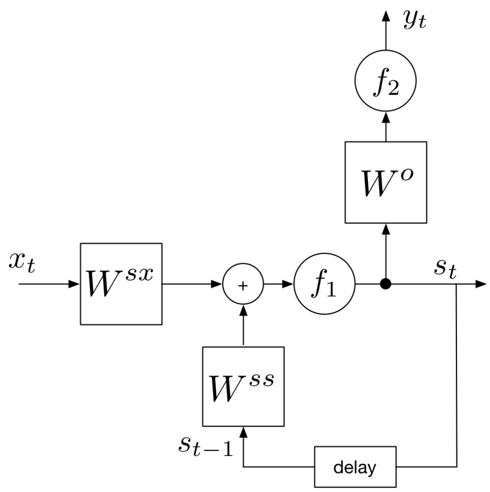
Double trouble

8. (7 points) One of the RNN architectures we studied was

$$s_t = f_1(W^{ss} s_{t-1} + W^{sx} x_t)$$
$$y_t = f_2(W^o s_t)$$

where W^{ss} is $m \times m$, W^{sx} is $m \times l$ and W^o is $n \times m$. Assume f_i can be any of our standard activation functions. We omit the offset parameters for simplicity (set them to zero).



(a) Suppose we modify the original architecture as follows:

$$s_t = f_1(W^{ss1} f_3(W^{ss2} s_{t-1}) + W^{sx} x_t)$$

i. Provide values for the original W^{ss} that make the original architecture equivalent to this one, or explain why none exist.

$W^{ss} =$ ____________________

Solution: This architecture can represent state machines that can't be represented by the original architecture, because the class of state transition functions that can be modeled in the modified architecture is bigger.

ii. Provide values for W^{ss2}, f_3 and W^{ss1} that make this new architecture equivalent to the original, or explain why none exist.

$f_3 =$ **linear**

$W^{ss1} =$ W^{ss}

$W^{ss2} =$ I

Solution: See above

(b) Now, we'll consider two strategies for making the RNN generate two output symbols for each input symbol. Assume the symbols are drawn from a vocabulary of size n.

Model A: We use a separate softmax output for each symbol, so

$$y_t^1 = \text{softmax}(W^{o1} s_t)$$
$$y_t^2 = \text{softmax}(W^{o2} s_t)$$

where W^{o1} and W^{o2} are $n \times m$.

Model B: We use a single softmax output, but it ranges over n^2 possible pairs of symbols, so

$$y_t^1, y_t^2 = \text{softmax}(W^{o3} s_t)$$

i. What would the dimension of W^{o3} need to be?

Solution: $n^2 \times m$

ii. Which of the following is true:

- ◯ Models A and B can express exactly the same set of RNN models.
- ◯ Model A is more expressive than model B.
- √ **Model B is more expressive than model A.**