6.036: Introduction to Machine Learning

Cambridge MA elections: early voting now! Election Day Tues Nov 2

Lecture start: Tuesdays 9:35am

Who's talking? Prof. Tamara Broderick

Questions? Ask on Piazza: "lecture (week) 7" folder

Materials: slides, video will all be available on Canvas

Live Zoom feed: https://mit.zoom.us/j/94238622313

Last Time(s)

I. Neural nets: hypothesis class

II. Neurals nets: function graph representation

Today's Plan

I. More activation functions

II. Derivatives in neural nets (for SGD or GD)

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$n^{(\ell)}$ is NOT the number of data points

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- How to compute the derivatives in (S)GD?

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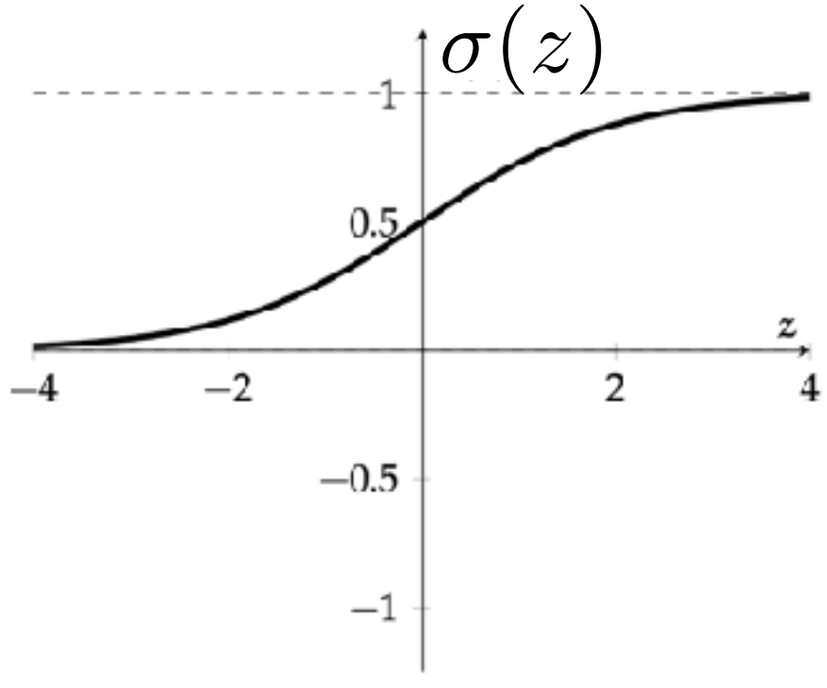

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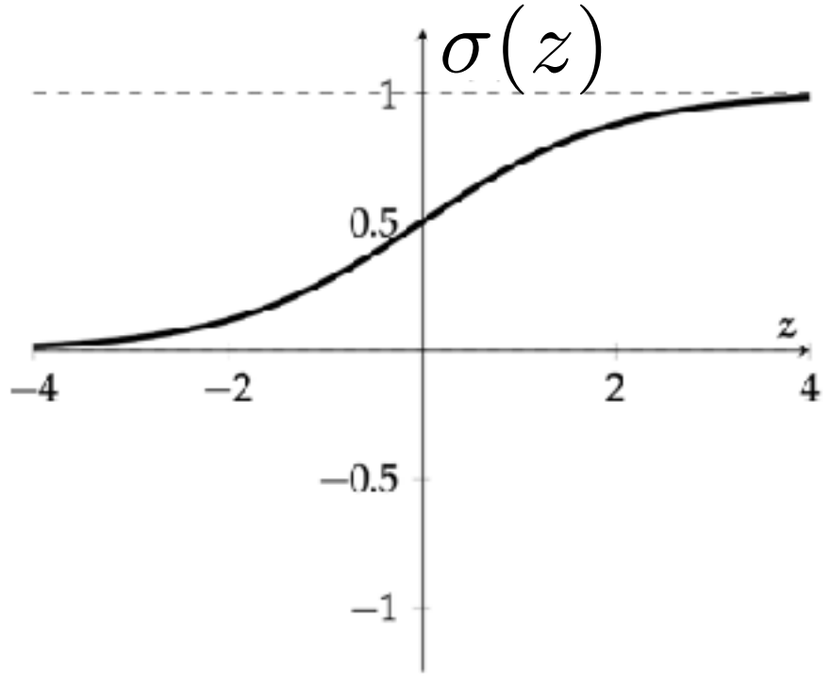

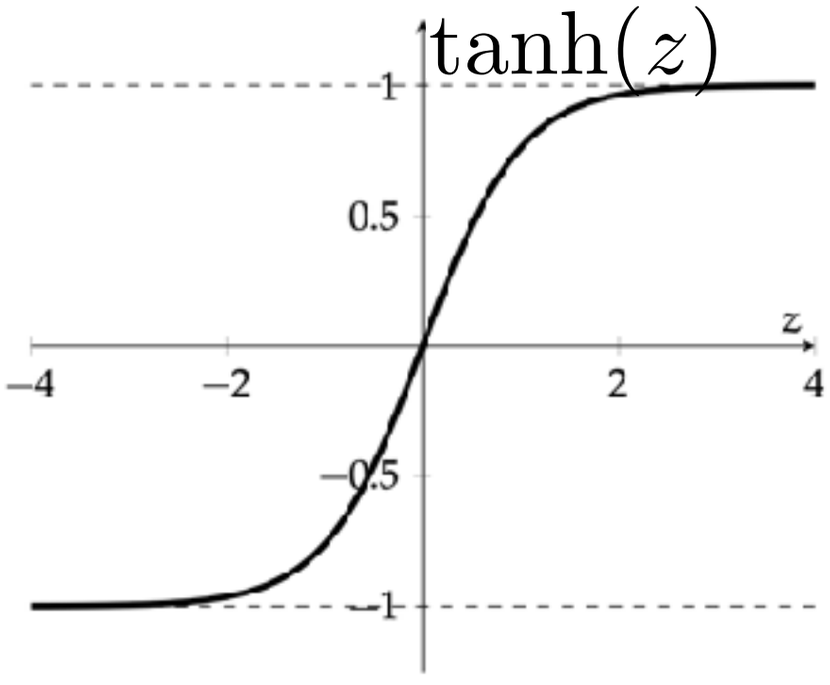

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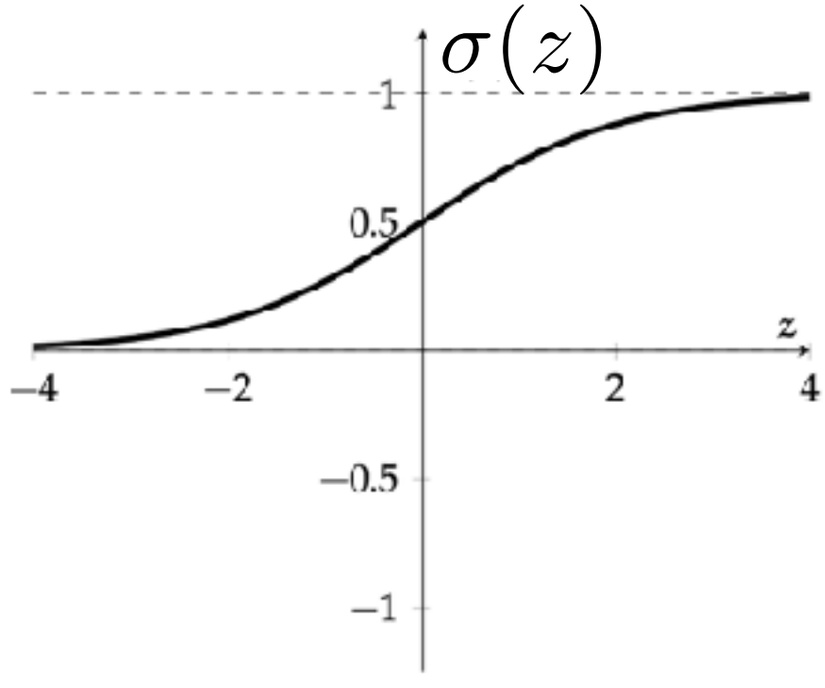

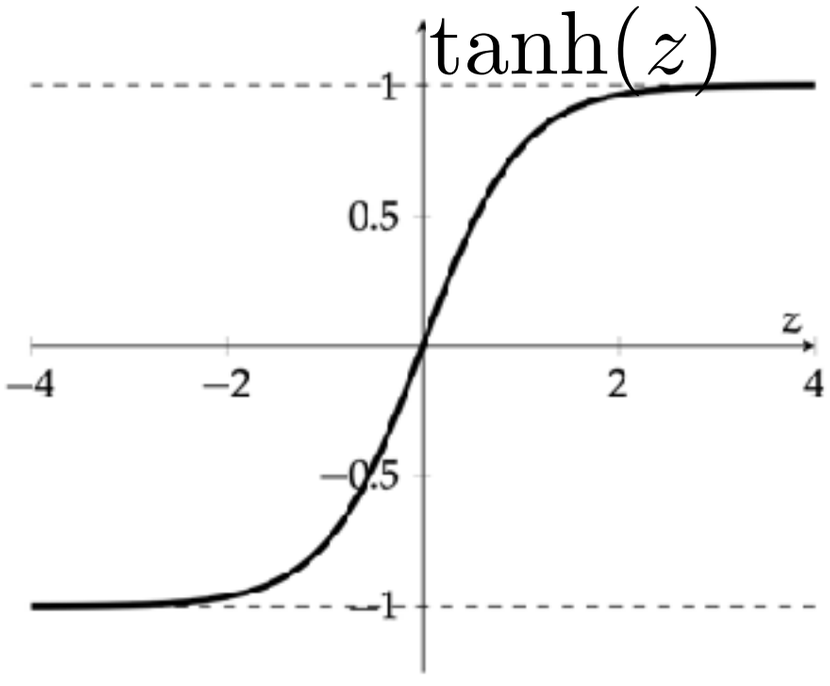

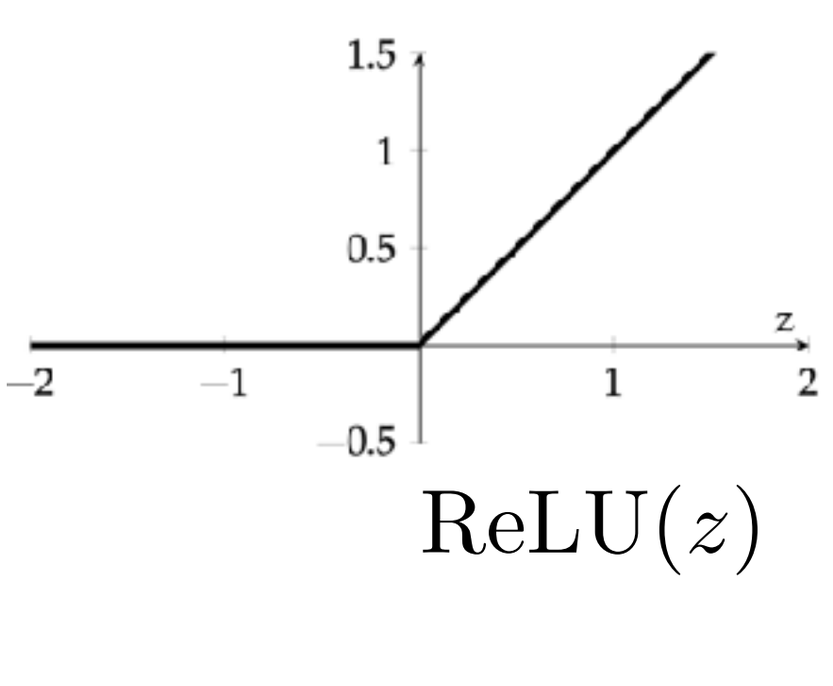

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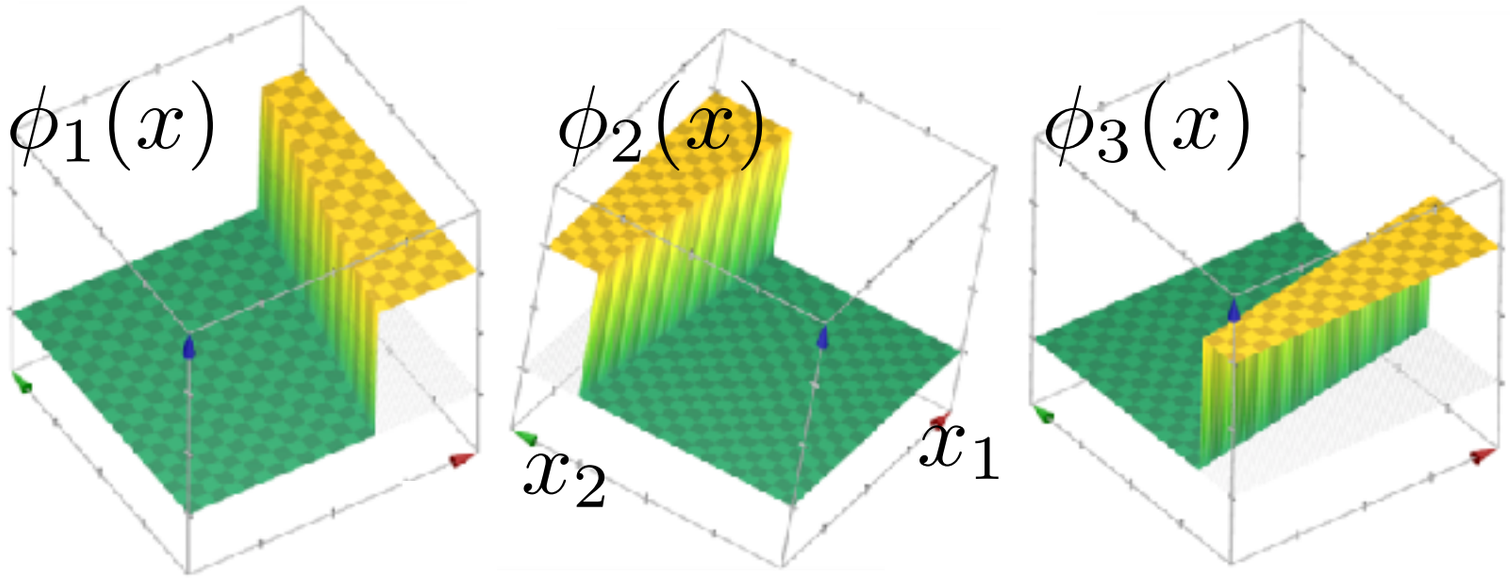

