MIT EECS
ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

# 6.036: Introduction to Machine Learning

Final lecture! Thanks for joining us on this adventure!

**Lecture start:** Tuesdays 9:35am

**Who's talking?** Prof. Tamara Broderick

**Questions?** Ask on Piazza: "lecture (week) 13" folder

**Materials:** slides, video will all be available on Canvas

**Live Zoom feed:** https://mit.zoom.us/j/94238622313

## Last Time(s)

I. State machines & MDPs
II. Actions change state of world and give reward
III. Choosing "best" actions

## Today's Plan

I. Back to supervised learning
II. Sequential data
III. Recurrent neural networks

# Review

# Review

expected reward

# Review

- $V_\pi^h(s)$: expected reward at horizon *h* with policy $\pi$ starting at *s*

# Review

- $V_{\pi}^{h}(s)$: expected reward at horizon *h* with policy $\pi$ starting at *s*

- $Q^{h}(s, a)$ : expected reward of starting at *s*, making action *a*, and then making the “best” action for the *h*-1 steps left

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- $V_\pi^h(s)$: expected reward at horizon *h* with policy $\pi$ starting at *s*
  - Can compute value of a policy even if not optimal
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- *Reinforcement learning* (RL): learning (to maximize rewards) by interacting with the world
  - Contrast with *supervised learning*
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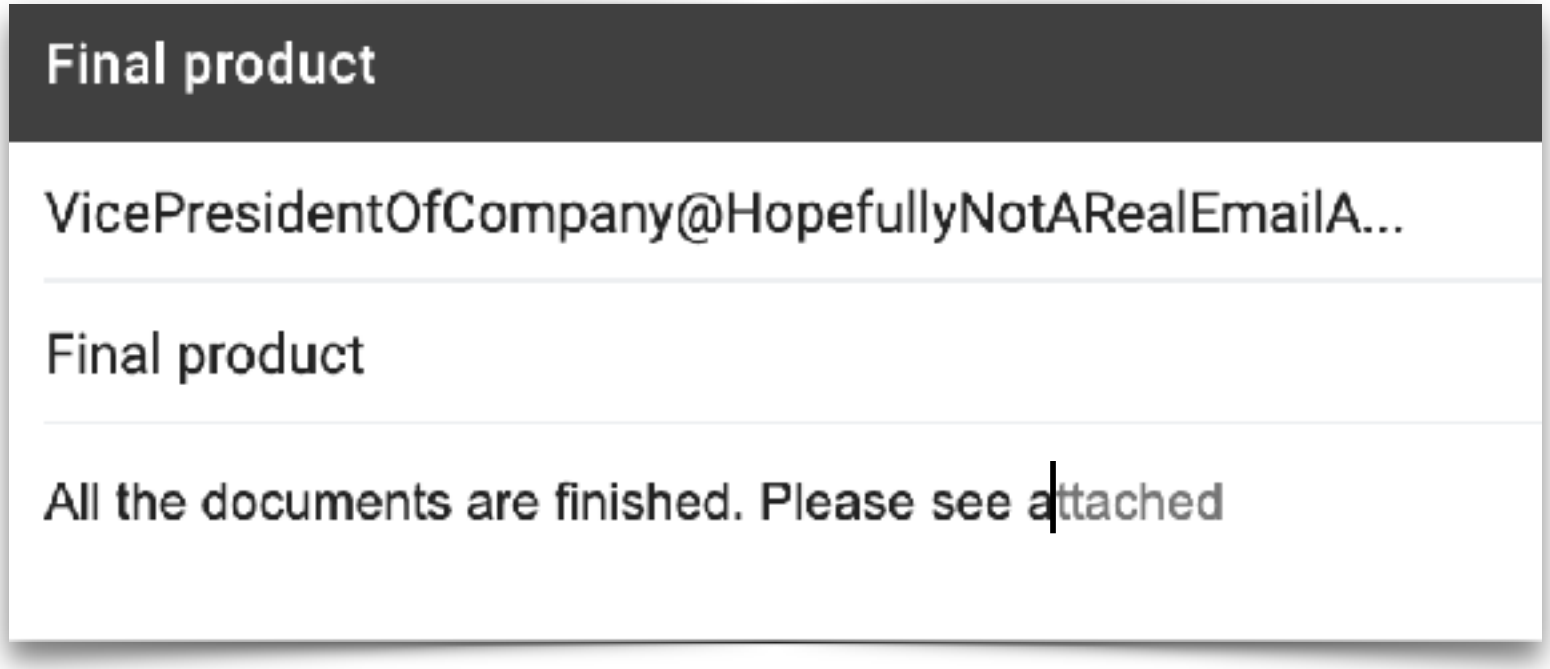
- *Reinforcement learning* (RL): learning (to maximize rewards) by interacting with the world
  - Contrast with *supervised learning*
  - Contrast with the $Q^h$ or $Q^*$ function (expected reward of starting at *s*, making action *a*, and then making the "best" action ever after)
  - Contrast with (any horizon) *value iteration*
- Today: can use state machines with supervised learning

# Text prediction

# Text prediction



**Final product**

VicePresidentOfCompany@HopefullyNotARealEmailA...

Final product

All the documents are finished. Please see attached

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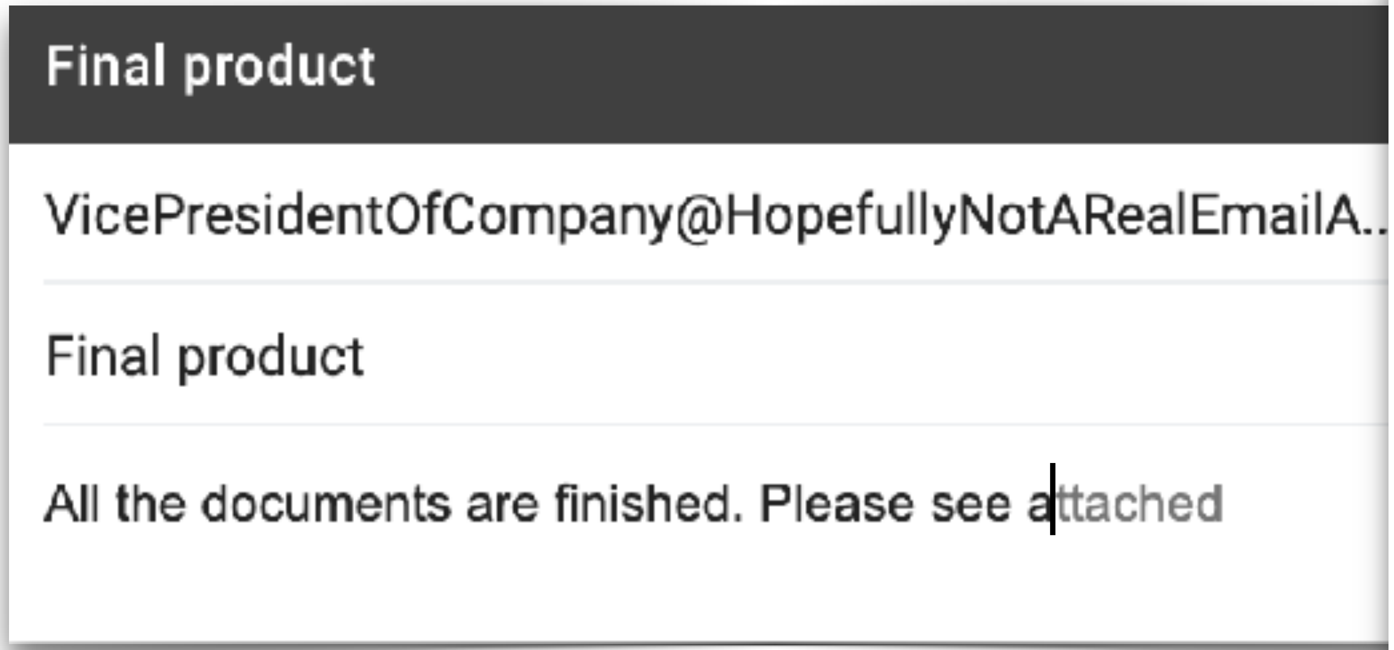

