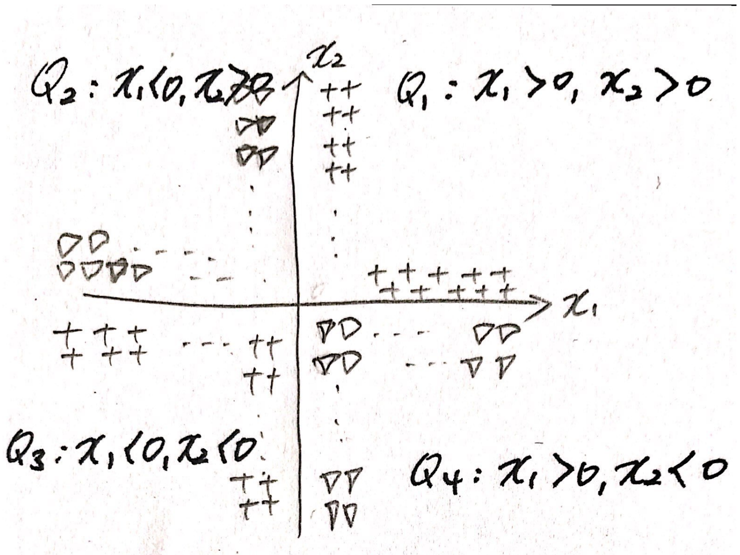
1 Spring 2016: Problem 4

4.1) (a) and (d) should be marked.

Explanation: Note that the points labeled '+' live in the first and third quadrants (Q1, Q3), while the points labeled $\triangledown$ live in the second and fourth quadrants (Q2, Q4).

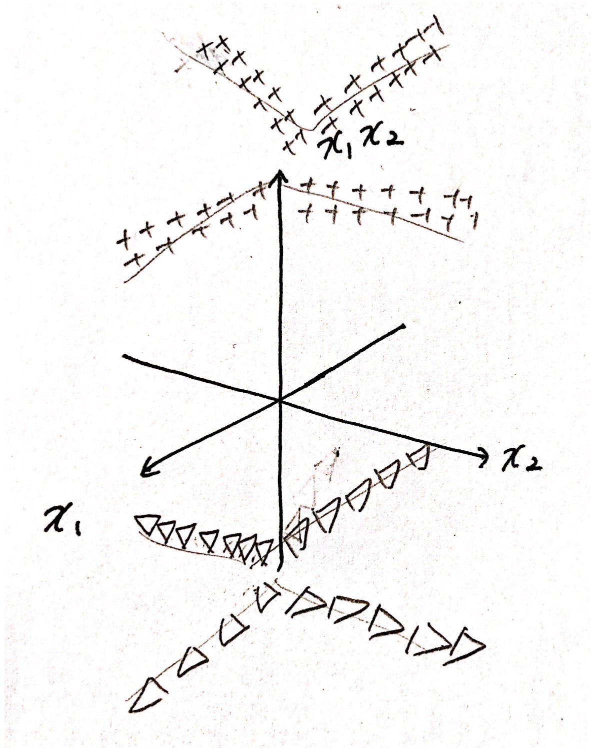


Also note that:

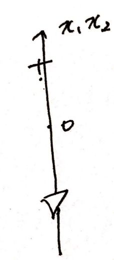
- In Q1: $x_1 > 0, x_2 > 0$ so $x_1 x_2 > 0$
- In Q2: $x_1 < 0, x_2 > 0$ so $x_1 x_2 < 0$
- In Q3: $x_1 < 0, x_2 < 0$ so $x_1 x_2 > 0$
- In Q4: $x_1 > 0, x_2 < 0$ so $x_1 x_2 < 0$

Thus, $x_1 x_2 > 0$ in Q1 and Q3 and $x_1 x_2 < 0$ in Q2 and Q4. Now let's go through each of the choices (a),(b),(c),(d) and see if data is linearly separable in each of the cases.

(a) We can see $[x_1, x_2, x_1 x_2]$ as the third dimension "height" added to the original picture above. Because $x_1 x_2 > 0$ in Q1 and Q3 and $x_1 x_2 < 0$ in Q2 and Q4, $[x_1, x_2, x_1 x_2]$ looks like the following:

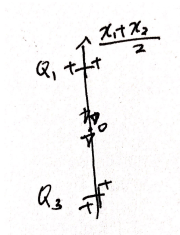


Or if we only look at the "height" dimension, like the following:



Thus, because of this "height" dimension created by xy, **there is a linear boundary (something like $xy = 0$) that linearly separates given data.**

(b) Note that $[x_1^2, x_2^2]$ maps all data to Q1. Because + and $\triangledown$ labeled points are (almost) symmetric with respect to the x_1, x_2 axis, + and $\triangledown$ labeled points may be mapped to (almost) same location in Q1. Thus, the first two dimensions do not help in any way for linear separability.
$\frac{x_1+x_2}{2}$ is always positive for points in Q1, always negative for points in Q3, can be either positive/negative/0 for points in Q2 and Q4.



As in the picture above, there is no linear boundary in the third dimension that linear separates data. Overall, data is **not linearly separable.**

(c) $tanh(x_1 + x_2)$ is always positive for points in Q1, always negative for points in Q3, can be either positive/negative/0 for points in Q2 and Q4. Similarly as in (b), data is **not linearly separable**.

(d) We saw in (a) that there exists a linear boundary in the direction of x_1x_2 that linearly separates given data. Thus, data is **linearly separable**.