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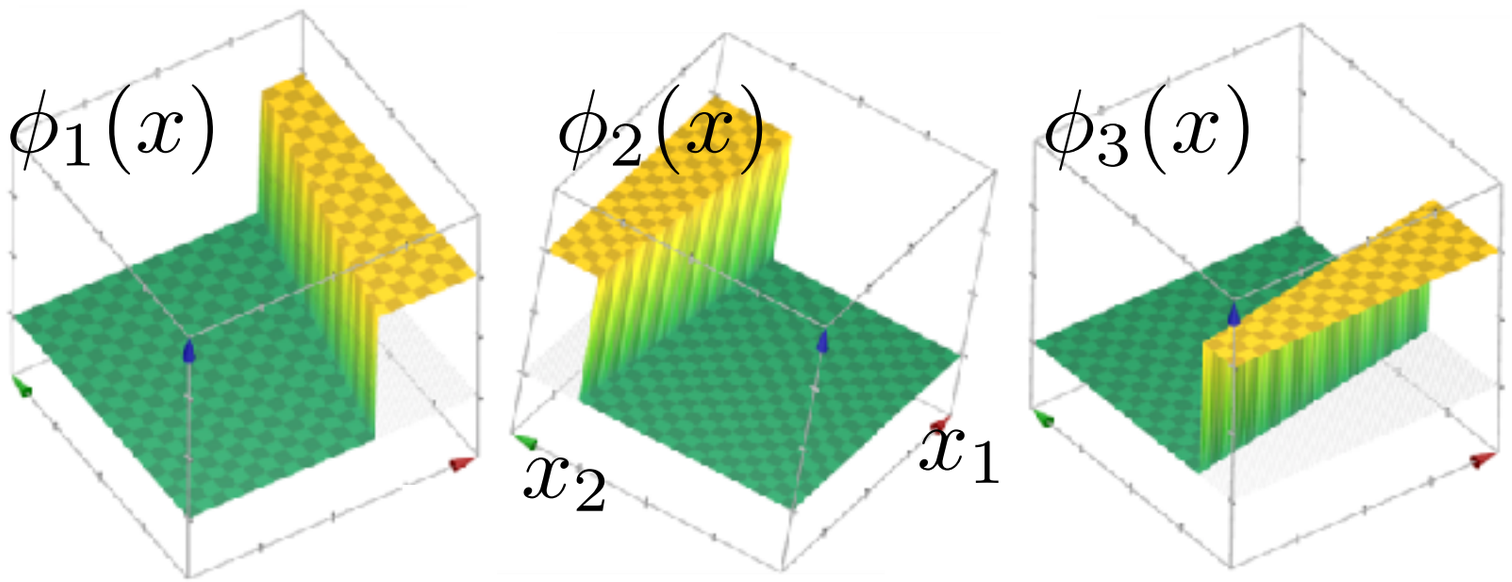

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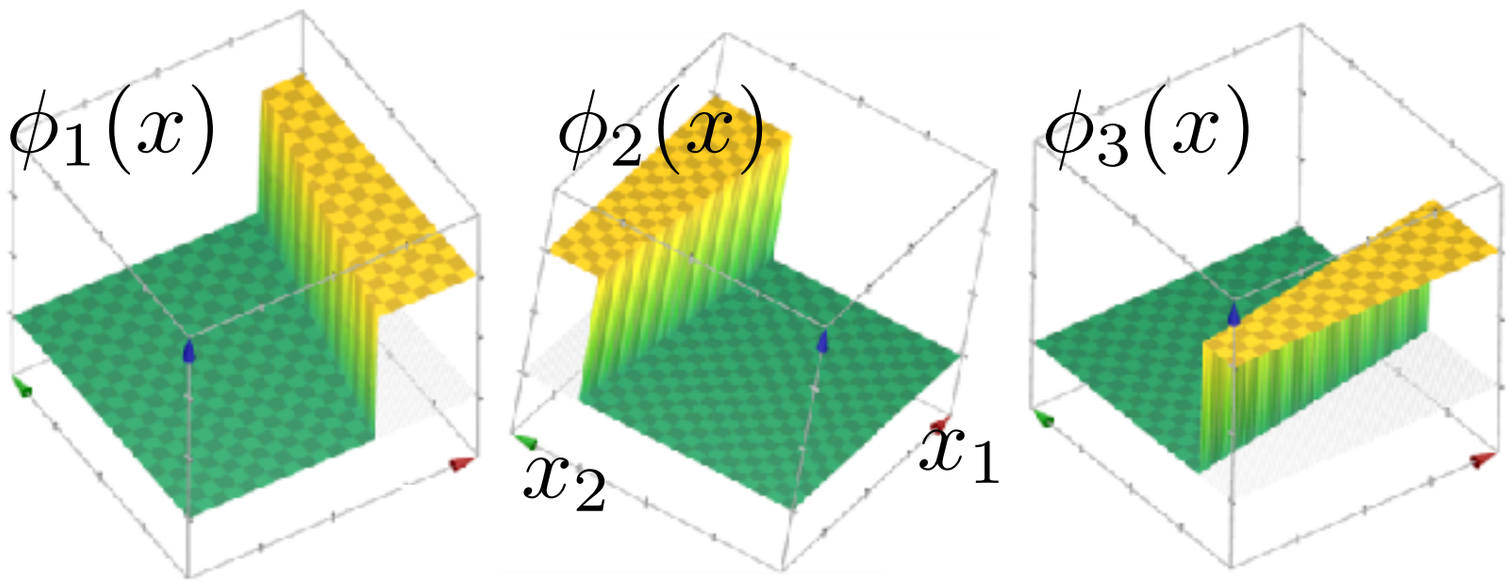

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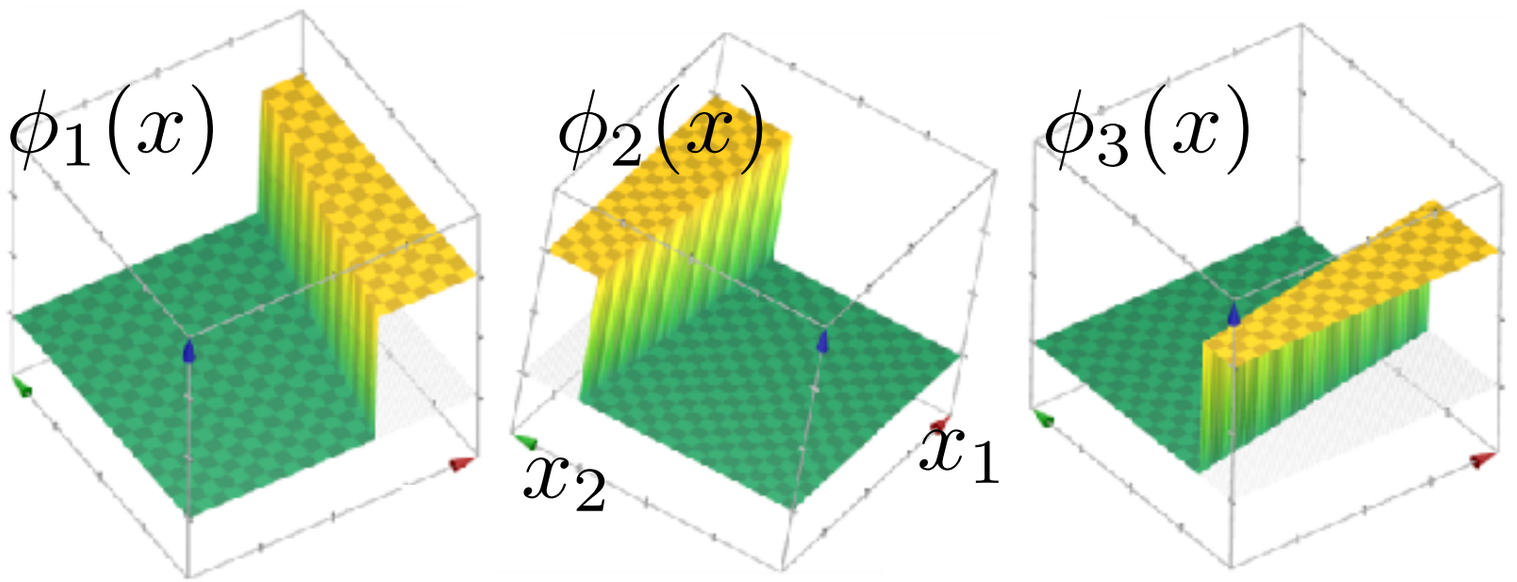

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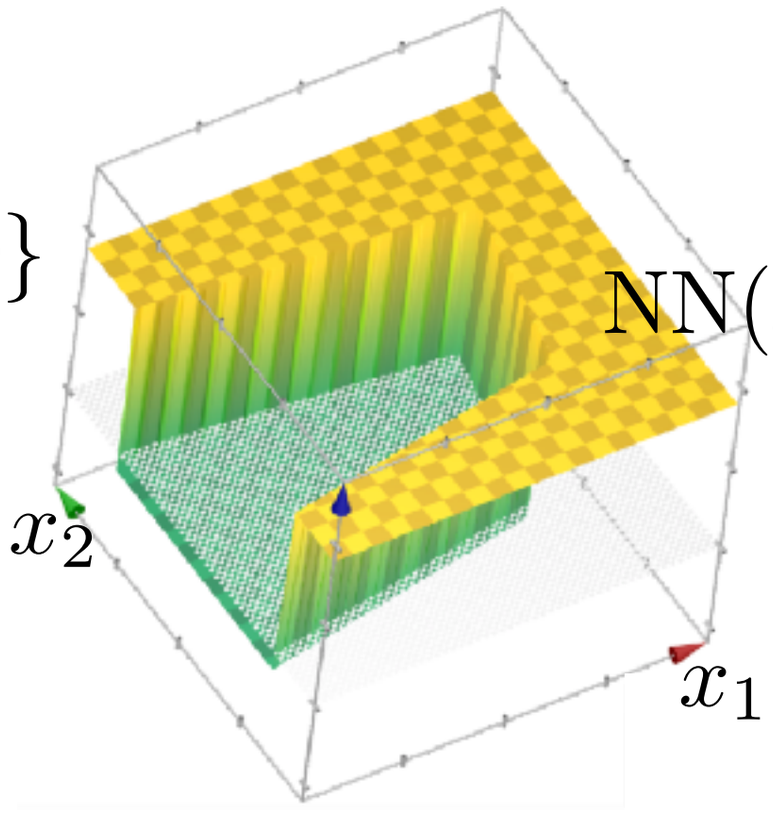

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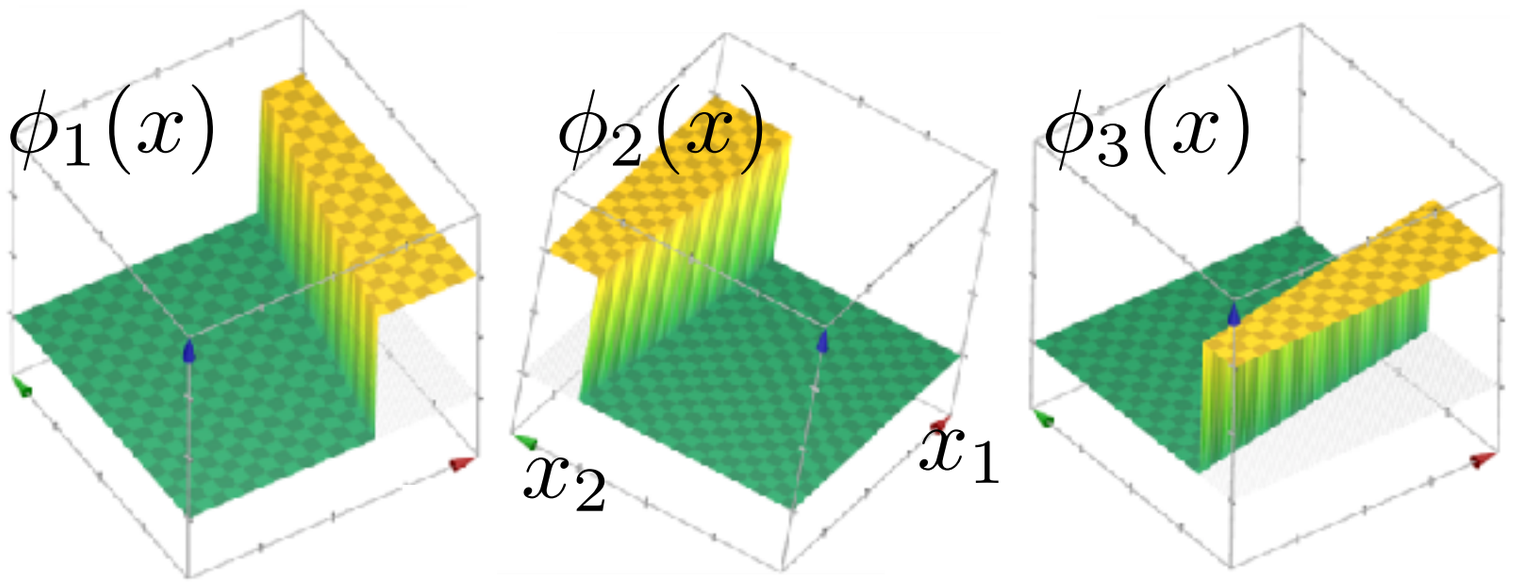

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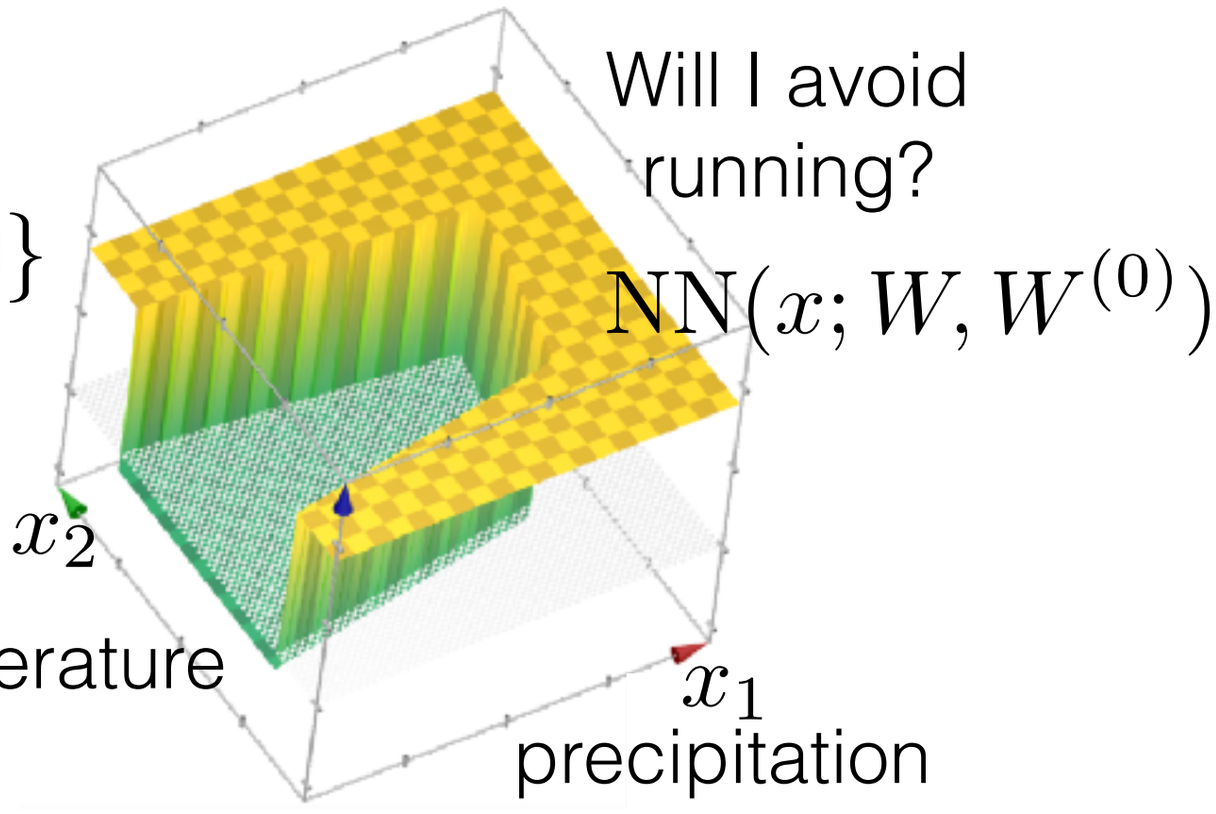