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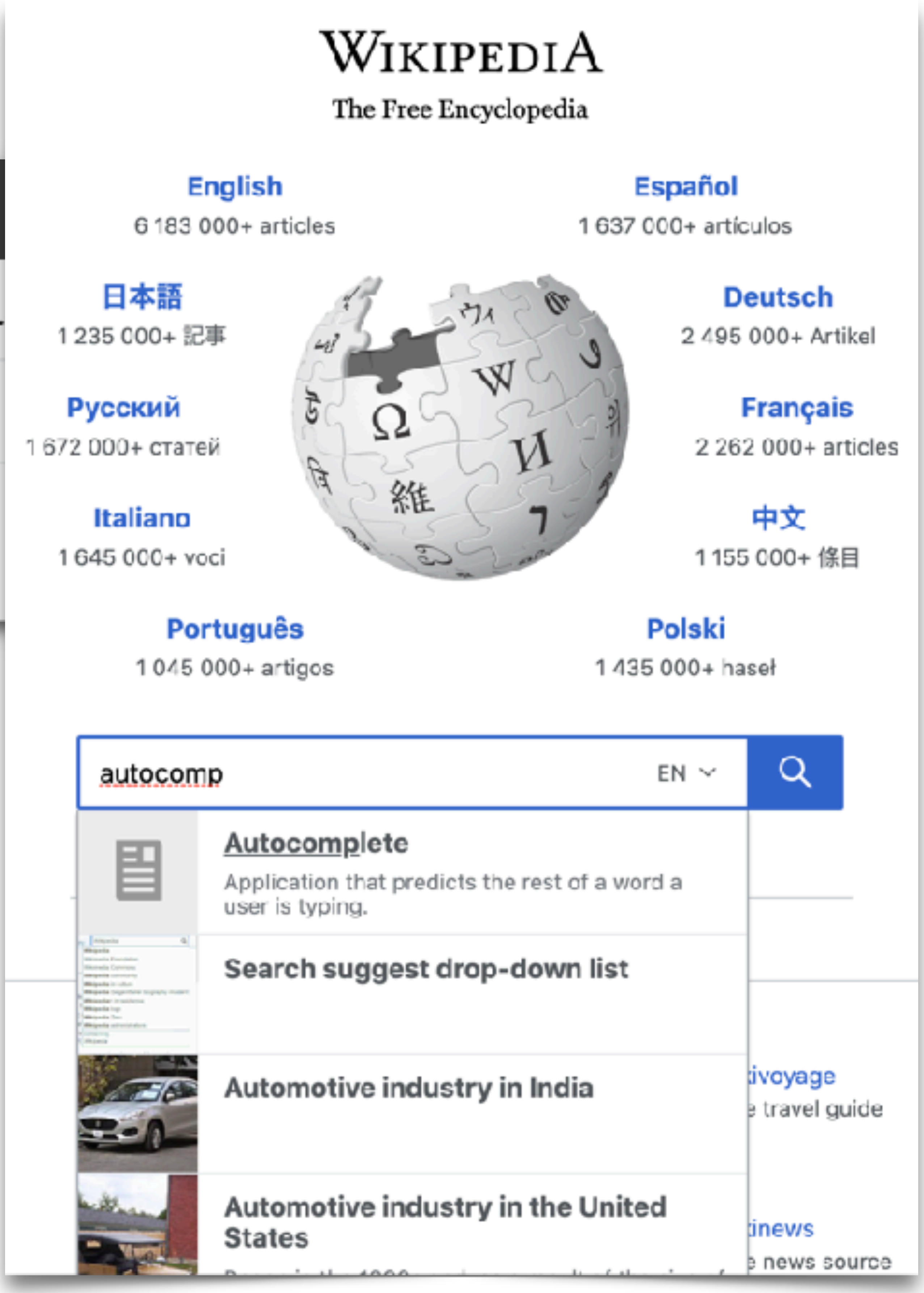

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autocomp EN

**Autocomplete**
Application that predicts the rest of a word a user is typing.

**Search suggest drop-down list**

**Automotive industry in India**

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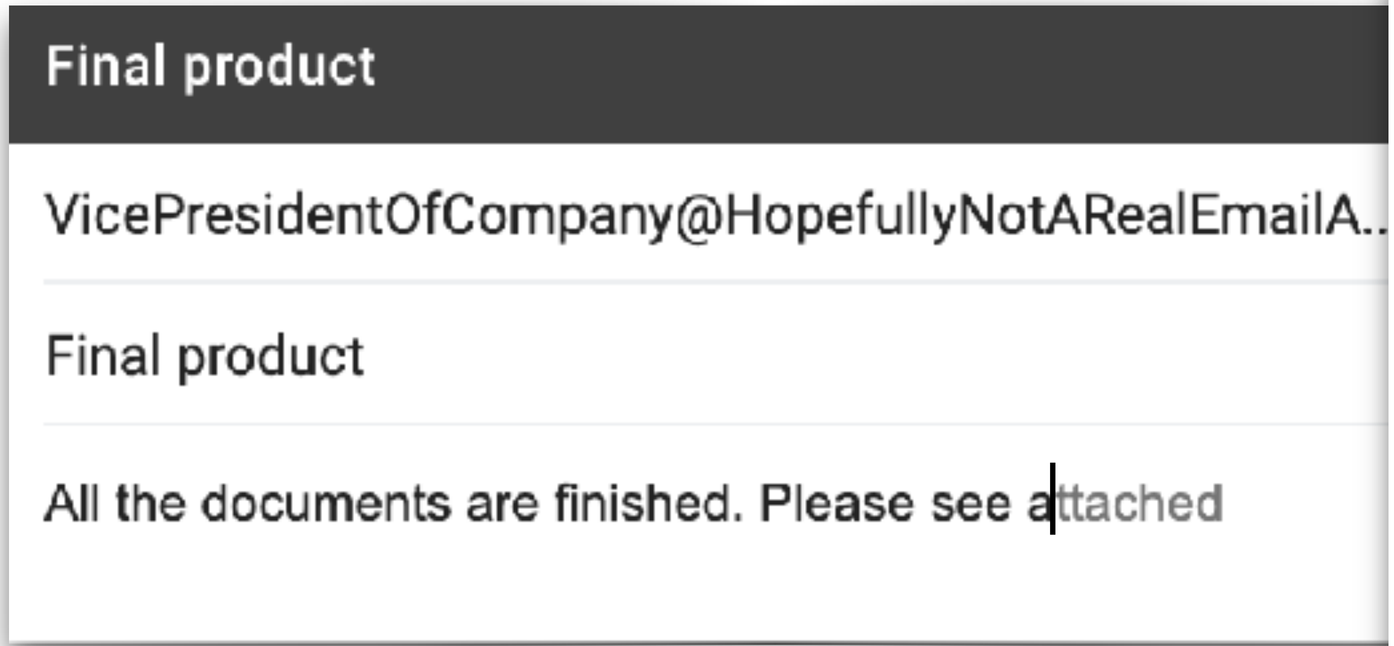
ivoyage
travel guide

