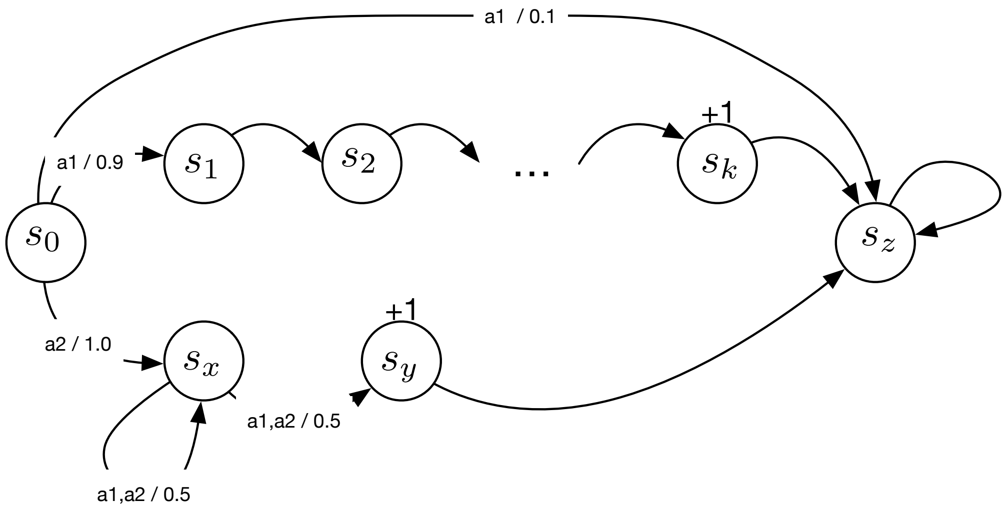
Name: ______________________________

Making Decisions Practical

5. (13 points) Consider the following MDP with $k+4$ states. There are two actions, a_1 and a_2. Arrows with no labels represent a transition for both actions with probability 1. Arrows labeled a/p make the transition on action a with probability p. States with no label have reward 0. Two states have reward $+1$, obtained when taking an action in that state. There are $k-2$ states between s_1 and s_k, with a deterministic transition on any action (so that once you are in s1 you are guaranteed to end up in s_k in $k-1$ steps).

   We are interested in the infinite-horizon discounted values of some states in this MDP.



   (a) What is $V(s_1)$ as a function of k when $\gamma = 0$? **0**

   (b) What is $V(s_1)$ as a function of k when $\gamma = 1$? **1**

   (c) What is $V(s_1)$ as a function of k when $0 < \gamma < 1$? γ^{k-1}

   (d) What is $V(s_x)$ when $\gamma = 0$? **0**

   (e) What is $V(s_x)$ when $\gamma = 1$? **1**

   (f) What is $V(s_x)$ when $0 < \gamma < 1$? $\gamma/(2-\gamma)$

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(g) Under what conditions on k and γ would we prefer to take action a_1 in state s_0? Write down a specific mathematical relationship.

Solution: When $(9/10)\gamma^{k-1} > \gamma/(2-\gamma)$.