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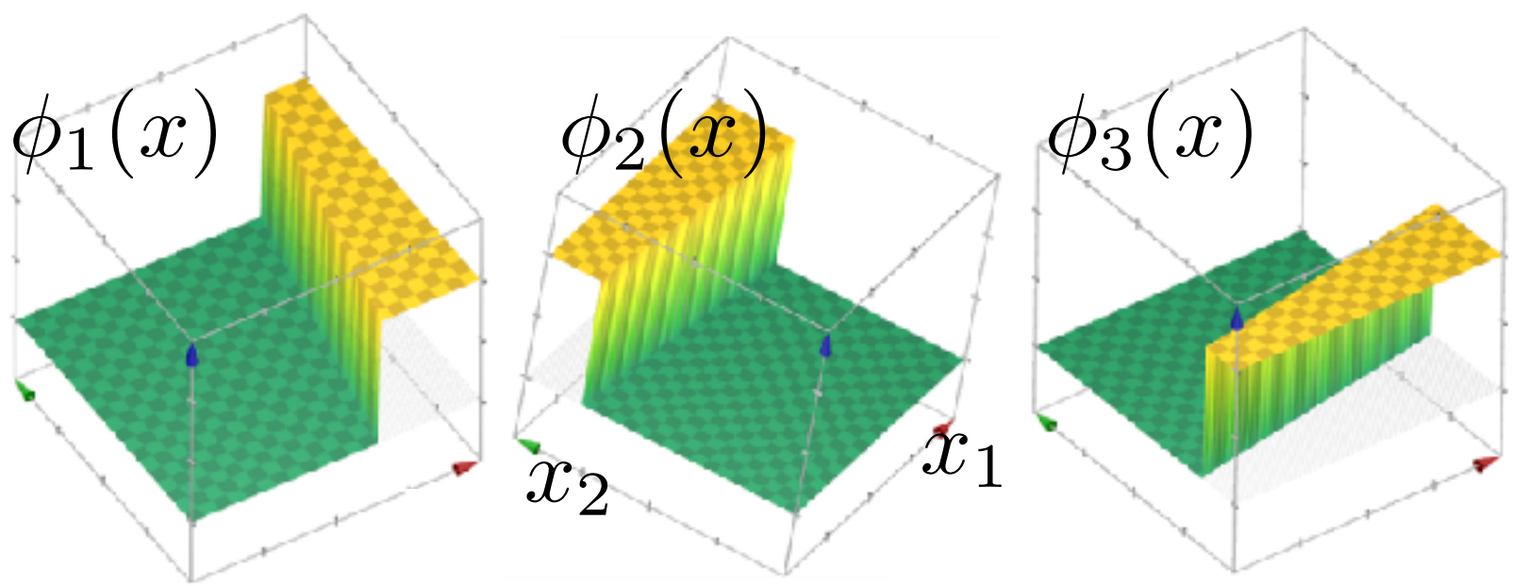

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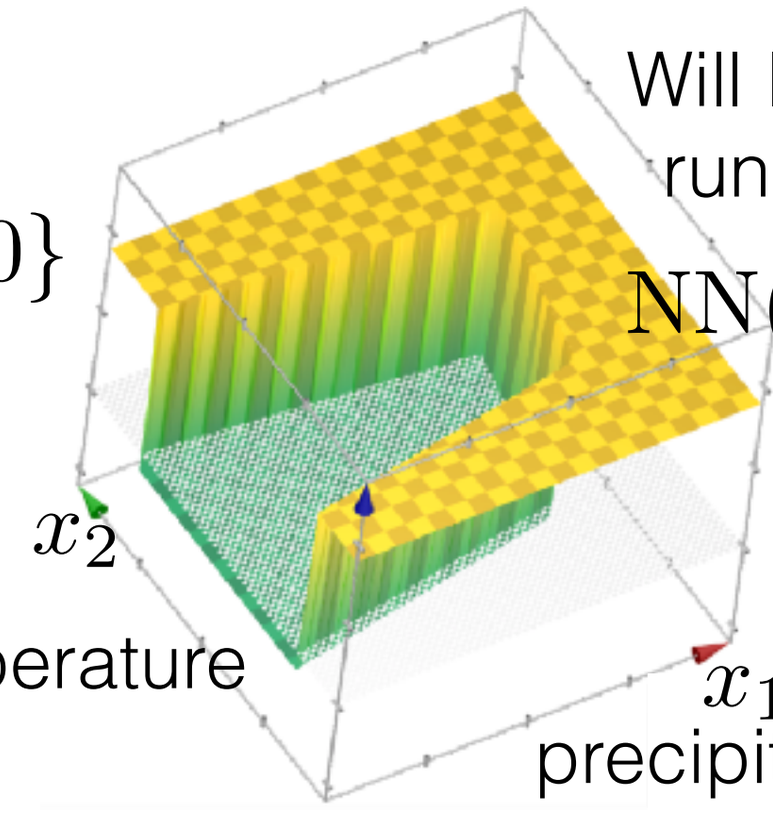

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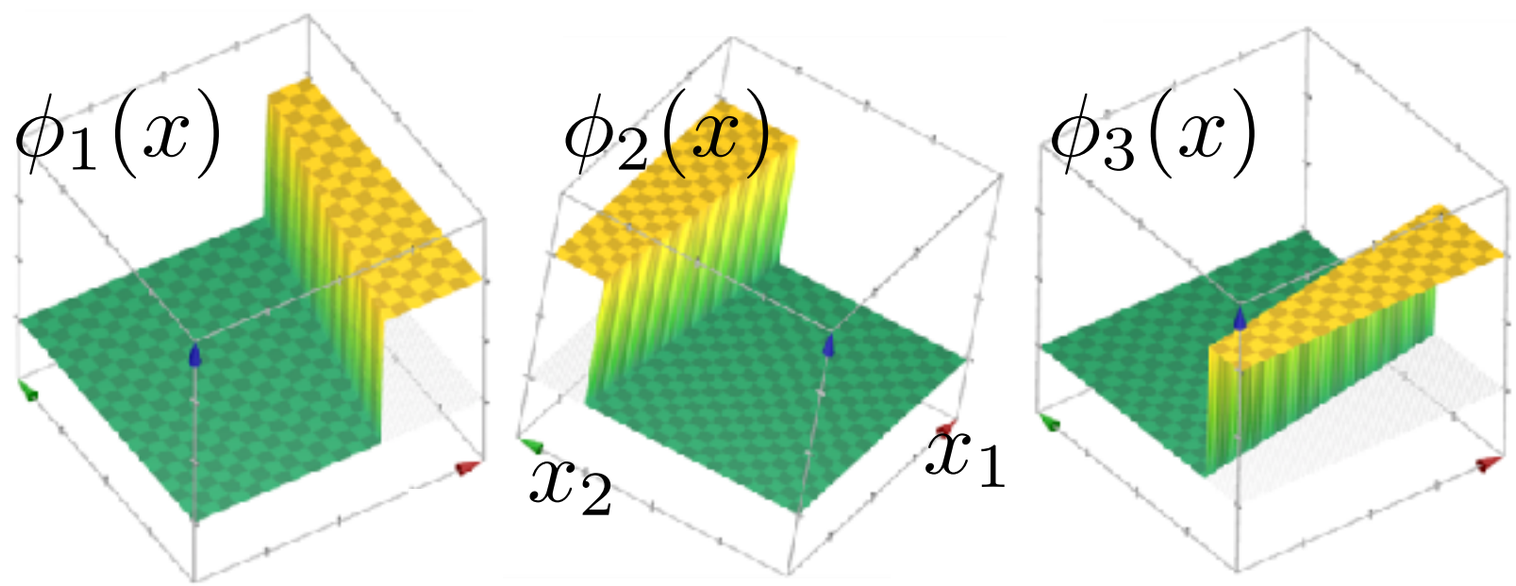

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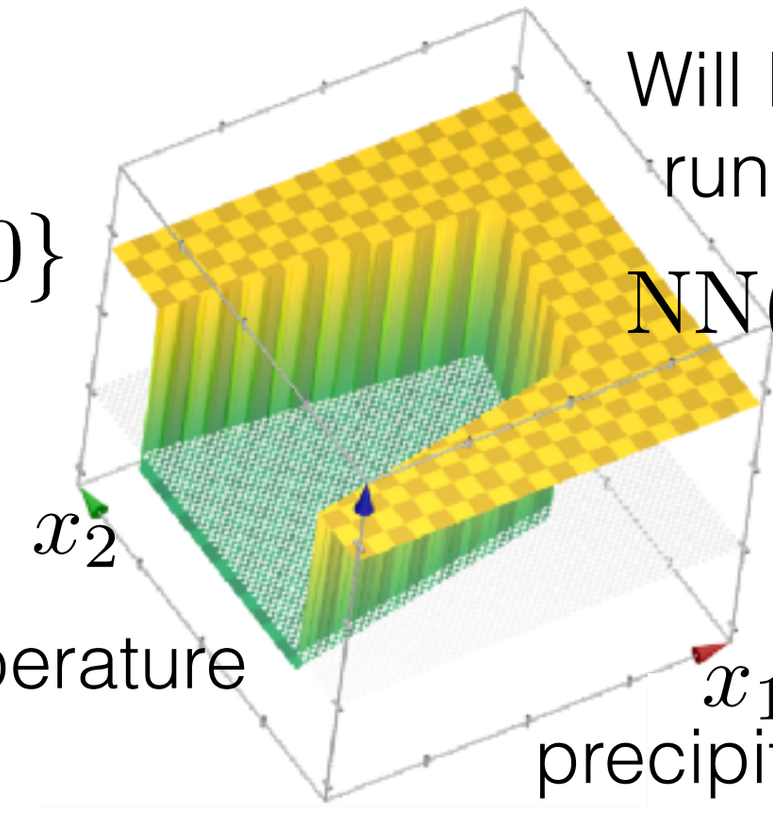

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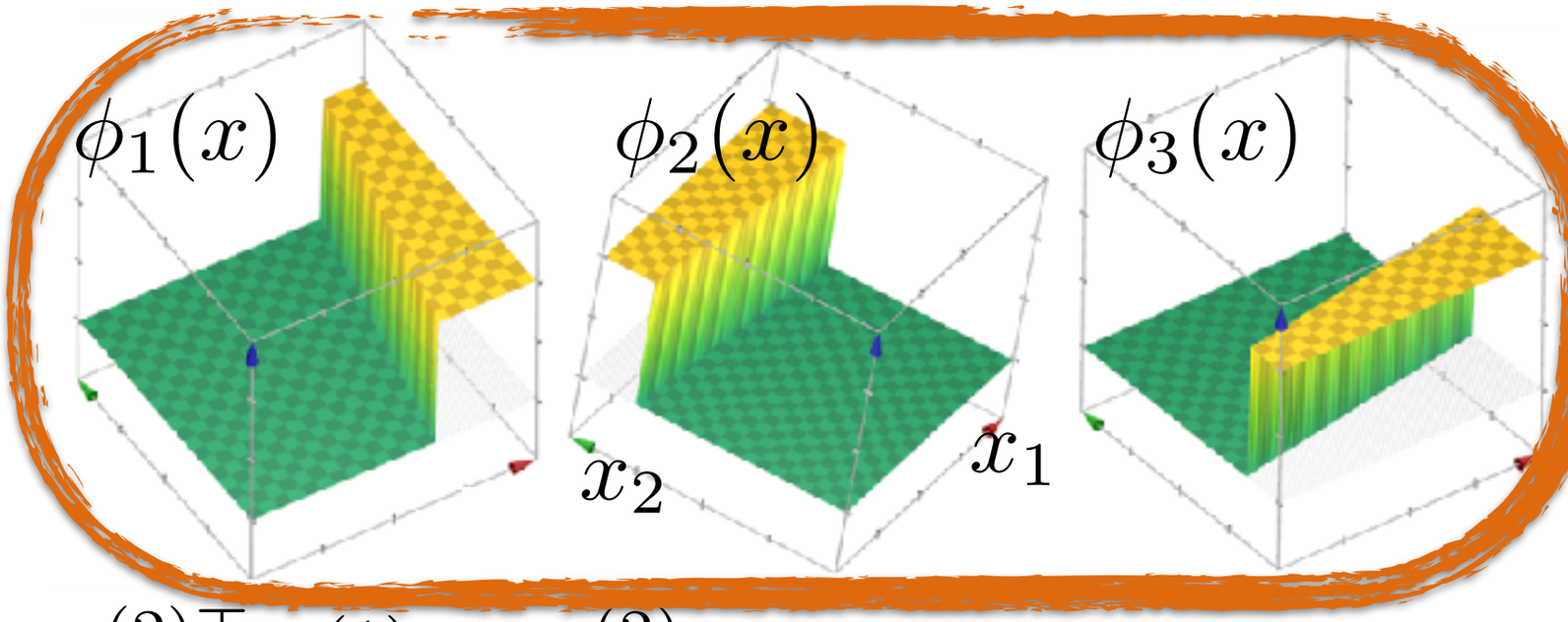

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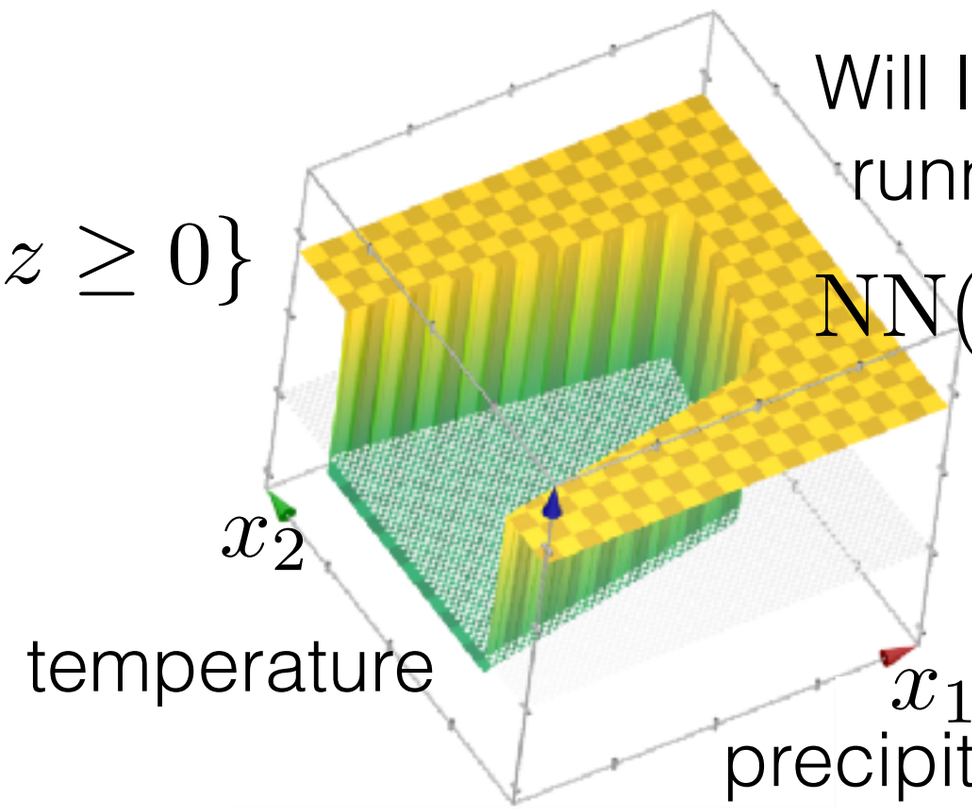