inews
news source

# Text prediction



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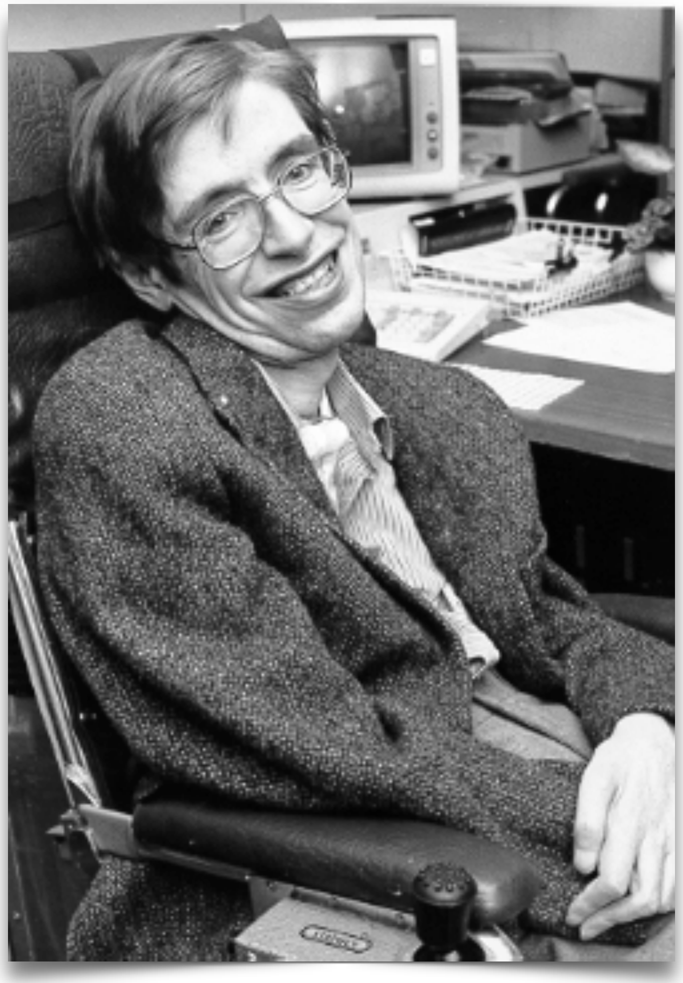

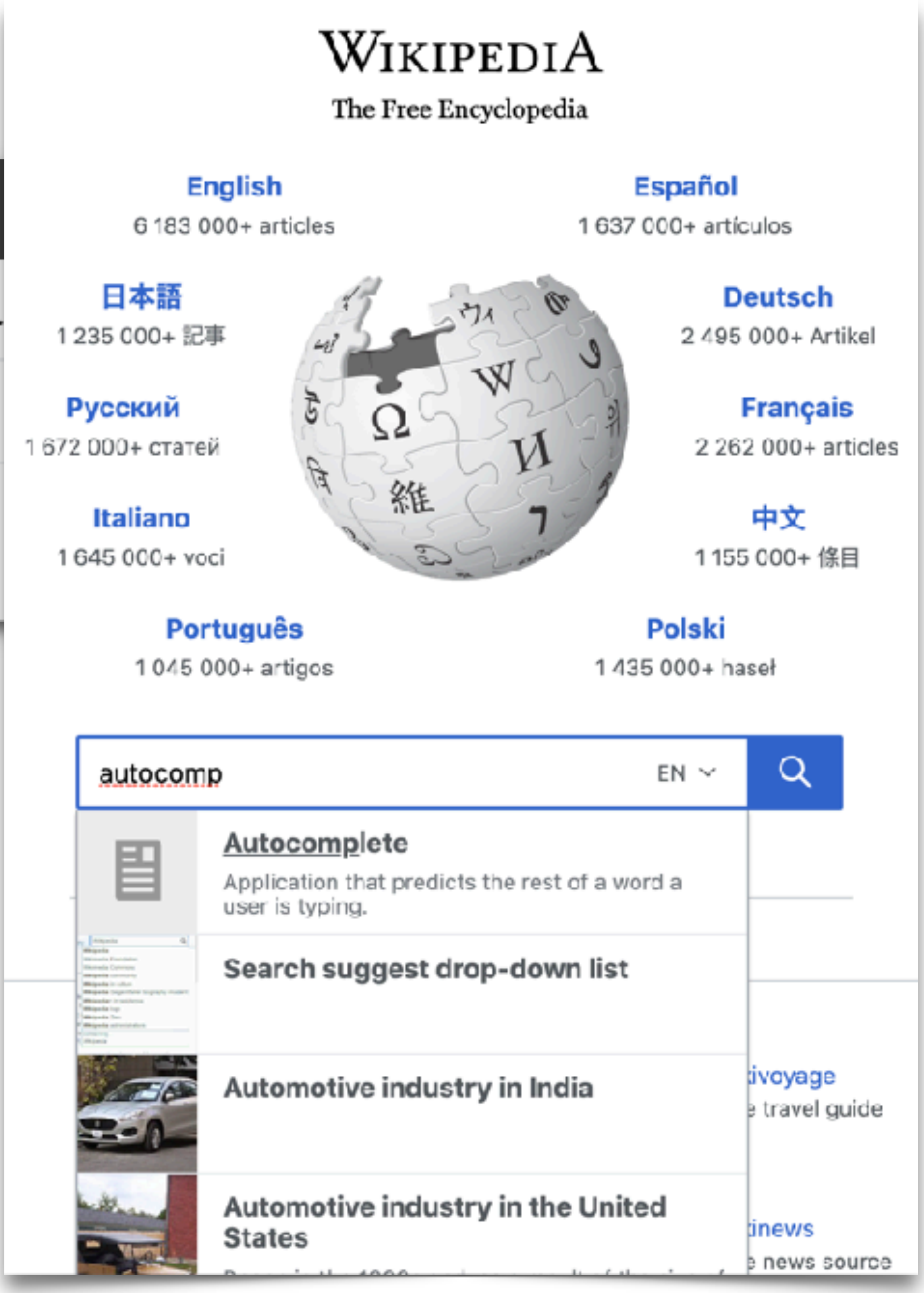

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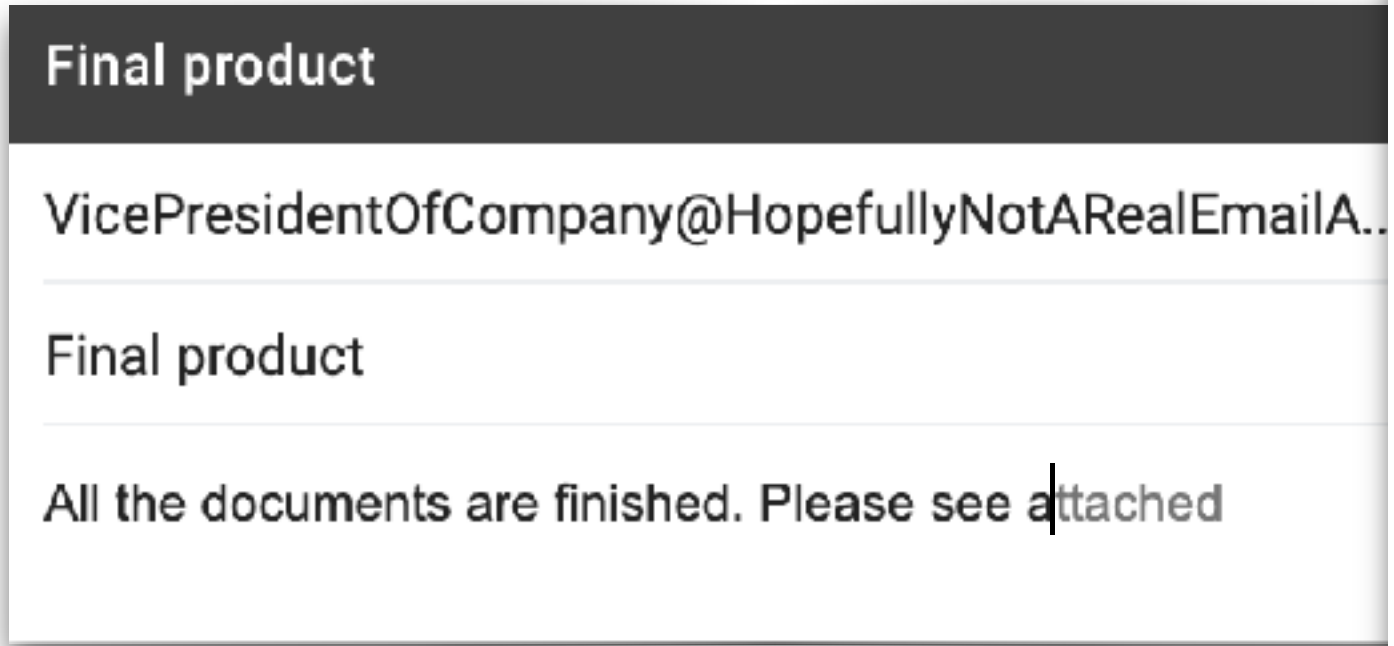

