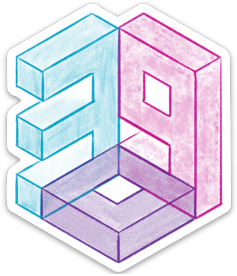


https://introml.mit.edu/

6.390 Intro to Machine Learning

Lecture 12: Reinforcement Learning

Shen Shen

Nov 20, 2025

11am, Room 10-250

Interactive Slides and Lecture Recording

Recall

MDP: Definition and terminologies

- $\mathcal{S}$: state space, contains all possible states s.
- $\mathcal{A}$: action space, contains all possible actions a.
- $\mathrm{T}(s, a, s')$: the probability of transition from state s to s' when action a is taken.
- $\mathrm{R}(s, a)$: reward, takes in a (state, action) pair and returns a reward.
- $\gamma \in [0, 1]$: discount factor, a scalar.
- $\pi(s)$: policy, takes in a state and returns an action.

  The goal of an MDP is to find a "good" policy.

In 6.390,

- $\mathcal{S}$ and $\mathcal{A}$ are small discrete sets, unless otherwise specified.
- s' and a' are short-hand for the next-timestep state and action.
- $\mathrm{R}(s, a)$ is deterministic and bounded.
- $\pi(s)$ is deterministic.

`Recall`

Use the definition and sum up expected rewards:

1. $$\mathrm{V}_h^\pi(s) := \mathbb{E}\left[\sum_{t=0}^{h-1} \gamma^t \mathrm{R}\left(s_t, \pi\left(s_t\right)\right) \mid s_0 = s, \pi\right]$$

Or, leverage the recursive structure:

2. **finite-**horizon Bellman recursions

$$\mathrm{V}_h^\pi(s) = \mathrm{R}(s, \pi(s)) + \gamma \sum_{s'} \mathrm{T}\left(s, \pi(s), s'\right) \mathrm{V}_{h-1}^\pi\left(s'\right)$$

3. **infinite-**horizon Bellman equations

$$\mathrm{V}_\infty^\pi(s) = \mathrm{R}(s, \pi(s)) + \gamma \sum_{s'} \mathrm{T}\left(s, \pi(s), s'\right) \mathrm{V}_\infty^\pi\left(s'\right)$$

`Recall`

horizon-h value in state s: the expected sum of discounted rewards, starting in state s and following policy π for h steps.

2️⃣ $$V_h^\pi(s) = R(s, \pi(s)) + \gamma \sum_{s'} T(s, \pi(s), s') V_{h-1}^\pi(s')$$

the immediate reward for taking the policy-prescribed action $\pi(s)$ in state s.

$(h-1)$ horizon future value at a next state s'

sum of future values weighted by the probability of reaching that next state s'

discounted by γ

`Recall`

the optimal state-action value functions $\mathrm{Q}_h^*(s,a)$:

the expected sum of discounted rewards, obtained by

- starting in state s
- take action a, for one step
- act **optimally** thereafter for the remaining $(h-1)$ steps

4️⃣ $\mathrm{V}_h^*(s) = \max_a \left[\mathrm{R}(s,a) + \gamma \sum_{s'} \mathrm{T}(s,a,s') \mathrm{V}_{h-1}^*(s')\right] = \max_a \left[\mathrm{Q}_h^*(s,a)\right]$

Q^* satisfies the Bellman recursion:

5️⃣ $$\mathrm{Q}_h^*(s,a) = \mathrm{R}(s,a) + \gamma \sum_{s'} \mathrm{T}\left(s,a,s'\right) \max_{a'} \mathrm{Q}_{h-1}^*\left(s',a'\right)$$

`Recall`

Value iteration: iteratively invoke

5️⃣ $$\mathrm{Q}_h^*(s,a) = \mathrm{R}(s,a) + \gamma \sum_{s'} \mathrm{T}(s,a,s') \max_{a'} \mathrm{Q}_{h-1}^*(s',a')$$

`Value Iteration`

1. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
2. $\quad \mathrm{Q}_{\text{old}}(\mathrm{s},\mathrm{a}) = 0$
3. **while** True:
4. $\quad$ **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
5. $\quad\quad \mathrm{Q}_{\text{new}}(s,a) \leftarrow \mathrm{R}(s,a) + \gamma \sum_{s'} \mathrm{T}(s,a,s') \max_{a'} \mathrm{Q}_{\text{old}}(s',a')$
6. $\quad$ **if** $\max_{s,a} |Q_{\text{old}}(s,a) - Q_{\text{new}}(s,a)| < \epsilon$:
7. $\quad\quad$ **return** Q_{new}
8. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$

if run this block h times and ***break,*** then the returns are Q_h^*

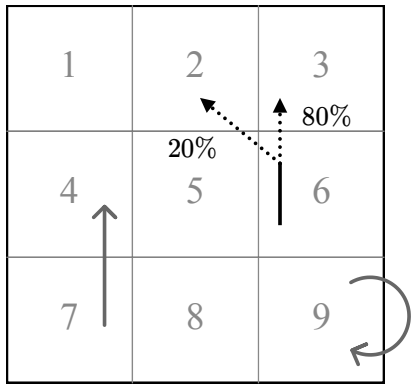
returns are Q_∞^*

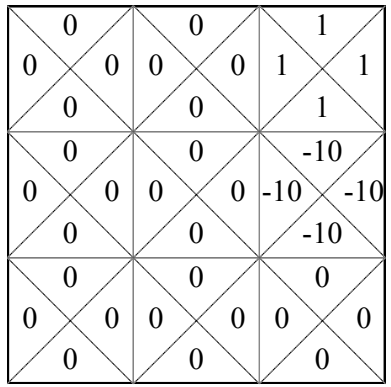
Outline

- Reinforcement learning setup
- Tabular Q-learning
  - exploration vs. exploitation
  - ϵ-greedy action selection
- Fitted Q-learning
- (Reinforcement learning setup again)

Mario in a grid-world *v1.0*

(Markov-decision-process version)





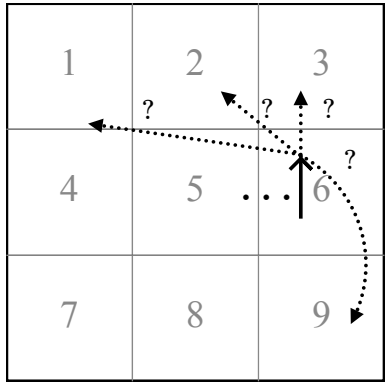
- 9 possible *states*
- 4 possible *actions*: {Up ↑, Down ↓, Left ←, Right →}
- *transition* probabilities are **known**

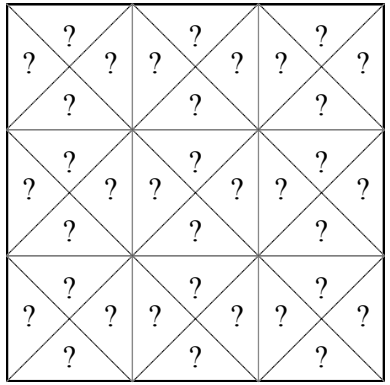
$$\text{e.g., } \mathrm{T}(7, \uparrow, 4) = 1 \quad \mathrm{T}(9, \rightarrow, 9) = 1 \quad \mathrm{T}(6, \uparrow, 3) = 0.8 \quad \mathrm{T}(6, \uparrow, 2) = 0.2$$

- *rewards* **known**
- *discount factor* $\gamma = 0.9$

Mario in a grid-world *v2.0*

(reinforcement learning version)





- 9 possible *states*
- 4 possible *actions*: {Up ↑, Down ↓, Left ←, Right →}
- *transition* probabilities are **unknown**

e.g., $\mathrm{T}(7, \uparrow, 4) = ?$ $\mathrm{T}(9, \rightarrow, 9) = ?$ $\mathrm{T}(6, \uparrow, 3) = ?$ $\mathrm{T}(6, \uparrow, 2) = ?$

- *rewards* **unknown**
- *discount factor* $\gamma = 0.9$

~~Markov Decision Processes~~ - Definition and terminologies

Reinforcement Learning

- $\mathcal{S}$: state space, contains all possible states s.
- $\mathcal{A}$: action space, contains all possible actions a.
- ~~T (s, a, s') : the probability of transition from state s to s' when action a is taken.~~
- ~~R(s, a) : reward, takes in a (state, action) pair and returns a reward~~.
- $\gamma \in [0, 1]$: discount factor, a scalar.

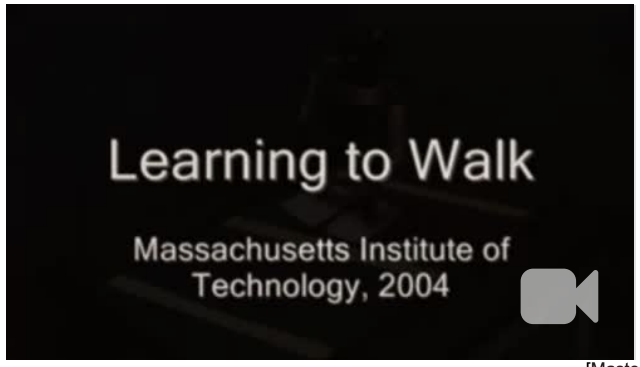
- $\pi(s)$: policy, takes in a state and returns an action.