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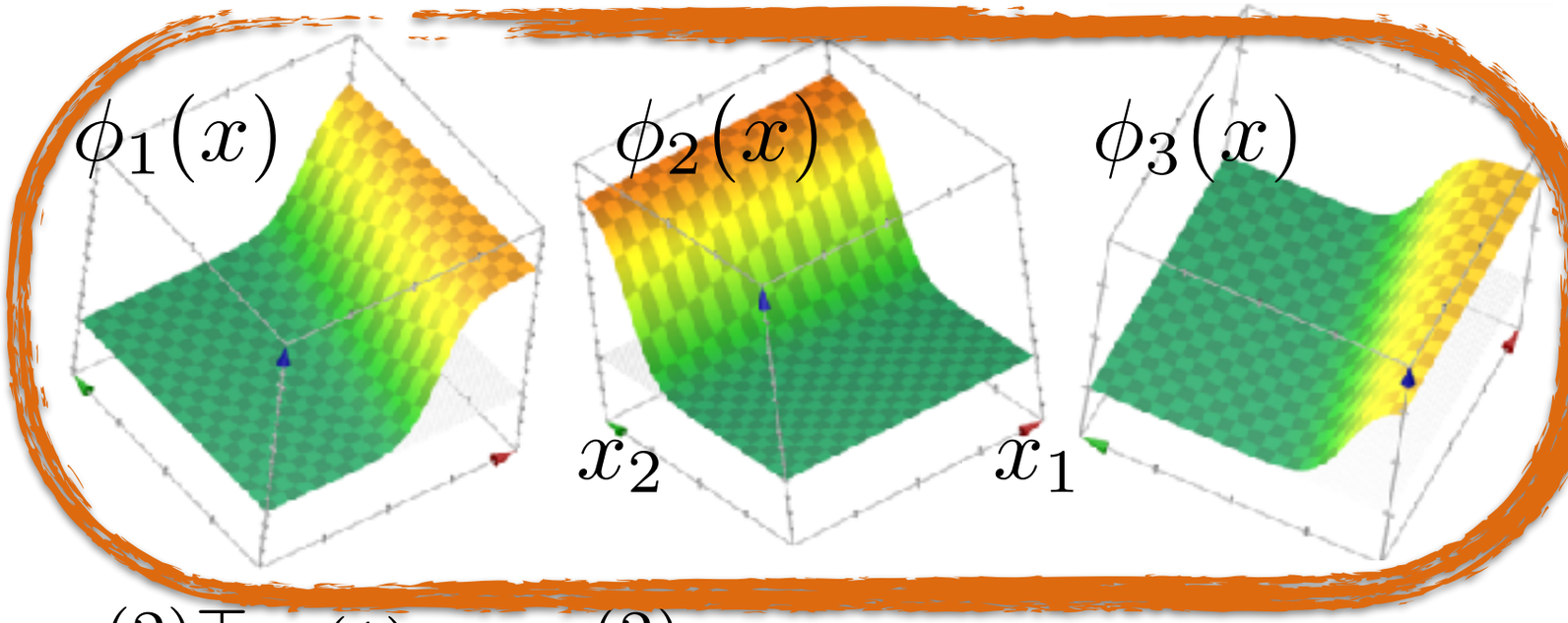

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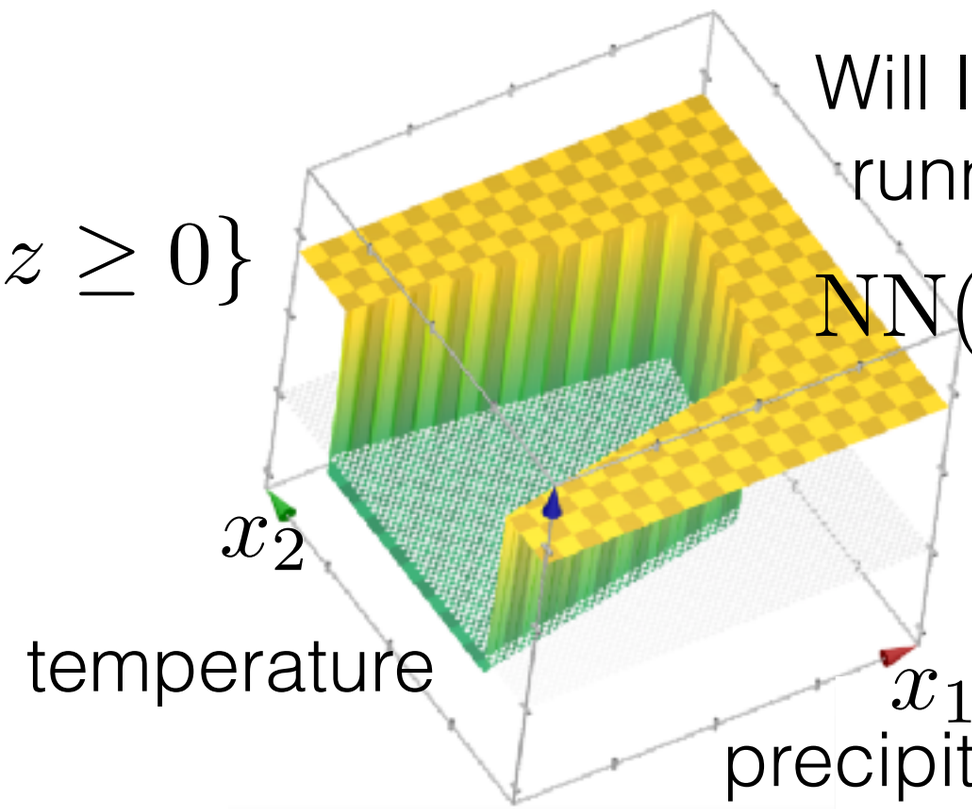

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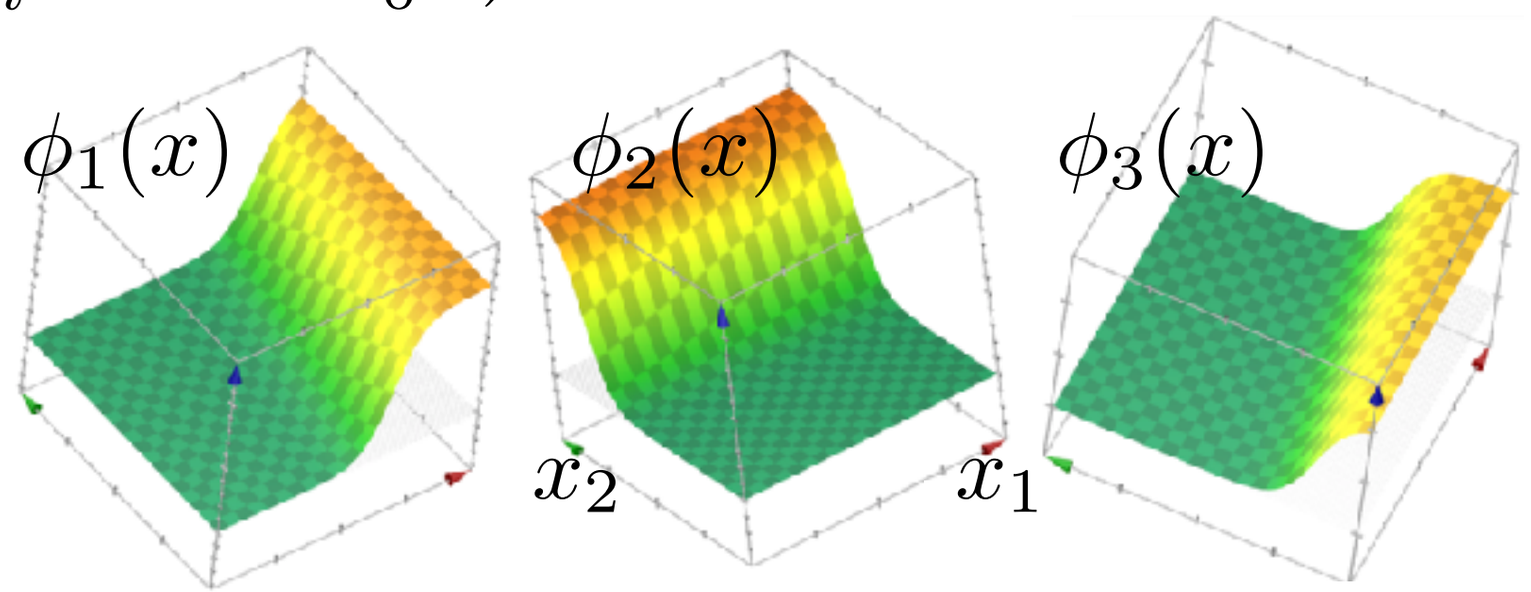

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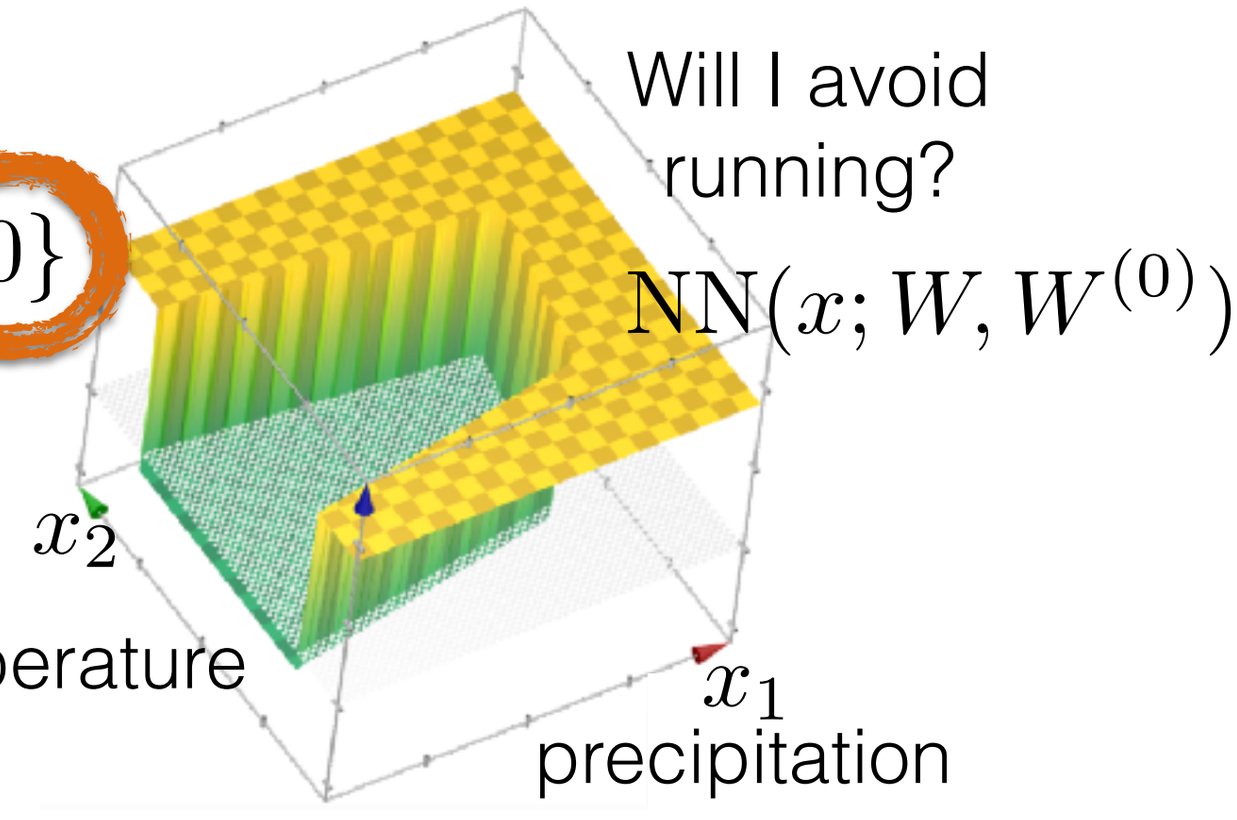

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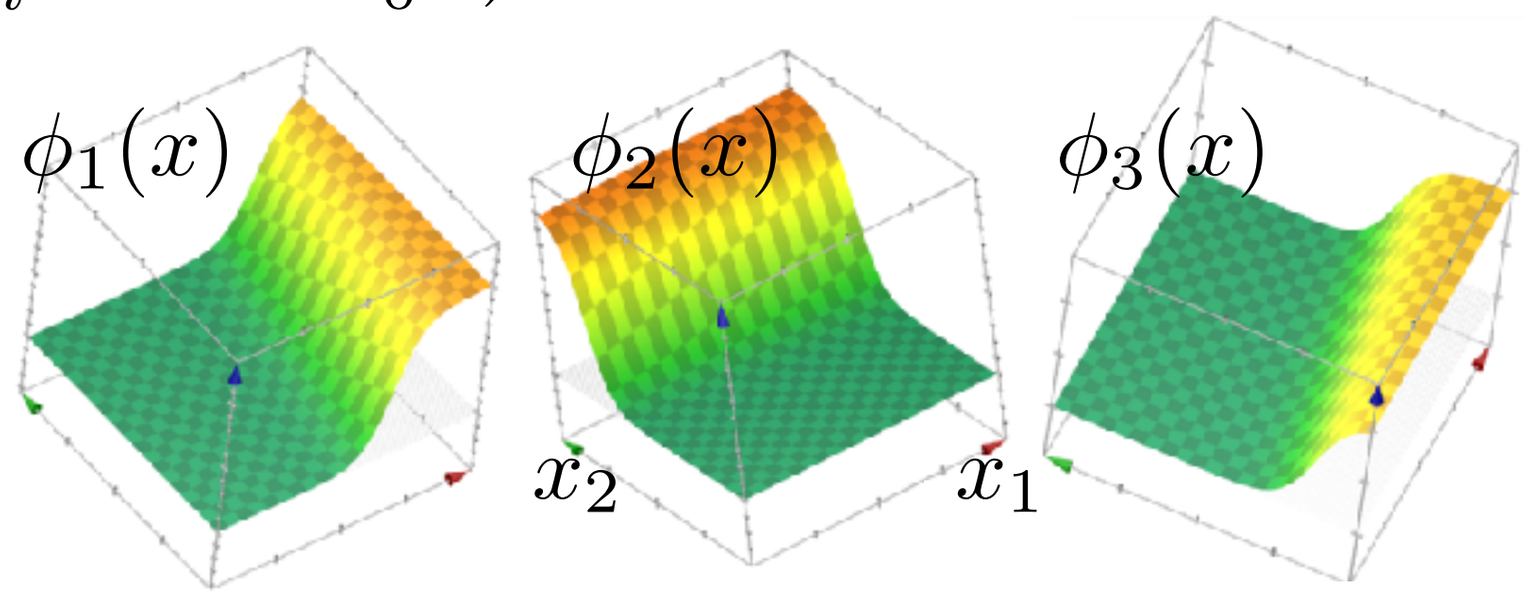

