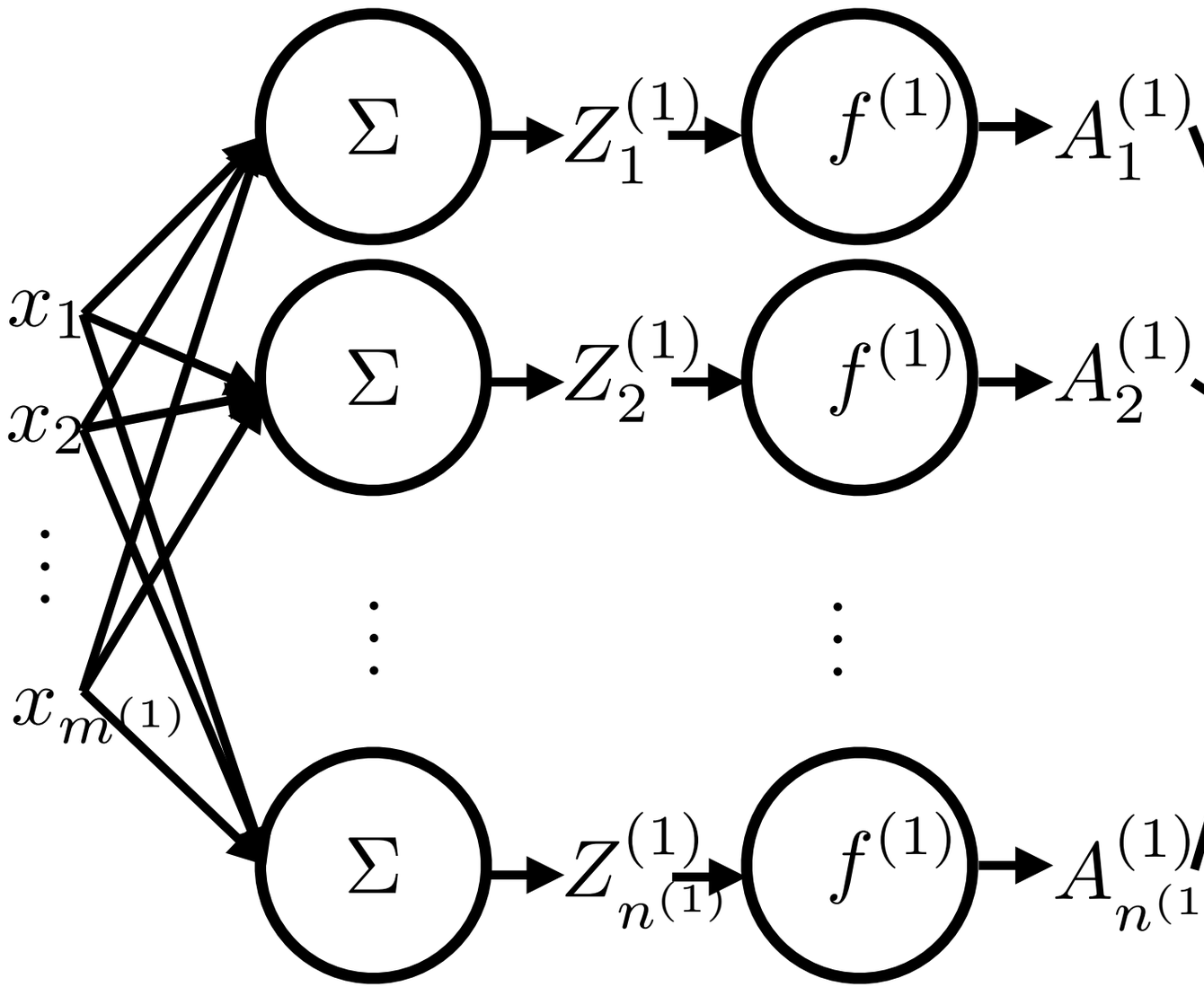


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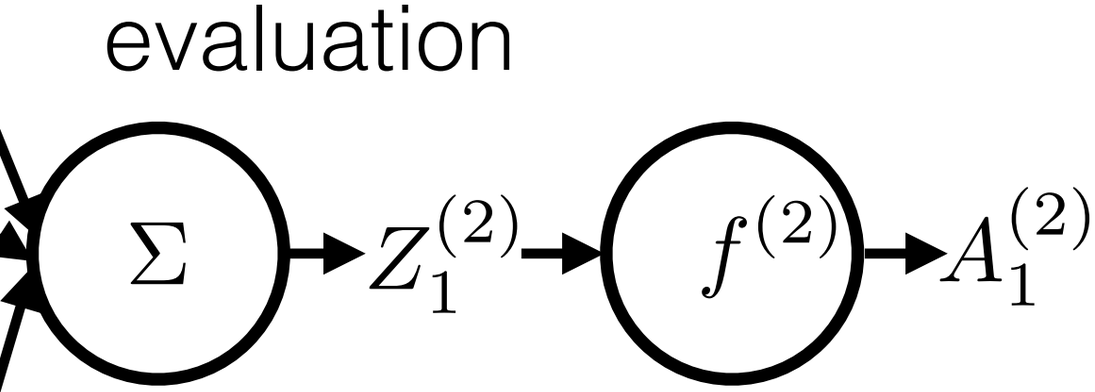

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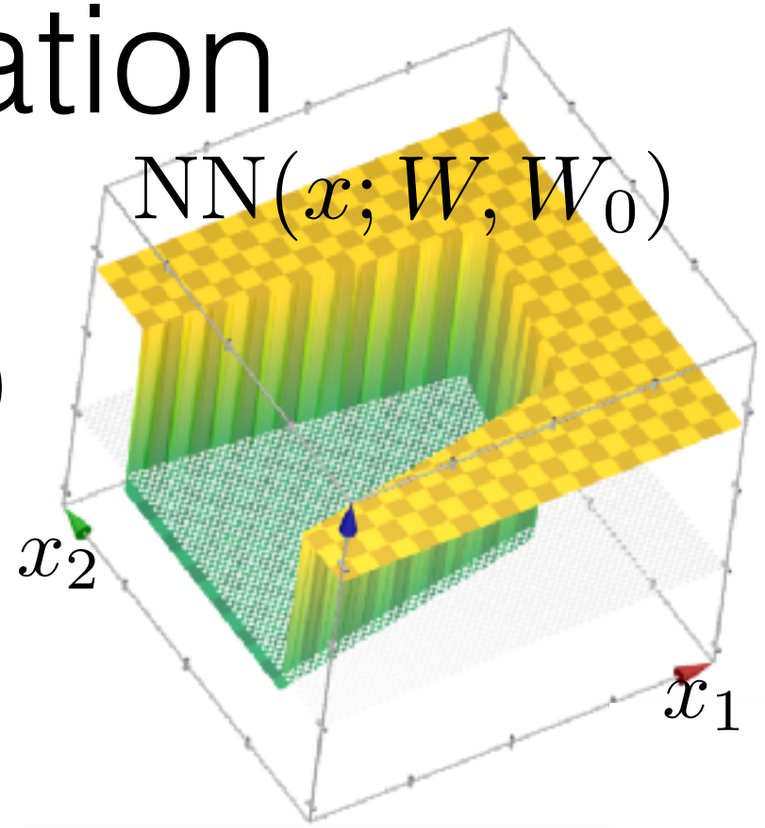


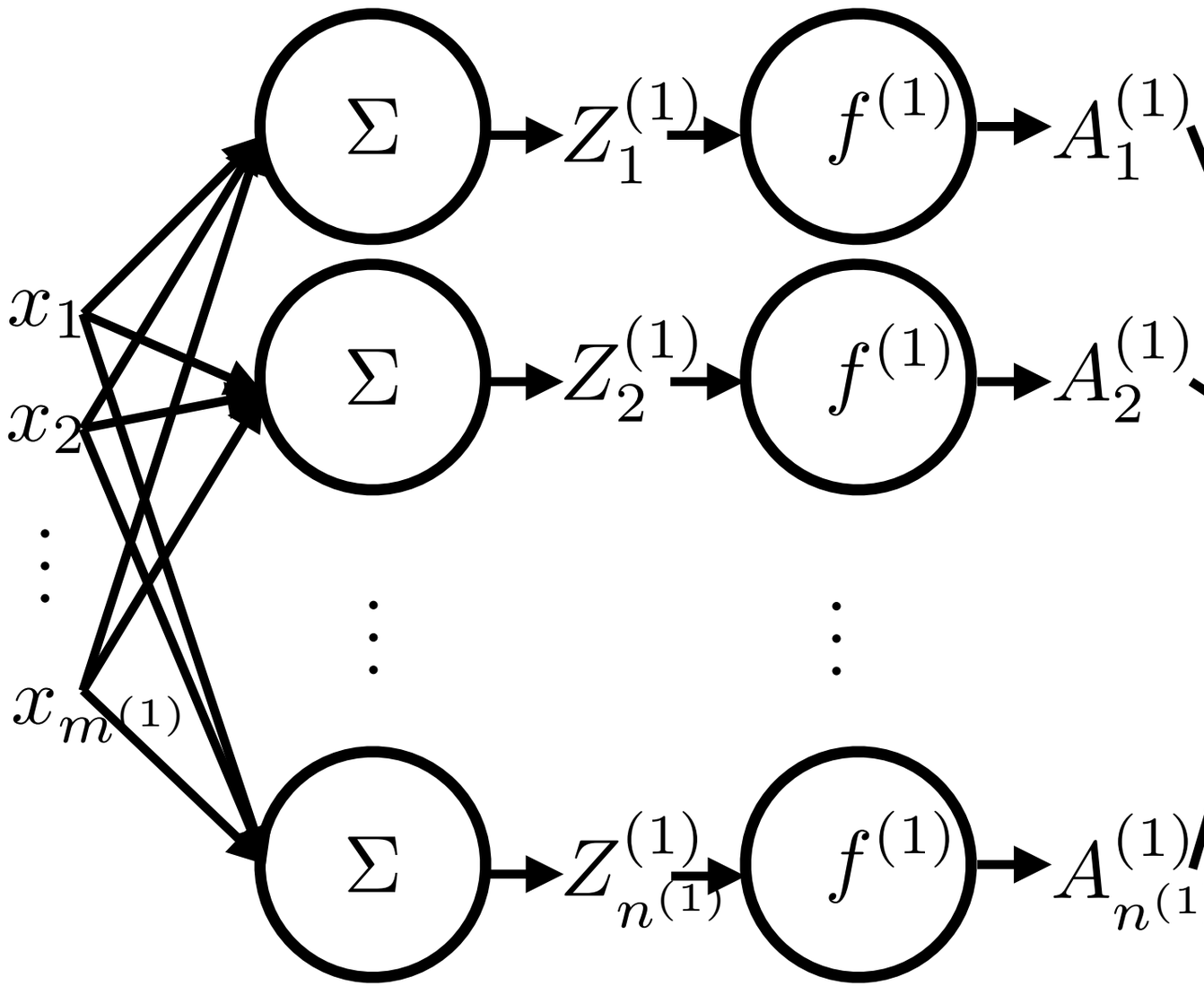


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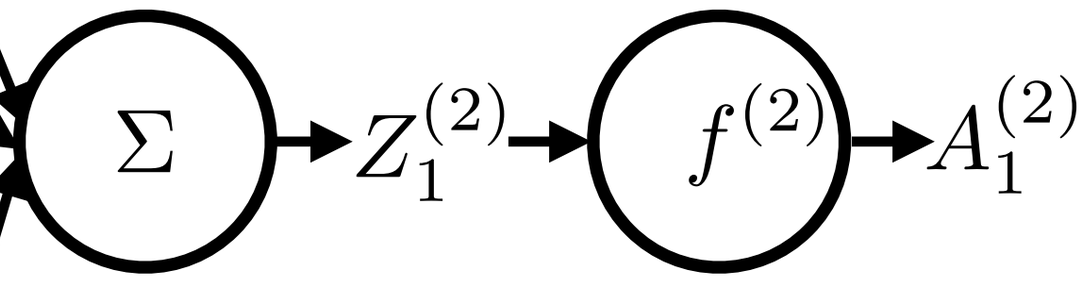


- Forward vs. backward

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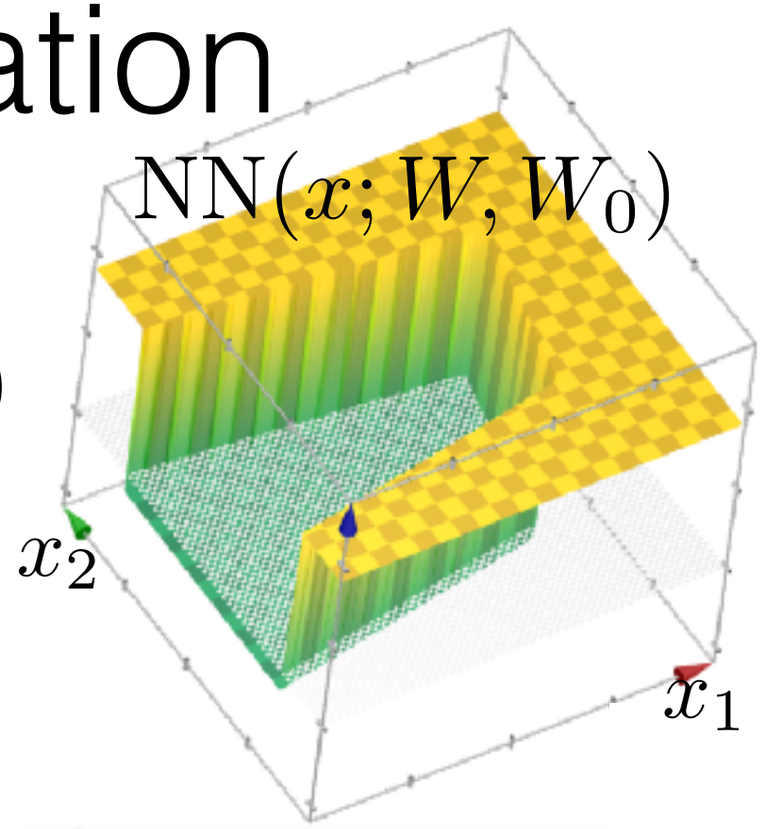


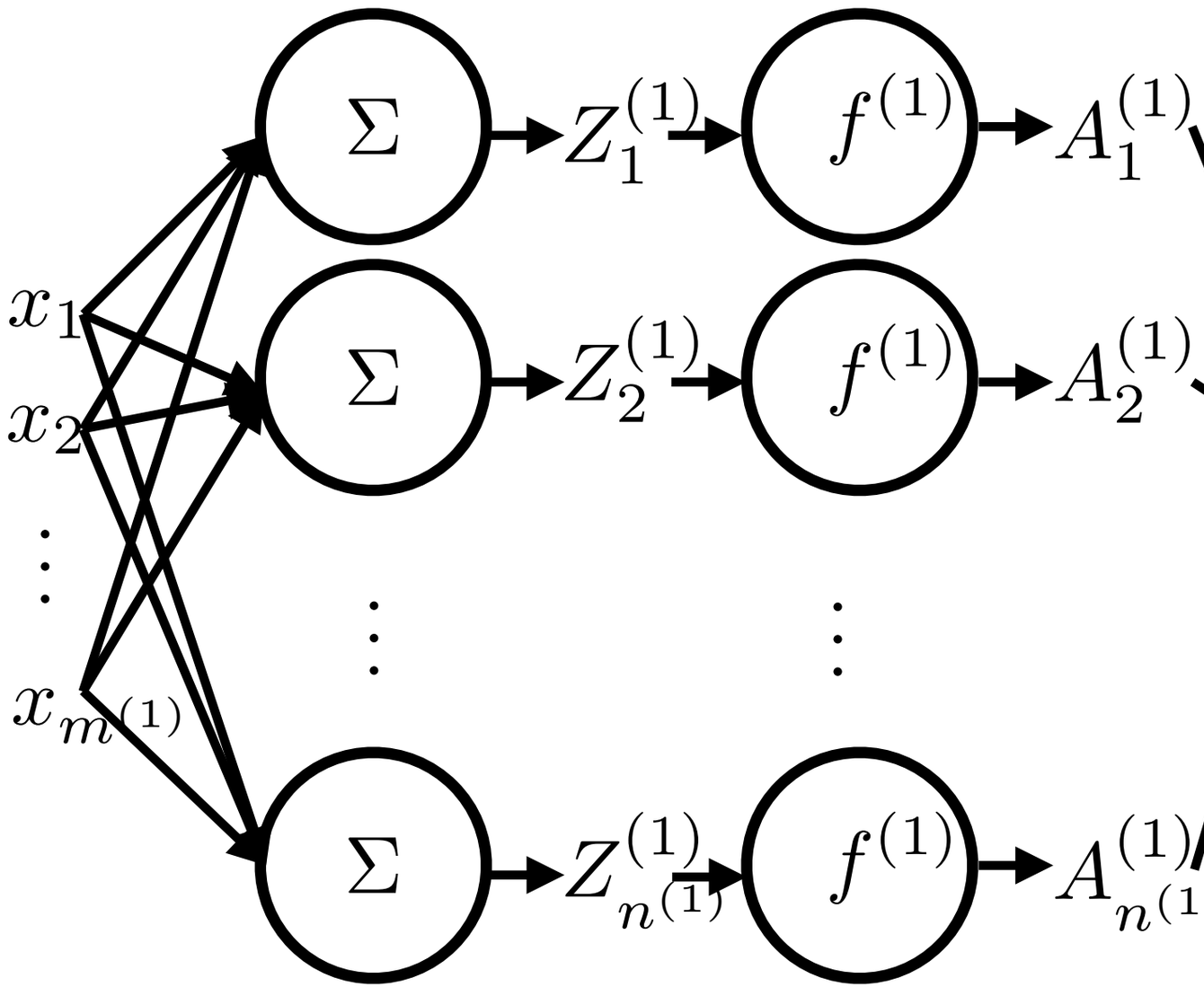


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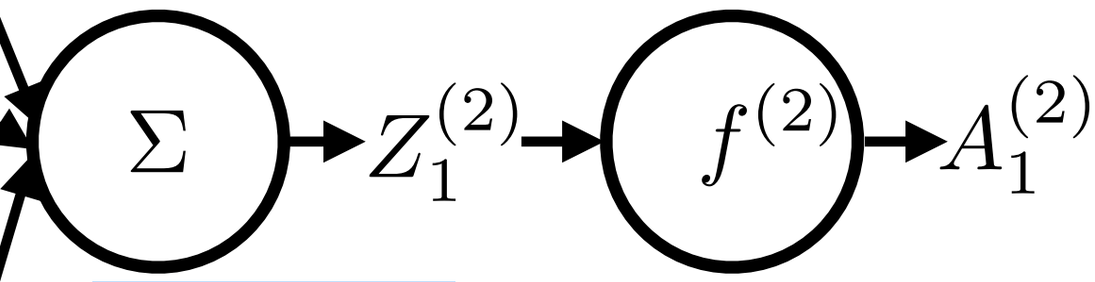


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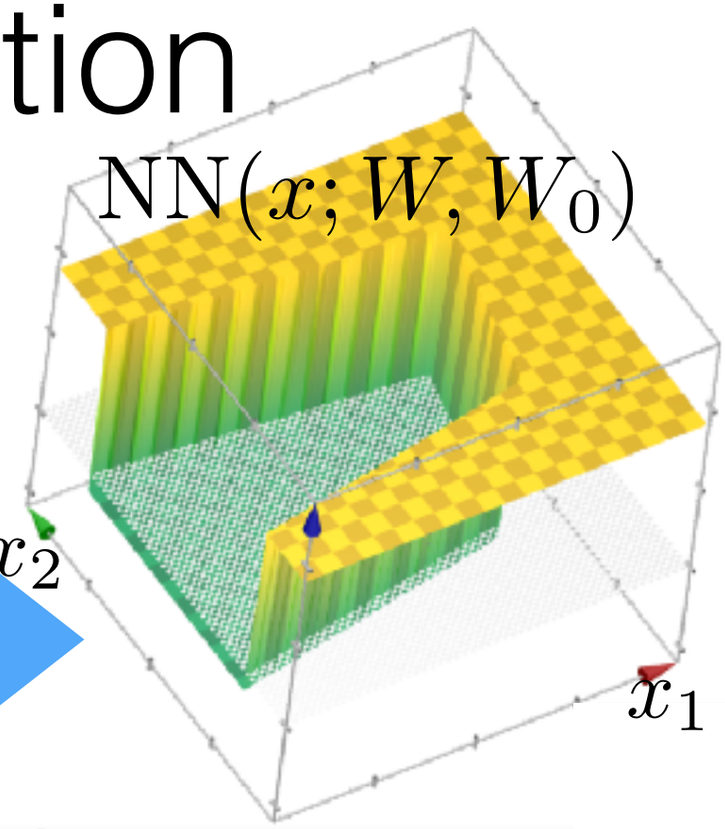