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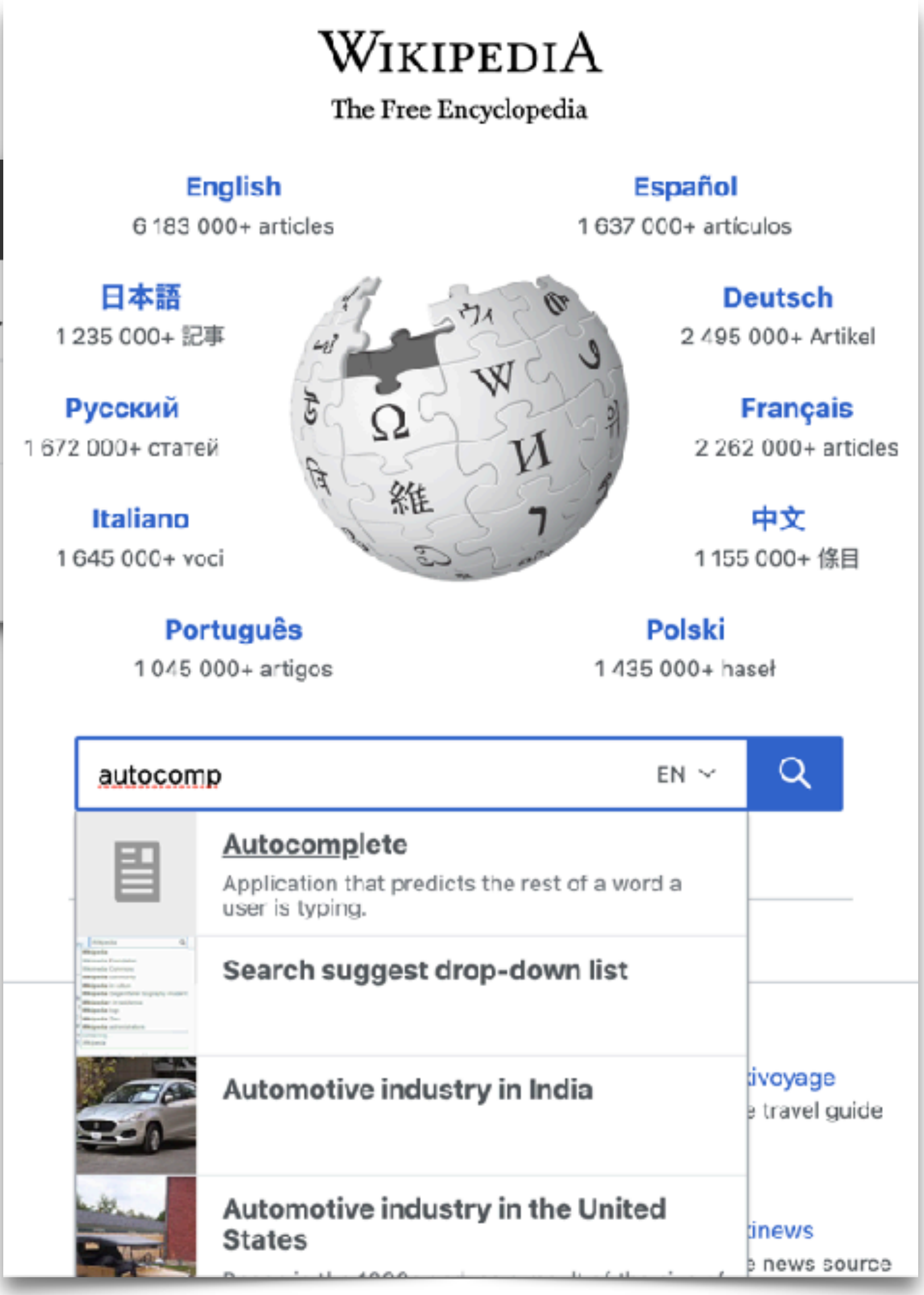

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# Text prediction: supervised learning

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- Training data: lots of text

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| **features** | **label** |
|---|---|
| w | h |

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| wh | a |

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| wha | t |

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|---|---|
| w | h |
| wh | a |
| wha | t |
| what | _ |
| what_ | h |
| what_h | a |
| what_ha | p |
| what_hap | p |
| what_happ | e |

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- Classification with 27 classes

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- How to featurize?

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- How to featurize?
- Idea: use all previous characters.

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- Idea: use last *m* characters

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|---|---|
| w | h |
| wh | a |
| wha | t |
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- Idea: use last $m = 3$ characters

# A state machine: writing & predicting text

“wha”

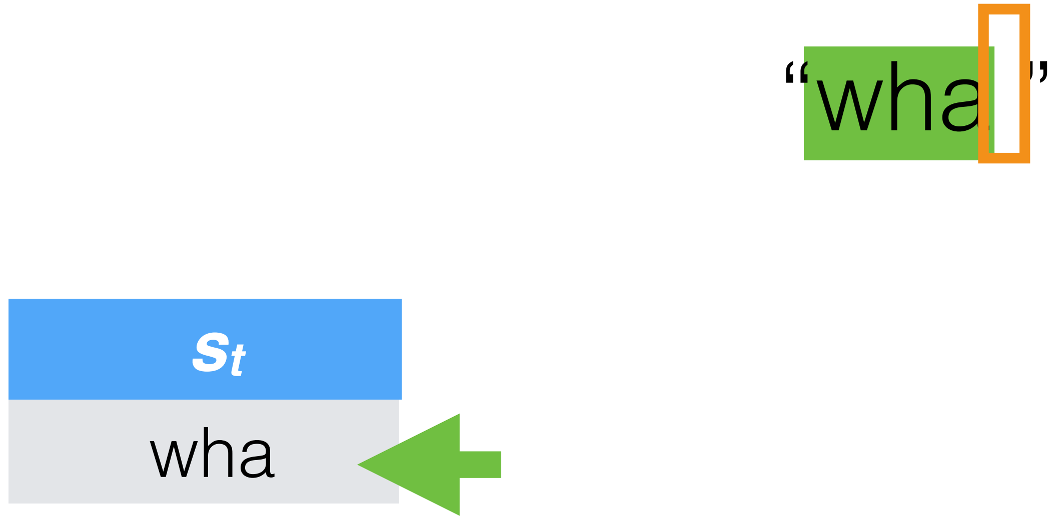
# A state machine: writing & predicting text