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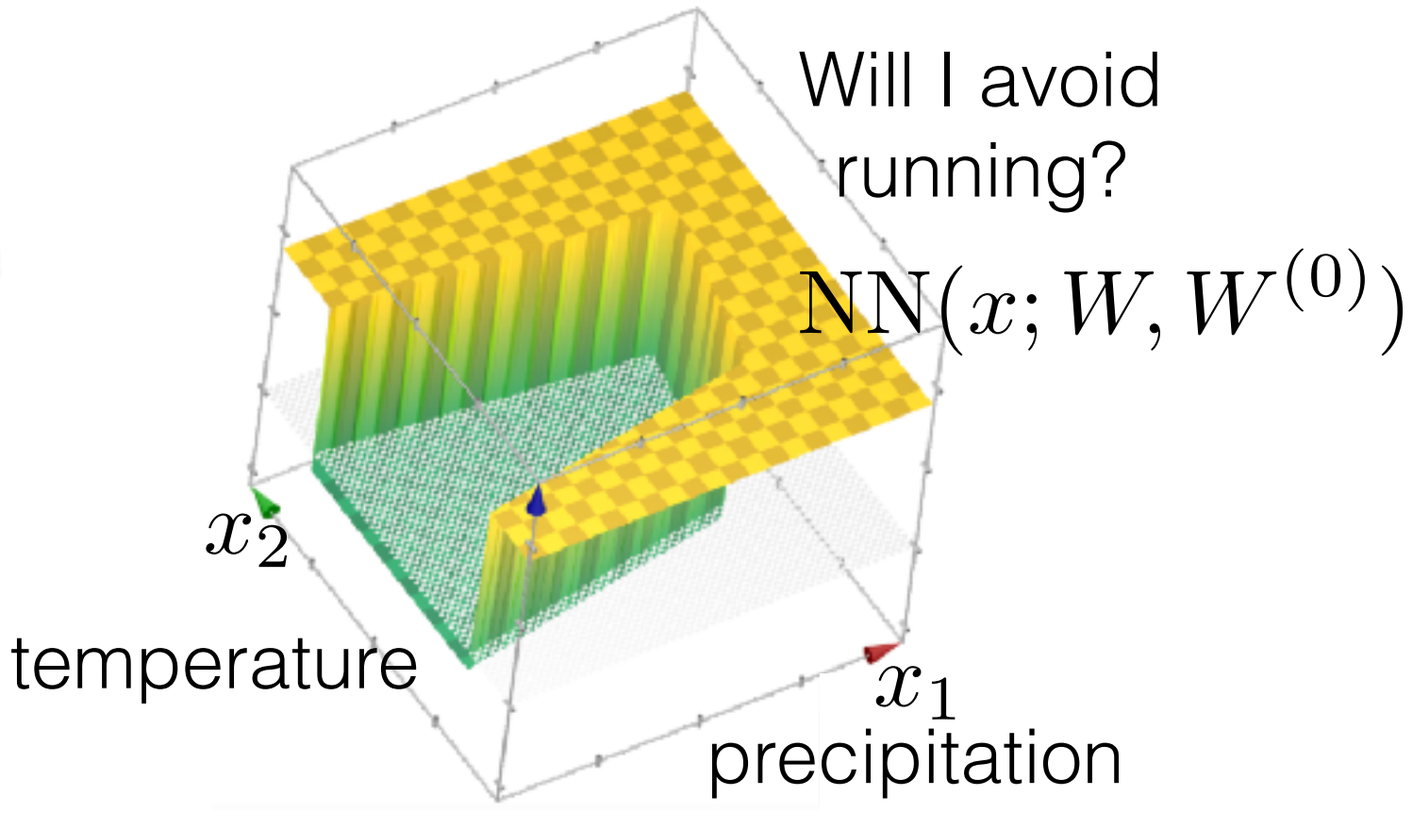

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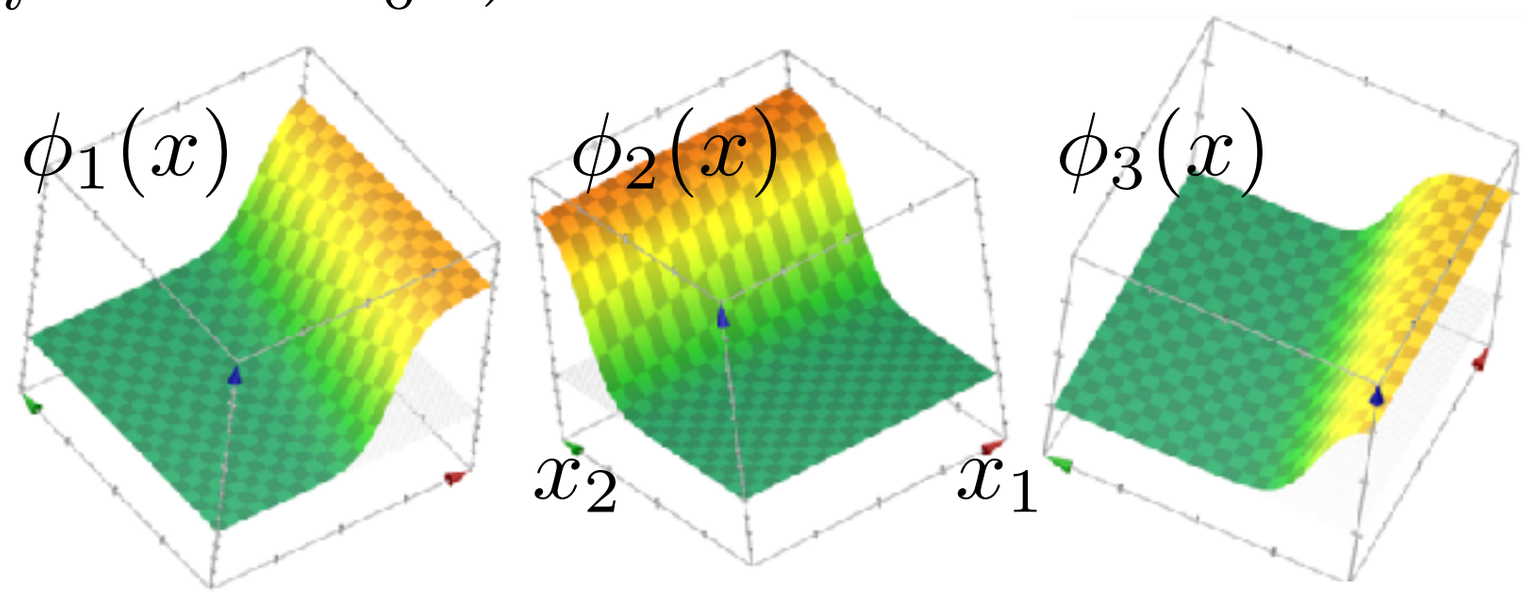

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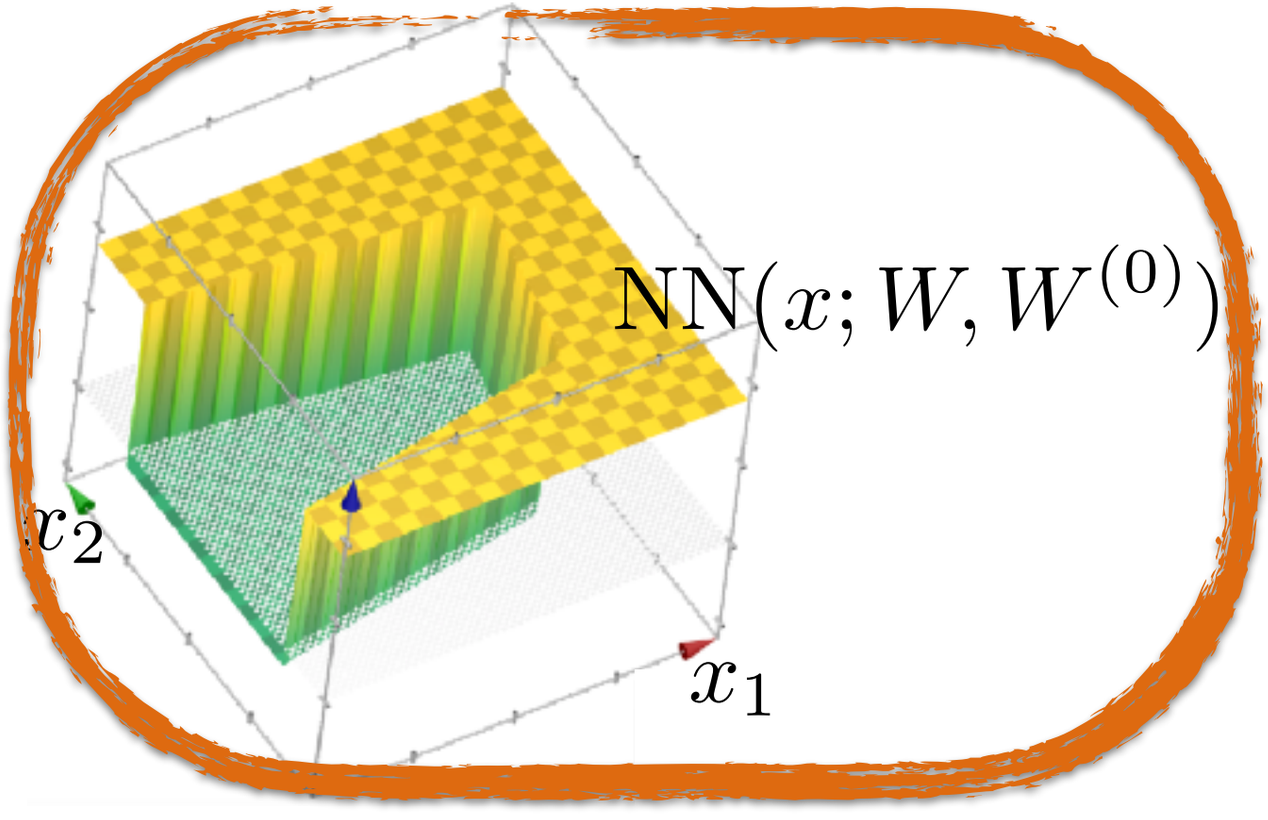

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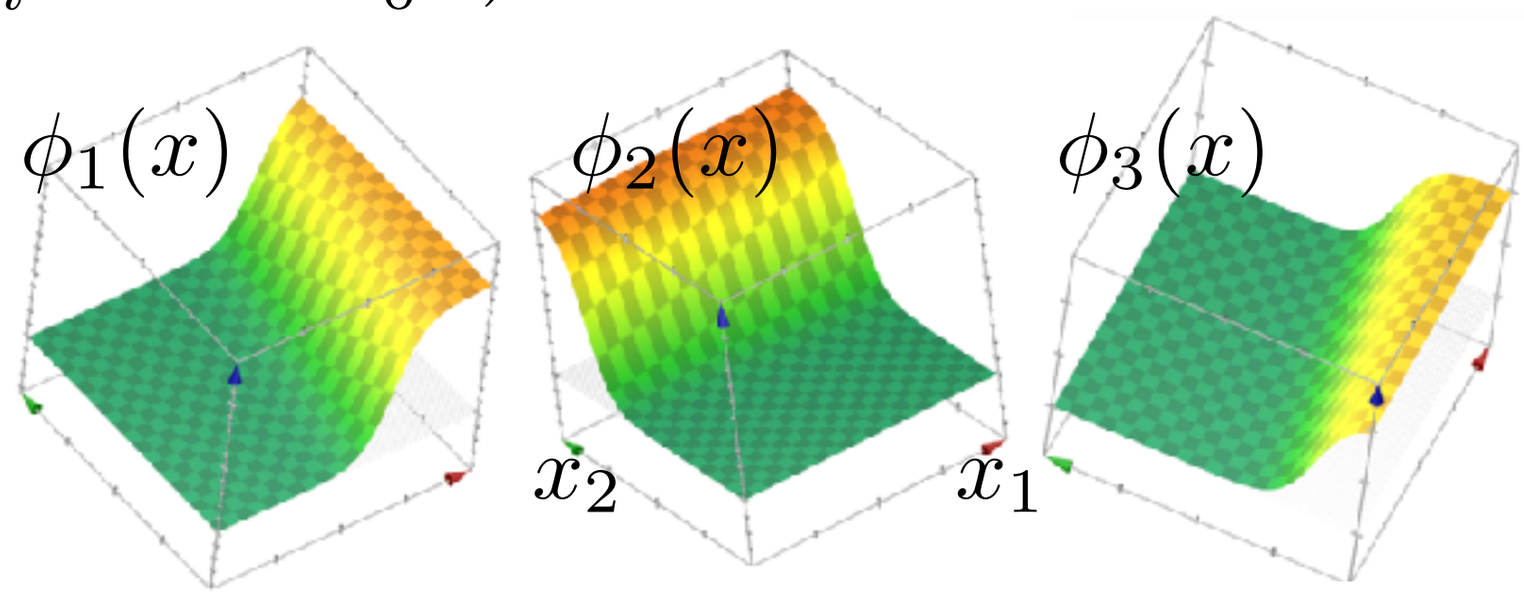

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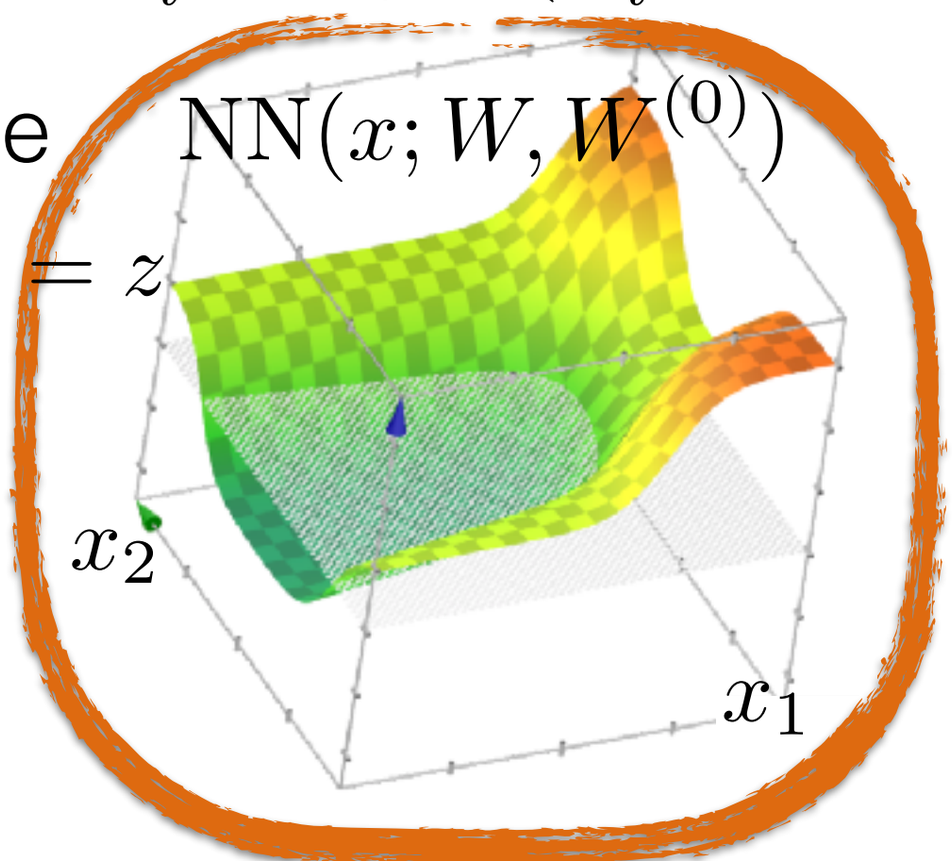

