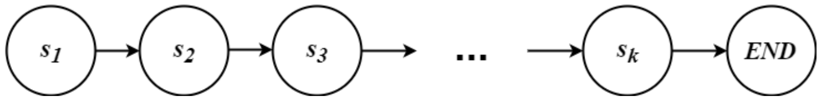
Go Positive, or Go Negative

5. (16 points) Consider the following simple MDP: **Positive Reward**



First consider the case where the MDP has positive reward. In this scenario, there is only one action ($next$); we name this decision policy π_A with $\pi_A(s) = next$ for all s. The reward is $R(s, next) = 0$ for all states s, except for state s_k where reward is $R(s_k, next) = 10$. We always start at state s_1 and each arrow indicates a deterministic transition probability $p = 1$. There is no transition out of the end state END, and 0 reward for any action from the end state.

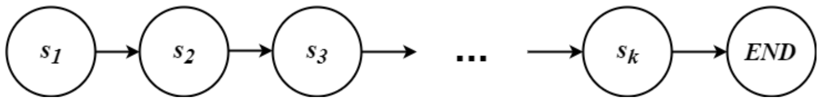
(a) Calculate $V_\pi(s)$ for each state in the finite-horizon case with horizon $h = 1$, $k = 4$, and discount factor $\gamma = 1$.

(b) Calculate $V_\pi(s)$ for each state in the infinite horizon case with $k = 4$ and discount factor $\gamma = 0.9$.

(c) Derive a formula for $V_\pi(s_1)$ that works for any value of (is expressed as a function of) k and γ for the above positive reward MDP, in the infinite horizon case.

Negative Reward

Now consider the case where this MDP has negative reward. In this scenario, the reward is $R(s, next) = -1$ for all states, except for state s_k where the reward is $R(s_k, next) = 0$. Again, there is only one action, $next$, and the decision policy remains $\pi_A(s) = next$ for all s. We always start at state s_1 and each arrow has a deterministic transition probability $p = 1$. There is no transition out of the end state END, and zero reward for any action from the end state, i.e., $R(END, next) = 0$.



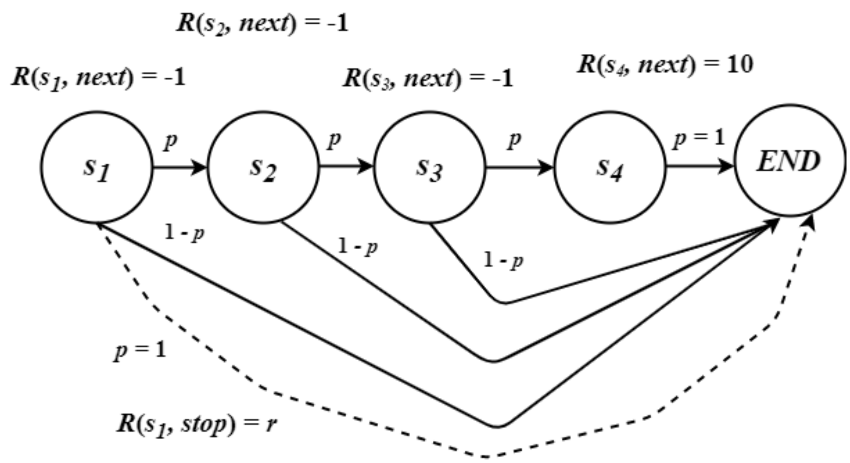
(d) Calculate $V_\pi(s)$ for each state in the finite-horizon case with horizon $h = 1$, $k = 4$, and discount factor $\gamma = 1$.

(e) Calculate $V_\pi(s)$ for each state in the infinite horizon case with $k = 4$ and discount factor $\gamma = 0.9$.

(f) Derive a formula for $V_\pi(s_1)$ that works for any value of (is expressed as a function of) k and γ for this negative reward MDP with infinite horizon. Recall that $\sum_{i=0}^{n} \gamma^i = \frac{(1-\gamma^{n+1})}{(1-\gamma)}$.

Positive and Negative Reward

Consider the MDP below with negative rewards for some $R(s, a)$ and positive rewards for others. Now there are two actions, *next* and *stop*. The solid arrows show the probabilities of state transitions under action *next*; the dashed arrows show the probability of state transitions under action *stop*. (If there is no dashed arrow from a state, that indicates a probability $p = 0$ of transitioning out of that state under action *stop*.) The corresponding rewards $R(s_i, a)$ are also indicated on the figure below. Note that the rewards are $R(s_i, next) = -1$ for all s_i, except for state s_4, where the reward is $R(s_4, next) = 10$. Finally, under action *stop*, we have reward $R(s_1, stop) = r$ (some unknown value r), and $R(s, stop) = 0$ for all other states. As before, we always start in state s_1. There is no transition out of the end state END, and zero reward for any action from the end state, i.e., $R(END, next) = R(END, go) = 0$. Assume discount factor γ and infinite horizon.



(g) We consider two possible policies: $\pi_A(s) = next$ for all s, and $\pi_B(s) = stop$ for all s. Your goal is to maximize your reward. When you start at s_1, you have reward 0 before taking any actions. Determine what r should be, so that it is best to run this MDP under policy π_B rather than policy π_A. Give your answer as an expression for r involving p and γ.