“wha▯”

# A state machine: writing & predicting text

“wha ”

# A state machine: writing & predicting text



“wha ”

$s_t$

wha

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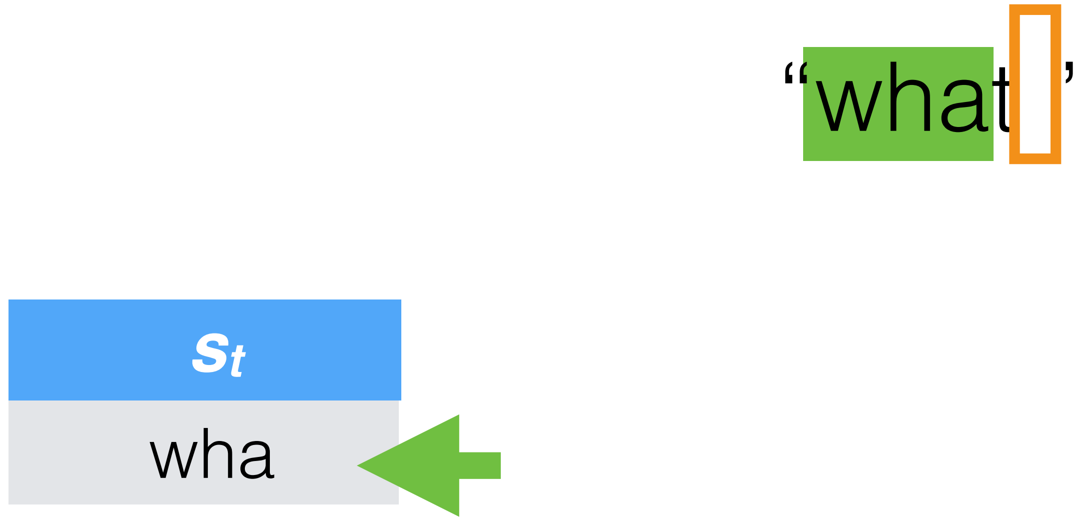

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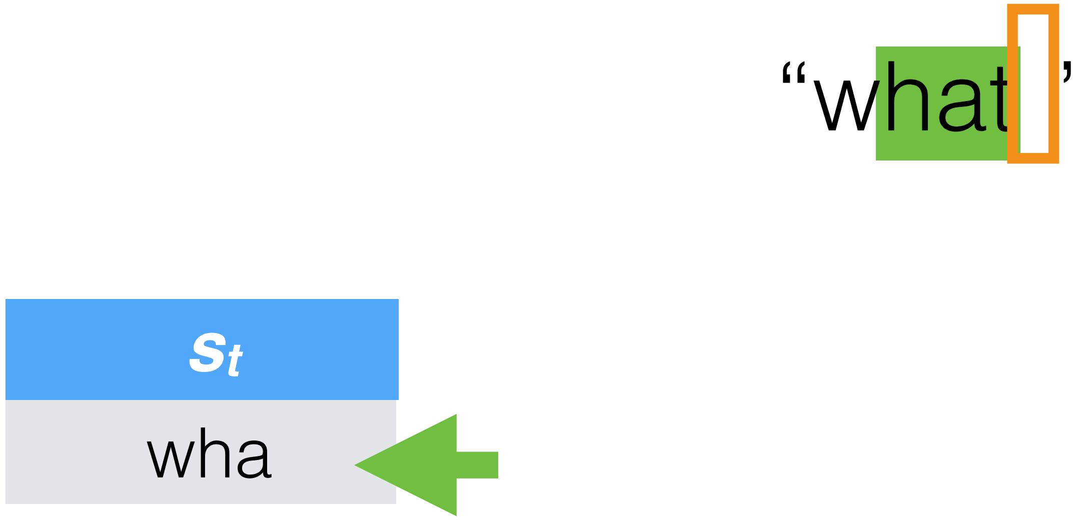


“what ”

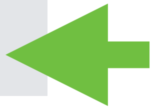
$s_t$

wha

# A state machine: writing & predicting text

“what”

| $x_t$ | $s_t$ |
|---|---|
|  | wha |



t

# A state machine: writing & predicting text

| $x_t$ | $s_t$ |
|---|---|
|  | wha |
| t | hat |

# A state machine: writing & predicting text

“what_ ”

| $x_t$ | $s_t$ |
|---|---|
|  | wha |
| t | hat |

# A state machine: writing & predicting text

“what_ ”

| $x_t$ | $s_t$ |
|---|---|
|  | wha |
| t | hat |

# A state machine: writing & predicting text

“what_ ”

| $x_t$ | $s_t$ |
|---|---|
|  | wha |
| t | hat |
| _ | at_ |

# A state machine: writing & predicting text

“what happens to a dream deferred”

| $x_t$ | $s_t$ |
|---|---|
| | wha |
| t | hat |
| _ | at_ |
| h | t_h |

# A state machine: writing & predicting text

- Recall state machines:
  - Set of possible states $\mathcal{S}$

“what happens to a dream deferred”

| $x_t$ | $s_t$ |
|---|---|
| | wha |
| t | hat |
| _ | at_ |
| h | t_h |

# A state machine: writing & predicting text

- Recall state machines:
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- Example:
  - Every sequence of 3 chars

“what happens to a dream deferred”

| $x_t$ | $s_t$ |
|---|---|
|  | wha |
| t | hat |
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  - Set of possible inputs $\mathcal{X}$
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- Example:
  - Every sequence of 3 chars
  - Every single character
  - $s_0 = \wedge\wedge\wedge$

“what happens to a dream deferred”

| $x_t$ | $s_t$ |
|---|---|
| | wha |
| t | hat |
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# A state machine: writing & predicting text

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  - Set of possible states $\mathcal{S}$
  - Set of possible inputs $\mathcal{X}$
  - Initial state
- Example:
  - Every sequence of 3 chars
  - Every single character
  - $s_0$ = ^^^

| $x_t$ | $s_t$ |
|---|---|
|  | ^^^ |
| w | ^^w |
| h | ^wh |
| a | wha |
| t | hat |
| _ | at_ |

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|---|---|
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- Example:
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  - $s_0$ = ^^^

| $t$ | $x_t$ | $s_t$ |
|---|---|---|
| 0 |  | ^^^ |
| 1 | w | ^^w |
| 2 | h | ^wh |
| 3 | a | wha |
| 4 | t | hat |
| 5 | _ | at_ |

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# A state machine: writing & predicting text

- Recall state machines:
  - Set of possible states $\mathcal{S}$
  - Set of possible inputs $\mathcal{X}$
  - Initial state
  - Transition function $f(s,x)$

- Example:
  - Every sequence of 3 chars
  - Every single character
  - $s_0$ = ^^^

| $t$ | $x_t$ | $s_t$ |
|---|---|---|
| 0 | | ^^^ |
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| $t$ | $x_t$ | $s_t$ |
|---|---|---|
| 0 | | ^^^ |
| 1 | w | ^^w |
| 2 | h | ^wh |
| 3 | a | wha |
| 4 | t | hat |
| 5 | _ | at_ |

"what happens to a dream deferred"

# A state machine: writing & predicting text

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previously “y”

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|---|---|---|
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“what happens to a dream deferred”

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- $x^{(1)}$: "what happens to a dream deferred"

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previously "y"

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- $x^{(1)}$: “what happens to a dream deferred”
- $x^{(2)}$: “if you can keep your head when all about you”
- $x^{(3)}$: “you may write me down in history”