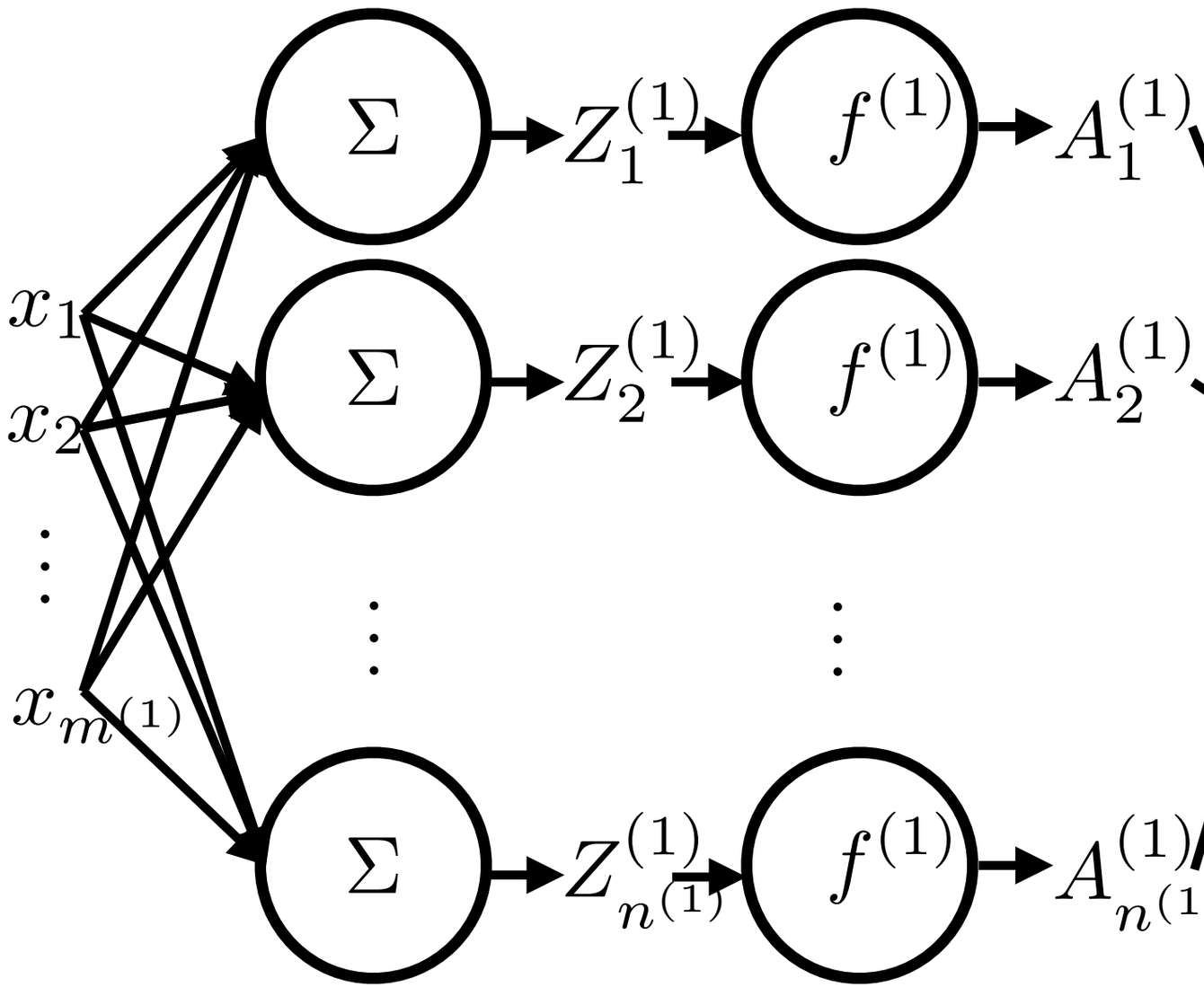


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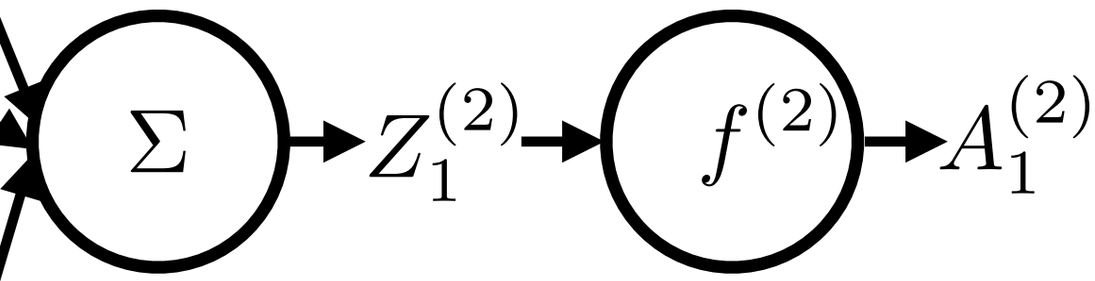


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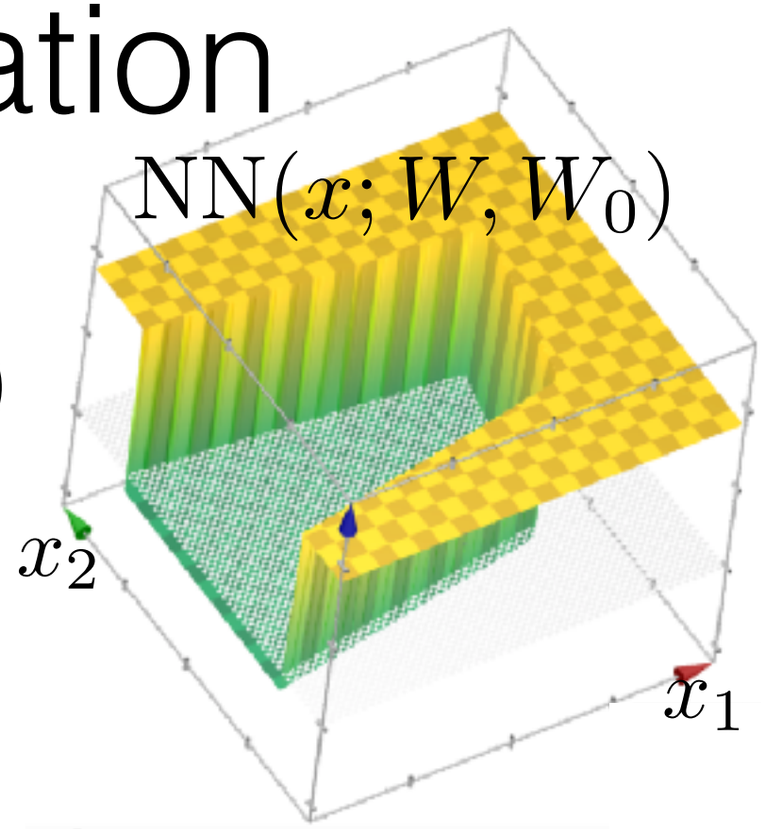


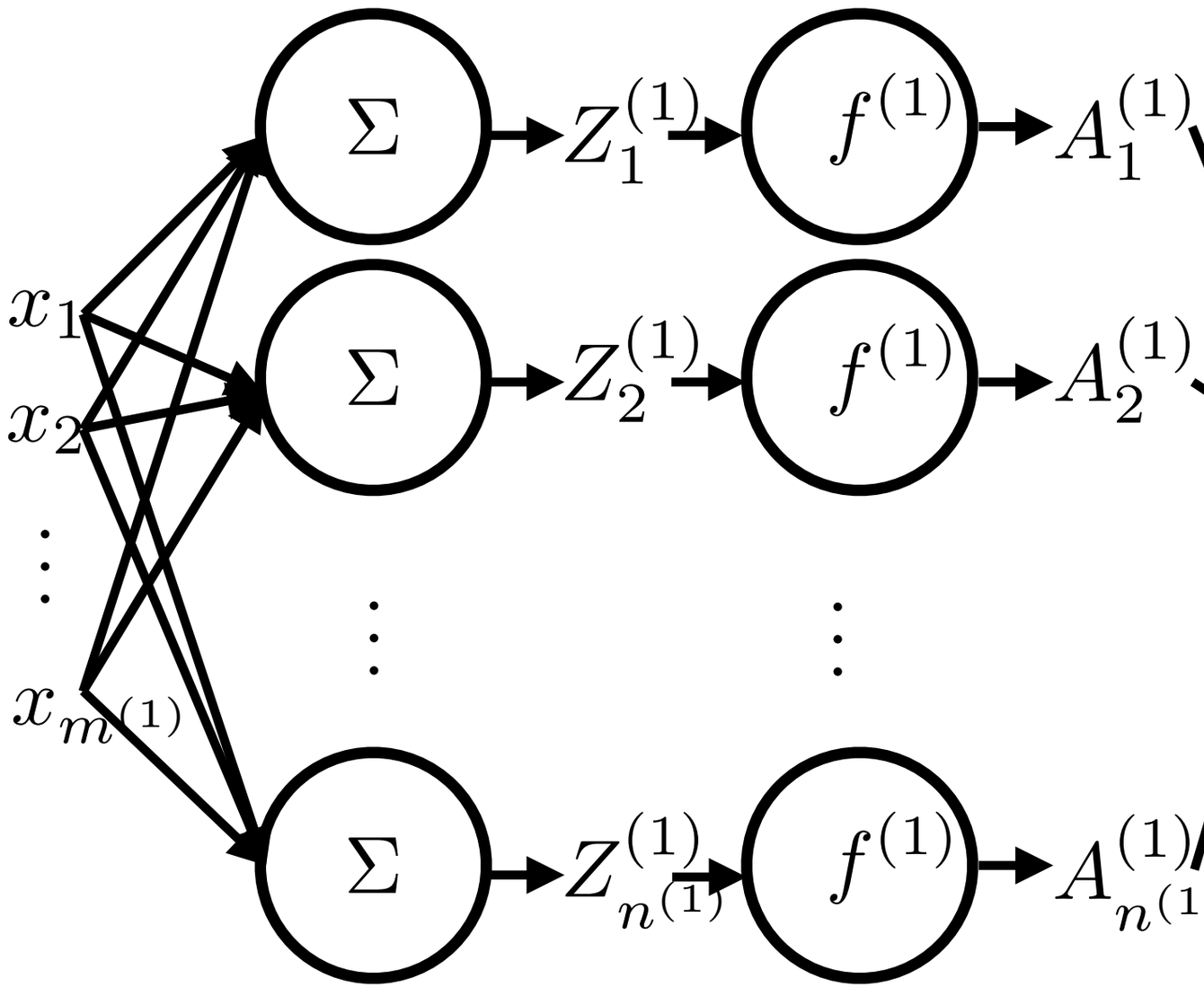


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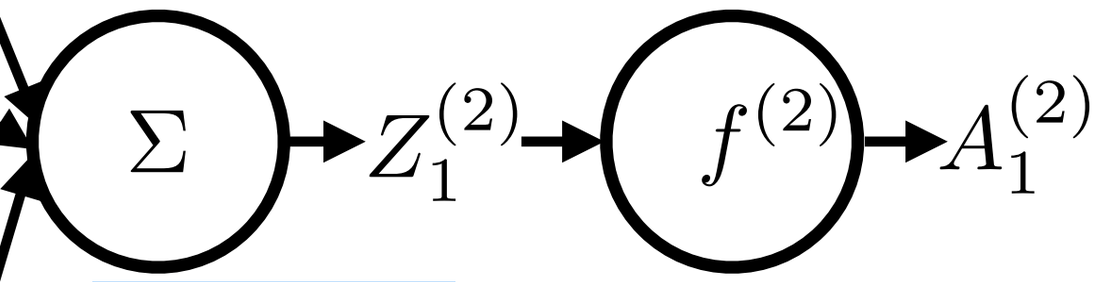


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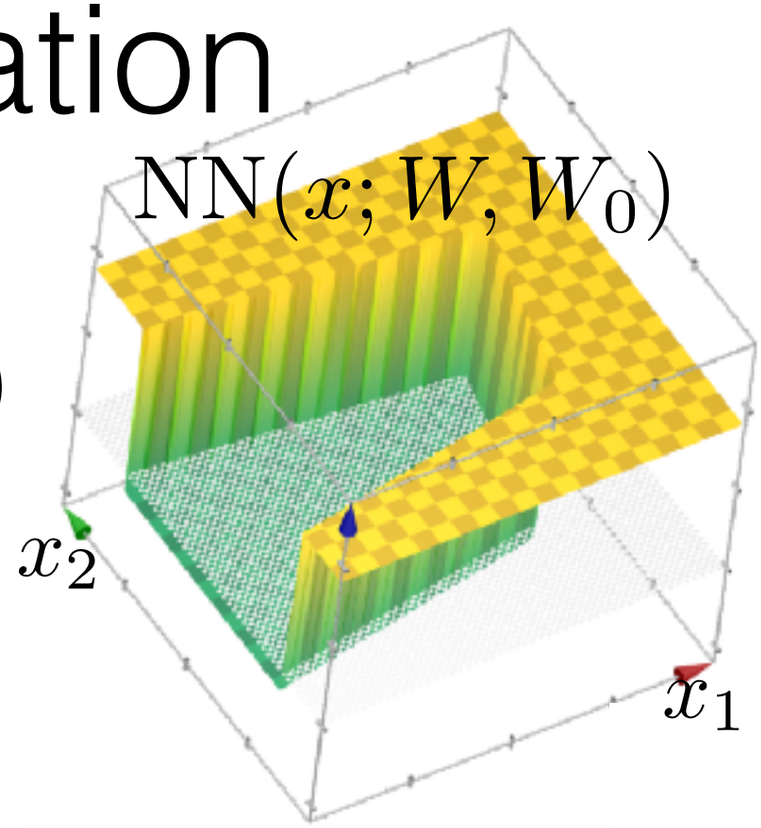


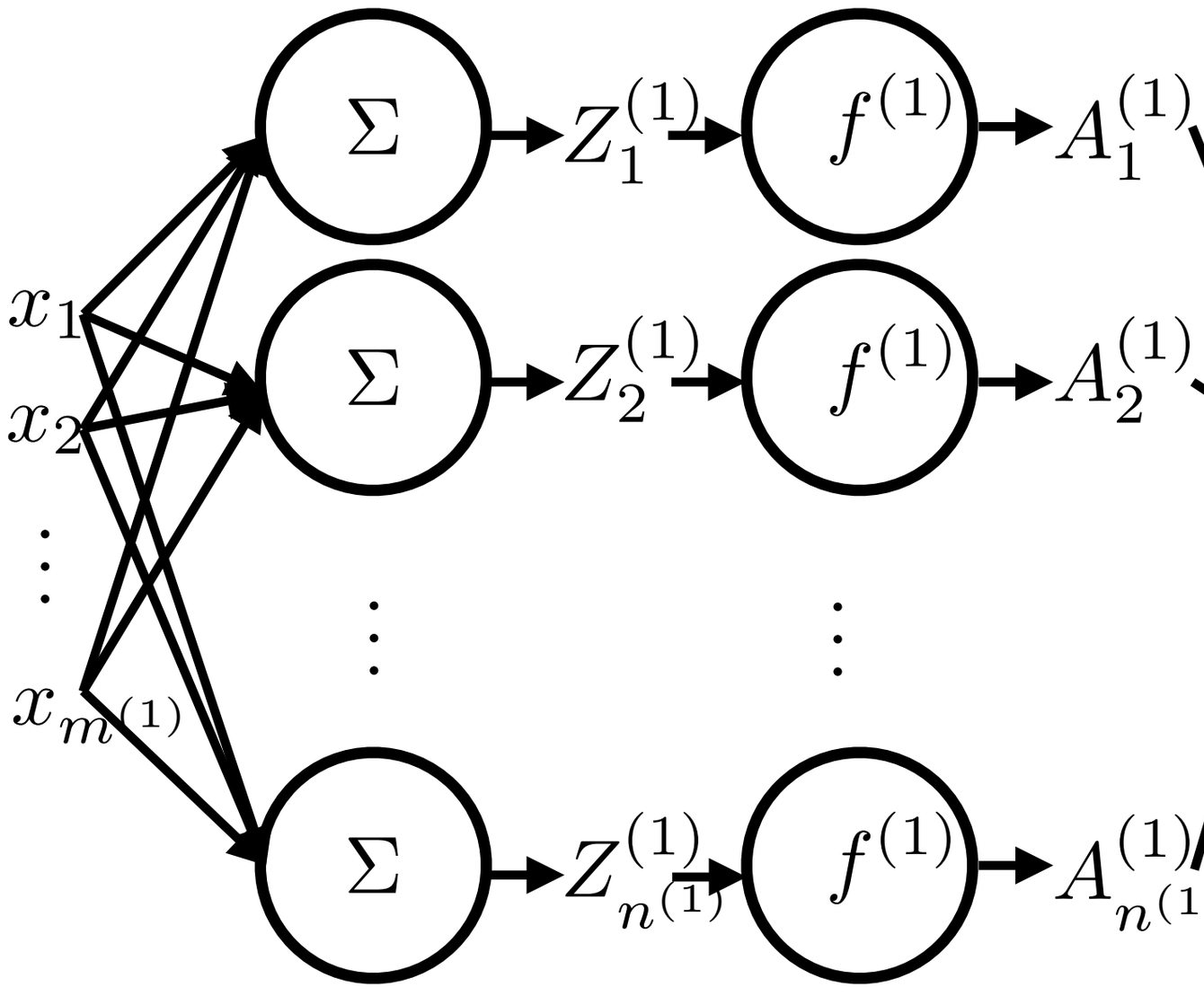


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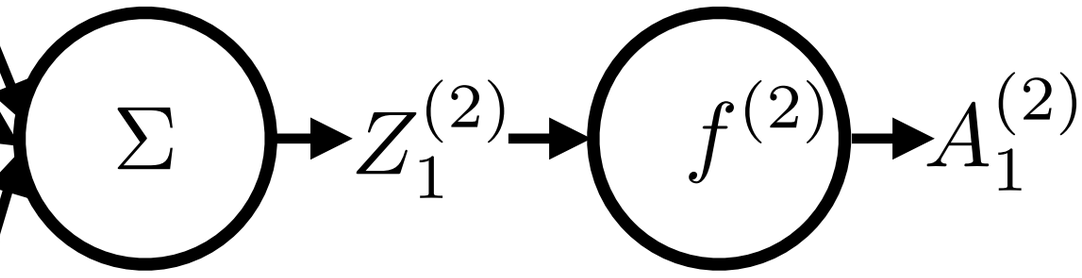


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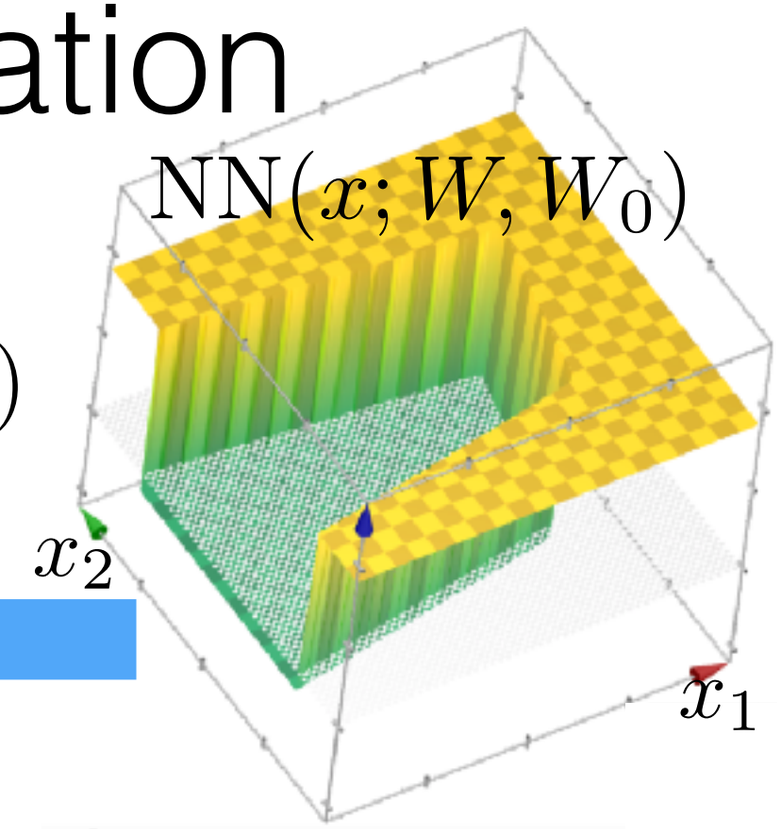