  The goal of an ~~MDP~~ problem is to find a "good" policy.

RL

Reinforcement learning is very *general*:

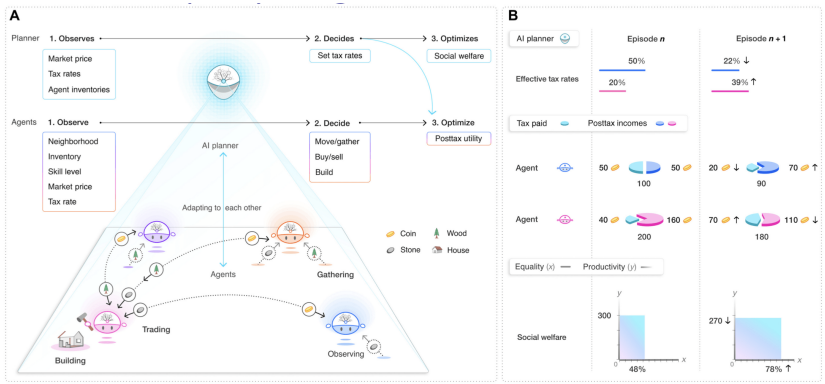
robotics



games

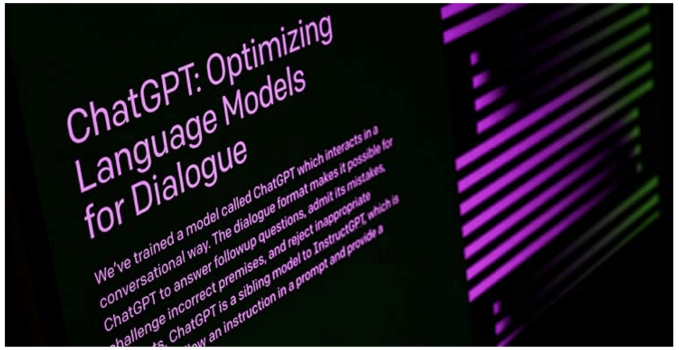
[Mastering the game of Go with deep neural networks and tree search. Silver et al. Nature 20

social sciences



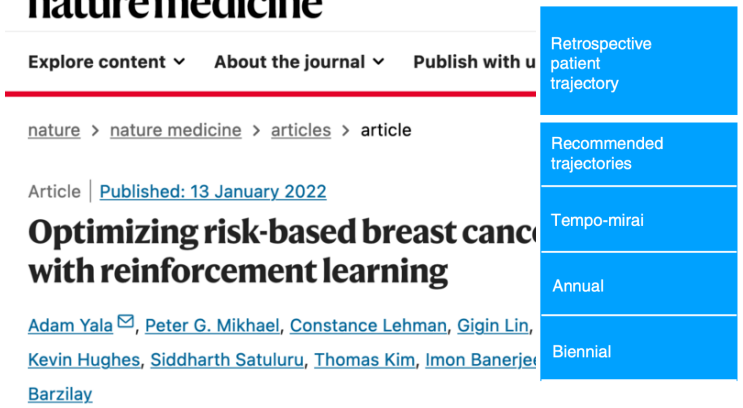
[The AI Economist: Taxation policy design via two-level deep multiagent RL. Zheng et al. Science 2022]

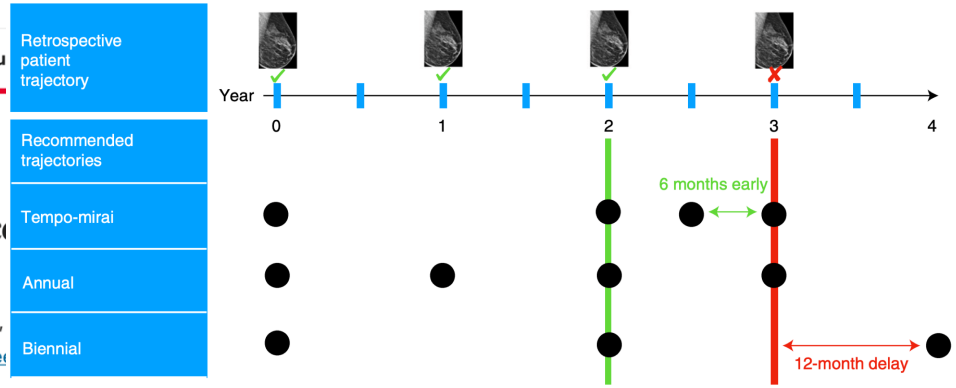
chatbot (RLHF)



[Aligning language models to follow instructions. Ouyang et al. 2022]

health care





...

Model-based RL: Learn the MDP tuple

1. Collect set of experiences (s, a, r, s')

2. Estimate $\hat{\mathrm{T}}$, $\hat{\mathrm{R}}$

e.g. $\hat{\mathrm{T}}(6, \uparrow, 2) \approx$

$$\frac{\text{observed } (6,\uparrow,2) \text{ count}}{\text{total gameplay count}}$$

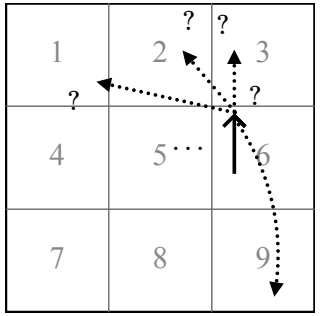
e.g. $\hat{\mathrm{R}}(6, \uparrow) =$

observed reward received from $(6, \uparrow)$

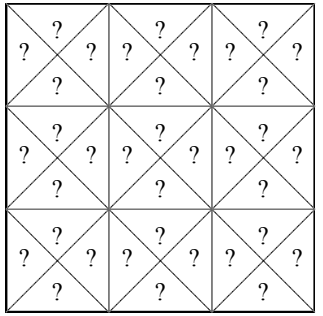
3. Solve $\langle \mathcal{S}, \mathcal{A}, \hat{\mathrm{T}}, \hat{\mathrm{R}}, \gamma \rangle$ via e.g. *Value Iteration*

$\gamma = 0.9$

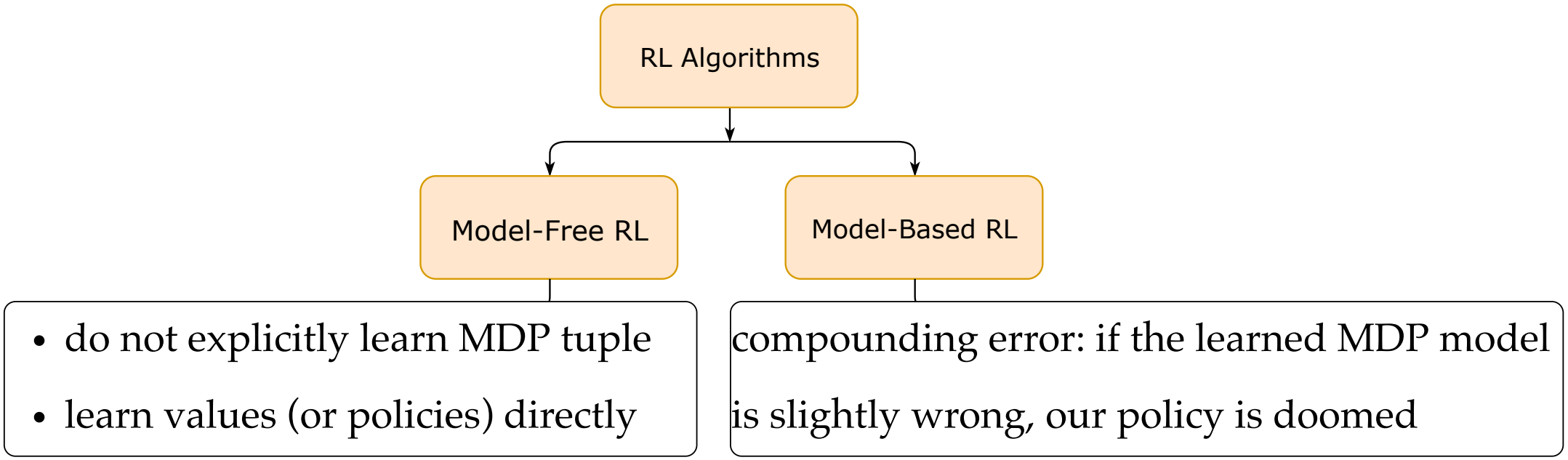
Unknow transition:



Unknown rewards



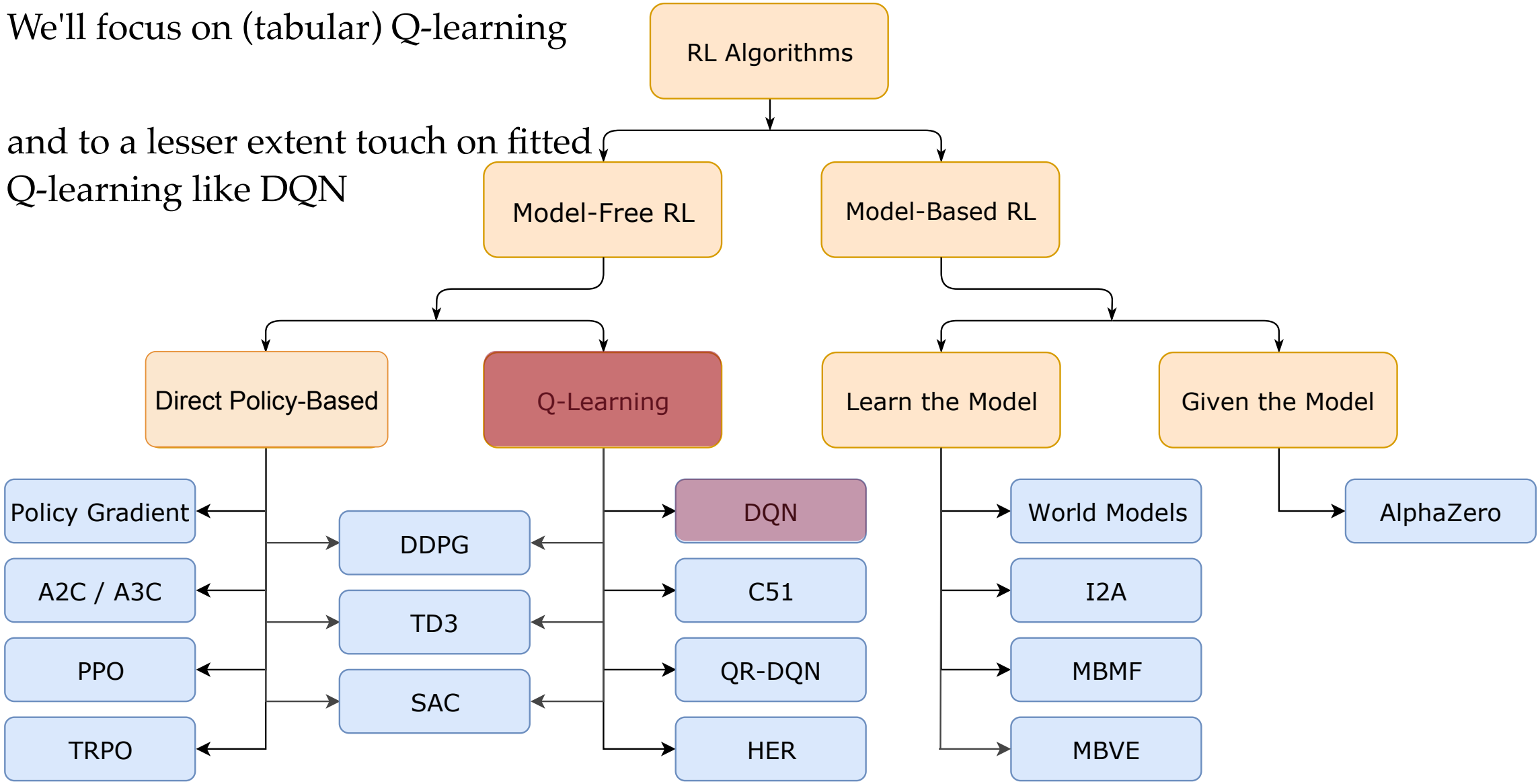
(s, a)	r	s'
$(1, \uparrow)$	0	1
$(1, \downarrow)$	0	4
	...	
$(3, \uparrow)$	1	3
$(3, \downarrow)$	1	6
	...	
$(6, \uparrow)$	-10	3
$(6, \uparrow)$	-10	2
	...	



- We rarely say *hypothesis* in RL; instead, we refer to what's learned as a *model*, *value*, or *policy*
- *Model* in MDP / RL typically means the MDP tuple $\langle \mathcal{S}, \mathcal{A}, \mathrm{T}, \mathrm{R}, \gamma \rangle$

We'll focus on (tabular) Q-learning

and to a lesser extent touch on fitted Q-learning like DQN



[A non-exhaustive, but useful taxonomy of algorithms in modern RL. Source]

Outline

- Reinforcement learning setup
- Tabular Q-learning
  - exploration vs. exploitation
  - ϵ-greedy action selection
- Fitted Q-learning
- (Reinforcement learning setup again)

Is it possible to get an optimal policy **without** learning transition or rewards explicitly?

Yes! We know one way already:

We switch to an infinite-horizon scenario. For our stochastic machine, here is the infinite-horizon Q^*_∞ function (computed via value iteration) for γ near 1.

```
              wash         paint      eject
dirty    [[ 2.32274541 -0.70048204  0.        ]
clean     [ 2.32274541  5.71581775  0.        ]
painted   [ 2.32274541  6.9        10.        ]
ejected   [ 0.          0.         0.        ]]
```

What is the optimal thing to do with a clean object?

What will you do if it becomes dirty?

Does this optimal policy make intuitive sense?

(Recall, from the MDP lab)

Optimal policy π^* easily extracted from Q^*:

6 $$\pi^*_h(s) = \arg\max_a Q^*_h(s,a)$$

But... didn't we arrive at Q^* by value iteration,

and doesn't value iteration rely on transition and rewards explicitly?

Value Iteration

1. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
2. $\quad \mathrm{Q}_{\text{old}}(\mathrm{s}, \mathrm{a}) = 0$
3. **while** True:
4. $\quad$ **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
5. $\quad\quad \mathrm{Q}_{\text{new}}(s, a) \leftarrow \mathrm{R}(s, a) + \gamma \sum_{s'} \mathrm{T}(s, a, s') \max_{a'} \mathrm{Q}_{\text{old}}(s', a')$
6. $\quad$ **if** $\max_{s,a} \left| Q_{\text{old}}(s, a) - Q_{\text{new}}(s, a) \right| < \epsilon$:
7. $\quad\quad$ **return** Q_{new}
8. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$

- Indeed, *value iteration* relies on having full access to R and T

$$\mathrm{Q}_{\text{new}}(s,a) \leftarrow \mathrm{R}(s,a) + \gamma \sum_{s'} \mathrm{T}(s,a,s') \max_{a'} \mathrm{Q}_{\text{old}}(s',a')$$

$\mathrm{R}(s,a)$: immediate reward

$\max_{a'} \mathrm{Q}_{\text{old}}(s',a')$: future value, starting in state s' and acting ***optimally*** for $(h-1)$ steps

$\sum_{s'} \mathrm{T}(s,a,s') \max_{a'} \mathrm{Q}_{\text{old}}(s',a')$: expected future value, weighted by the chance of landing in that particular future state s'

- Without R and T, how about: execute (s,a), observe r and s', and update:

$$\mathrm{Q}_{\text{new}}(s,a) \leftarrow r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s',a')$$

target

(we will see this idea has issues)

$$Q_{\text{new}}(s,a) \leftarrow r + \gamma \max_{a'} Q_{\text{old}}(s',a')$$

$Q_{\text{old}}(s,a)$

0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0

$Q_{\text{new}}(s,a)$

0 0 0 0 0	0 0 0 0 0	1 1 1 1 1
0 0 0 0 0	0 0 0 0 0	-10 -10 -10 -10 -10
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0

$\gamma = 0.9$

(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	1
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
	...	

$$Q_{new}(s,a) \leftarrow r + \gamma \max_{a'} Q_{old}(s',a')$$

$\gamma = 0.9$

$Q_{old}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-10 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

$Q_{new}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-9.1 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

(s,a)	r	s'
	...	
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
	...	

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(3,a')$$

$$= -10 + 0.9 * 1 = -9.1$$

$$Q_{new}(s,a) \leftarrow r + \gamma \max_{a'} Q_{old}(s',a')$$

$\gamma = 0.9$

$Q_{old}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-9.1 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

$Q_{new}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-10 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

(s,a)	r	s'
	...	
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
	...	

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(2,a')$$

$$= -10 + 0.9 * 0 = -10$$

$$Q_{new}(s,a) \leftarrow r + \gamma \max_{a'} Q_{old}(s',a')$$

$\gamma = 0.9$

$Q_{old}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-10 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

$Q_{new}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-9.1 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

(s,a)	r	s'
	...	
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
	...	

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(3,a')$$

$$= -10 + 0.9 * 1 = -9.1$$

$$Q_{new}(s,a) \leftarrow r + \gamma \max_{a'} Q_{old}(s',a')$$

$\gamma = 0.9$

$Q_{old}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-9.1 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

$Q_{new}(s,a)$

0 / 0 0 0 / 0	0 / 0 0 0 / 0	1 / 1 1 1 / 1
0 / 0 0 0 / 0	0 / 0 0 0 / 0	-10 / -10 -10 -10 / -10
0 / 0 0 0 / 0	0 / 0 0 0 / 0	0 / 0 0 0 / 0

(s,a)	r	s'
	...	
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
	...	