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- Example with alphabet {0,1} ($v$=2); state is last $m$ = 3 chars

$$s_t = f(s_{t-1}, x_t) = \boxed{?}\; x_t + \boxed{?}\; s_{t-1}$$

# A state machine: writing & predicting text

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$$s_t = f(s_{t-1}, x_t) = \boxed{?}\; x_t + \boxed{?}\; s_{t-1}$$

3x1

# A state machine: writing & predicting text

- Recall state machines:
    - Set of possible states $\mathcal{S}$
    - Set of possible inputs $\mathcal{X}$
    - Initial state
    - Transition function *f*(*s*,*x*)
    - Set of possible outputs
    - Output function *p* = *g*(*s*)
- Example:
    - Every sequence of 3 chars
    - Every single character
    - $s_0$
    - E.g. $f\left(\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, x\right) = \begin{bmatrix} c_2 \\ c_3 \\ x \end{bmatrix}$
    - Output: *v* prediction probs
    - E.g. $p = g\left(\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}\right)$
- Example with alphabet {0,1} (*v*=2); state is last *m* = 3 chars

$$\underset{3\text{x}1}{s_t} = f(s_{t-1}, x_t) = \boxed{?}\ \underset{1\text{x}1}{x_t} + \boxed{?}\ s_{t-1}$$

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- Recall state machines:
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- Example:
  - Every sequence of 3 chars
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- Example with alphabet {0,1} ($v$=2); state is last $m$ = 3 chars

$$\underset{3\text{x}1}{s_t} = f(s_{t-1}, x_t) = \boxed{?}\; \underset{1\text{x}1}{x_t} + \boxed{?}\; \underset{3\text{x}1}{s_{t-1}}$$

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- Example with alphabet {0,1} ($v$=2); state is last $m$ = 3 chars

$$\underset{3x1}{s_t} = f(s_{t-1}, x_t) = \underset{3x1}{\boxed{?}}\ \underset{1x1}{x_t} + \boxed{\quad ? \quad}\ \underset{3x1}{s_{t-1}}$$

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$$\underset{\text{3x1}}{s_t} = f(s_{t-1}, x_t) = \underset{\text{3x1}}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \underset{\text{1x1}}{x_t} + \underset{\text{3x3}}{\boxed{?}} \underset{\text{3x1}}{s_{t-1}}$$

# A state machine: writing & predicting text

- Recall state machines:
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$$p_t = g(s_t)$$

$$=$$

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# A state machine: writing & predicting text

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$$s_t = \begin{bmatrix} w_1^{sx} \\ w_2^{sx} \\ w_3^{sx} \end{bmatrix} x_t + \begin{bmatrix} w_{11}^{ss} & w_{12}^{ss} & w_{13}^{ss} \\ w_{21}^{ss} & w_{22}^{ss} & w_{23}^{ss} \\ w_{31}^{ss} & w_{32}^{ss} & w_{33}^{ss} \end{bmatrix} s_{t-1} + \begin{bmatrix} w_{0,1}^{ss} \\ w_{0,2}^{ss} \\ w_{0,3}^{ss} \end{bmatrix}$$

$$\begin{aligned} p_t &= g(s_t) \\ &= f_2(W^o s_t + W_0^o) \end{aligned}$$

same as final layer in all of our neural networks so far, except for notation

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componentwise; activation functions

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**Put it all together:**

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**Put it all together:**

$$p^{(i)} = \mathrm{RNN}(x^{(i)}; W, W_0)$$

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"recurrent neural network"

# A state machine: writing & predicting text

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$\begin{bmatrix} c_2 \\ c_3 \\ x \end{bmatrix}$

tion probs

Recall:

- In 2-layer, fully-connected NNs, we learned the features.
- In CNNs, we learned the filters.
- Analogous idea here.

- Example with alphabet {0, 1} ($V$=2), state is last $m$ = 3 chars

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# Recurrent Neural Networks

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Recall: familiar pattern

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sequence $L_{\text{seq}}(p^{(i)}, y^{(i)}) = \sum_{t=1}^{n^{(i)}} L_{\text{elt}}(p_t^{(i)}, y_t^{(i)})$ element

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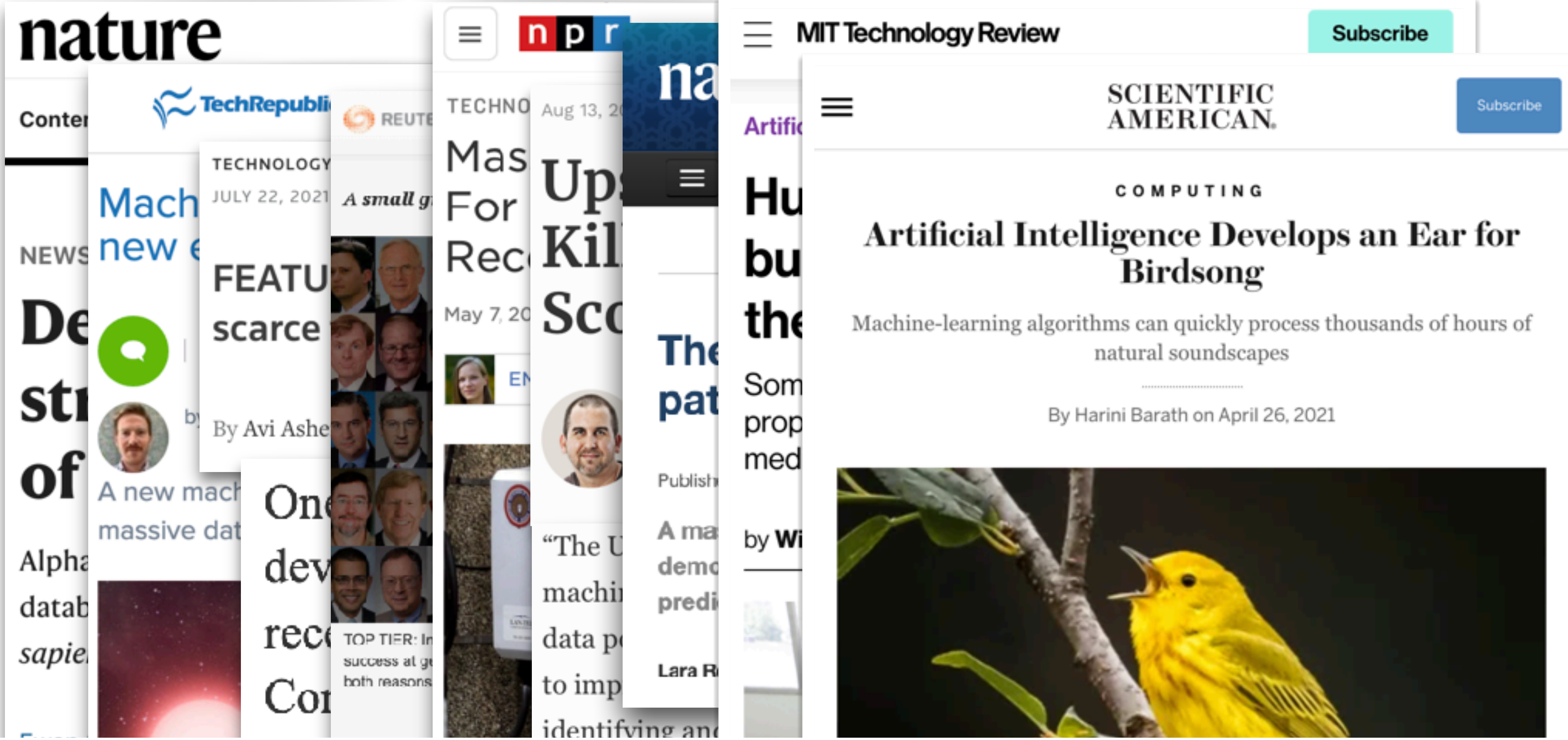
3. Stochastic gradient descent