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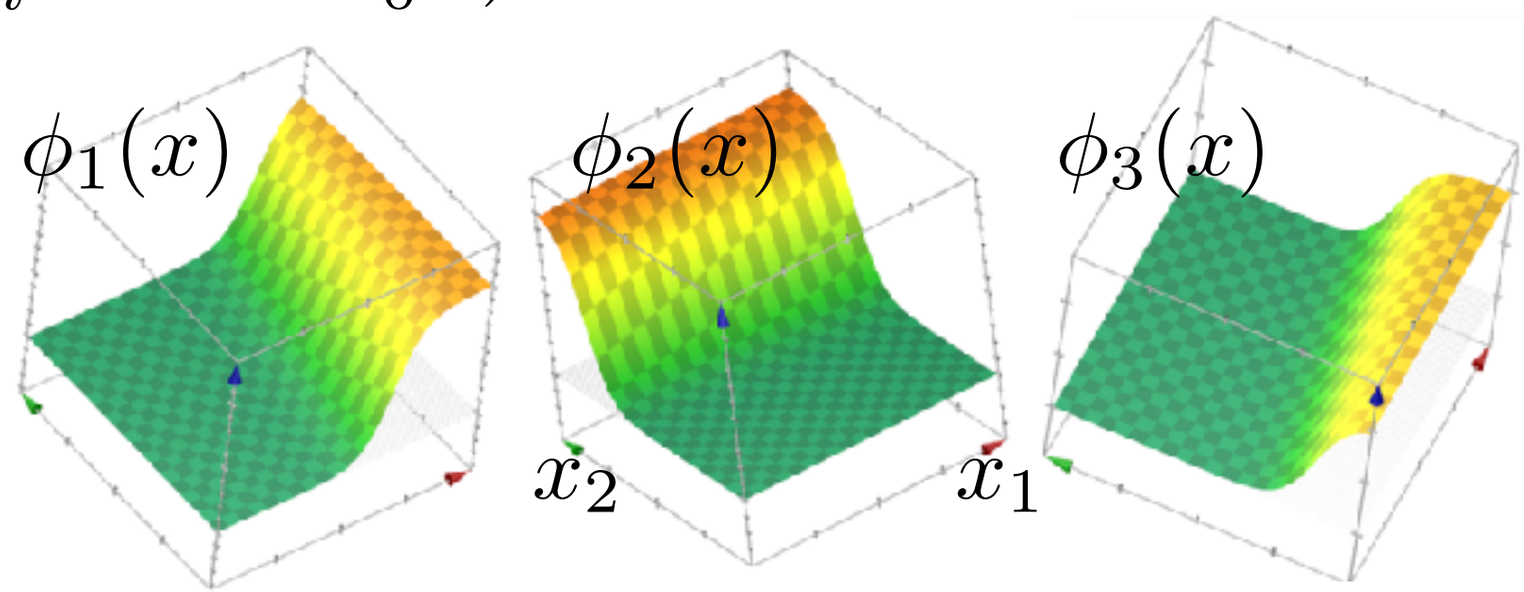

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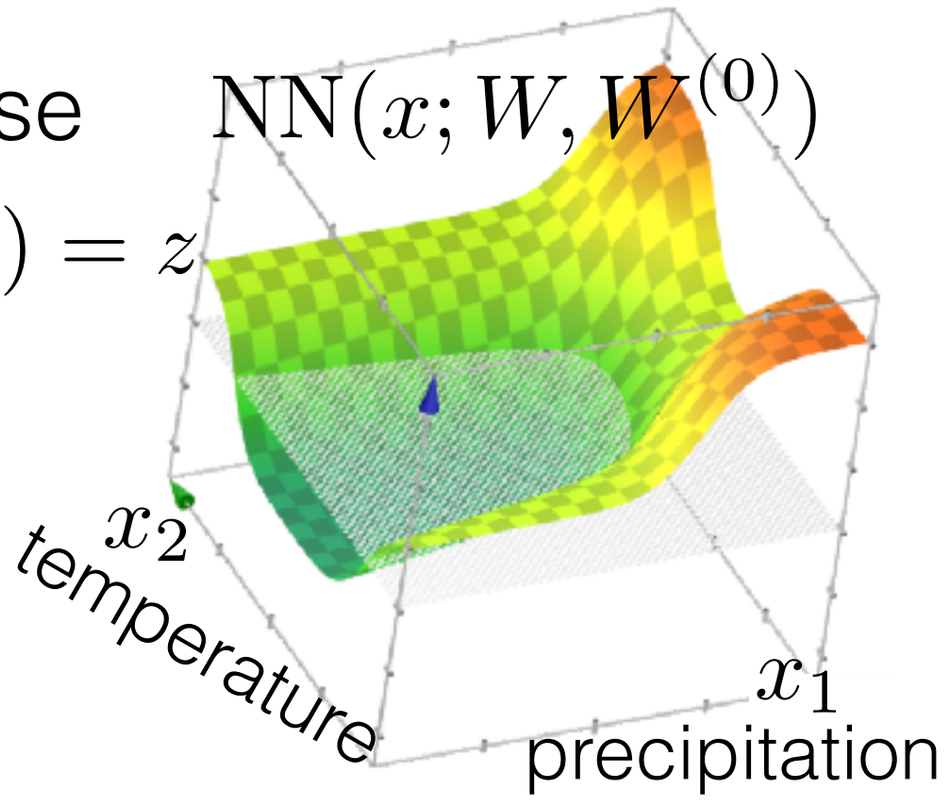

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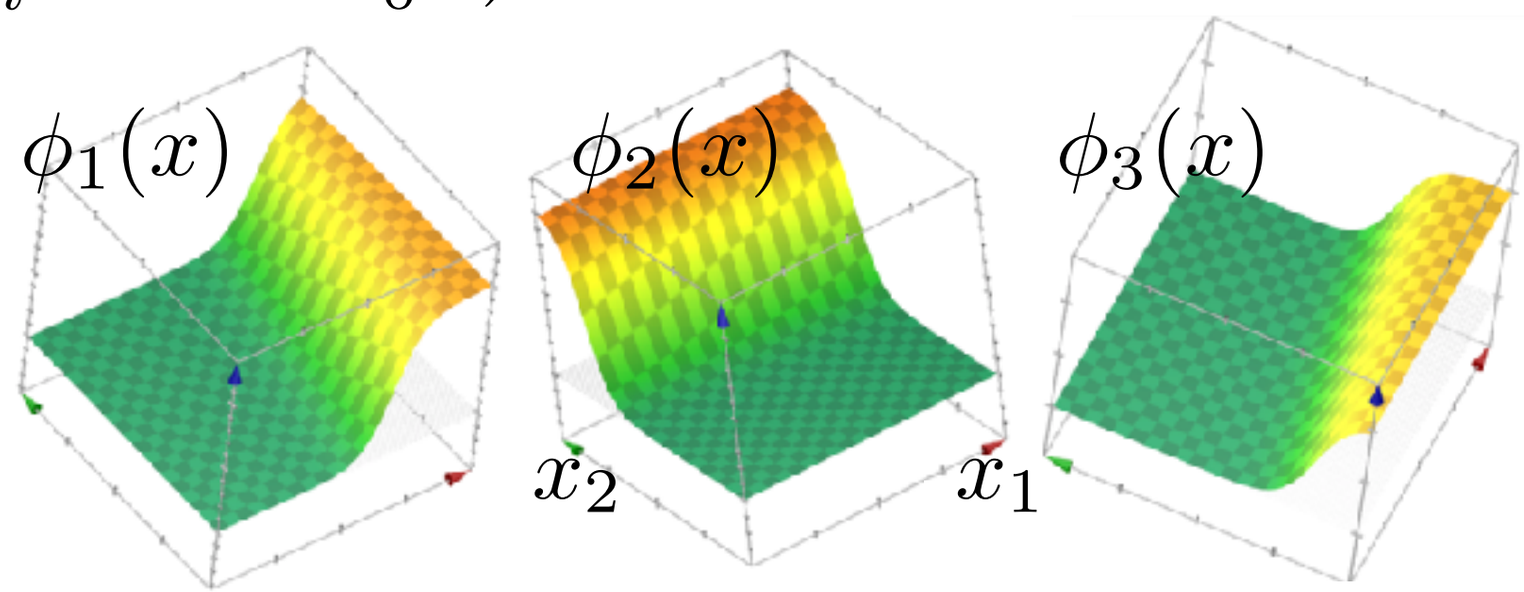

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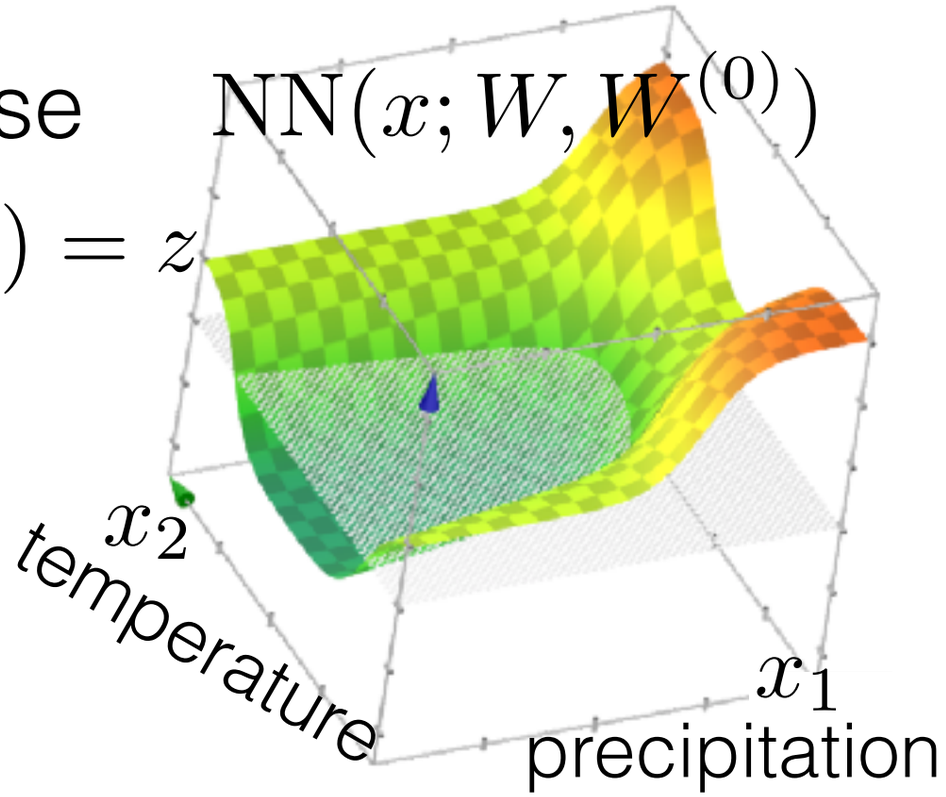
$\mathrm{NN}(x; W, W^{(0)})$

How much will I dislike running today?



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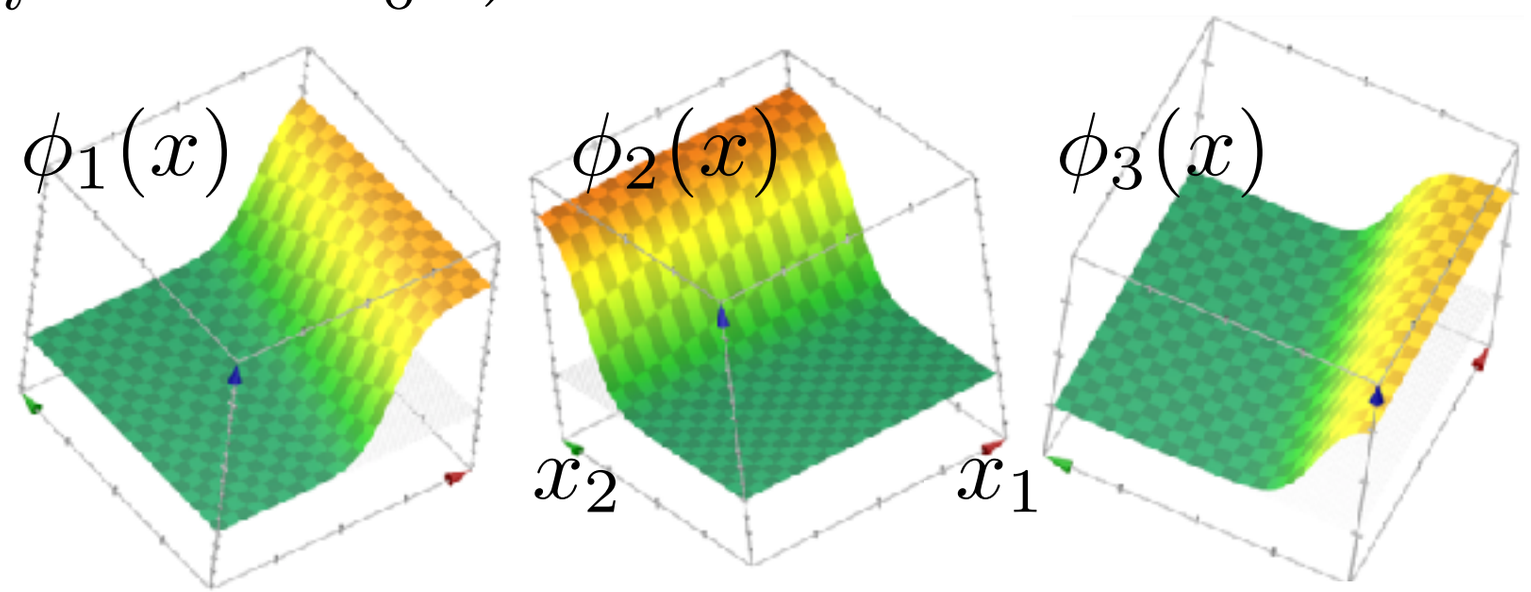

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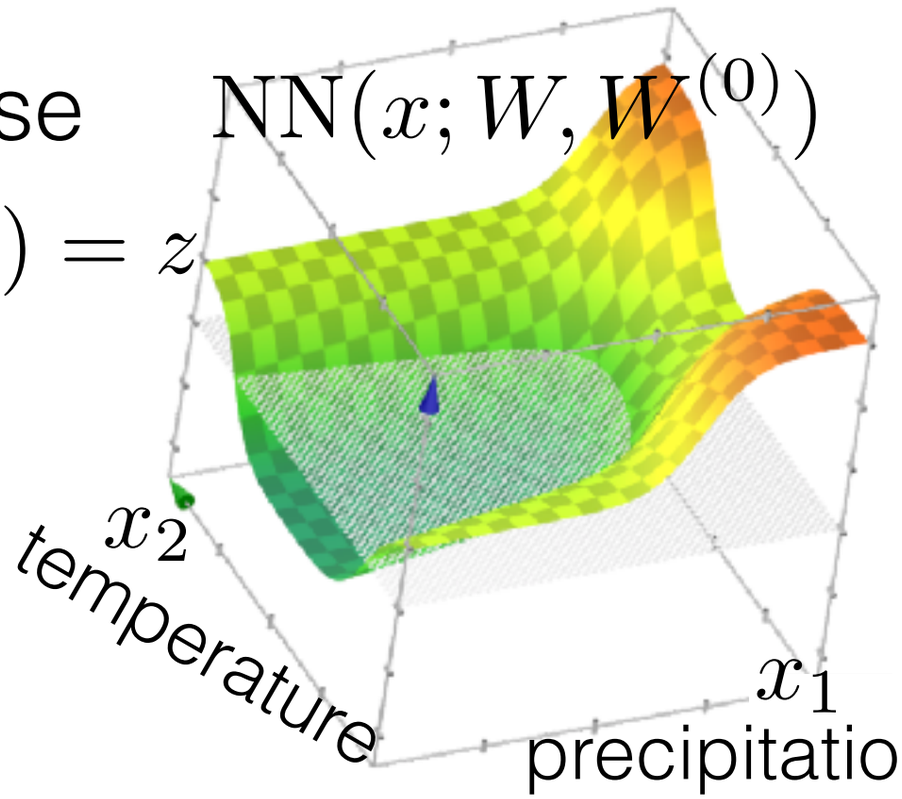