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(2,a')$$

$$= -10 + 0.9 * 0 = -10$$

$$Q_{new}(s,a) \leftarrow r + \gamma \max_{a'} Q_{old}(s',a')$$

target

Whenever observe $(6, \uparrow)$, -10, 3:

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(3,a')$$

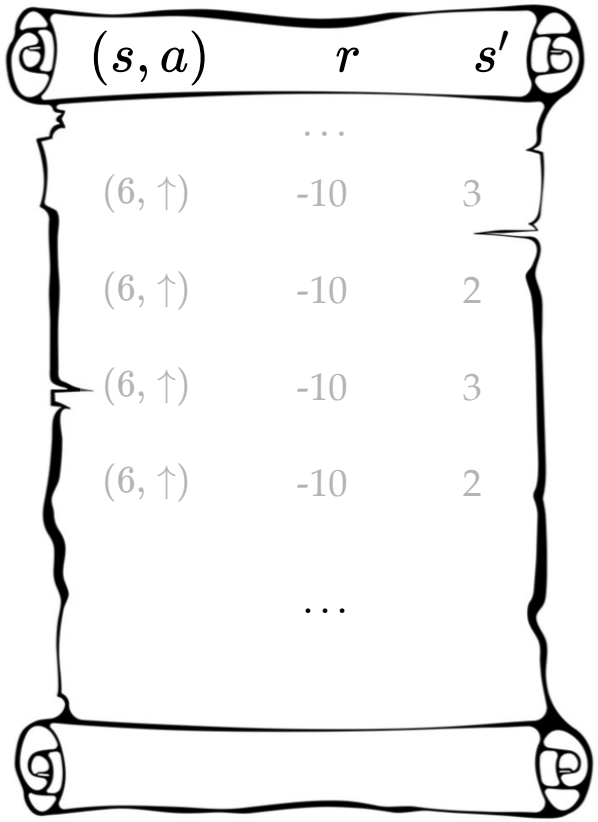
$$= -10 + 0.9 * 1 = -9.1$$

Whenever observe $(6, \uparrow)$, -10, 2:

$$Q_{new}(6,\uparrow) \leftarrow -10 + \gamma \max_{a'} Q_{old}(2,a')$$

$$= -10 + 0.9 * 0 = -10$$

$\gamma = 0.9$



🥺 Simply commit to the new target keeps "washing away" the old belief

😍 merge *old belief* and *target*

$$(1-\text{learning rate})\ \text{old belief} \quad + \quad \text{learning rate}\ \ \text{target}$$

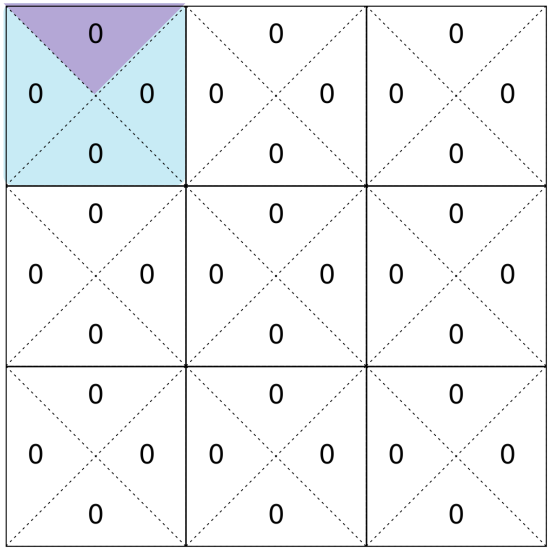
$$Q_{\text{new}}(s,a) \leftarrow (1-\alpha)\, Q_{\text{old}}(s,a) + \alpha \left(r + \gamma \max_{a'} Q_{\text{old}}(s',a') \right)$$

- Core update rule of *Q-learning*
- $\alpha \in [0,1]$, hyperparameter, trades off how much we trust new evidence versus old belief
- What we saw was $\alpha = 1$, trusting new experience entirely
- Let's see an example using $\alpha = 0.7$

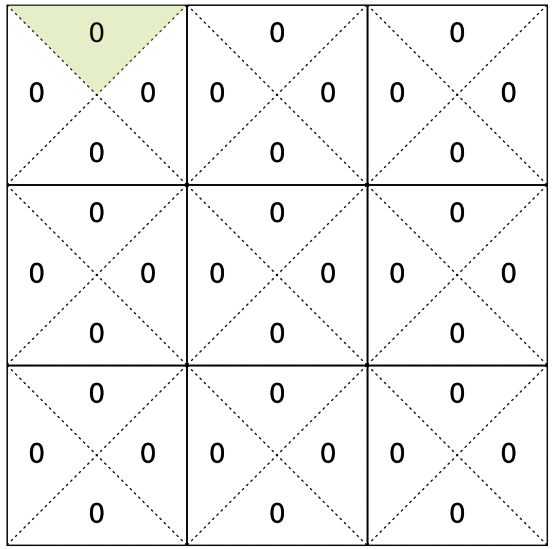
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	

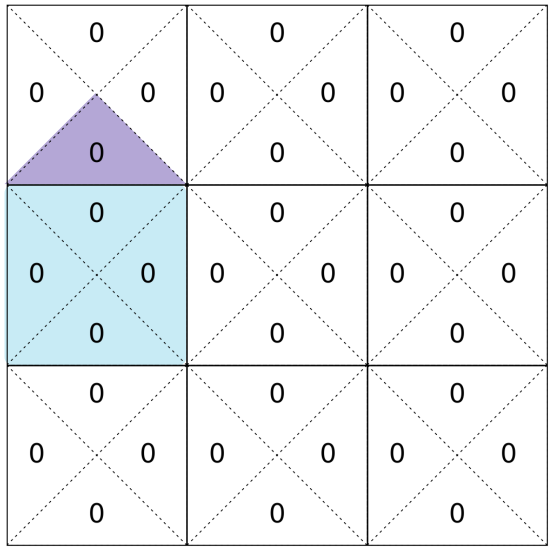
$$Q_{new}(1,\uparrow) \leftarrow (1-0.7)Q_{old}(1,\uparrow) + 0.7\left(0 + 0.9 \max_{a'} Q_{old}(1,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (0 + 0.9 * 0) = 0$$

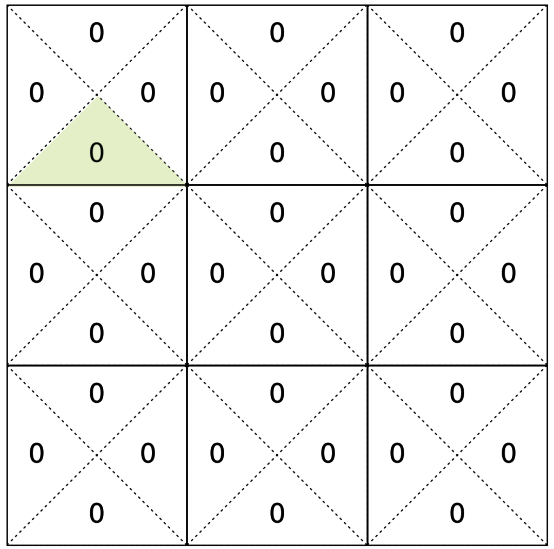
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	

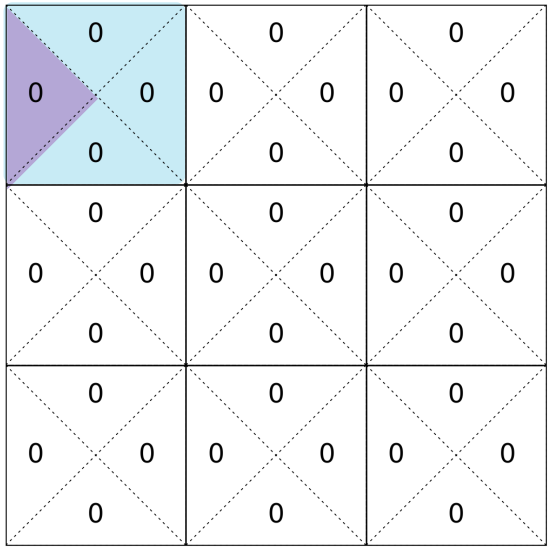
$$Q_{new}(1,\downarrow) \leftarrow (1-0.7)Q_{old}(1,\downarrow) + 0.7\left(0 + 0.9 \max_{a'} Q_{old}(4,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (0 + 0.9 * 0) = 0$$

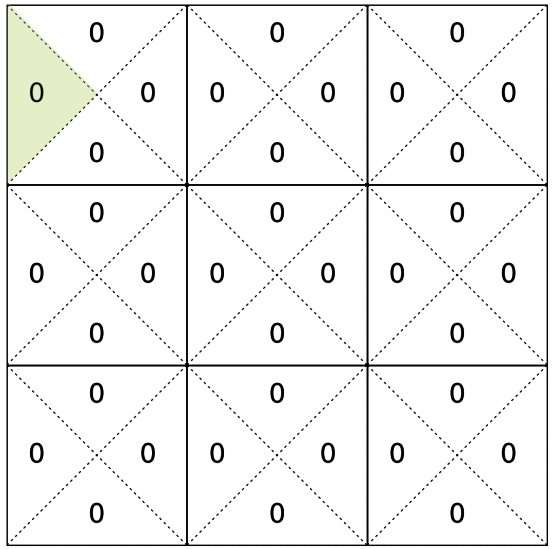
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	

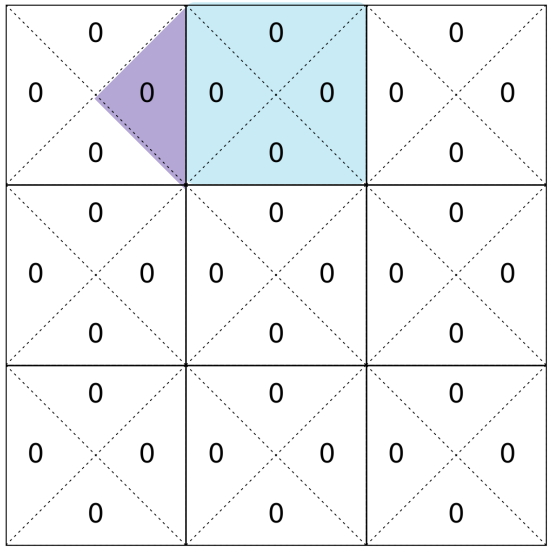
$$Q_{new}(1,\leftarrow) \leftarrow (1-0.7)Q_{old}(1,\leftarrow) + 0.7\left(0 + 0.9 \max_{a'} Q_{old}(1,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (0 + 0.9 * 0) = 0$$