# Machine learning (ML): why & what



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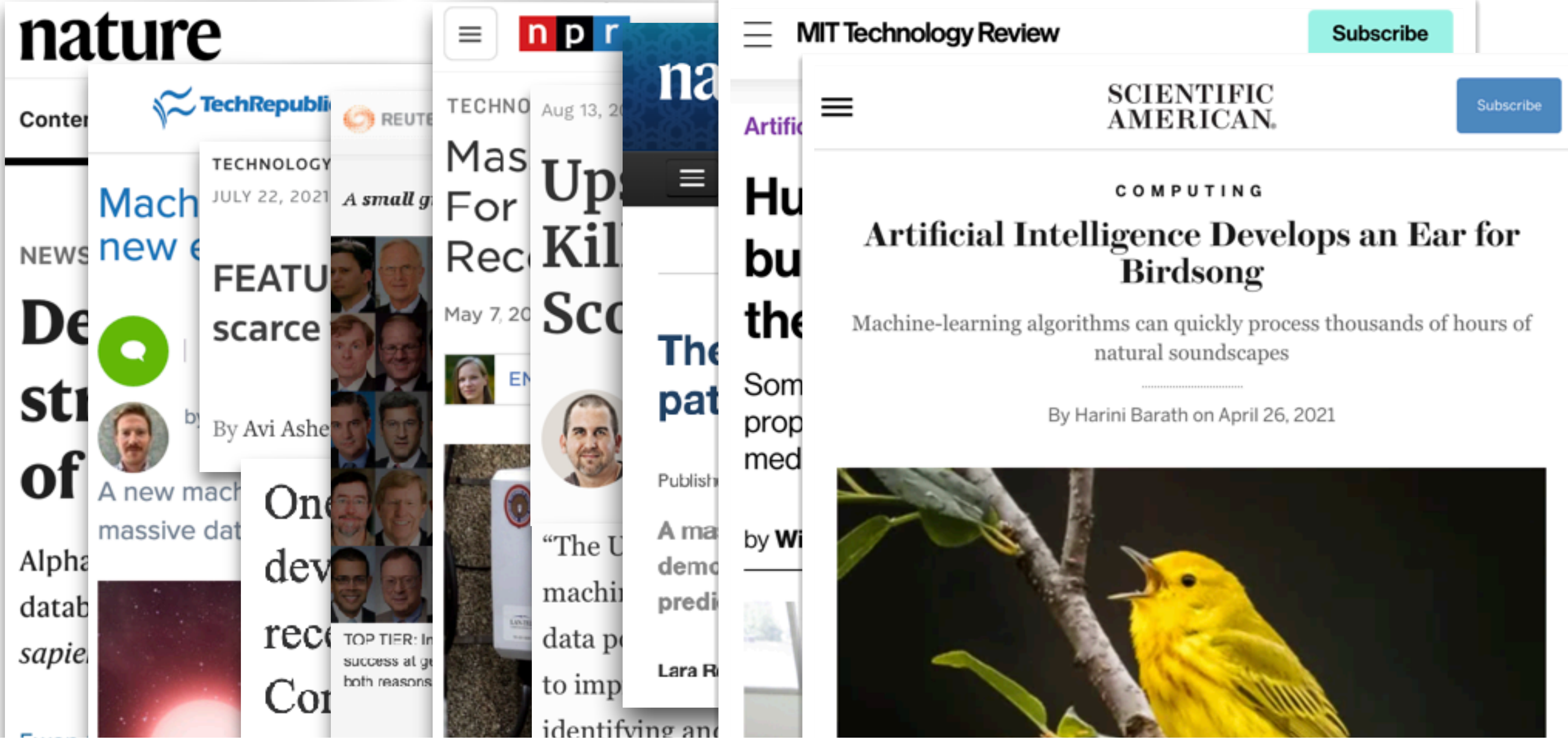
- **What is ML?**

# Machine learning (ML): why & what



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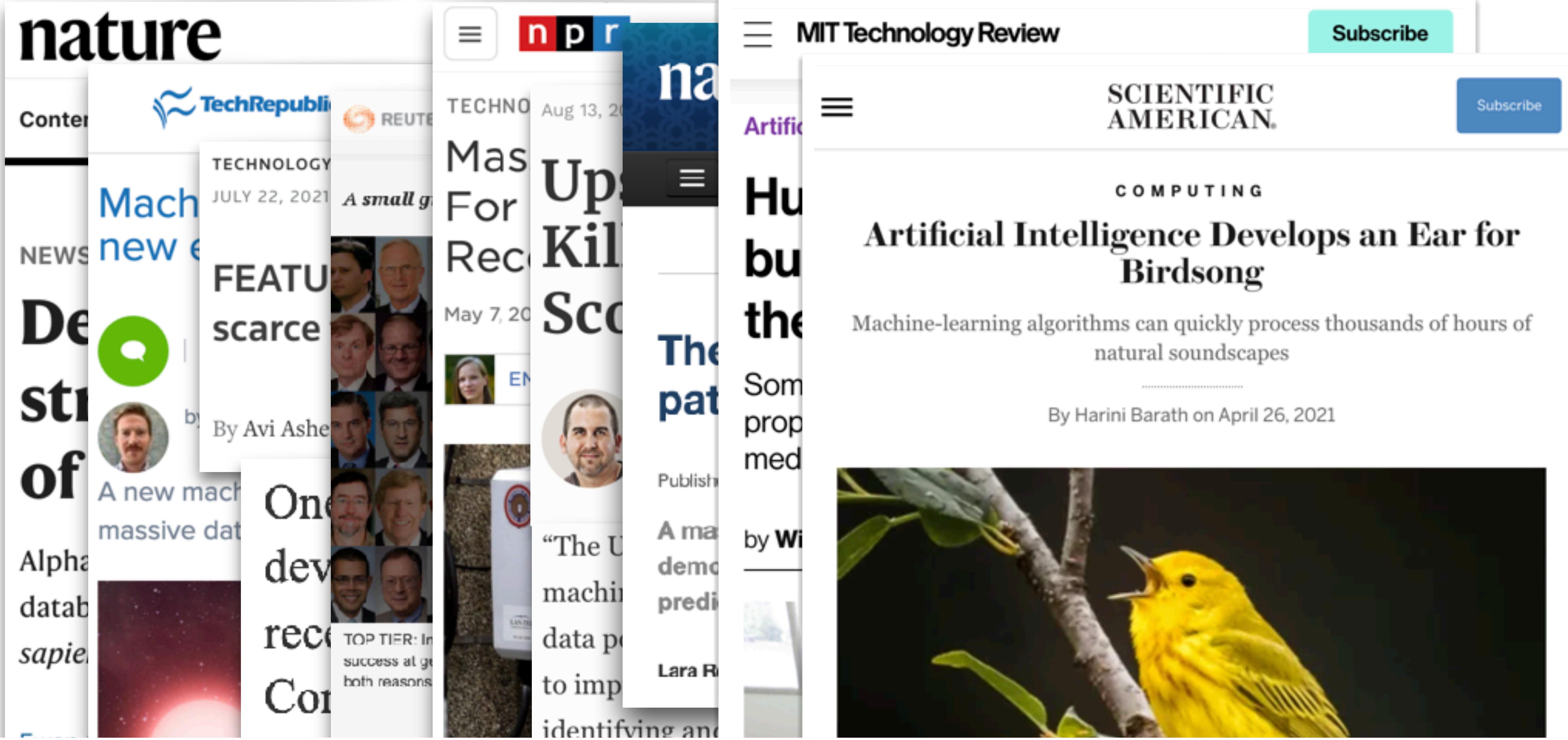