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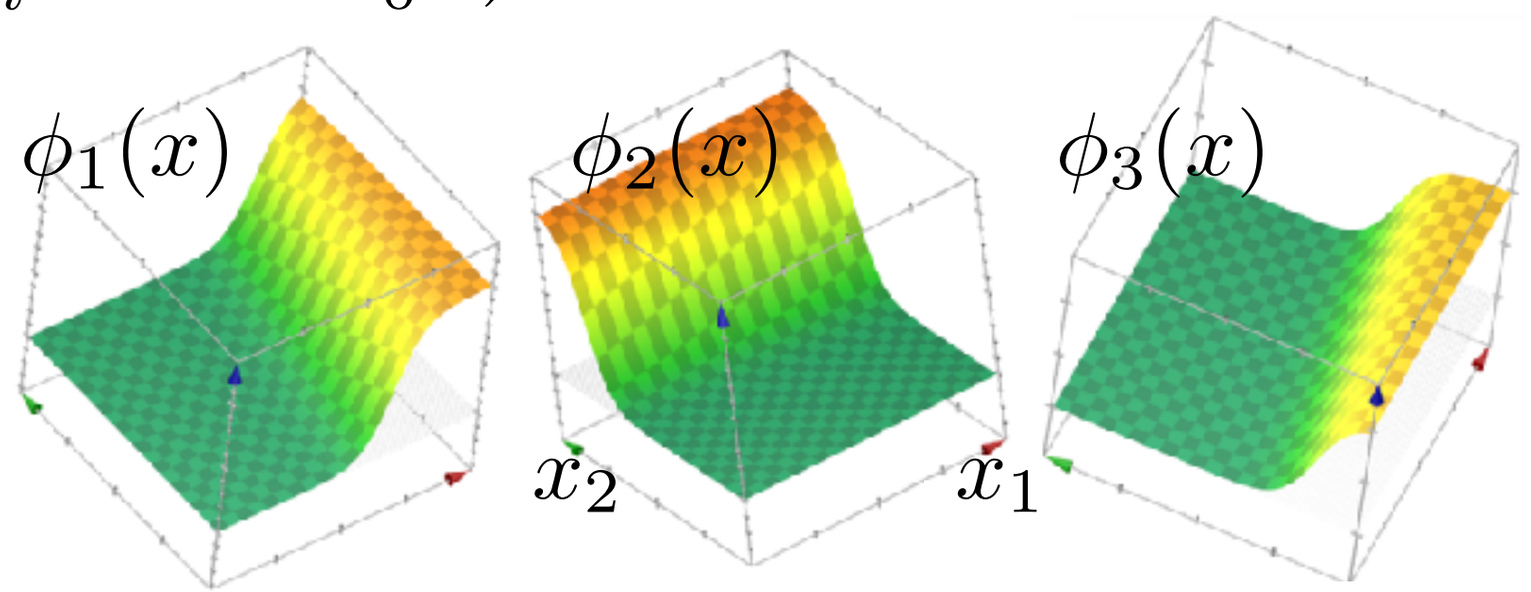

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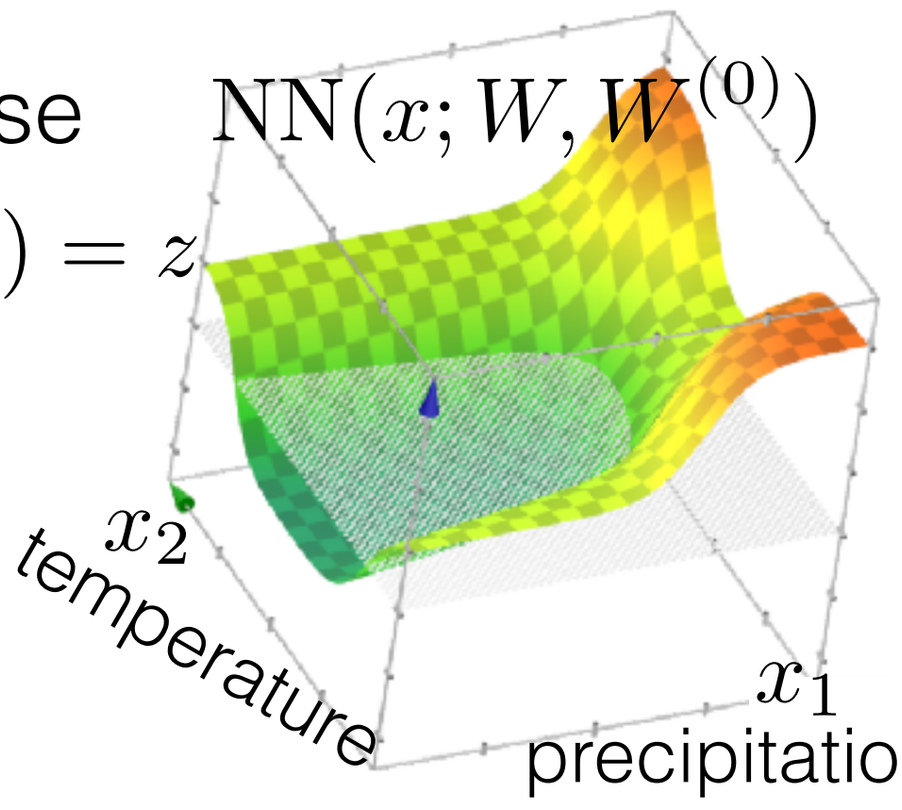

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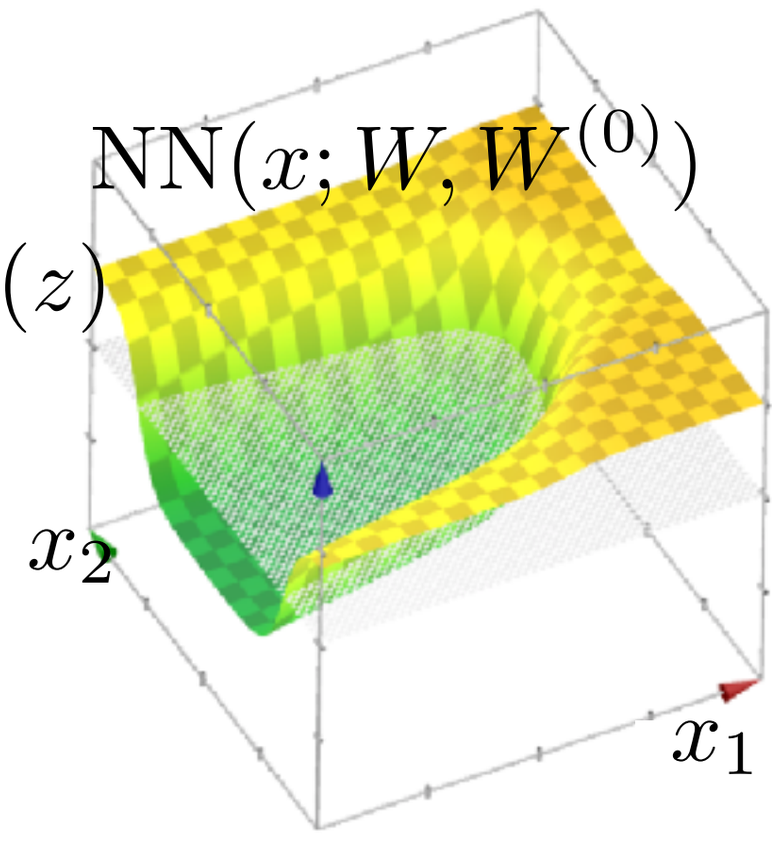

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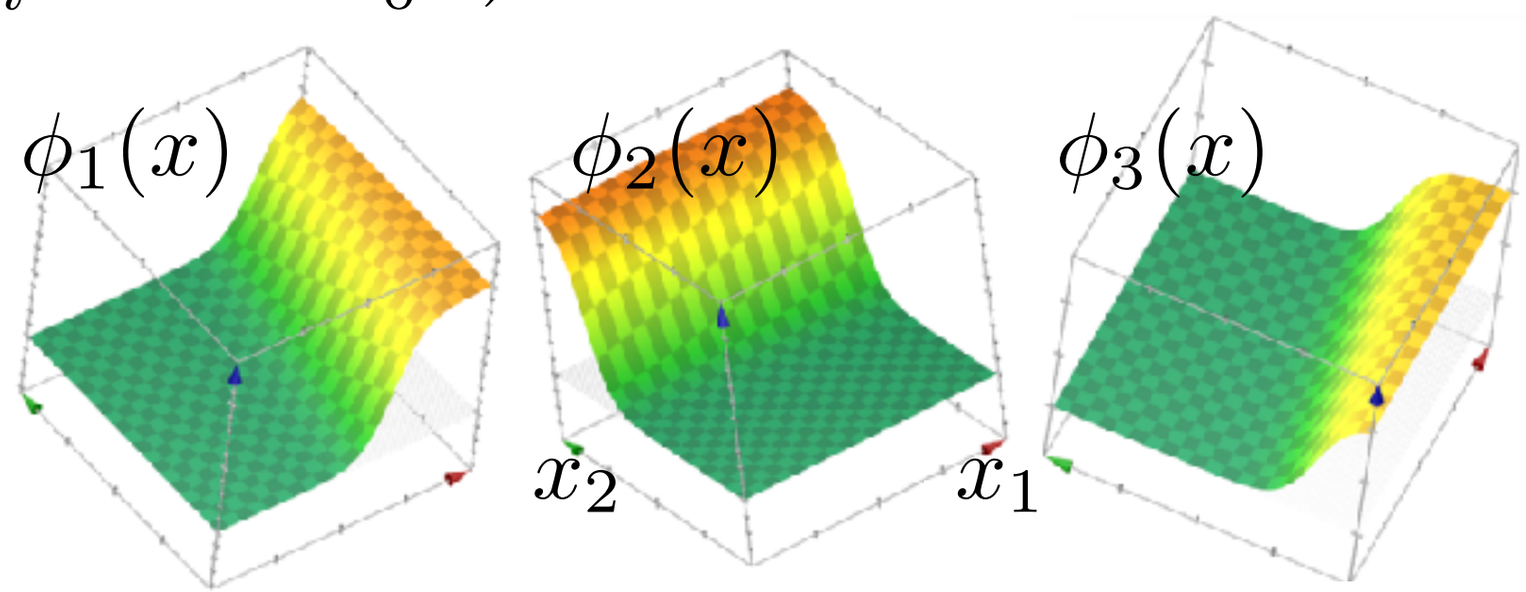

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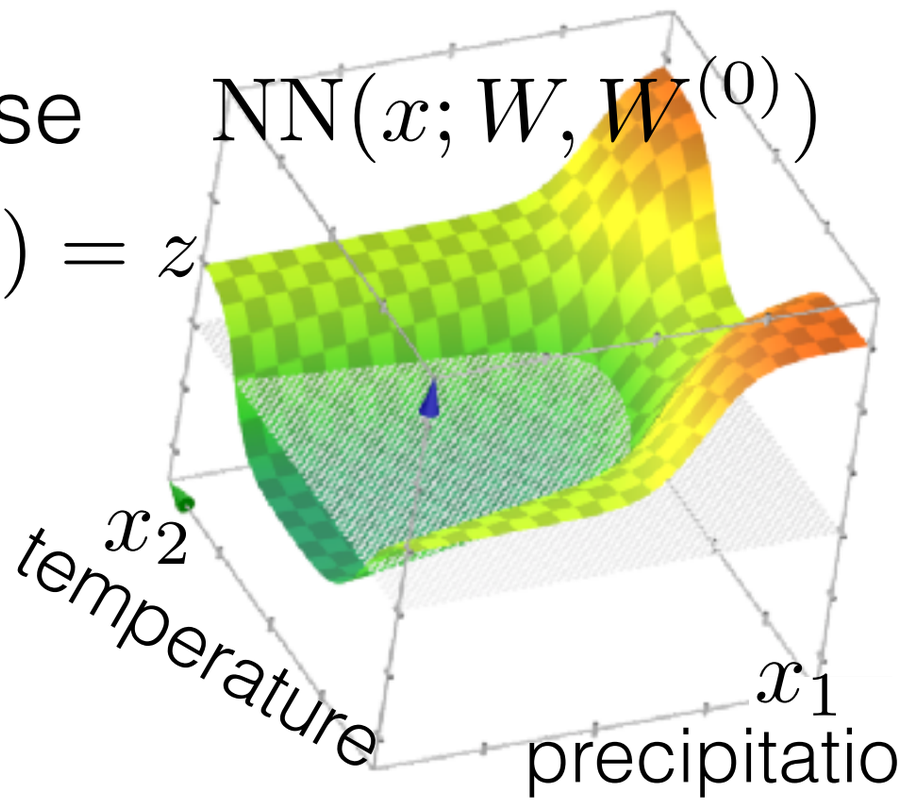

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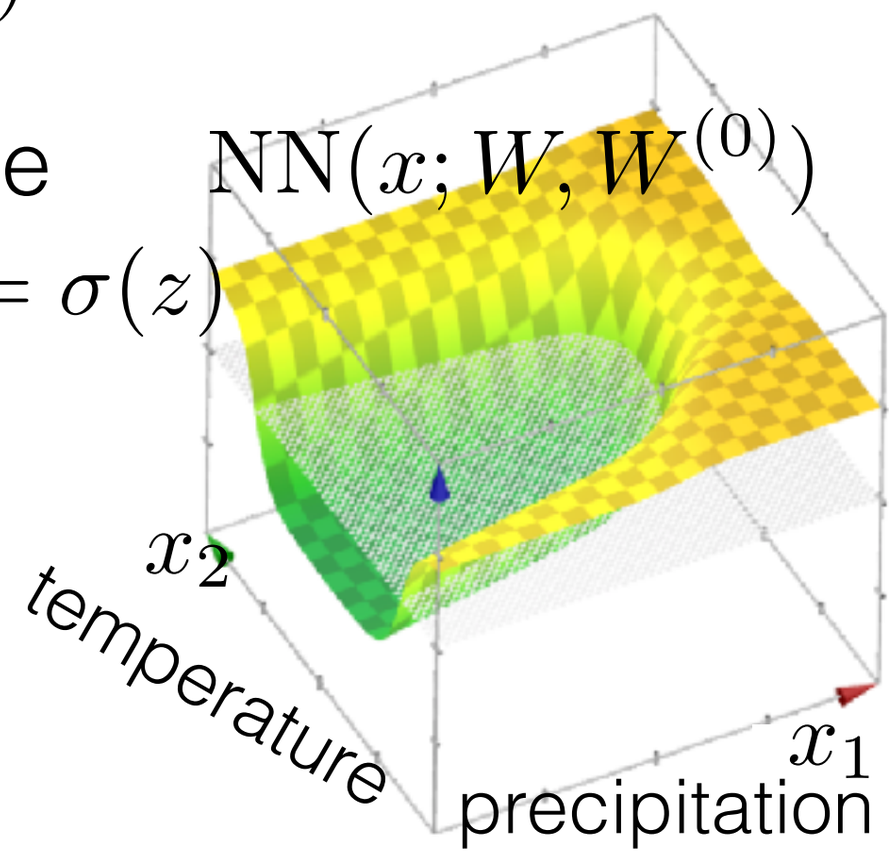