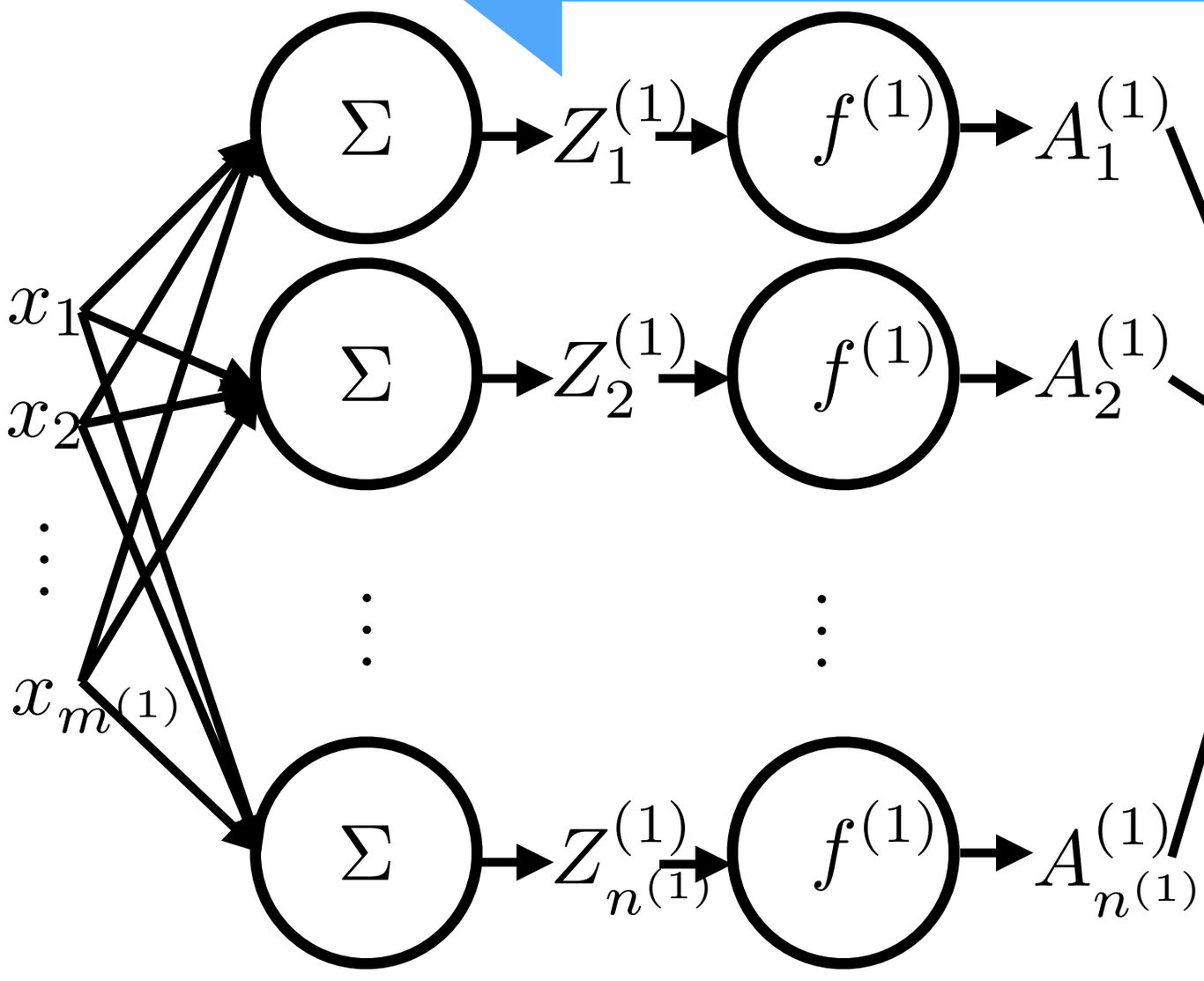


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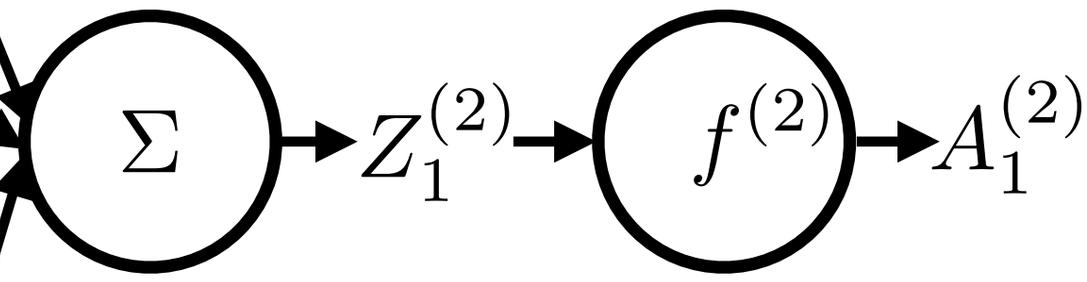


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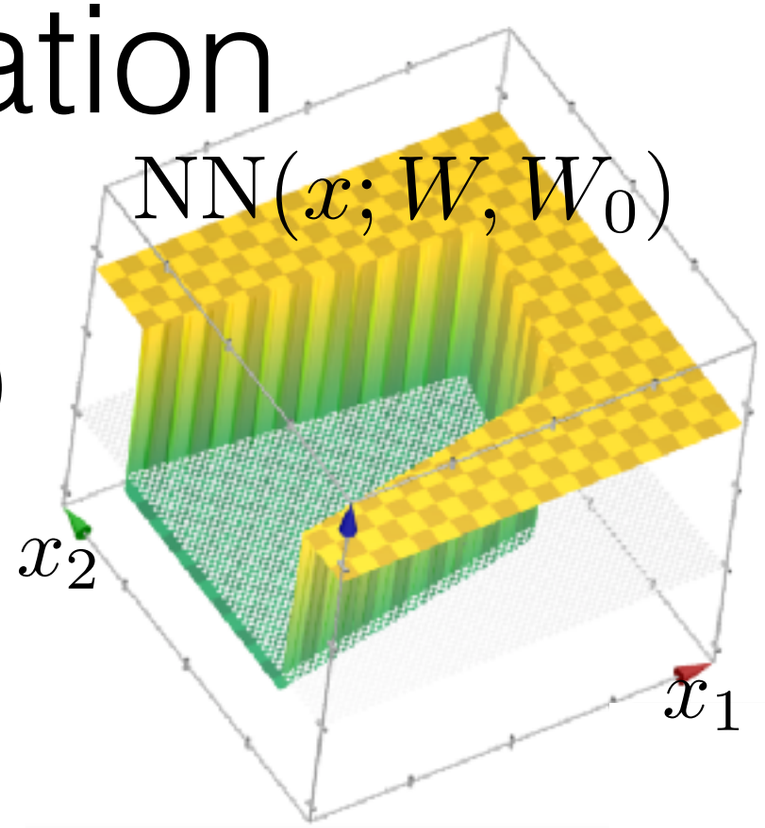


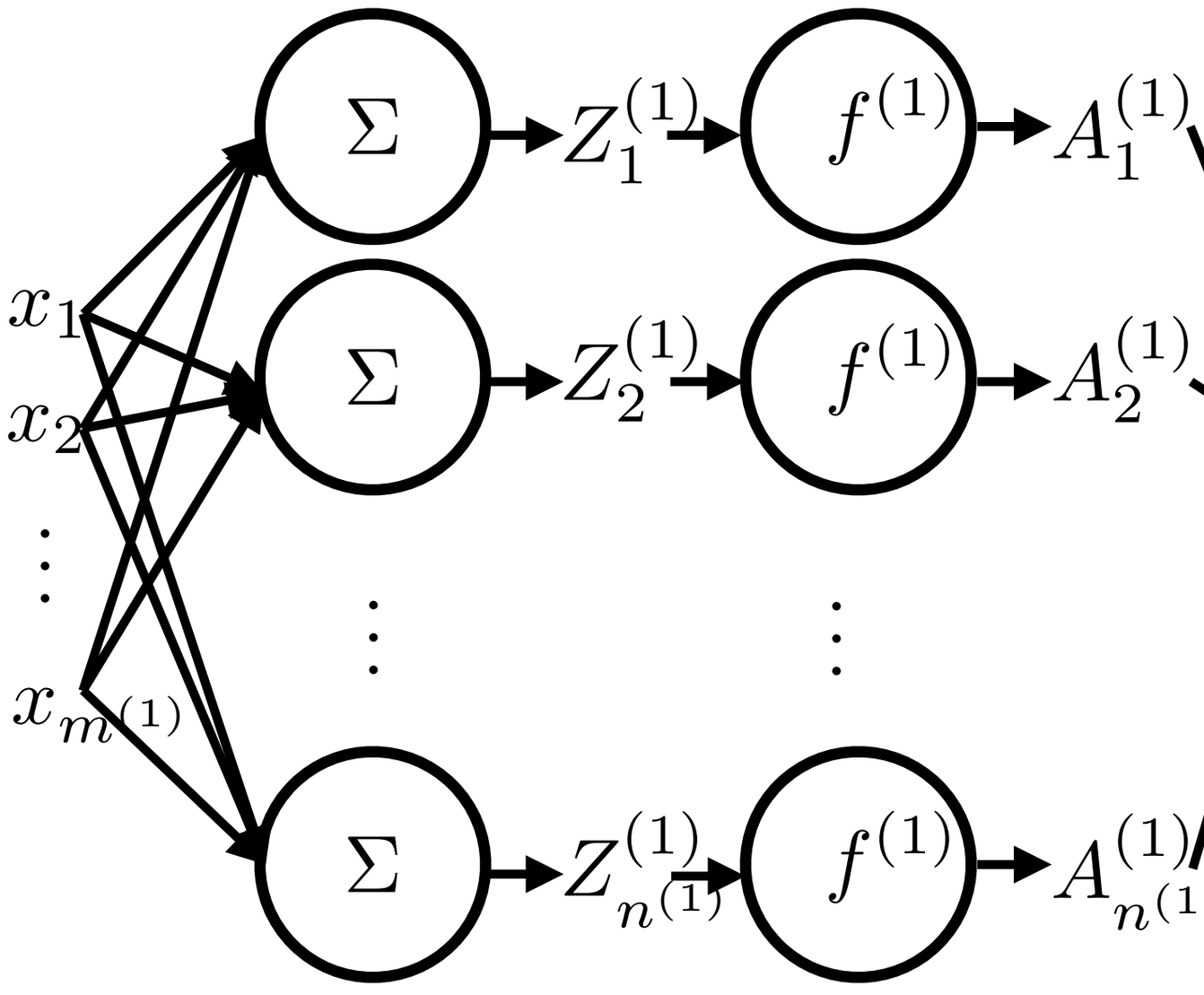


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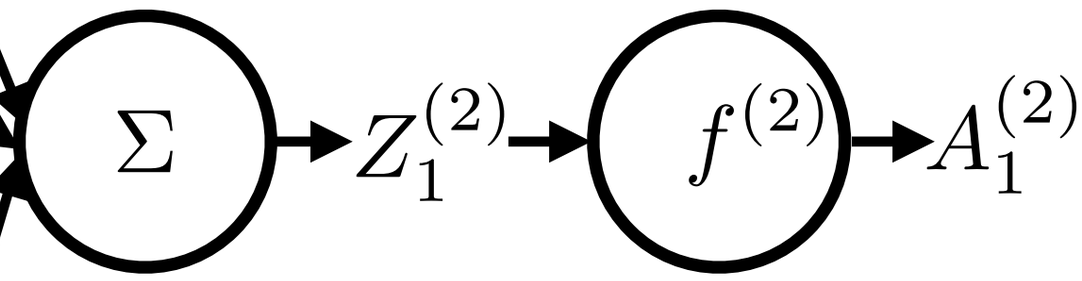


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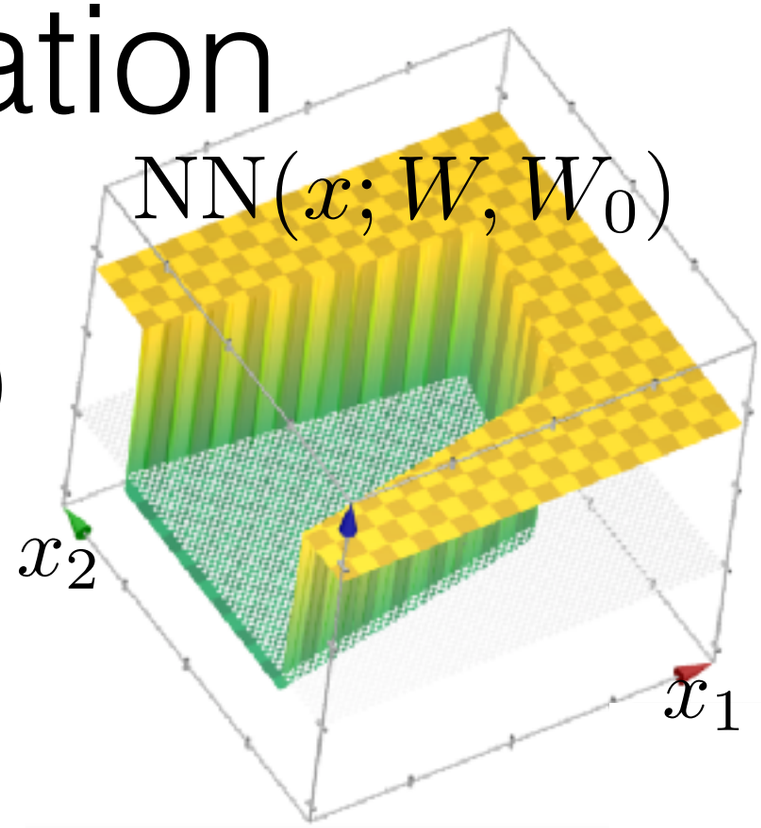


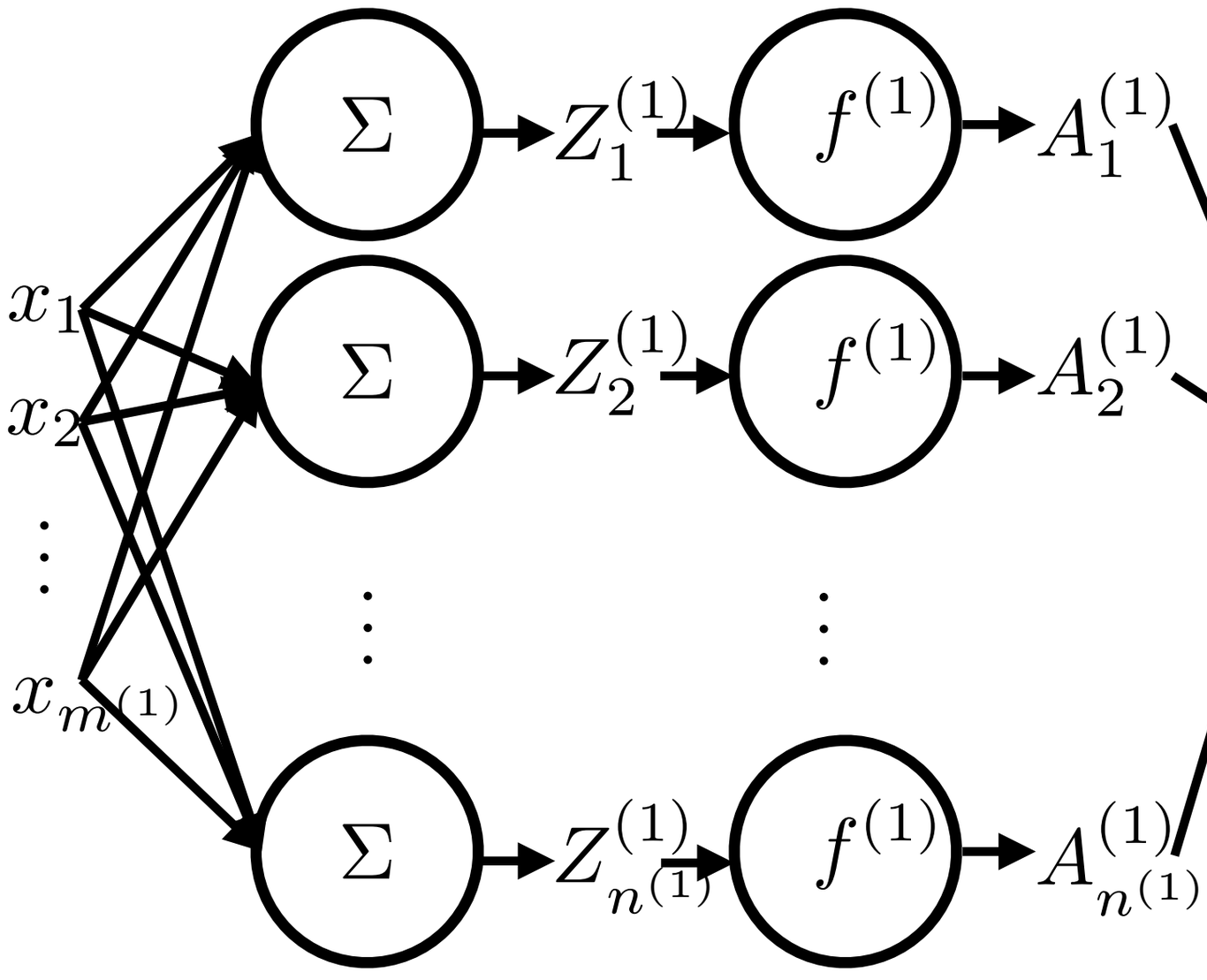


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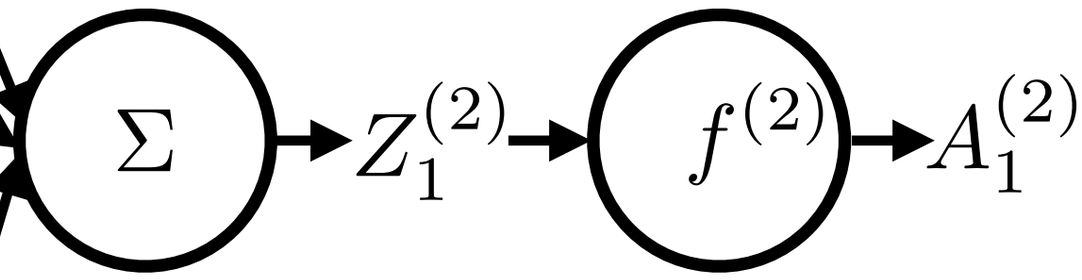


