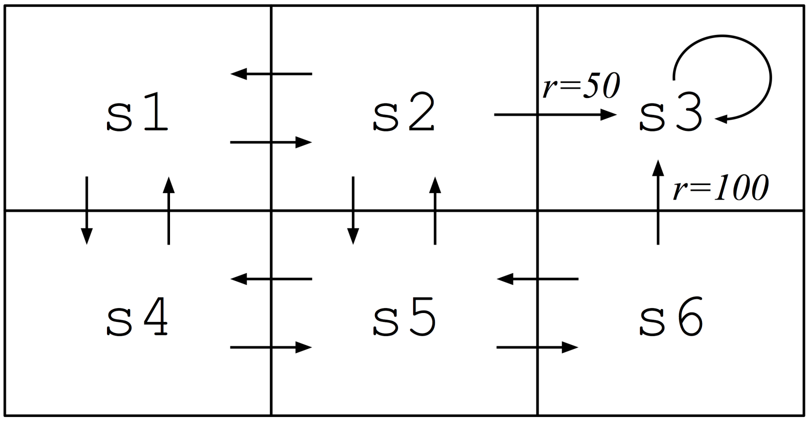
Robots

5. (16 points) Consider the following deterministic Markov Decision Process (MDP), describing a simple robot grid world. Notice that the values of the immediate rewards r for **two** transitions are written next to them; the other transitions, with no value written next to them, have an immediate reward of $r = 0$. **Assume the discount factor γ is 0.8.**



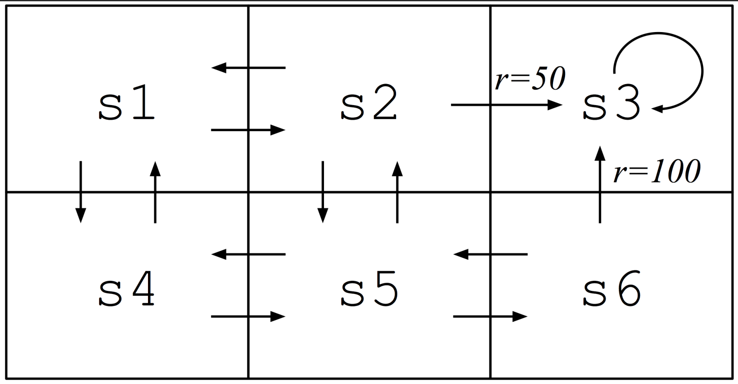
(a) For states $s \in \{s6, s5, s2\}$, write the value for $V_{\pi^*}(s)$, the discounted infinite horizon value of state s using an optimal policy π^*. It is fine to write a numerical expression—you don't have to evaluate it—but it shouldn't contain any variables.

i. $V_{\pi^*}(s6) =$

ii. $V_{\pi^*}(s5) =$

iii. $V_{\pi^*}(s2) =$

(b) For each state in the state diagram below, circle exactly one outgoing arrow, indicating an optimal action $\pi^*(s)$ to take from that state. If there is a tie, it is fine to select any action with optimal value.



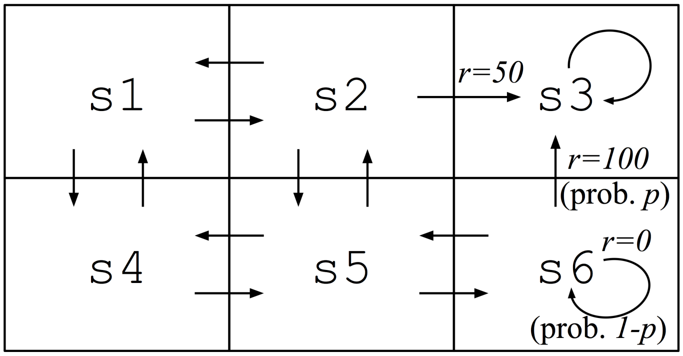
(c) Give a value for γ (constrained by $0 < \gamma < 1$) that results in a different optimal policy, and describe the resulting policy by indicating which $\pi^*(s)$ values (i.e., which policy actions) change.

New value for γ:

Changed policy action:

(d) Is it possible to change the immediate reward for each state in such a way that V_{π^*} changes but the optimal policy π^* remains unchanged? If yes, provide a new reward function, and explain how the resulting V_{π^*} changes but π^* does not. Otherwise, explain in at most two sentences why this is impossible.

When winter comes, snow also appears on one path in the grid world, making exactly one of the actions non-deterministic. The resulting MDP is shown below. Specifically, the change is that now the result of the action "go north" from state `s6` results in one of two outcomes. With probability p, the robot succeeds in transitioning to state `s3` and receives immediate reward 100. However, with probability $(1-p)$ it slips on the ice, and remains in state `s6` with 0 immediate reward. **Assume again that the discount factor** $\gamma = 0.8$.



(e) Assume $p = 0.75$. For each of the states $s \in \{s2, s5, s6\}$, write the value for $V_{\pi^*}(s)$. It is fine to write a numerical expression, but it shouldn't contain any variables.

i. $V_{\pi^*}(s6) =$

ii. $V_{\pi^*}(s5) =$

iii. $V_{\pi^*}(s2) =$

(f) How bad does the ice have to get before the robot will prefer to completely avoid the ice? Let us answer the question by giving a value for p for which the optimal policy chooses actions that completely avoid the ice, i.e., choosing the action "go left" over "go up" when the robot is in the state `s6`. Approach this in four parts. The answer to each of the first three parts can be a numerical expression; the answer to the last part can be an expression involving numbers and p.

i. What is the value V of going right in state `s2`?

ii. What is the value V of going up in state `s5`, if you're going to go right in state `s2`?

iii. What is the value V of going left in state `s6`, if you're going to go up in state `s5` and right in state `s2`?

iv. Under what condition on p is it better to go left in state `s6` (then up in state `s5` and right in state `s2`) than it is to go up in state `s6`?