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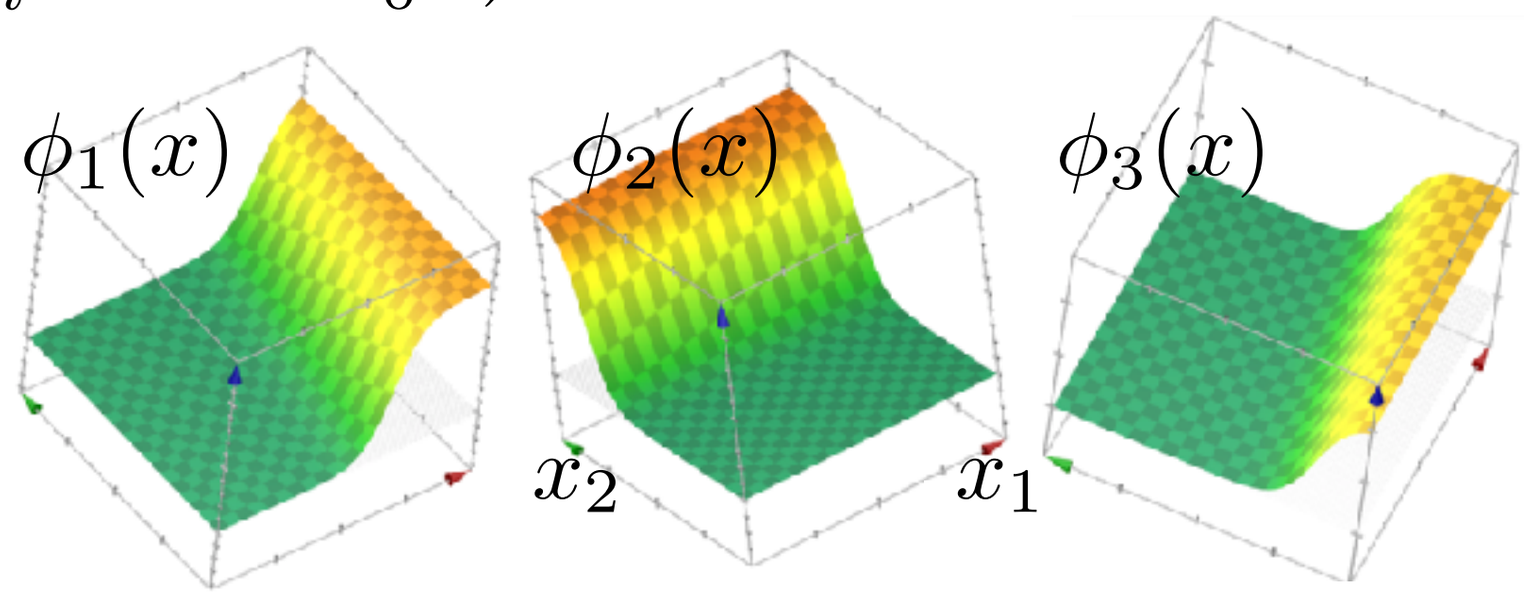

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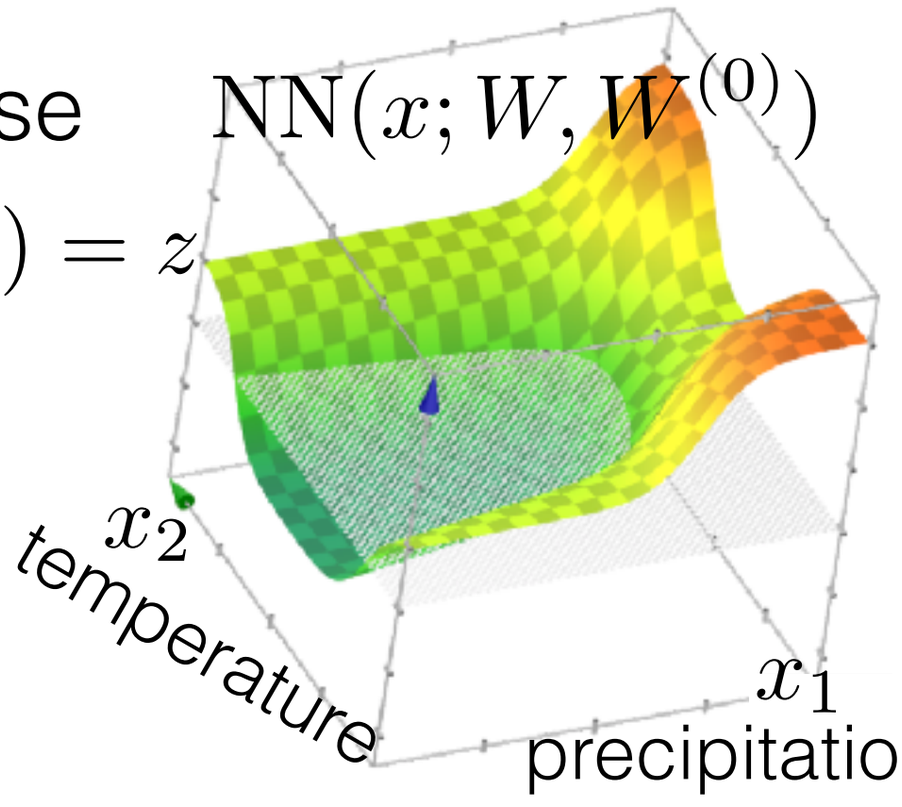

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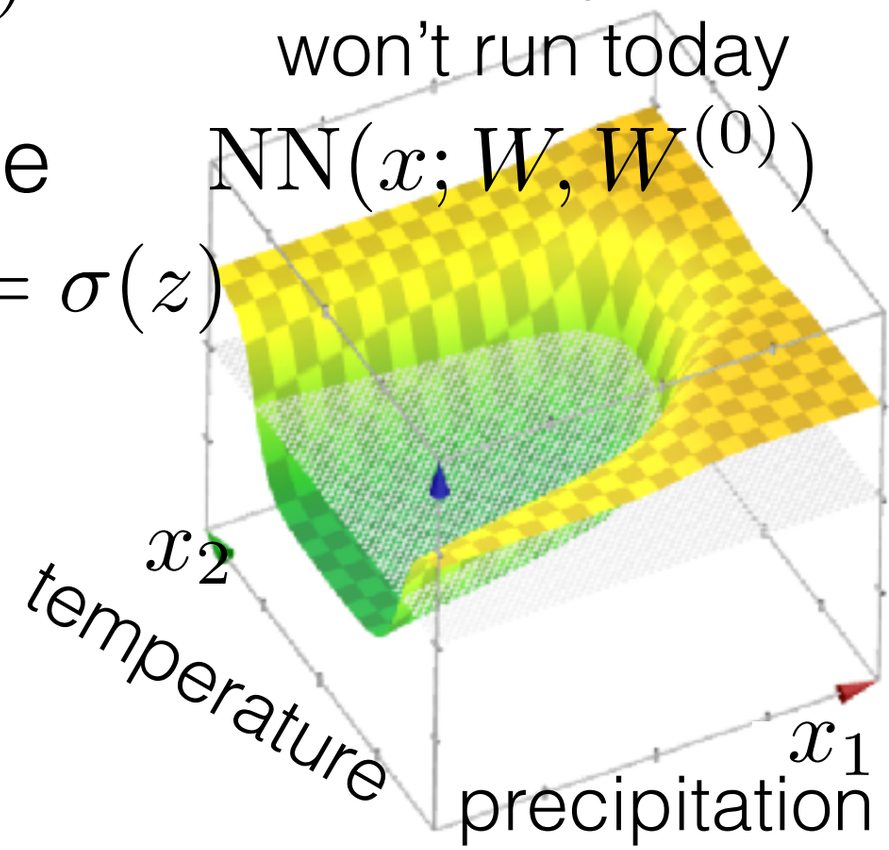
Probability that I won't run today



Learning the parameters

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Exercise: check that when these conventions overlap, they agree

Taking derivatives for SGD

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- Example 1-layer NN

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Notice: order of the terms

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Exercise: check that you can't just switch the order and get the same result

Taking derivatives for SGD

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Taking derivatives for SGD

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  - data dimension $m^{(1)} = 4$; # outputs $n^{(1)} = 3$

$$A^{(1)} = \underbrace{W^{(1)^\top}}_{4\text{x}3} x + \underbrace{W_0^{(1)}}_{3\text{x}1}$$

3x1 4x1

1x1 $$\text{loss} = L(A^{(1)}, y) = \sum_{k=1}^{3} (A_k^{(1)} - y_k)^2$$

- Part of SGD step *t*: randomly choose (*x*,*y*) and update:

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$$W_{0,j}^{(1)} \leftarrow W_{0,j}^{(1)} - \eta(t) \frac{\partial \text{loss}}{\partial W_{0,j}^{(1)}}$$

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notice: we're dropping *i*

all values on this side are from step *t*-1

- Use the scalar chain rule from calculus:

$$\underset{1\text{x}1}{\frac{\partial \text{loss}}{\partial W_{0,j}^{(1)}}} = \sum_{k=1}^{3} \underset{1\text{x}1}{\frac{\partial \text{loss}}{\partial A_k^{(1)}}} \underset{1\text{x}1}{\frac{\partial A_k^{(1)}}{\partial W_{0,j}^{(1)}}} = \sum_{k=1}^{3} \frac{\partial A_k^{(1)}}{\partial W_{0,j}^{(1)}} \frac{\partial \text{loss}}{\partial A_k^{(1)}}$$

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Exercise: compute this matrix

$$\frac{\partial \text{loss}}{\partial W_{0,j}^{(1)}} = \frac{\partial A^{(1)}}{\partial W_{0,j}^{(1)}} \frac{\partial \text{loss}}{\partial A^{(1)}} \qquad \frac{\partial \text{loss}}{\partial W_0^{(1)}} = \frac{\partial A^{(1)}}{\partial W_0^{(1)}} \frac{\partial \text{loss}}{\partial A^{(1)}}$$

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Notice: We can't just substitute the weight **matrix** in the final formula. (Why not?)

$$\frac{\partial \text{loss}}{\partial W_0^{(1)}} = \frac{\partial A^{(1)}}{\partial W_0^{(1)}} \frac{\partial \text{loss}}{\partial A^{(1)}}$$

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Taking derivatives for SGD

- Example 1-layer NN
  - data dimension $m^{(1)} = 4$; # outputs $n^{(1)} = 3$

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4x3

Taking derivatives for SGD

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Taking derivatives for SGD

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- So: $$\underset{4\text{x}3}{\frac{\partial \text{loss}}{\partial W^{(1)}}} = \underset{4\text{x}1}{x} \underbrace{\left(\frac{\partial \text{loss}}{\partial A^{(1)}} \right)^\top}_{3\text{x}1}$$

Taking derivatives for SGD

Taking derivatives for SGD

- General case: NN with *L* layers

Taking derivatives for SGD

- General case: NN with *L* layers
  - Layer ℓ has $m^{(l)}$ inputs

Taking derivatives for SGD

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  - Layer ℓ has $m^{(l)}$ inputs and $n^{(l)} = m^{(l+1)}$ outputs

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Taking derivatives for SGD

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$$\frac{\partial \text{loss}}{\partial W_{j,k}^{(\ell)}} =$$

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1

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1x1 1x1 1x1 1x1 1x1 $m^{\ell} \times n^{\ell}$

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1 $m^{\ell} \times n^{\ell}$ $m^{\ell} \times 1$

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1x1 1x1 1x1 1x1 1x1 $m^{\ell} \times n^{\ell}$ $m^{\ell} \times 1$ $1 \times n^{\ell}$

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1x1 1x1 1x1 1x1 1x1 m^{ℓ}xn^{ℓ} m^{ℓ}x1 1xn^{ℓ}

  - It remains to find $\partial \text{loss}/\partial Z^{(\ell)}$

$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} =$$

n^{ℓ}x1

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1 m^{ℓ}xn^{ℓ} m^{ℓ}x1 1xn^{ℓ}

  - It remains to find $\partial \text{loss}/\partial Z^{(\ell)}$

$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} = \qquad \frac{\partial \text{loss}}{\partial A^{(L)}}$$

n^{ℓ}x1 n^{L}x1

Taking derivatives for SGD

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$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} = \cdot \frac{\partial A^{(L)}}{\partial Z^{(L)}} \frac{\partial \text{loss}}{\partial A^{(L)}}$$

n^{ℓ}x1 n^{L}xn^{L} n^{L}x1

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1 m^{ℓ}xn^{ℓ} m^{ℓ}x1 1xn^{ℓ}

  - It remains to find $\partial \text{loss}/\partial Z^{(\ell)}$

$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} = \frac{\partial Z^{(L)}}{\partial A^{(L-1)}} \frac{\partial A^{(L)}}{\partial Z^{(L)}} \frac{\partial \text{loss}}{\partial A^{(L)}}$$

n^{ℓ}x1 n^{L-1}xn^{L} n^{L}xn^{L} n^{L}x1

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1 m^{ℓ}xn^{ℓ} m^{ℓ}x1 1xn^{ℓ}

  - It remains to find $\partial \text{loss}/\partial Z^{(\ell)}$

$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} = \frac{\partial A^{(L-1)}}{\partial Z^{(L-1)}} \frac{\partial Z^{(L)}}{\partial A^{(L-1)}} \frac{\partial A^{(L)}}{\partial Z^{(L)}} \frac{\partial \text{loss}}{\partial A^{(L)}}$$

n^{ℓ}x1 n^{L-1}xn^{L-1} n^{L-1}xn^{L} n^{L}xn^{L} n^{L}x1

Taking derivatives for SGD

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1x1 1x1 1x1 1x1 1x1 $m^{\ell} \times n^{\ell}$ $m^{\ell} \times 1$ $1 \times n^{\ell}$

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$$\frac{\partial \text{loss}}{\partial Z^{(\ell)}} = \frac{\partial A^{(\ell+1)}}{\partial Z^{(\ell+1)}} \cdots \frac{\partial A^{(L-1)}}{\partial Z^{(L-1)}} \frac{\partial Z^{(L)}}{\partial A^{(L-1)}} \frac{\partial A^{(L)}}{\partial Z^{(L)}} \frac{\partial \text{loss}}{\partial A^{(L)}}$$

$n^{\ell} \times 1$ $n^{\ell+1} \times n^{\ell+1}$ $n^{L-1} \times n^{L-1}$ $n^{L-1} \times n^{L}$ $n^{L} \times n^{L}$ $n^{L} \times 1$

Taking derivatives for SGD

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- Can similarly show: $\partial \text{loss}/\partial W_0^{(\ell)} = \partial \text{loss}/\partial Z^{(\ell)}$

Taking derivatives for SGD

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Exercise: Check that you agree with these formulas!

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1x1 1x1 1x1 1x1 1x1 m^{ℓ}xn^{ℓ} m^{ℓ}x1 1xn^{ℓ}

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Taking derivatives for SGD

Exercise: use these formulas to show that the final derivatives w.r.t. W^{ℓ} are 0 if any layer above ℓ has step function activations

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- ers
- nd
- $A^{(\ell-1)} + W_0^{(\ell)}$ with $A^{(0)} = x$
- ff the "final" weight derivative

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