- Forward vs. backward
- A feed-forward neural network

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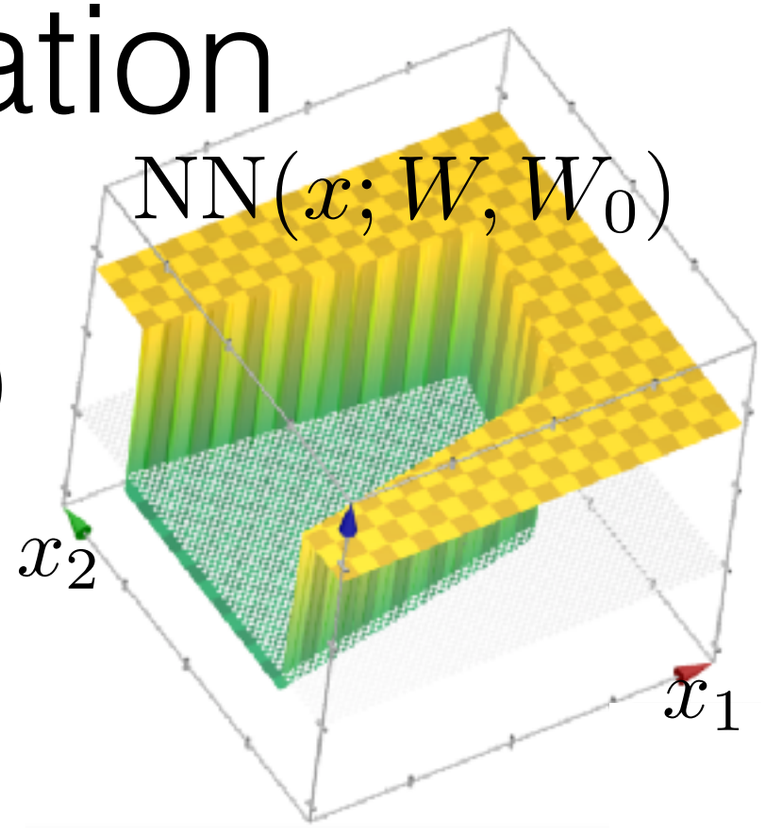


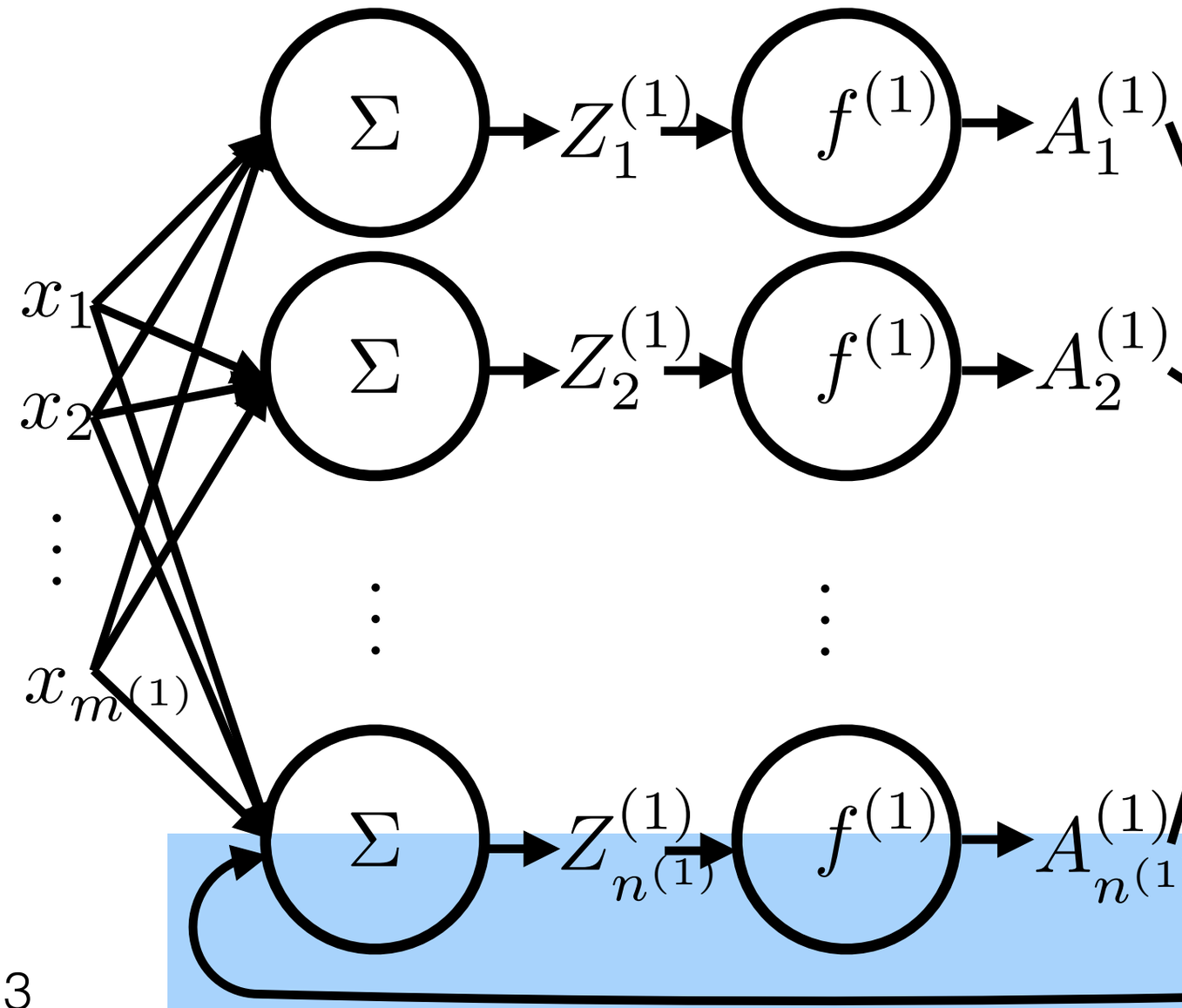


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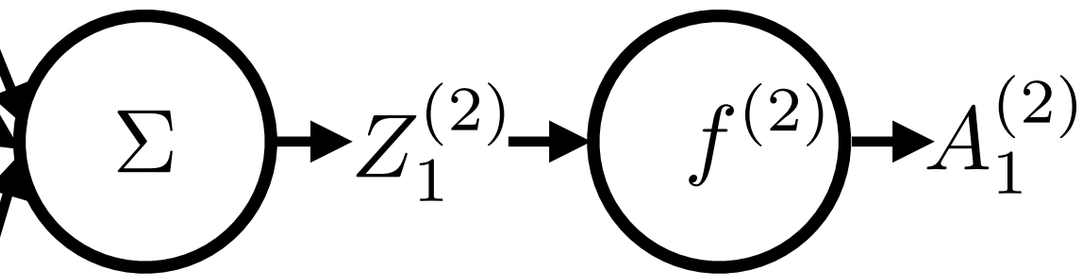


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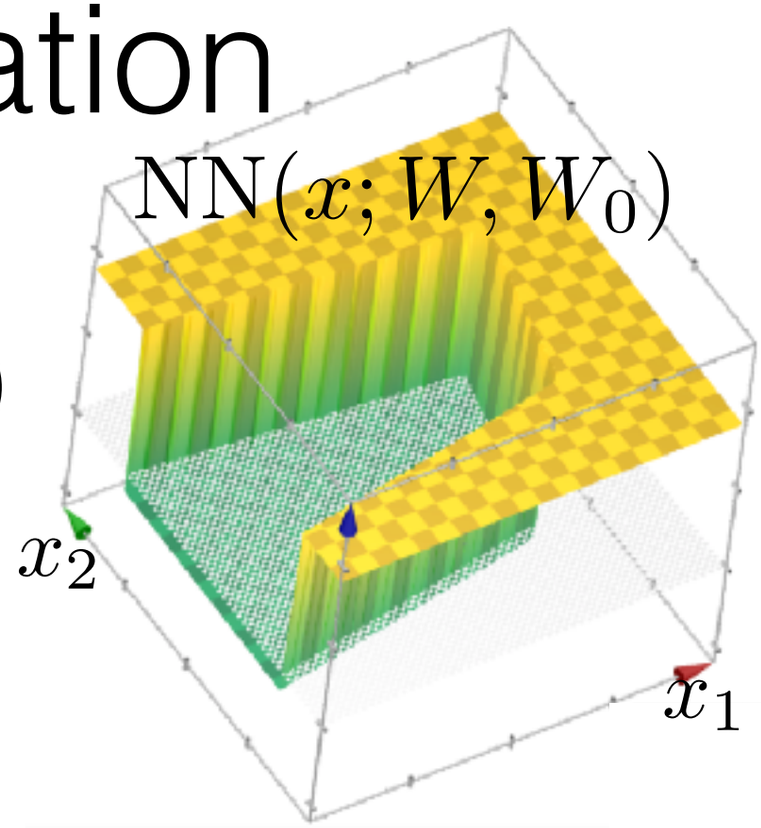


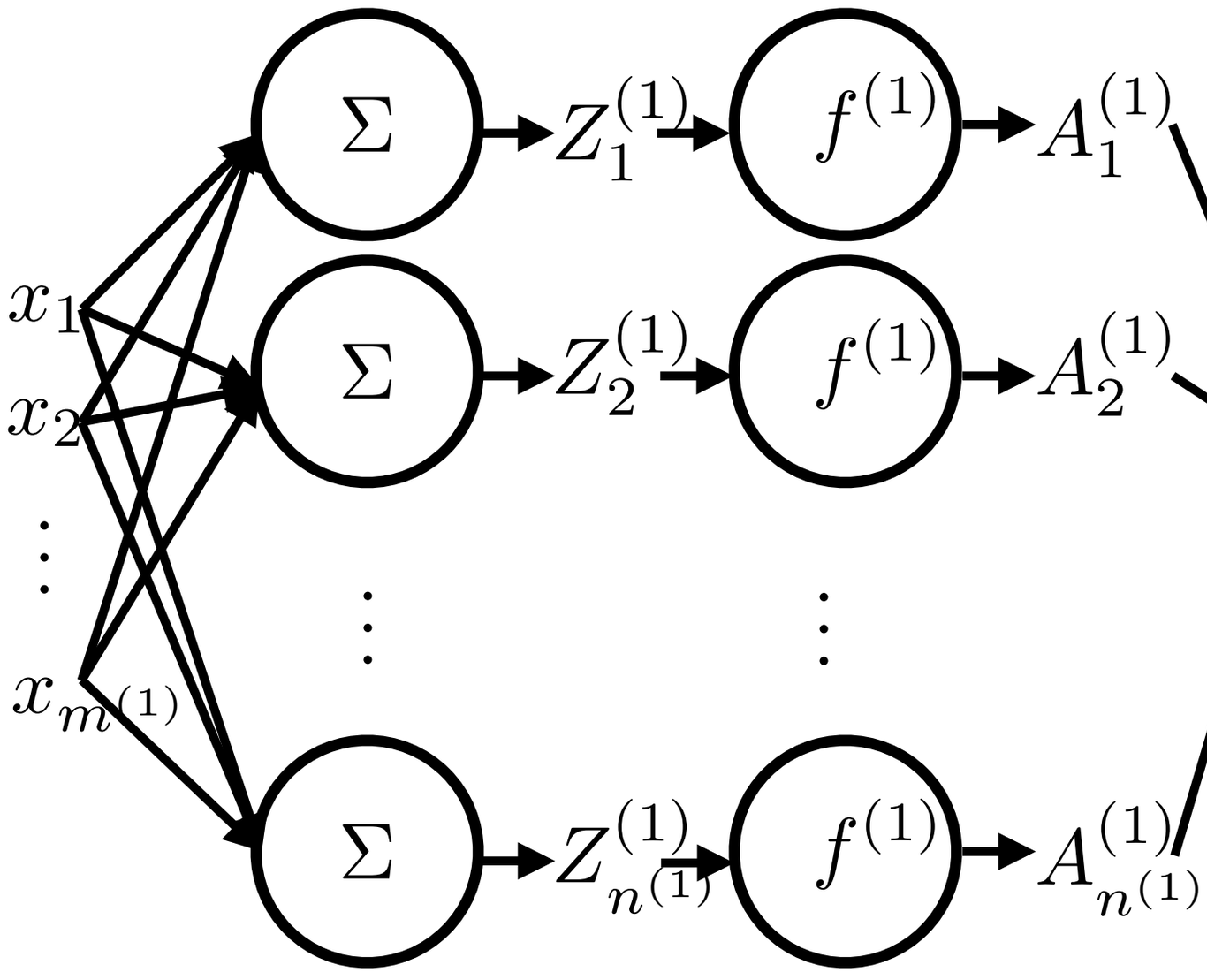


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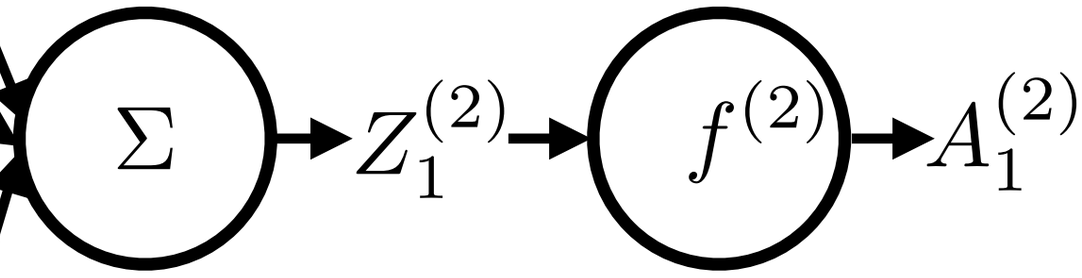


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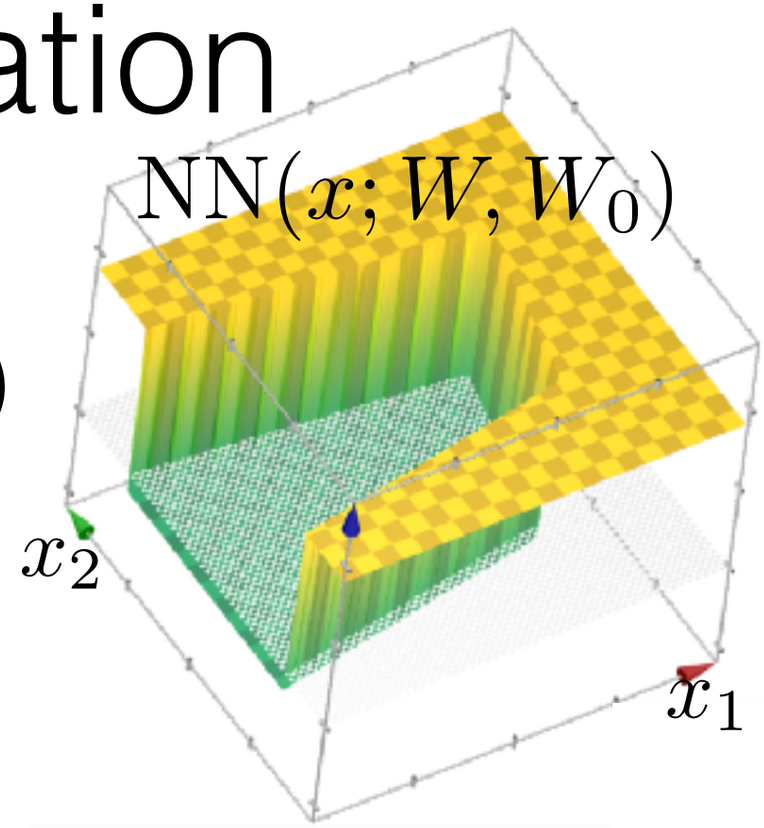


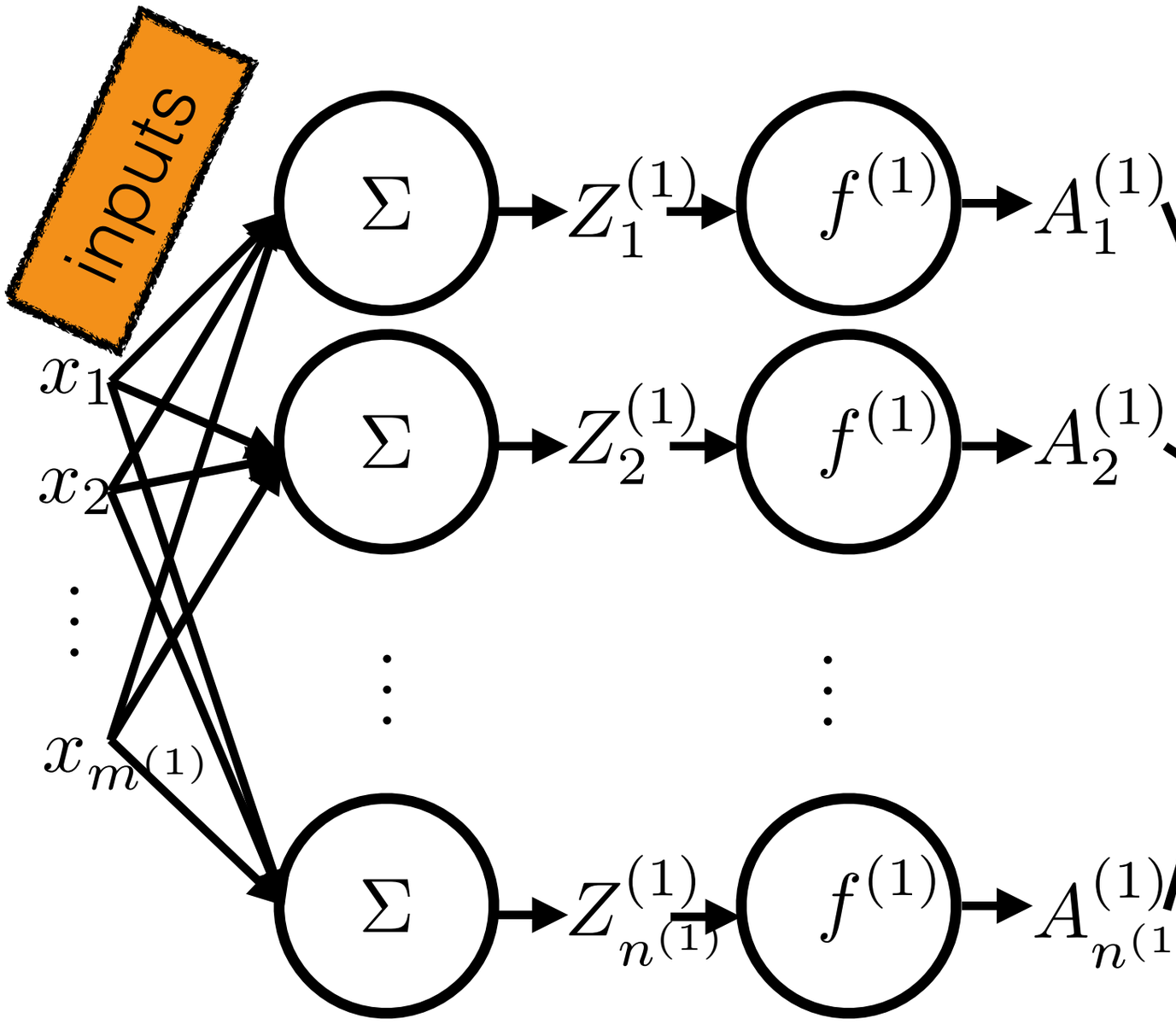


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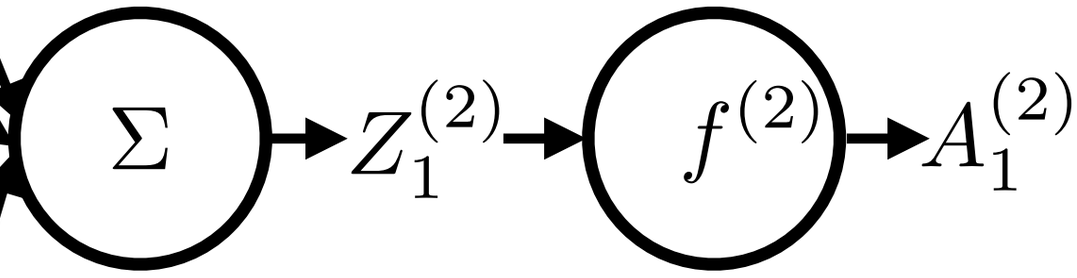