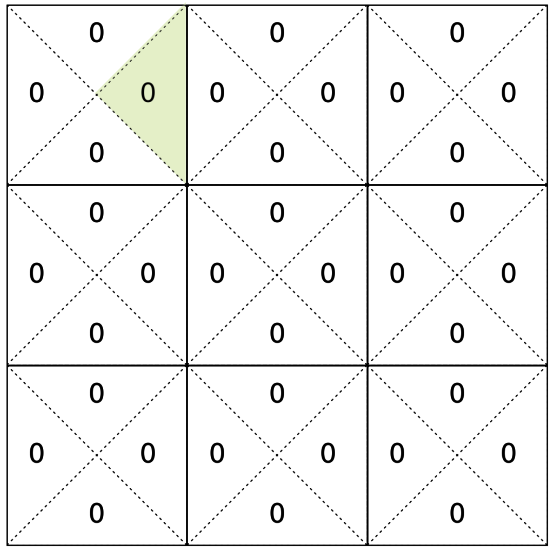
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
...		
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
...		

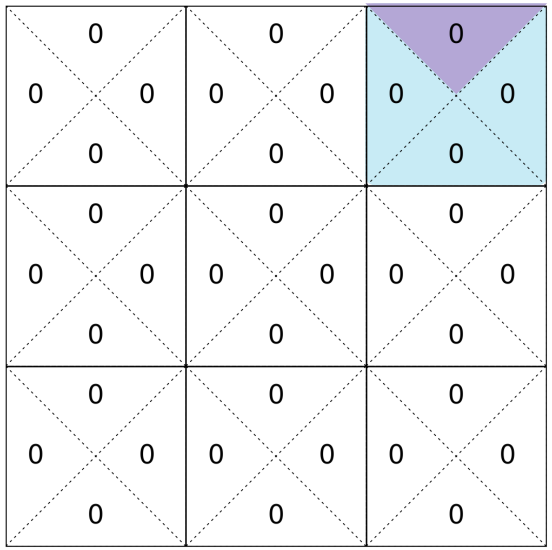
$$Q_{new}(1,\rightarrow) \leftarrow (1-0.7)Q_{old}(1,\rightarrow) + 0.7\left(0 + 0.9 \max_{a'} Q_{old}(2,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (0 + 0.9 * 0) = 0$$

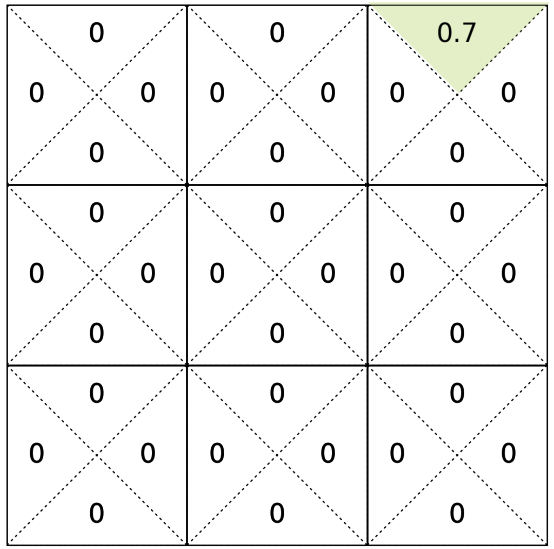
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	

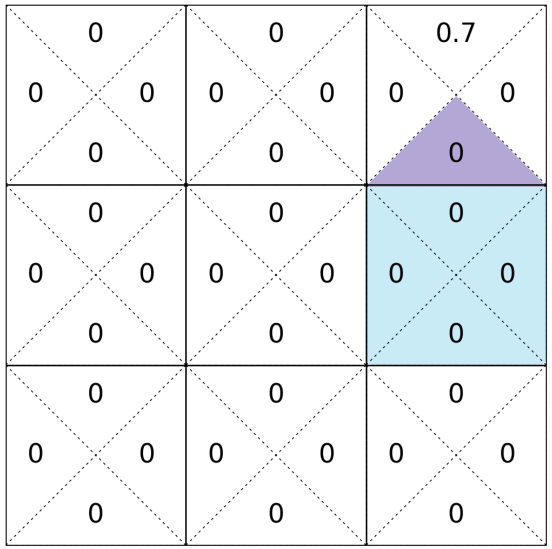
$$Q_{new}(3,\uparrow) \leftarrow (1-0.7)Q_{old}(3,\uparrow) + 0.7\left(1 + 0.9 \max_{a'} Q_{old}(3,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (1 + 0.9 * 0) = 0.7$$

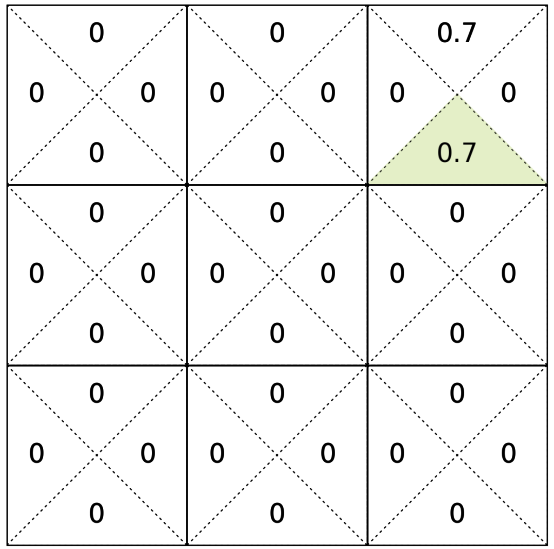
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
$(1,\uparrow)$	0	1
$(1,\downarrow)$	0	4
$(1,\leftarrow)$	0	1
$(1,\rightarrow)$	0	2
	...	
$(3,\uparrow)$	1	3
$(3,\downarrow)$	1	6
	...	

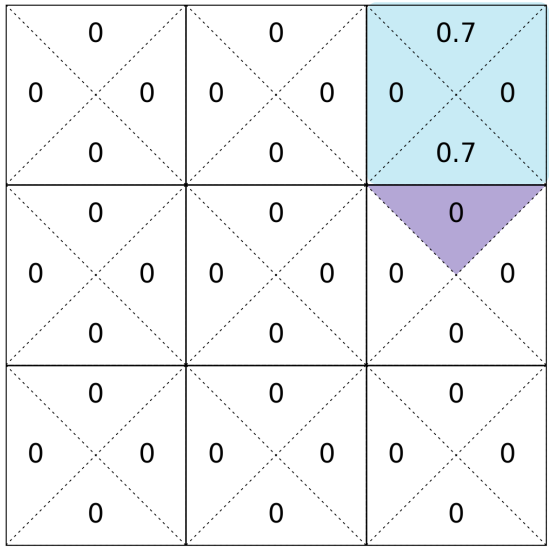
$$Q_{new}(3,\downarrow) \leftarrow (1-0.7)Q_{old}(3,\downarrow) + 0.7\left(1 + 0.9 \max_{a'} Q_{old}(6,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (1 + 0.9 * 0) = 0.7$$

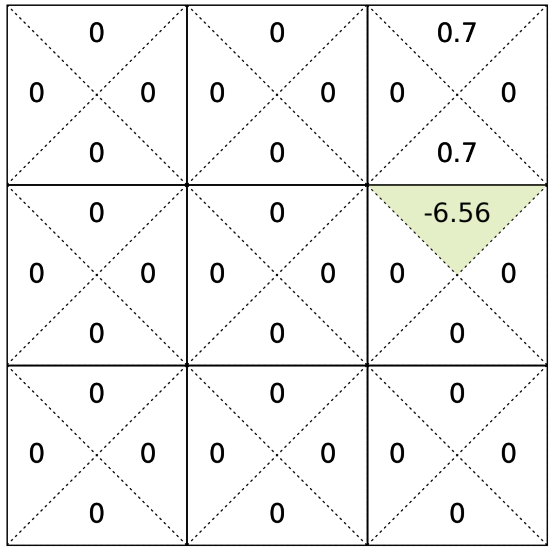
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
...		
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
...		

