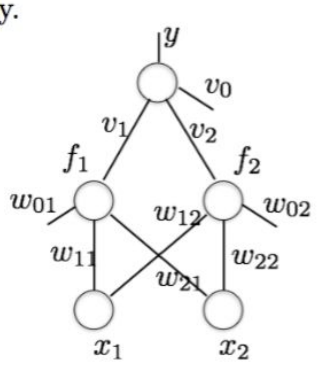
PROBLEM 10

Problem 2 We would like to build a neural network model that can detect whether two inputs are the same or not. To this end, we assume two input coordinates x_1 and x_2, two hidden units f_1 and f_2, and a single output neurons y. All units except input units involve ReLU non-linearity.



(2.1) **(6 points)** Specify the parameter values in the network such that output $y = 1$ if $x_1 = x_2$ and $y = 0$ if $|x_1 - x_2| \geq \epsilon$. Two values of the parameter matrix are pre-specified.

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1/\epsilon \\ -1/\epsilon \end{bmatrix}, \quad \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & 1 \end{bmatrix}$$

(2.2) **(3 points)** Suppose now for simplicity that x_t is binary 0/1 and that we receive $x_1, x_2, \ldots,$ in a sequence. Our goal is to detect whether successive pairs of inputs are identical using a recurrent neural network. We do so by feeding the RNN state as an input to the above feed-forward detector. Which of the following state update equations would be suitable for this purpose? You can assume that $s^0 = [0, 0]^T$

$$(\ \mathbf{X}\ ) \quad s^t = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} s^{t-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_t$$

$$(\ \ ) \quad s^t = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} s^{t-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_t$$

$$(\ \ ) \quad s^t = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} s^{t-1} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} x_t$$

10) This problem uses similar strategies to the minimal network for separating XOR-type distributions. Note that all non-input nodes have ReLU activations.