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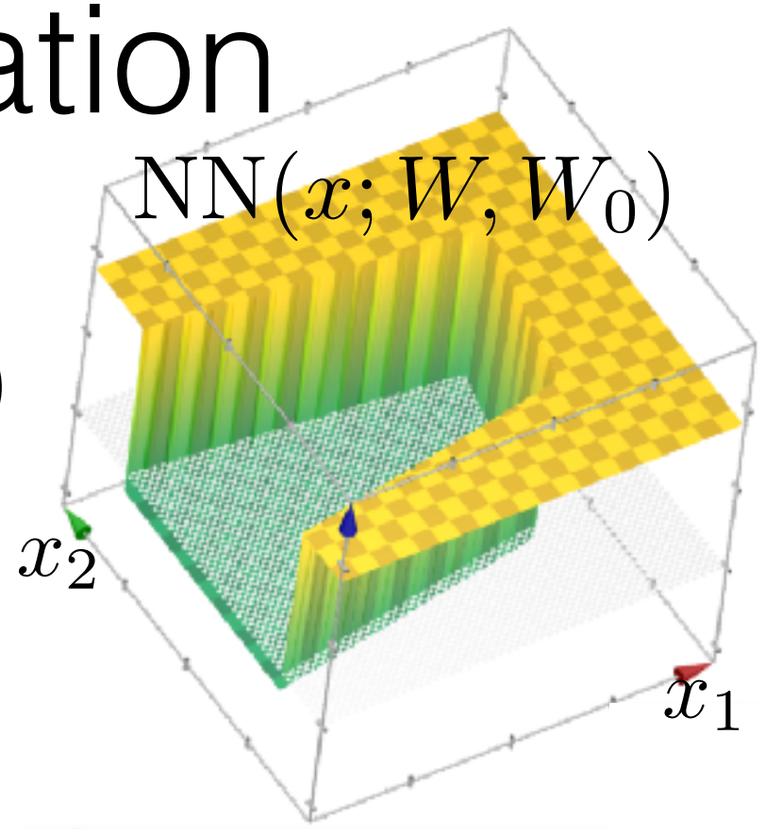
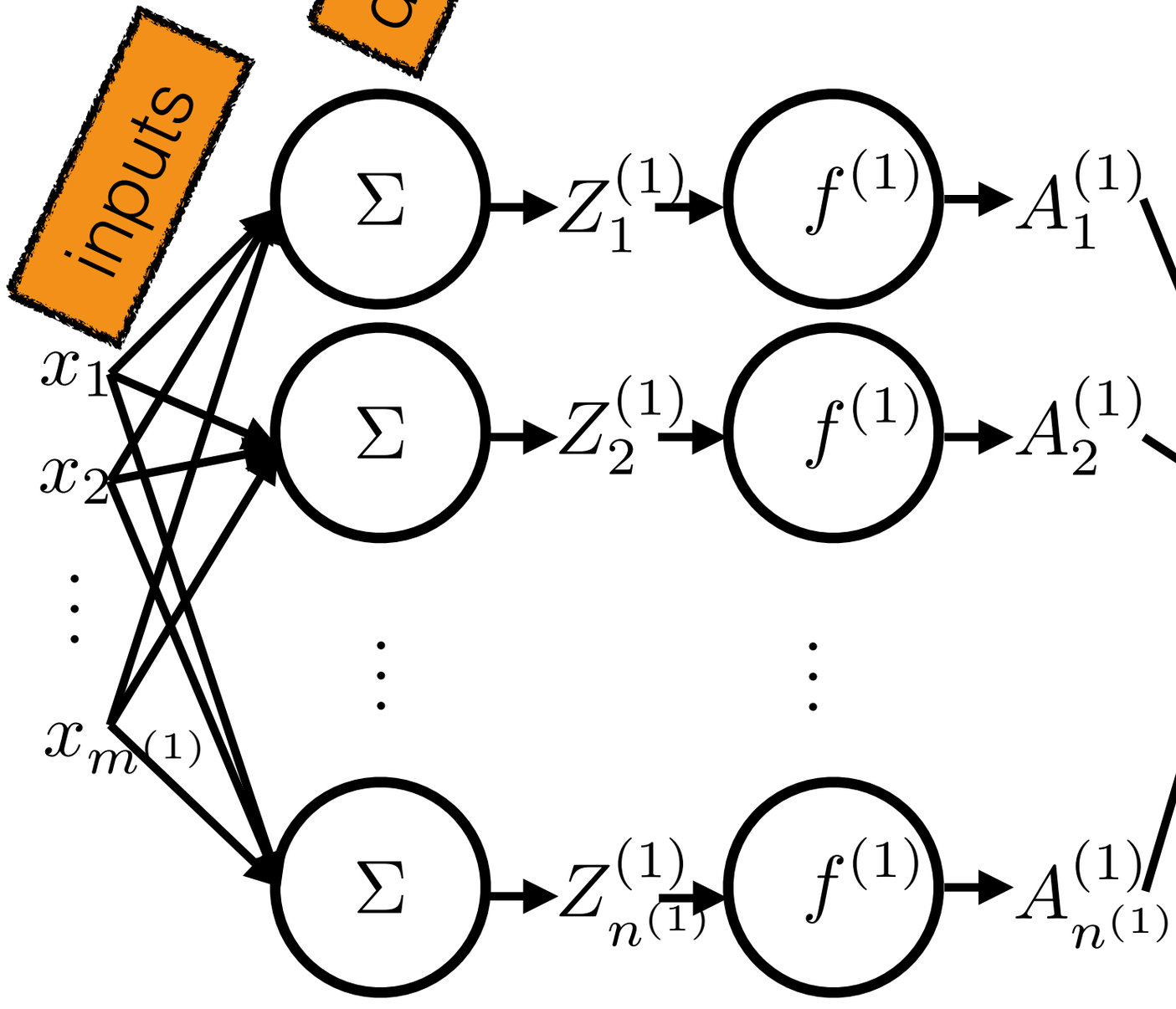
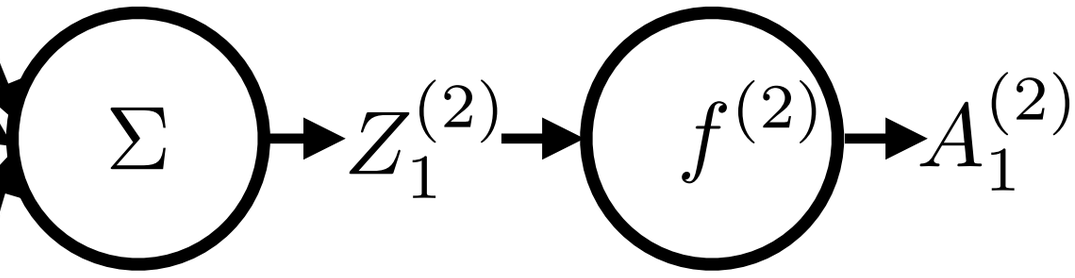



dot product

inputs




- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

# Function graph representation

- 1st layer: $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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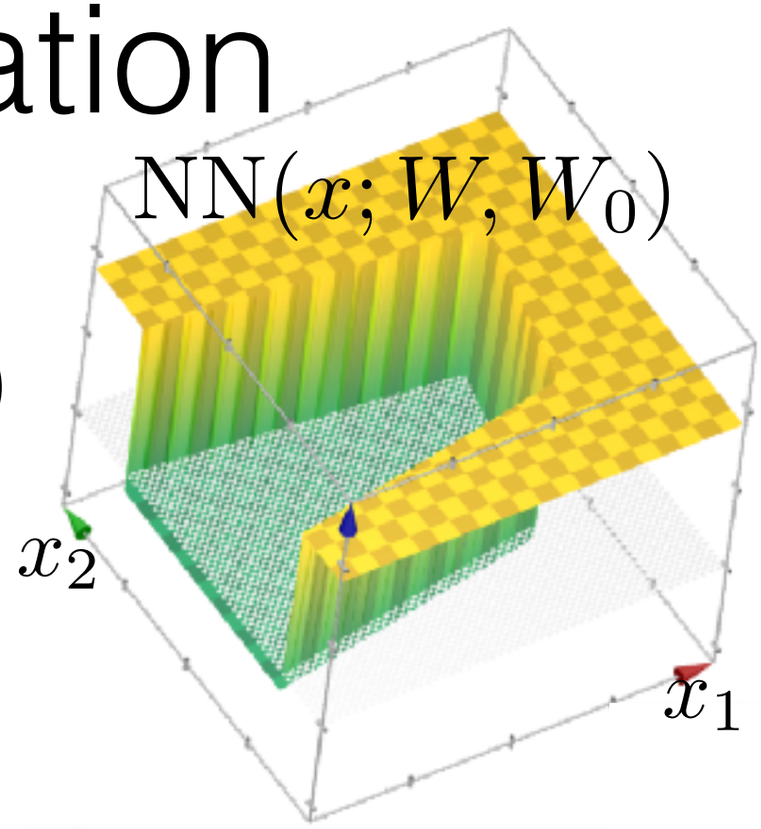
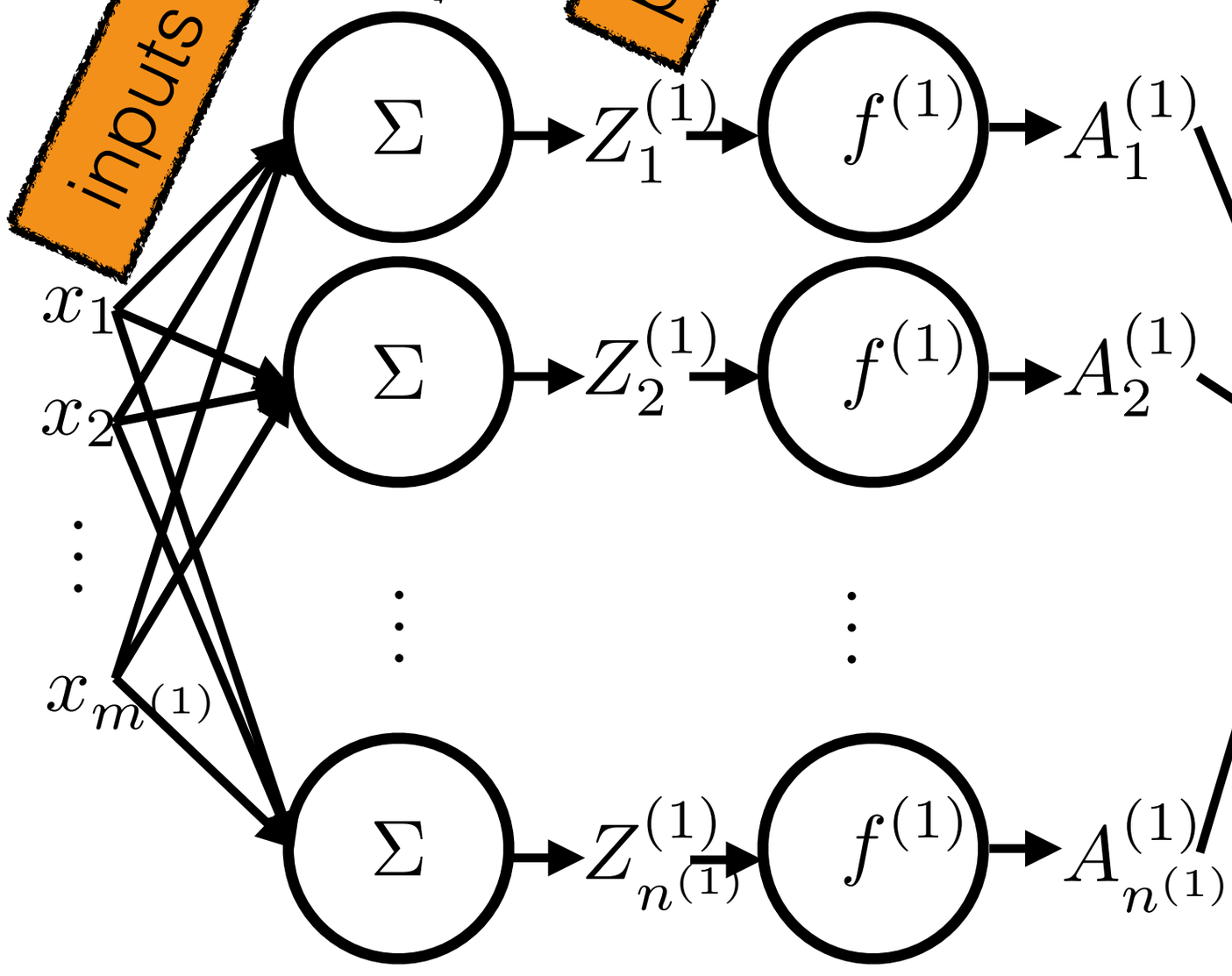


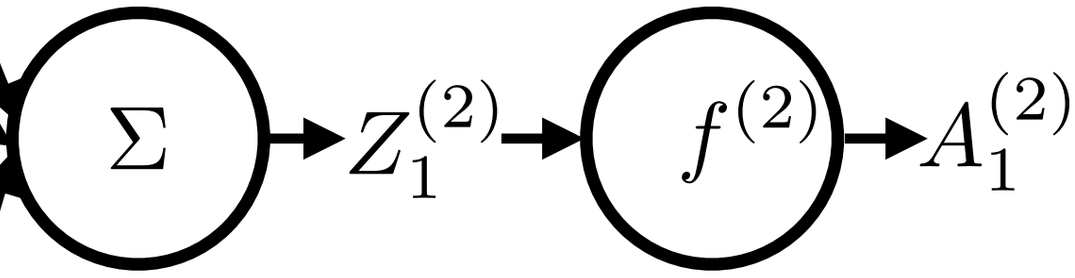

dot product

pre-activation

inputs




- Circle: function evaluation




- Forward vs. backward
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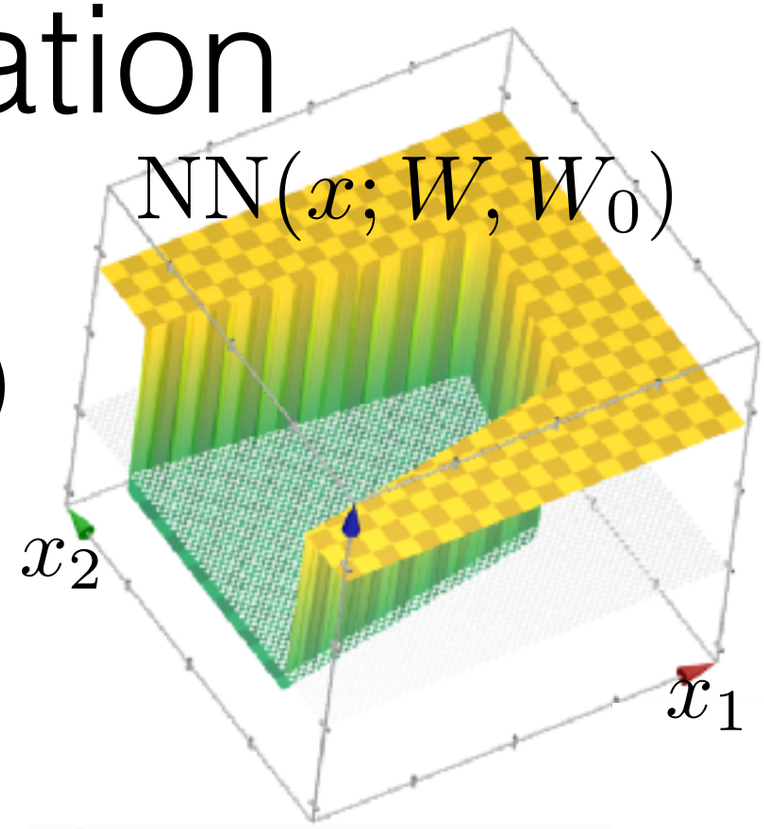
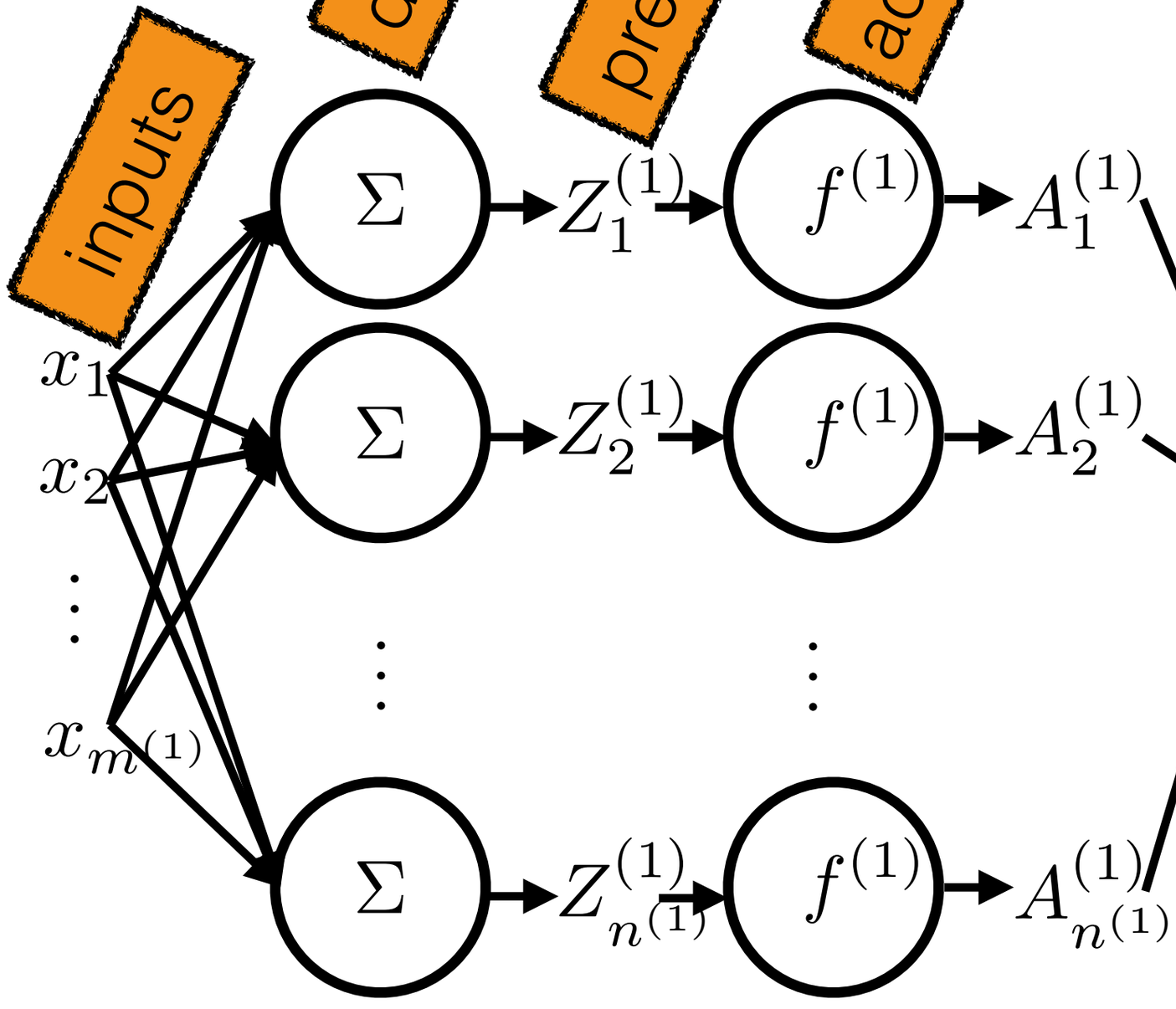
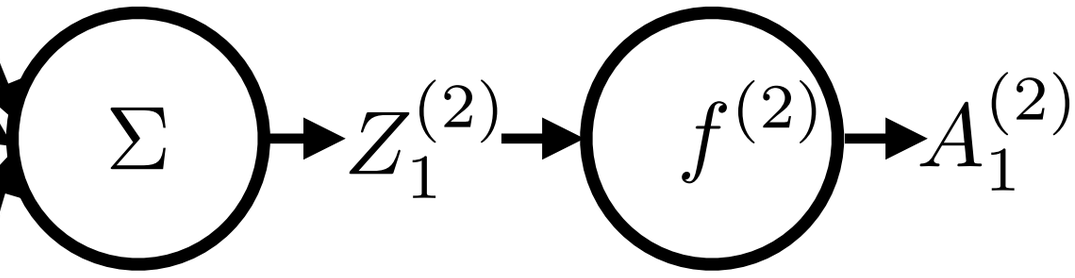



- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

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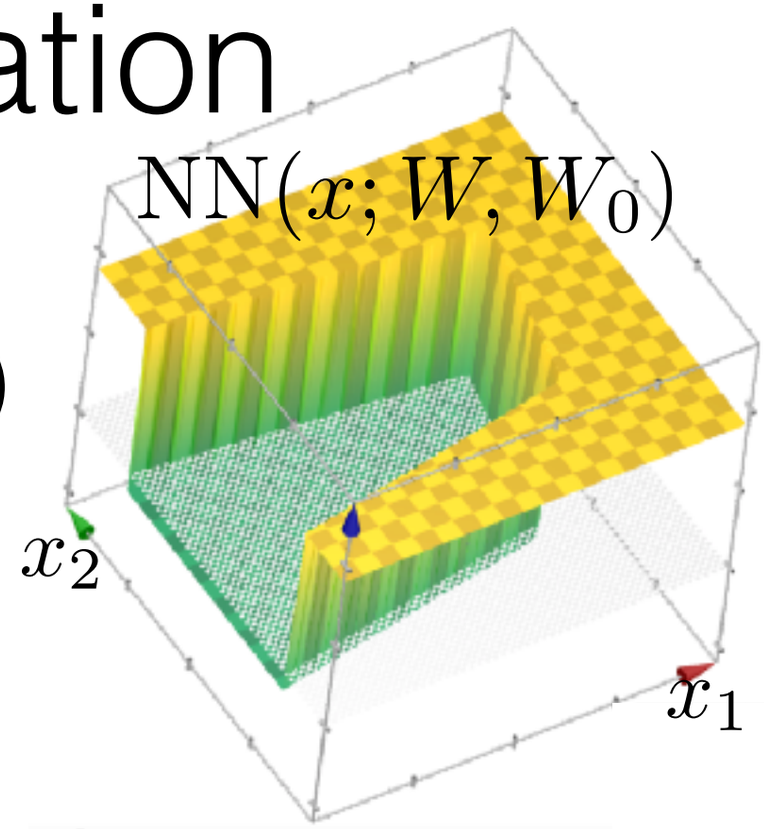
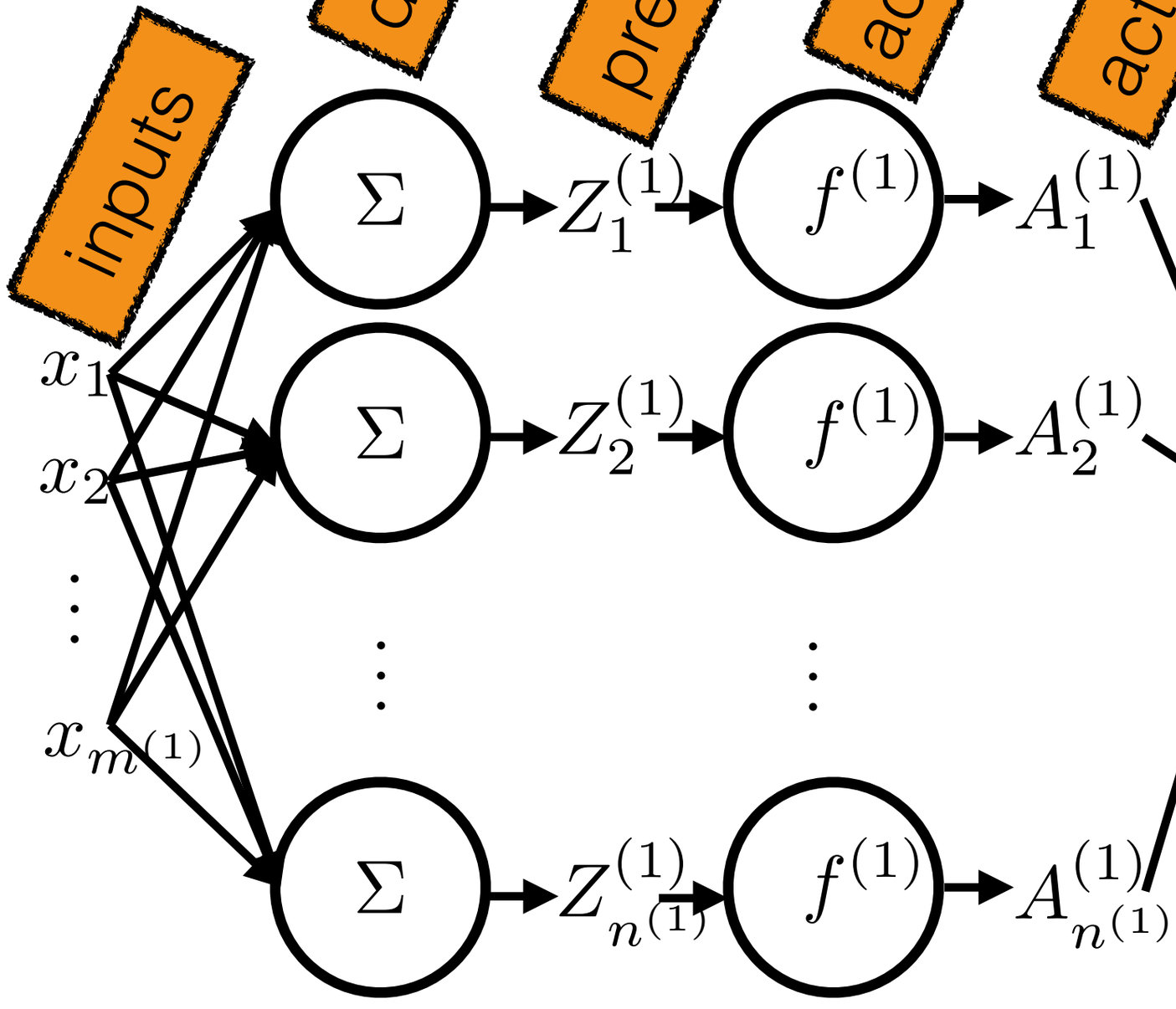
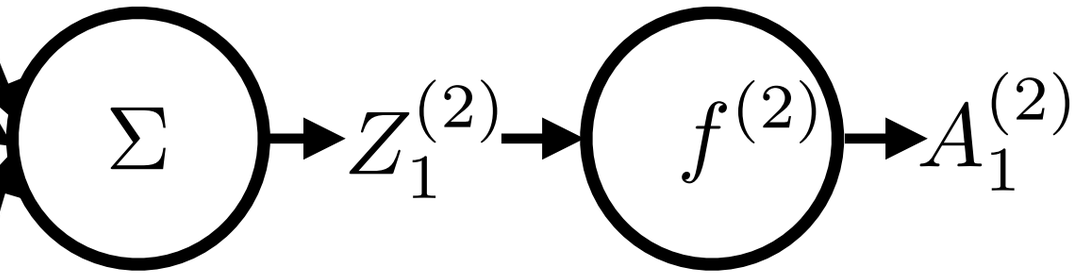



- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

# Function graph representation

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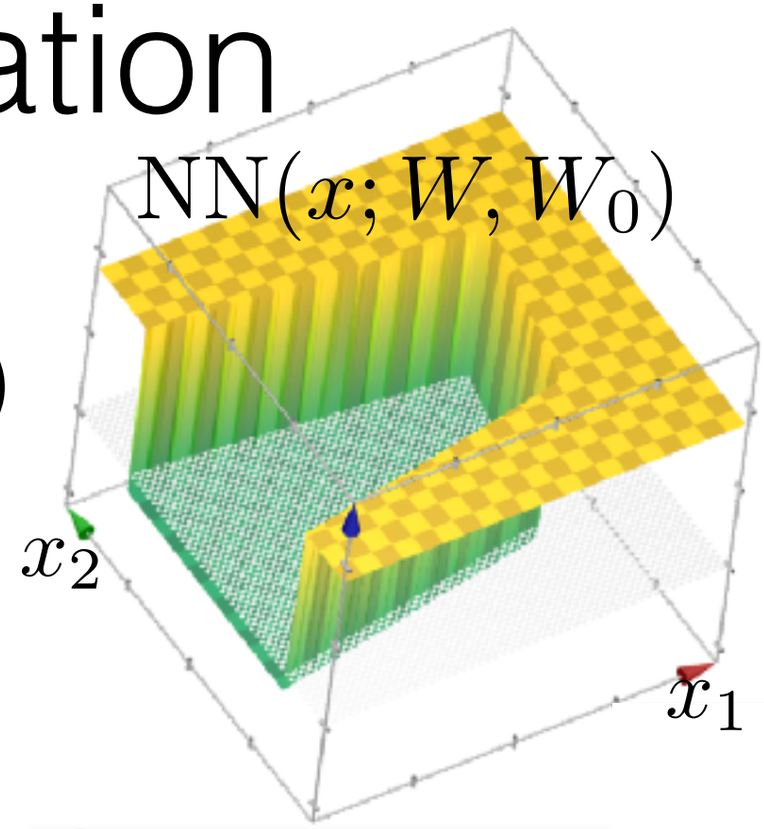


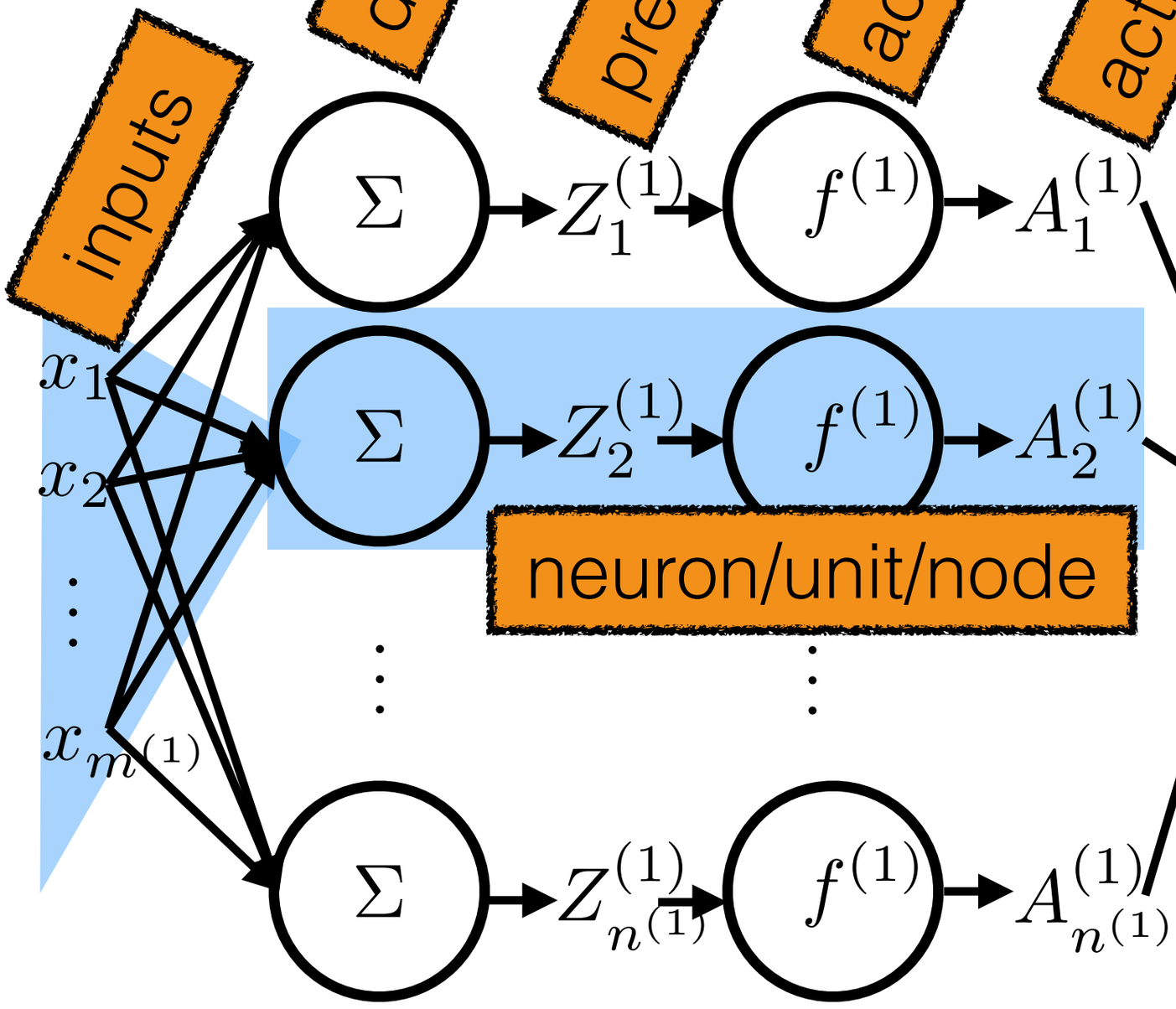


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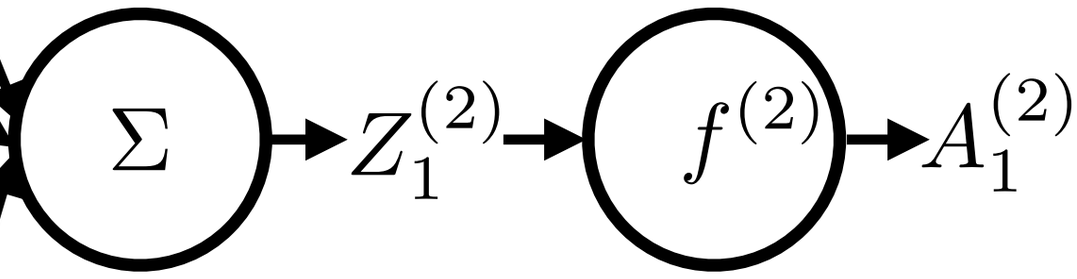
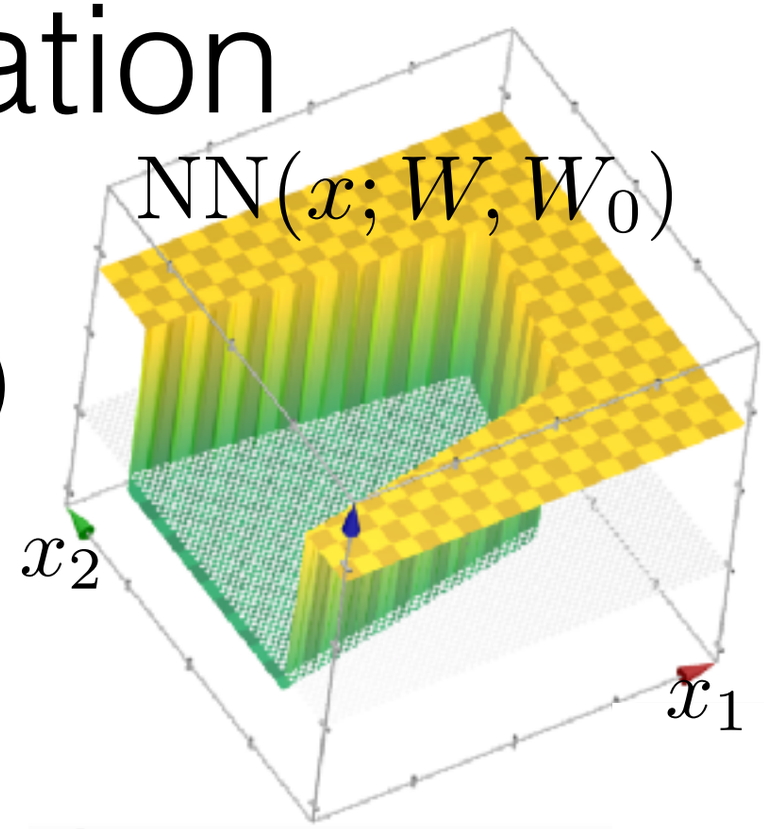
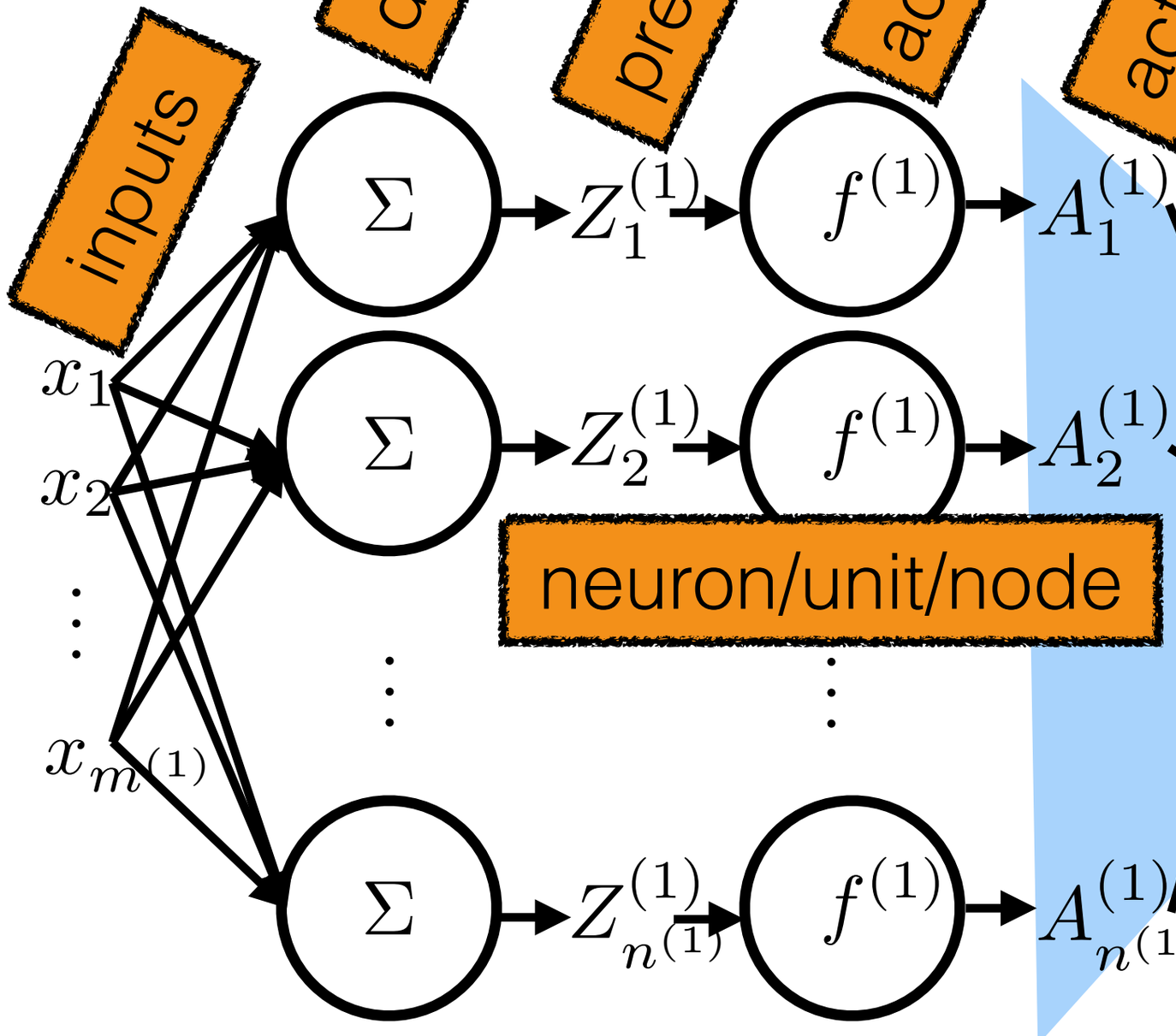



- Forward vs. backward
- A feed-forward neural network

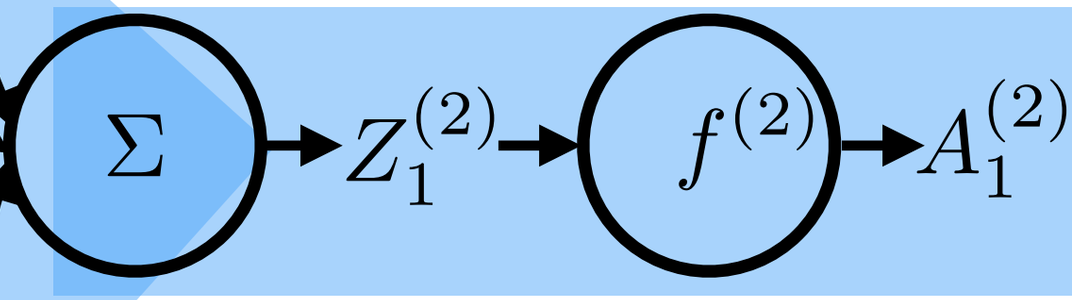
# Function graph representation

- 1st layer: $A^{(1)} = f^{(1)}(\underbrace{W^{(1)} x}_{\text{dot product}} + \underbrace{W_0^{(1)}}_{\text{pre-activation}})$
- 2nd layer: $A^{(2)} = f^{(2)}(\underbrace{W^{(2)} A^{(1)}}_{\text{activation function}} + \underbrace{W_0^{(2)}}_{\text{activation}})$
- Whole thing: $A^{(2)} = \mathrm{NN}(x; W, W_0)$




- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

# Function graph representation

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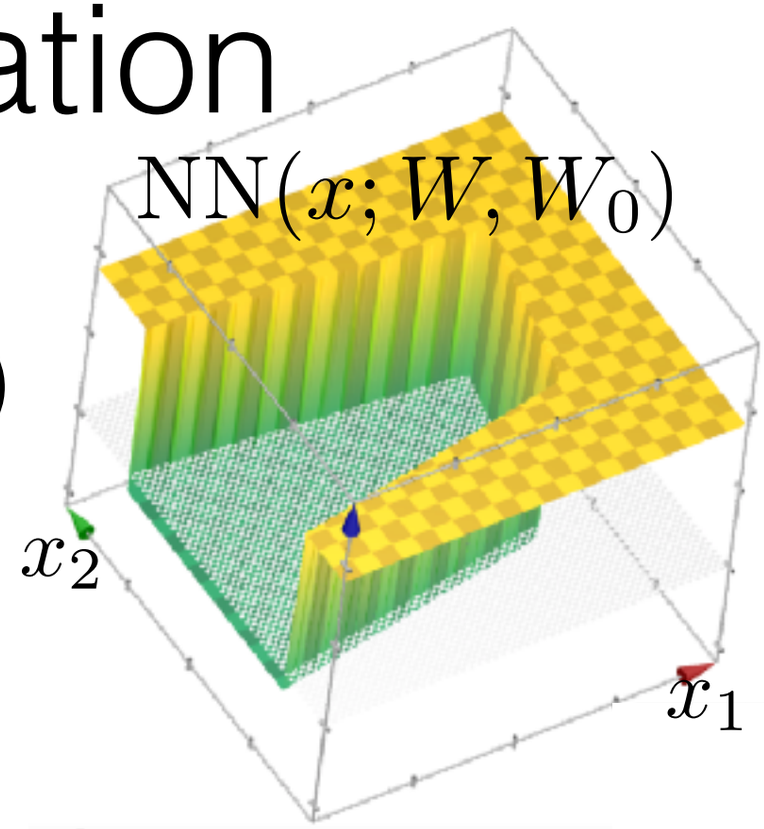
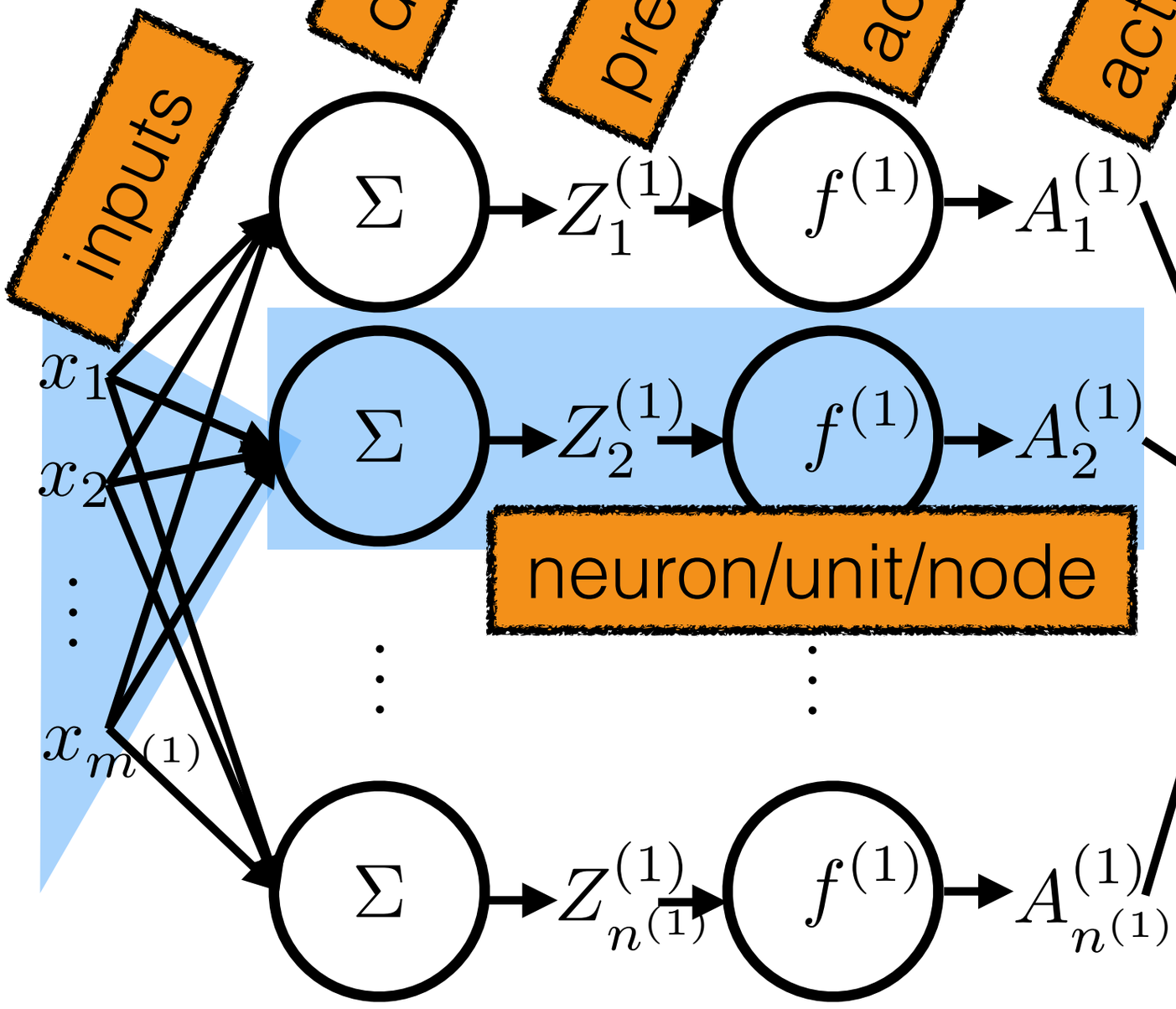
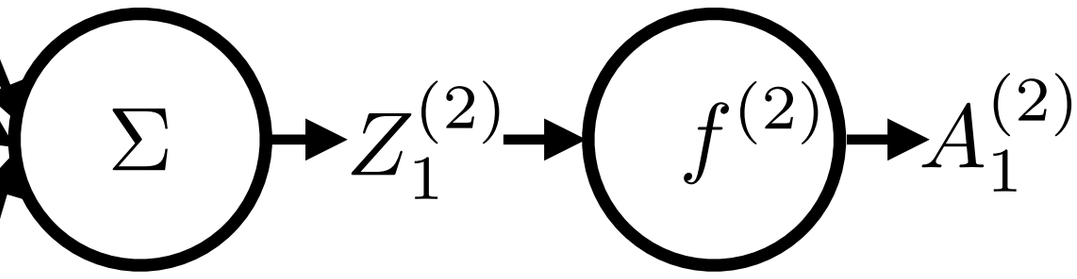



- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

# Function graph representation

- 1st layer: $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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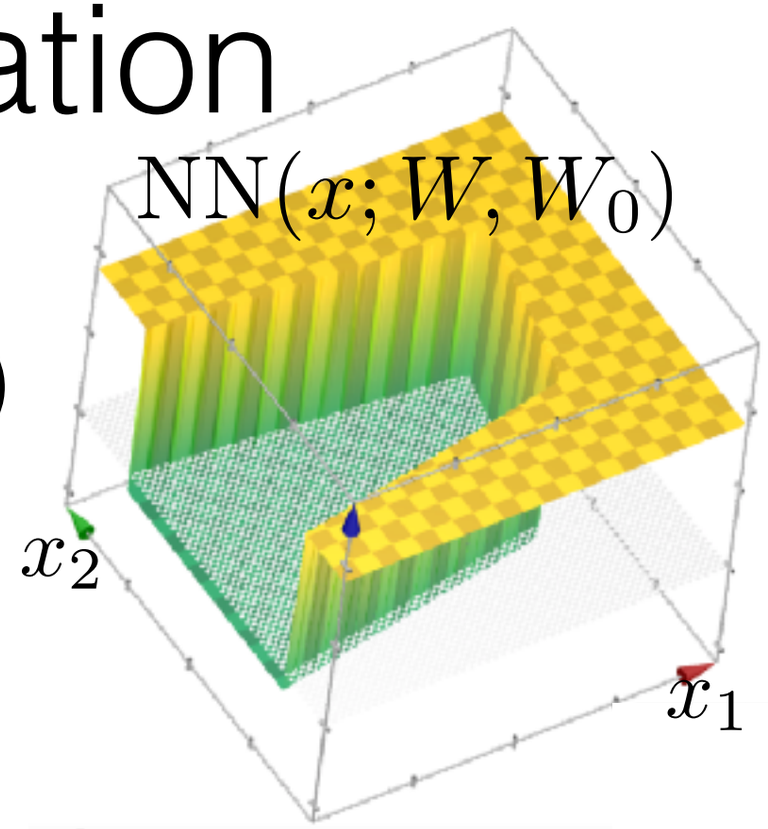
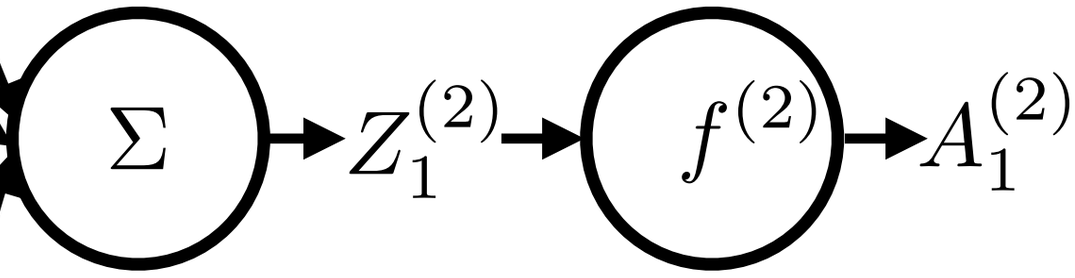






- Circle: function evaluation




- Forward vs. backward
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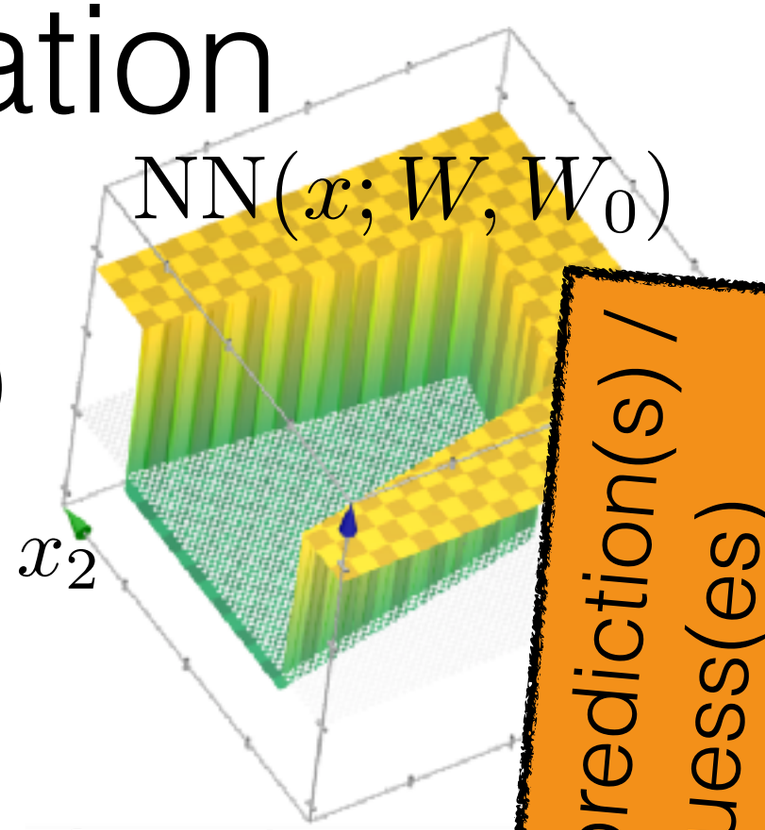


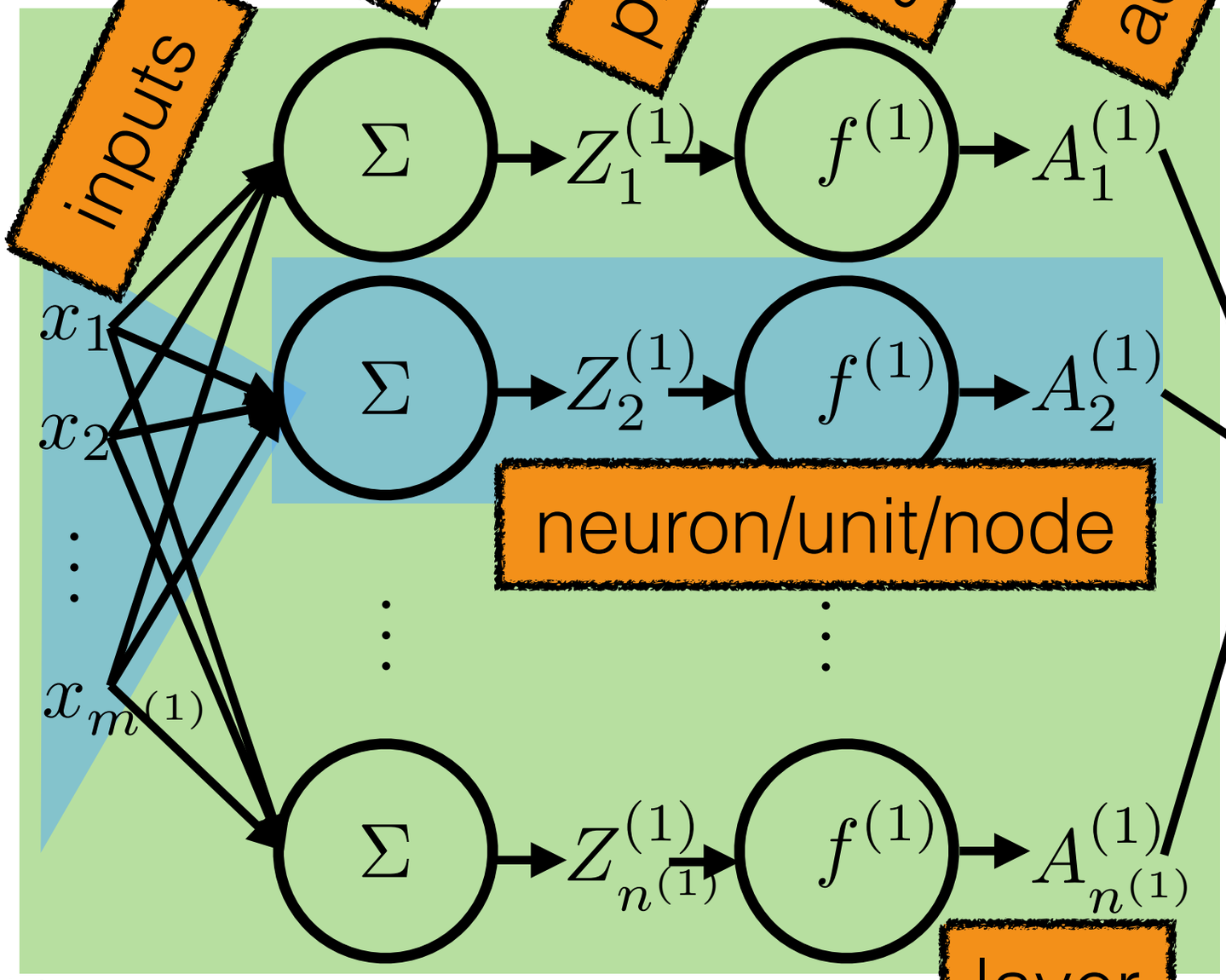

dot product

pre-activation

activation function

activation

inputs

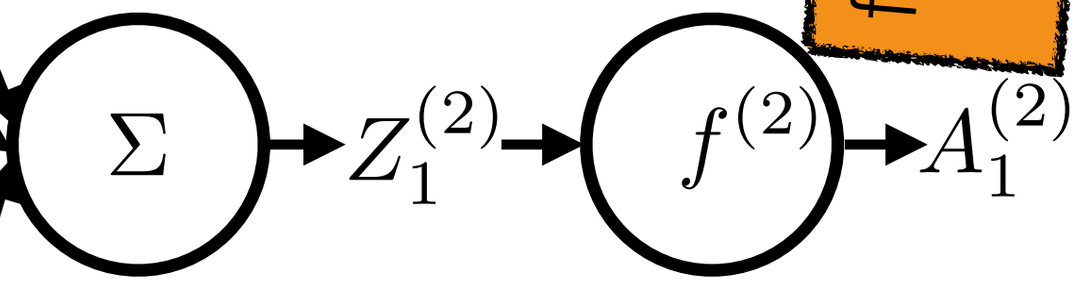



neuron/unit/node

layer

final prediction(s) / guess(es)

- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network

# Function graph representation

- 1st layer: $Z^{(1)} = W^{(1)}(\cdot) + W_0^{(1)}$
- 2nd layer: $Z^{(2)} = W^{(2)}(\cdot)^\top A^{(\cdot)} + W_0^{(2)}$
- Whole thing: $A^{(2)} = \mathrm{NN}(x; W, W_0)$