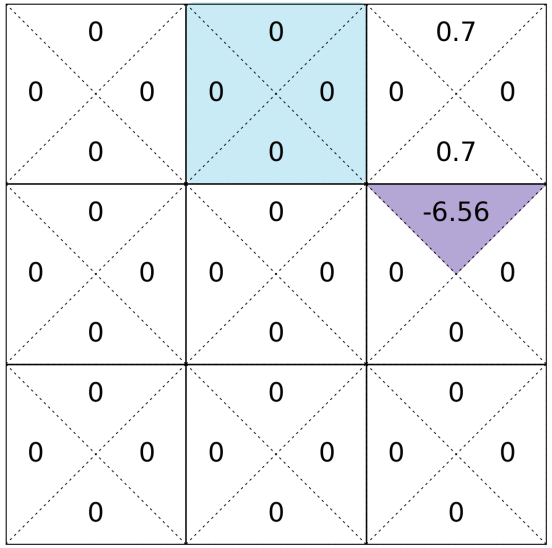
$$Q_{new}(6,\uparrow) \leftarrow (1-0.7)Q_{old}(6,\uparrow) + 0.7\left(-10 + 0.9 \max_{a'} Q_{old}(3,a')\right)$$

$$= (1-0.7) * 0 + 0.7 * (-10 + 0.9 * 0.7) = -6.56$$

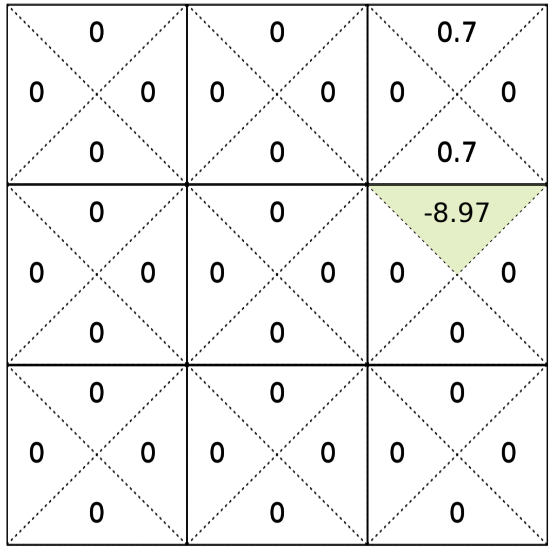
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
...		
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
...		

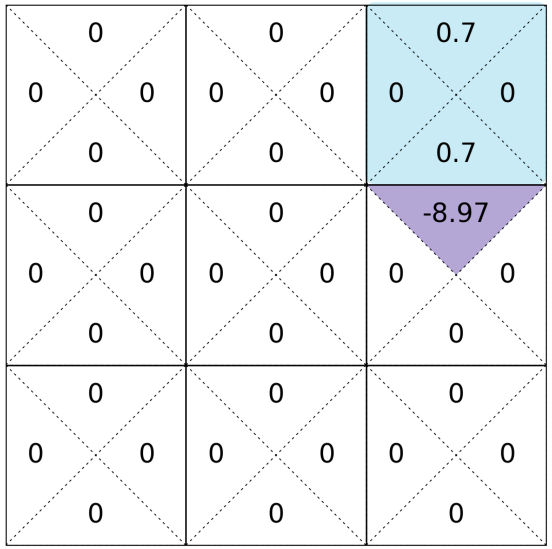
$$Q_{new}(6,\uparrow) \leftarrow (1-0.7)Q_{old}(6,\uparrow) + 0.7\left(-10 + 0.9 \max_{a'} Q_{old}(2,a')\right)$$

$$= (1-0.7) * -6.56 + 0.7 * (-10 + 0.9 * 0) = -8.97$$

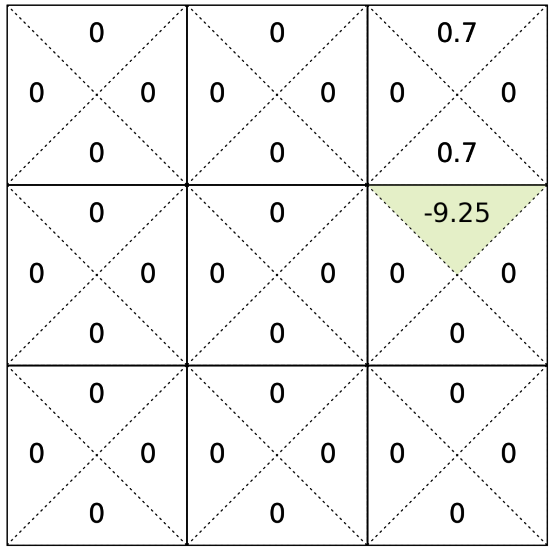
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$



(s,a)	r	s'
...		
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
$(6,\uparrow)$	-10	3
$(6,\uparrow)$	-10	2
...		

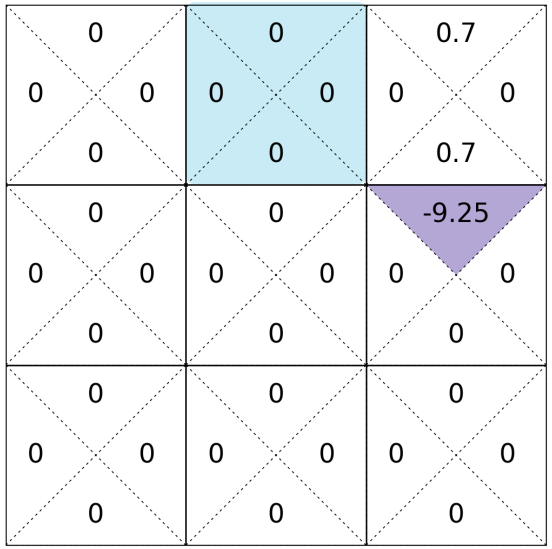
$$Q_{new}(6,\uparrow) \leftarrow (1-0.7)Q_{old}(6,\uparrow) + 0.7\left(-10 + 0.9 \max_{a'} Q_{old}(3,a')\right)$$

$$= (1-0.7) * -8.97 + 0.7 * (-10 + 0.9 * 0.7) = -9.25$$

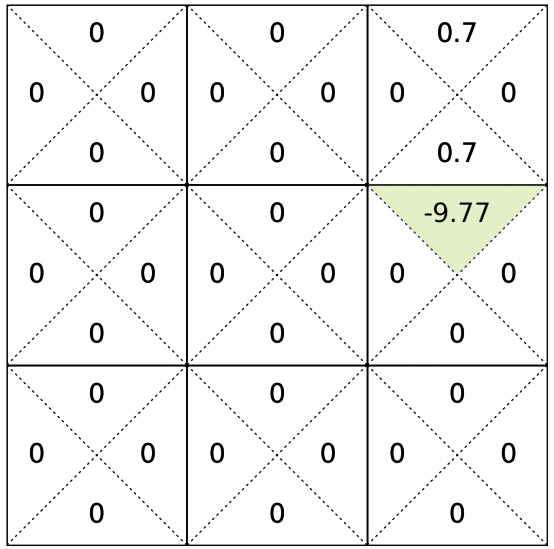
$$Q_{new}(s,a) \leftarrow (1-\alpha)Q_{old}(s,a) + \alpha\left(r + \gamma \max_{a'} Q_{old}(s',a')\right)$$

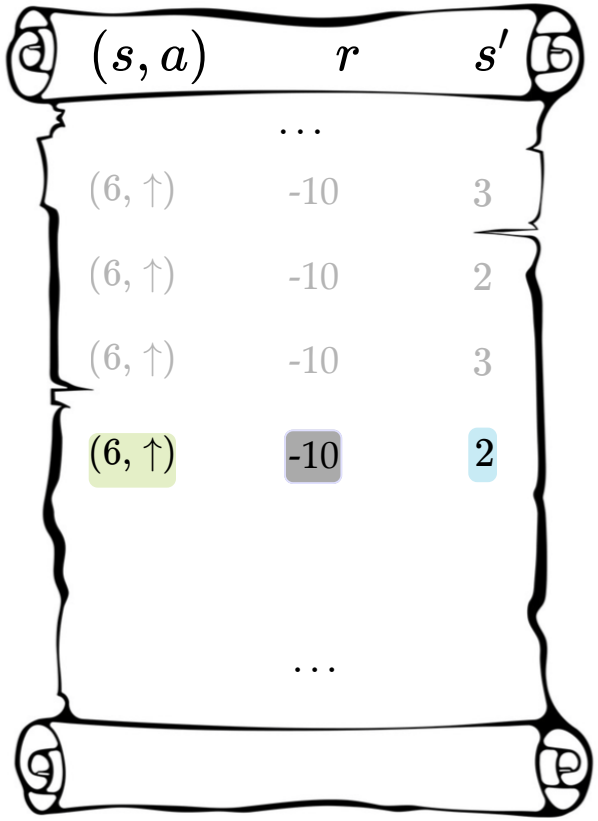
$\gamma = 0.9$ $\alpha = 0.7$

$Q_{old}(s,a)$



$Q_{new}(s,a)$





$$Q_{new}(6,\uparrow) \leftarrow (1-0.7)Q_{old}(6,\uparrow) + 0.7\left(-10 + 0.9 \max_{a'} Q_{old}(2,a')\right)$$

$$= (1-0.7) * -9.25 + 0.7 * (-10 + 0.9 * 0) = -9.77$$

$$\mathrm{Q}_{\text{new}}\,(s,a) \leftarrow (1-\alpha)\mathrm{Q}_{\text{old}}\,(s,a) + \alpha\,(r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}\,(s',a'))$$

- To update a particular Q(s, a), we need experience from actually taking that a in that s.
- If we never try a move, its Q-value never changes.
- So if we only get one "play" quota left, which (s,a) should we try?

  Depends on

  - are we trying to "win" the game, or
  - are we trying to get more accurate Q estimates

This is the fundamental dilemma in reinforcement learning... and in life!
Whether to *exploit* what we've already learned or *explore* to discover something better.