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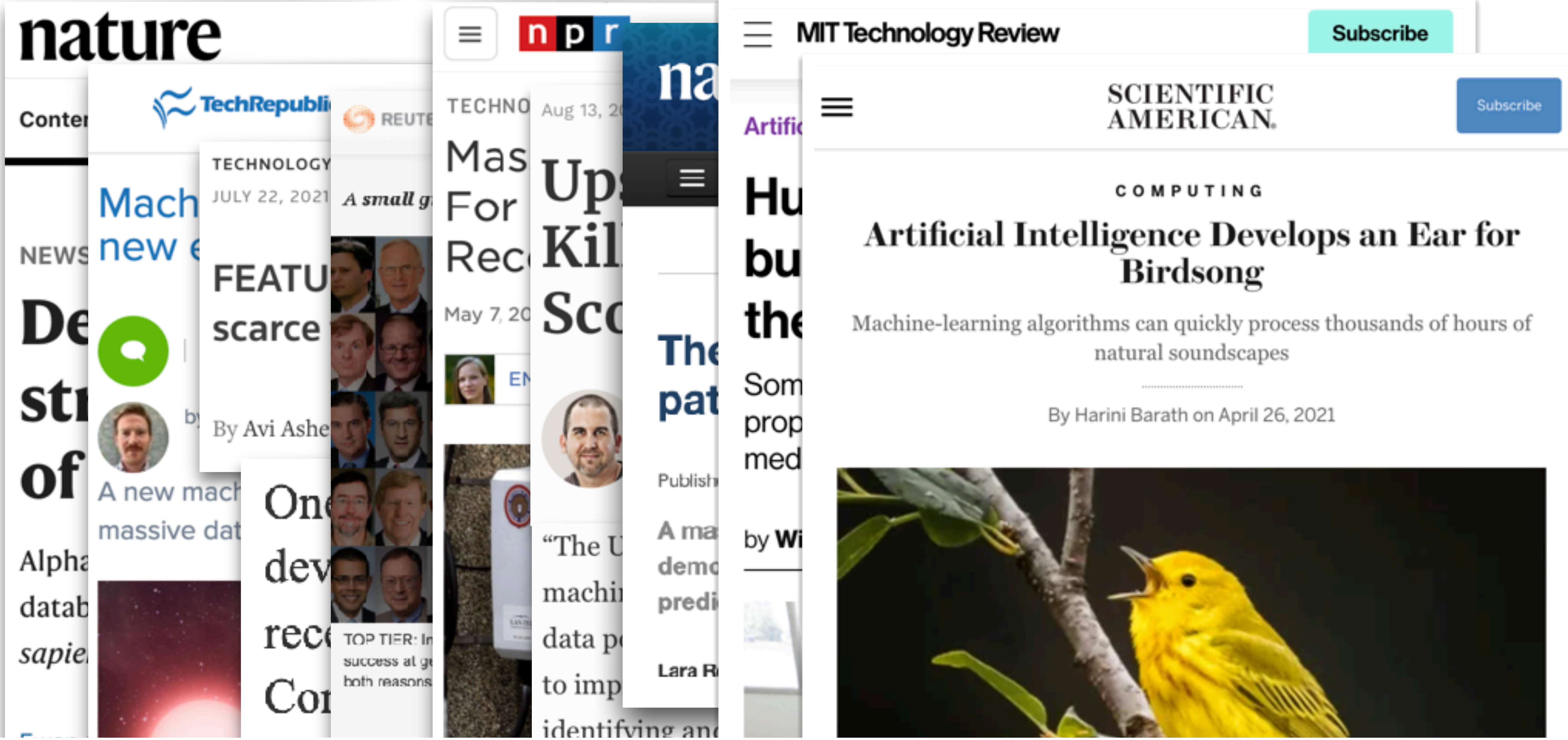

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- **Why study ML?** To apply; to understand; to evaluate
- **Notes:** ML is a tool with pros & cons. ML is built on math

MIT EECS
ELECTRICAL ENGINEERING
AND COMPUTER SCIENCE

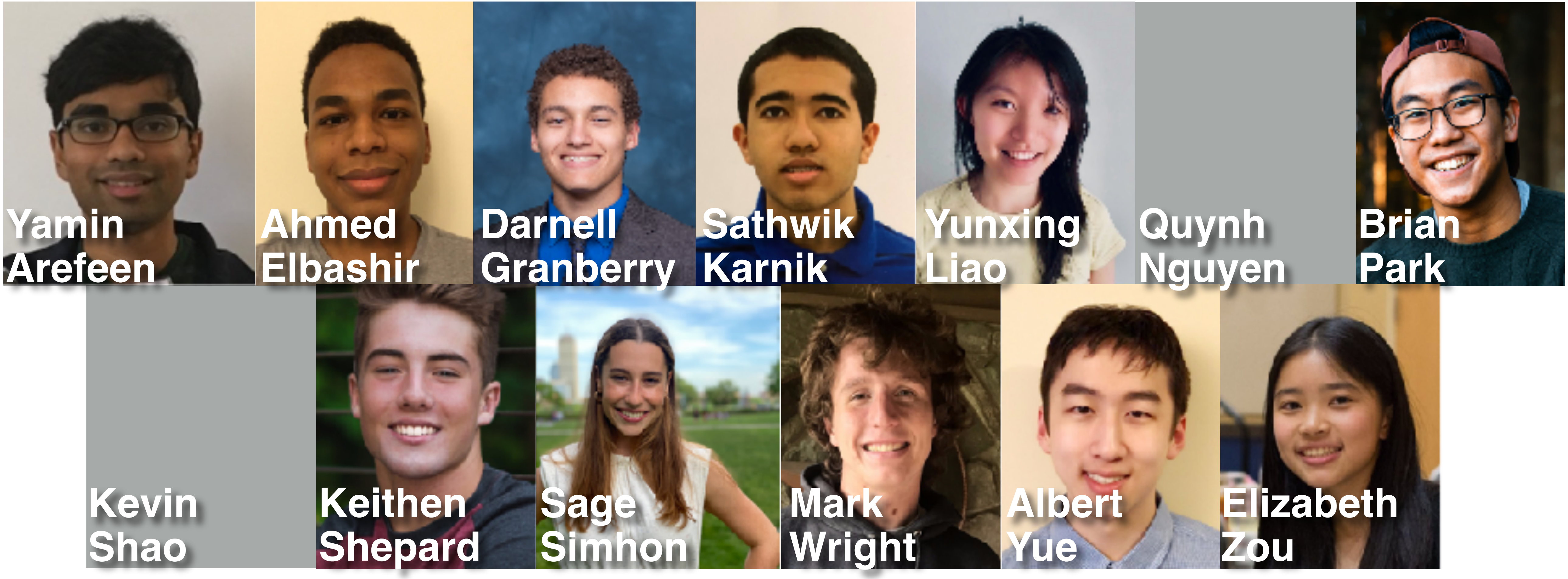
# 6.036: Staff

Big thanks to all of our staff and to the MIT AV team!

Instructors:

Jehangir Amjad
Tamara Broderick
Ike Chuang
Iddo Drori
David Sontag
Tess Smidt

Teaching Assistants:



Yamin Arefeen
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Darnell Granberry
Sathwik Karnik
Yunxing Liao
Quynh Nguyen
Brian Park
Kevin Shao
Keithen Shepard
Sage Simhon
Mark Wright
Albert Yue
Elizabeth Zou

And Lab Assistants!

# Thank you!