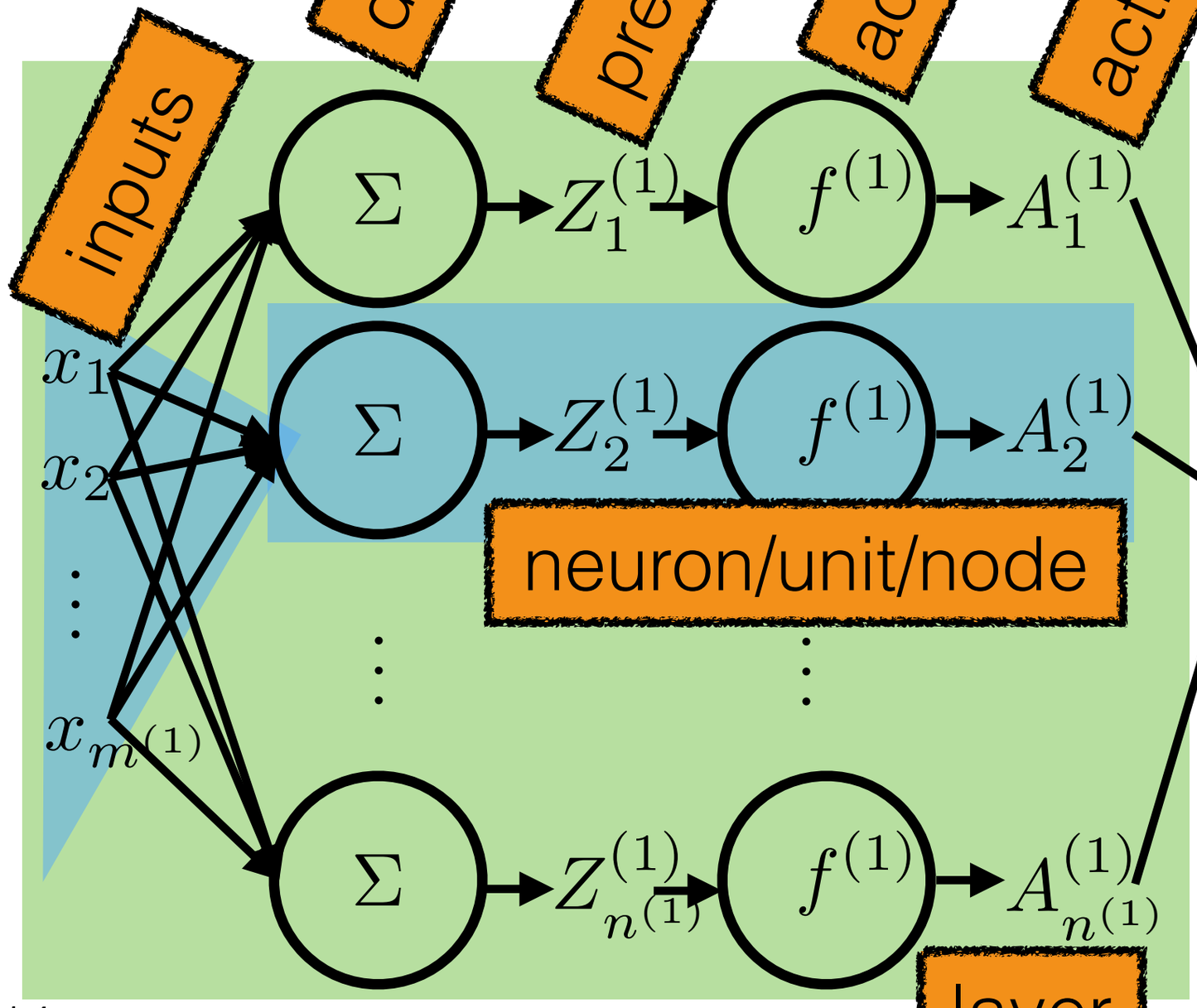
dot product

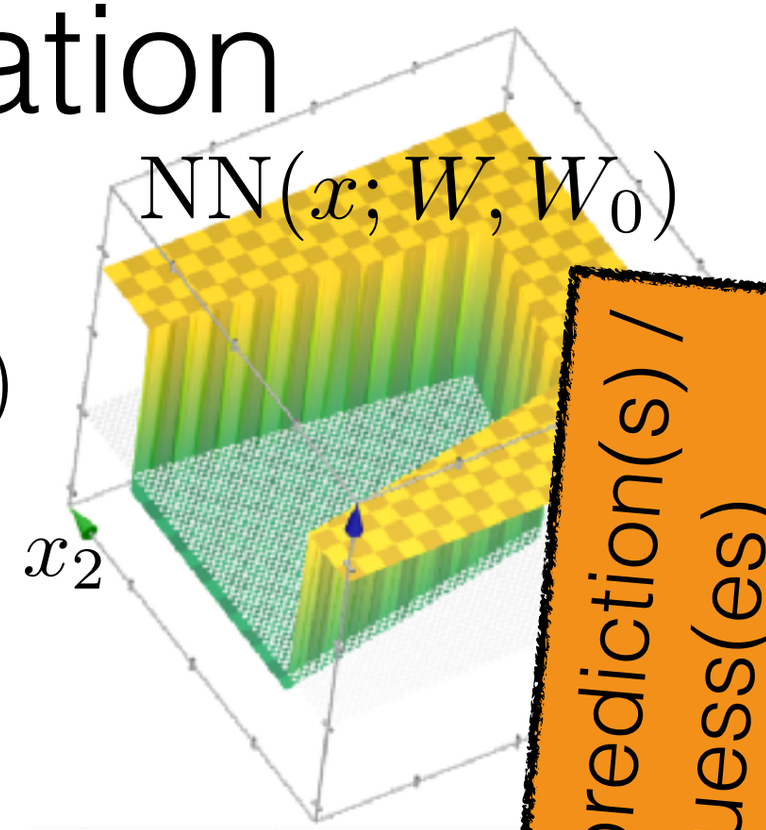
pre-activation

activation function

activation

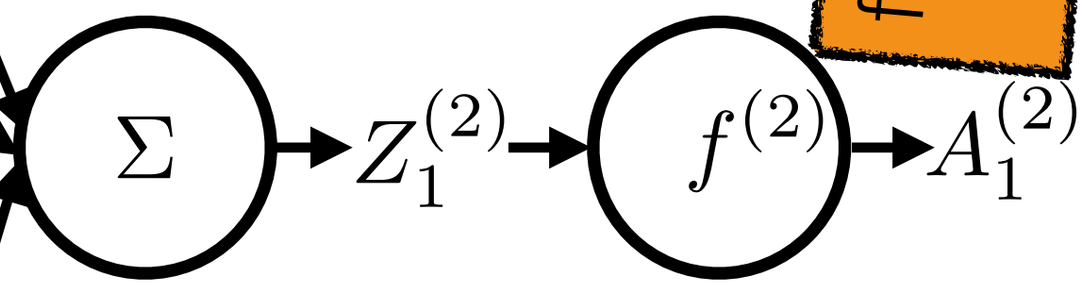
inputs




final prediction(s) / guess(es)

- Circle: function evaluation




- Forward vs. backward
- A feed-forward neural network
- Fully connected

# Function graph representation

- 1st layer: $A^{(1)} = f^{(1)}(Z^{(1)}) = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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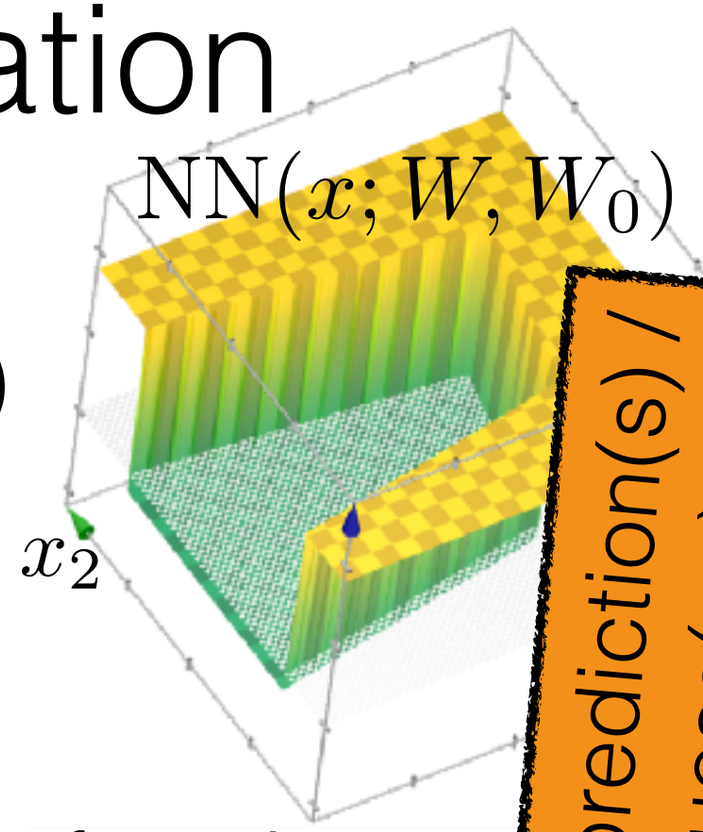


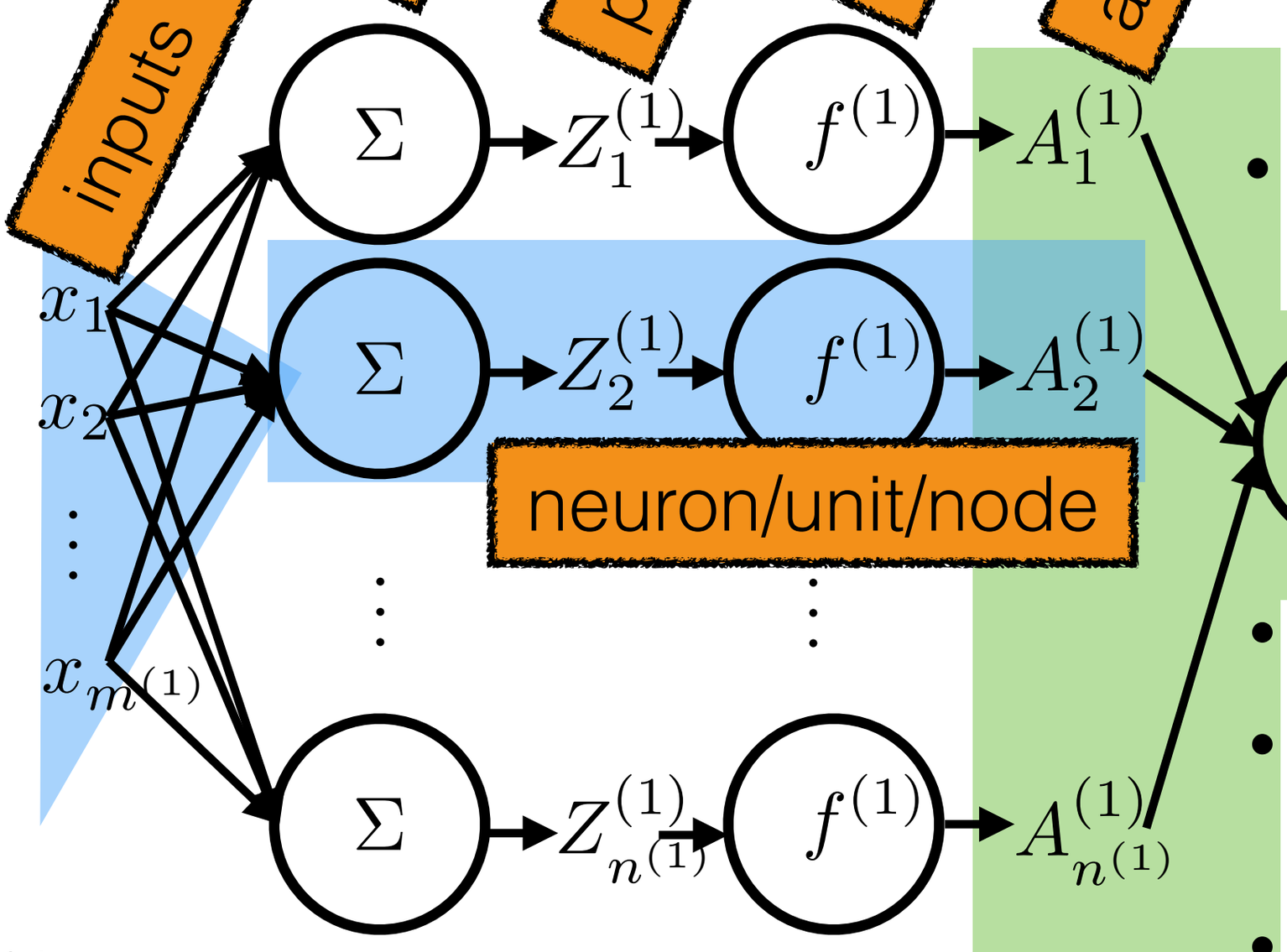


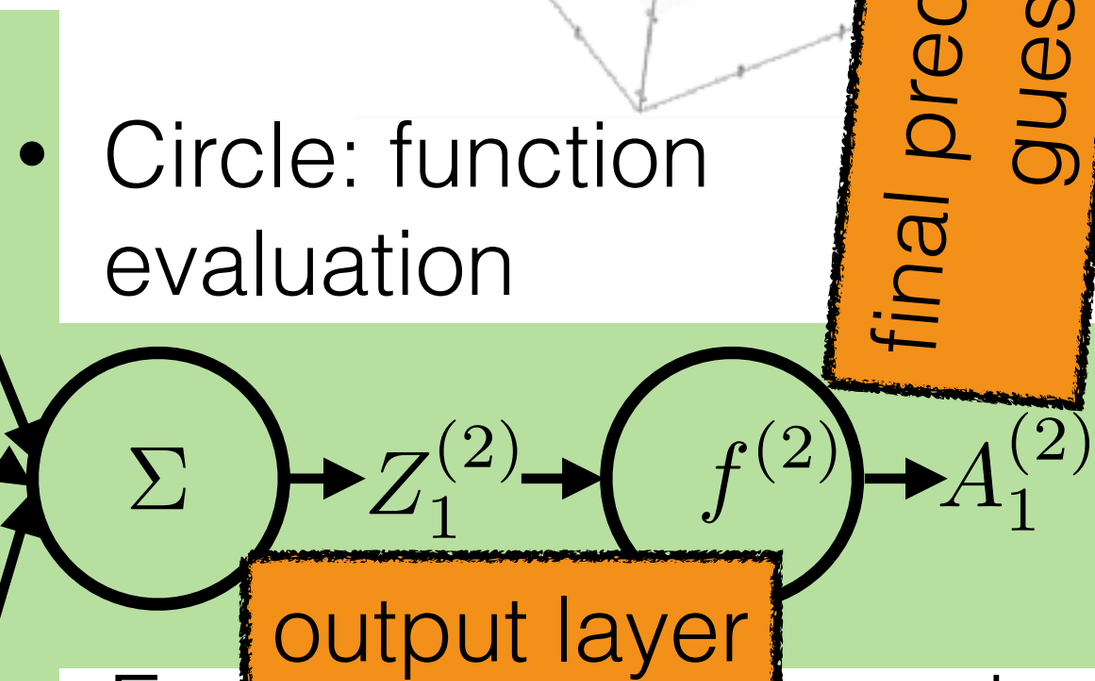


- Circle: function evaluation
- Forward vs. backward
- A feed-forward neural network
- Fully connected

# Function graph representation

- 1st layer: $Z^{(1)} = W^{(1)}(\cdot) + W_0^{(1)})$
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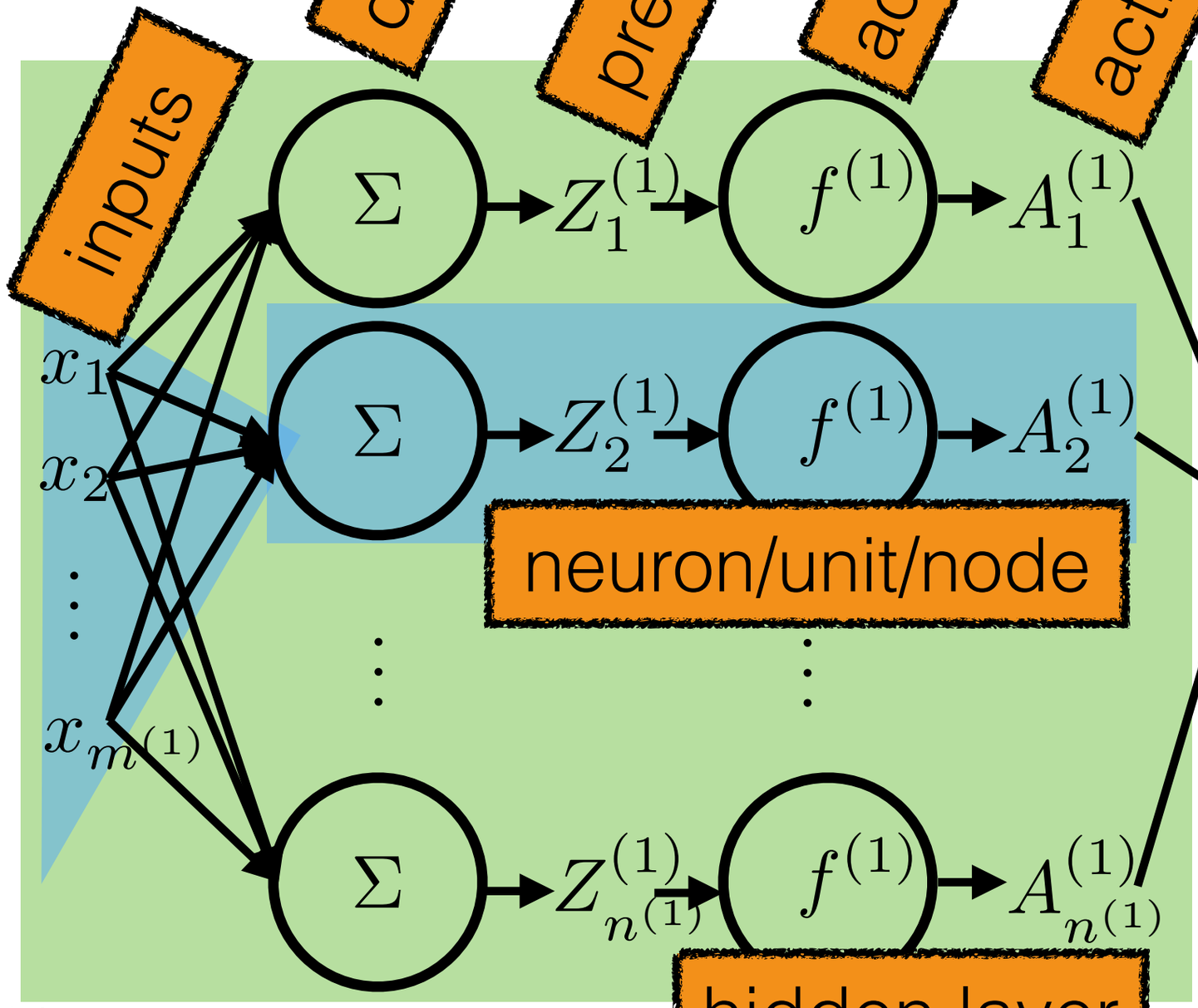
dot product

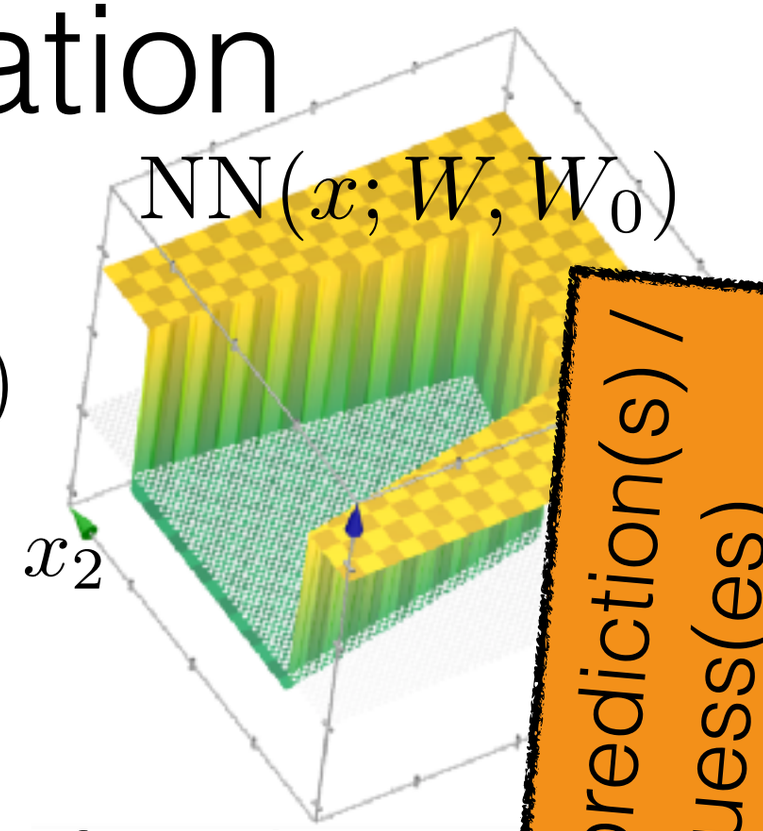
pre-activation

activation function

activation

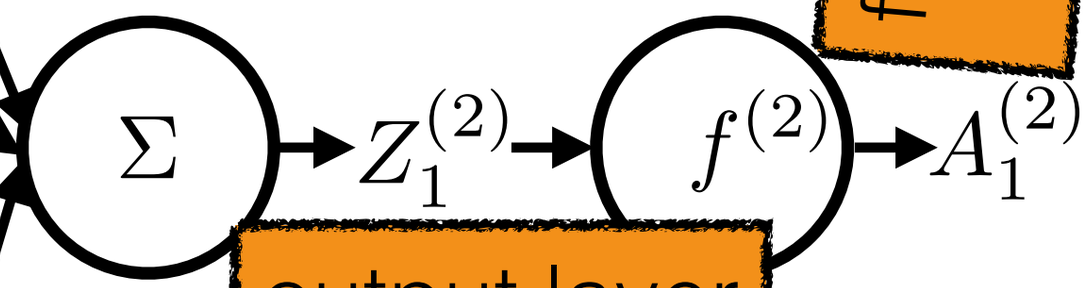
inputs




final prediction(s) / guess(es)

- Circle: function evaluation




output layer

- Forward vs. backward
- A feed-forward neural network
- Fully connected