ϵ-greedy action selection strategy

- with probability ϵ, choose an action $a \in \mathcal{A}$ uniformly at random

  During learning, especially in early stages, we'd like to explore, and observe diverse (s, a) consequences.

- with probability $(1 - \epsilon)$, choose $\arg\max_{\mathrm{a}} \mathrm{Q}_{\mathrm{old}}(s, \mathrm{a})$

  During later stages, can act more greedily w.r.t. the current estimated Q values

ϵ controls the trade-off between *exploration vs. exploitation.*

Value Iteration($\mathcal{S}, \mathcal{A}, \mathrm{T}, \mathrm{R}, \gamma, \epsilon$)

1. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
2. $\quad \mathrm{Q}_{\text{old}}(\mathrm{s}, \mathrm{a}) = 0$
3. **while** True:
4. $\quad$ **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
5. $\quad\quad \mathrm{Q}_{\text{new}}(s, a) \leftarrow \mathrm{R}(s, a) + \gamma \sum_{s'} \mathrm{T}(s, a, s') \max_{a'} \mathrm{Q}_{\text{old}}(s', a')$
6. $\quad$ **if** $\max_{s,a} |Q_{\text{old}}(s, a) - Q_{\text{new}}(s, a)| < \epsilon$:
7. $\quad\quad$ **return** Q_{new}
8. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$

"calculating"

Q-Learning ($\mathcal{S}, \mathcal{A}, \gamma, \alpha, s_0$ max-iter)

1. $i = 0$
2. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
3. $\quad \mathrm{Q}_{\text{old}}(s, a) = 0$
4. $s \leftarrow s_0$
5. **while** $i <$ max-iter :
6. $\quad a \leftarrow \text{select\_action}(s, \mathrm{Q}_{\text{old}}(s, a))$
7. $\quad r, s' \leftarrow \text{execute}(a)$
8. $\quad \mathrm{Q}_{\text{new}}(s, a) \leftarrow (1 - \alpha)\mathrm{Q}_{\text{old}}(s, a) + \alpha(r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s', a'))$
9. $\quad s \leftarrow s'$
10. $\quad i \leftarrow (i + 1)$
11. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$
12. **return** Q_{new}

"learning" (estimating)

Q-Learning ($\mathcal{S}, \mathcal{A}, \gamma, \alpha, s_0$ max-iter)

1. $i = 0$
2. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
3. $\quad \mathrm{Q}_{\text{old}}(s, a) = 0$
4. $s \leftarrow s_0$
5. **while** $i <$ max-iter :
6. $\quad a \leftarrow \text{select\_action}(s, \mathrm{Q}_{\text{old}}(s, a))$
7. $\quad r, s' \leftarrow \text{execute}(a)$
8. $\quad \mathrm{Q}_{\text{new}}(s, a) \leftarrow (1-\alpha)\mathrm{Q}_{\text{old}}(s, a) + \alpha(r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s', a'))$
9. $\quad s \leftarrow s'$
10. $\quad i \leftarrow (i+1)$
11. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$
12. **return** Q_{new}

"learning"

- Remarkably, 👈 *can* converge to the true Q^*_∞
- Once converged, act greedily w.r.t Q^* again.
- But convergence can be extremely slow, and often not practical:
  - We might only be interested in part of a (state, action) space
  - We might not have enough resources to visit all (state, action) pairs—even those we care about.

[1] given we visit all s, a infinitely often, and satisfy a decaying condition on the learning rate α.

Q-Learning ($\mathcal{S}, \mathcal{A}, \gamma, \alpha, s_0$ max-iter)

1. $i = 0$
2. **for** $s \in \mathcal{S}, a \in \mathcal{A}$:
3. $\quad \mathrm{Q}_{\text{old}}(s, a) = 0$
4. $s \leftarrow s_0$
5. **while** $i <$ max-iter :
6. $\quad a \leftarrow \text{select\_action}(s, \mathrm{Q}_{\text{old}}(s, a))$
7. $\quad r, s' \leftarrow \text{execute}(a)$
8. $\quad \mathrm{Q}_{\text{new}}(s, a) \leftarrow (1 - \alpha)\mathrm{Q}_{\text{old}}(s, a) + \alpha(r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s', a'))$
9. $\quad s \leftarrow s'$
10. $\quad i \leftarrow (i + 1)$
11. $\quad \mathrm{Q}_{\text{old}} \leftarrow \mathrm{Q}_{\text{new}}$
12. **return** Q_{new}

"learning"

- ϵ-greedy controls what actions to explore, versus exploiting current estimate

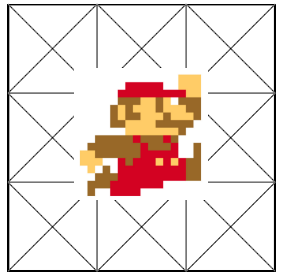
- three scalars in Q-learning:
  - discount factor γ
  - learning rate α
  - greedy factor ϵ

each between 0 and 1, controls some trade-off

Outline

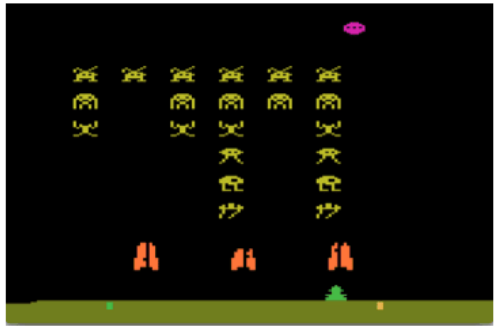
- Reinforcement learning setup
- Tabular Q-learning
  - exploration vs. exploitation
  - ϵ-greedy action selection
- Fitted Q-learning
- (Reinforcement learning setup again)

Fitted Q-learning: from table to functions

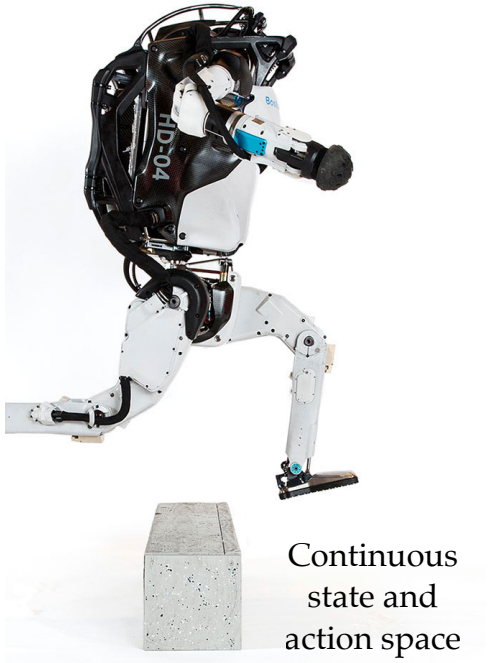


- So far, Q-learning is sensible for (small) tabular setting.
- What if $\mathcal{S}$ and/or $\mathcal{A}$ are large, or even continuous?

Game	Board size	State space
Go	19 x 19	10^{172}
Chess	8 x 8	10^{50}



10^{16992} (pixels) states



Continuous state and action space

- We can't keep a table — we must approximate $Q(s, a)$ with a function $Q_\theta(s, a)$.

- Notice that the core update rule of Q-learning:

$$\mathrm{Q}_{\text{new}}(s,a) \leftarrow (1-\alpha)\mathrm{Q}_{\text{old}}(s,a) + \alpha\left(r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s',a')\right)$$

  is equivalently:

$$\mathrm{Q}_{\text{new}}(s,a) \leftarrow \mathrm{Q}_{\text{old}}(s,a) + \alpha([r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s',a')] - \mathrm{Q}_{\text{old}}(s,a))$$

- Does this

$$\text{new belief} \leftarrow \text{old belief} + \text{learning rate}\ (\text{target} - \text{old belief})$$

  remind you of something?

- Yes! Recall:

$$\theta_{\text{new}} \leftarrow \theta_{\text{old}} + \eta(\text{target} - \text{guess}_\theta)\nabla_\theta \text{guess}$$

Gradient update rule when minimizing $(\text{target} - \text{guess}_\theta)^2$

- Same idea as before — we adjust our belief toward a target.

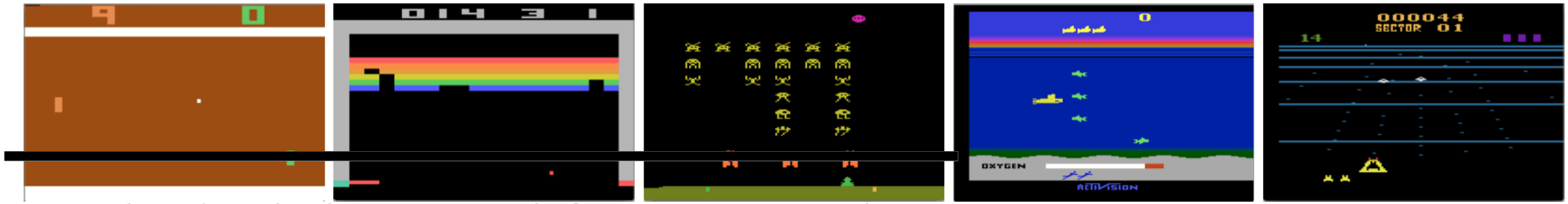
$$\mathrm{Q}_{\text{new}}(s,a) \leftarrow \mathrm{Q}_{\text{old}}(s,a) + \alpha([r + \gamma \max_{a'} \mathrm{Q}_{\text{old}}(s',a')] - \mathrm{Q}_{\text{old}}(s,a))$$

$$\text{new belief} \leftarrow \text{old belief} + \text{learning rate}(\text{target} - \text{old belief})$$

- Now Q is a function (e.g. a neural network) and θ are its weights.
- Generalize tabular Q-learning for continuous state/action space:

1. parameterize $\mathrm{Q}_\theta(s,a)$
2. execute (s,a), observe(r,s'), construct the target $r + \gamma \max_{a'} \mathrm{Q}_\theta(s',a')$
3. regress $\mathrm{Q}_\theta(s,a)$ against the target, i.e. update θ via gradient-descent to minimize

$$(\text{target} - \mathrm{Q}_\theta(s,a))^2$$



Playing Atari with Deep Reinforcement Learning

Volodymyr Mnih **Koray Kavukcuoglu** **David Silver** **Alex Graves** **Ioannis Antonoglou**

Daan Wierstra **Martin Rie**

DeepMind Technologie

{vlad,koray,david,alex.graves,ioannis,daan,mart

Abstract

We present the first deep learning model to successfully
rectly from high-dimensional sensory input using reinf
model is a convolutional neural network, trained with
whose input is raw pixels and whose output is a value fu
rewards. We apply our method to seven Atari 2600 game
ing Environment, with no adjustment of the architecture c
find that it outperforms all previous approaches on six of
a human expert on three of them.

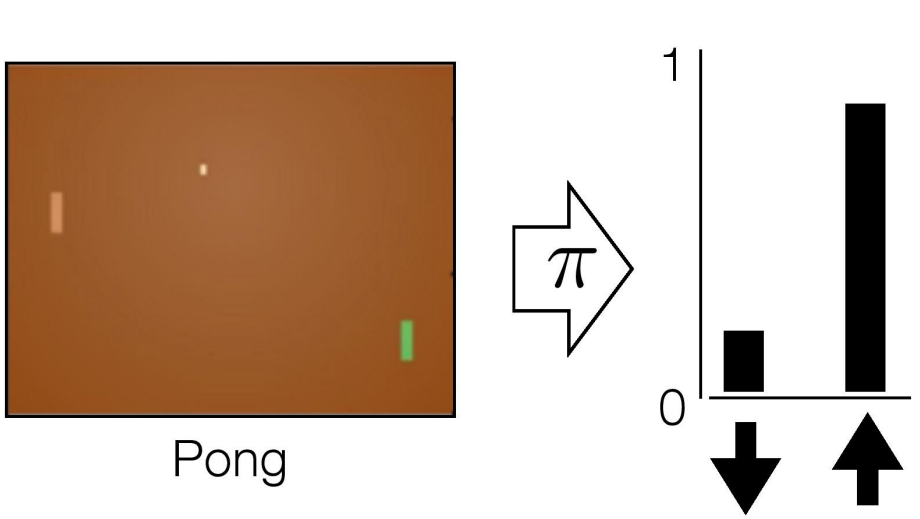
Algorithm 1 Deep Q-learning with Experience Replay

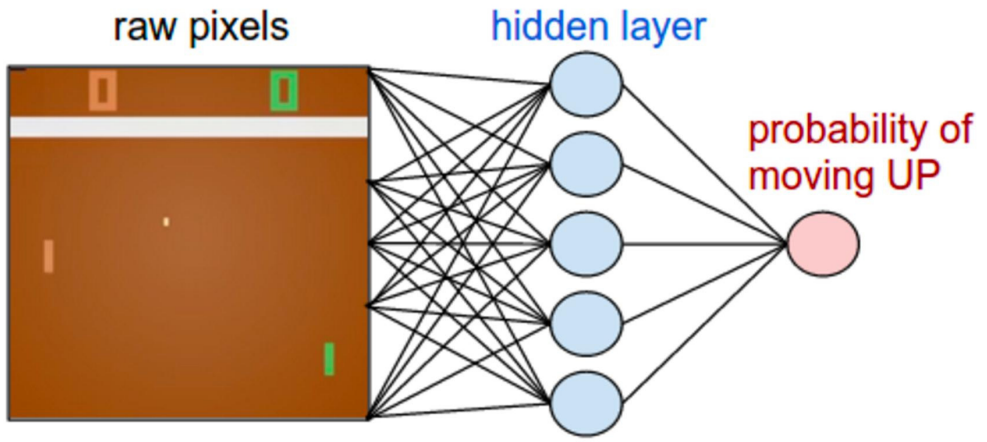
Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Outline

- Reinforcement learning setup
- Tabular Q-learning
  - exploration vs. exploitation
  - ϵ-greedy action selection
- Fitted Q-learning
- (Reinforcement learning setup again)

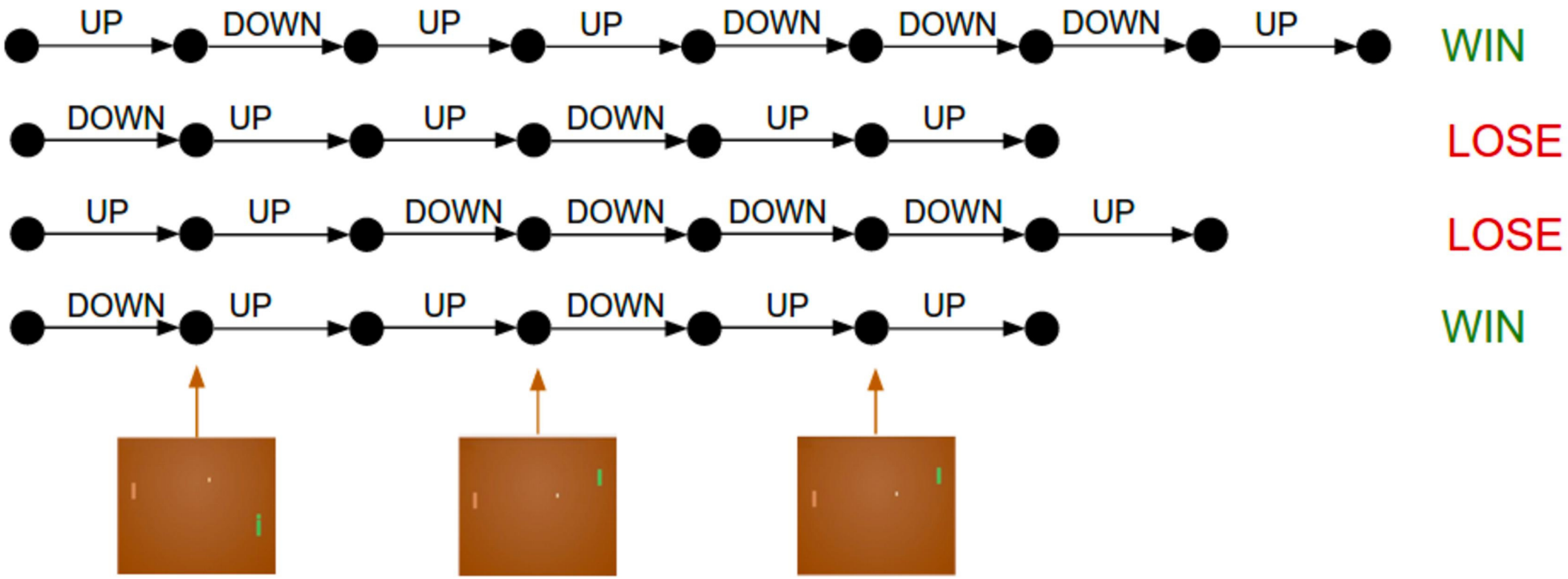
- If explicit "good" state-action pair is given, also supervised learning.
- Behavior cloning or imitation learning.
- But what if no explicit guide?





[Adapted from Andrej Karpathy: http://karpathy.github.io/2016/05/31/rl/]

- If no direct supervision is available?
- Strictly RL setting. Interact, observe, get data, use rewards as "coy" supervision signal.



[Adapted from Andrej Karpathy: http://karpathy.github.io/2016/05/31/rl/]

How Much Information is the Machine Given during Learning?

Y. LeCun

- **"Pure" Reinforcement Learning (cherry)**
  - The machine predicts a scalar reward given once in a while.
  - **A few bits for some samples**
- **Supervised Learning (icing)**
  - The machine predicts a category or a few numbers for each input
  - Predicting human-supplied data
  - **10→10,000 bits per sample**
- **Self-Supervised Learning (cake génoise)**
  - The machine predicts any part of its input for any observed part.
  - Predicts future frames in videos
  - **Millions of bits per sample**

[Slide Credit: Yann LeCun]

Summary

- We saw, last week, how to find good policy in a known MDP: these are policies with high cumulative expected reward.
- In reinforcement learning, we assume we are interacting with an *unknown* MDP, but we still want to find a good policy. We will do so via estimating the Q value function.
- One problem is how to select actions to gain good reward while learning. This "exploration vs exploitation" problem is important.
- Q-learning, for discrete-state problems, will converge to the optimal value function (with enough exploration).
- "Deep Q learning" can be applied to continuous-state or large discrete-state problems by using a parameterized function to represent the Q-values.

https://forms.gle/YLZKTx8T4h42Pf4E7

We'd love to hear
your thoughts.

Thanks!