6.036 Fall 2018 Midterm Review Solutions

1 Spring 2013: Problem 1

1.1a) Here are plots of θ and the decision boundary $\theta \cdot x = 0$, obtained by simply running the perceptron algorithm on each point sequentially, with the initial value of θ as 0.

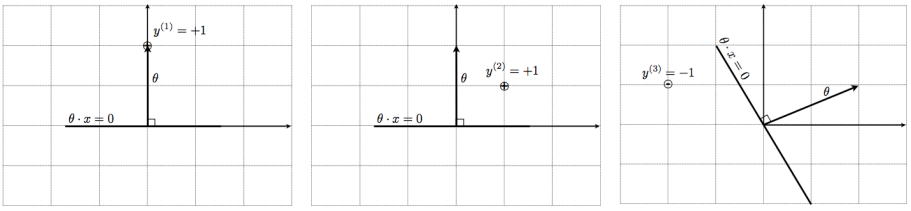

Figure 1: Problem 1.1a

1.1b) Here is one possible assignment of labels to the points such that the desired properties are satisfied.

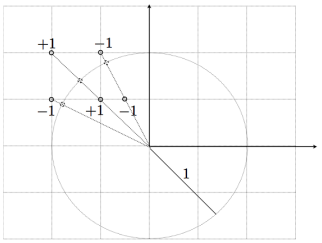

Figure 2: Problem 1.1b

2 Spring 2013: Problem 3

3.1a) Statements 1, 3, and 4 should be marked (TRUE).

3.1b) A good classifier here would be $y = 1$ iff $\hat{\theta} \cdot \phi(x) \geq 0.5$.

3.1c) The predictions will tend towards 0.

3 Spring 2014: Problem 3

3.1a) Yes.

3.1b) No.

3.1c)+3.1di) Here's a plot of the featurized data, $\hat{\theta}$, and separator.

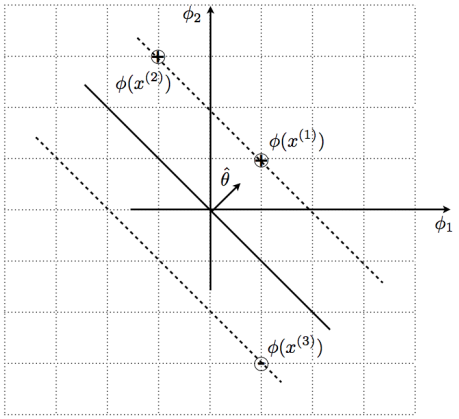

Figure 3: Problems: 3.1c and 3.1d(i)

3.1dii) The value of the margin is $\sqrt{2}$.

3.1diii) $\|\hat{\theta}\| = \frac{1}{\sqrt{2}}$.

3.1e) Please see the plot.

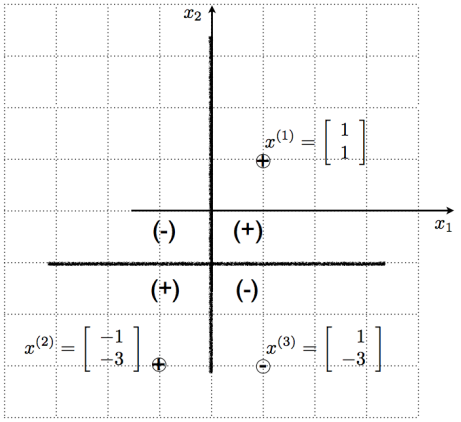

Figure 4: Problem: 3.1e

4 Spring 2016: Problem 1

1.1) No.

1.2) No.

1.3) Objective Function $= 2 + \frac{1}{3}$.
Yes there is a solution which has lower loss and smaller $||\theta||$.

1.4) The third option (labelled as (4) in the list) should be marked.

5 Spring 2016: Problem 2

2.1) $\hat{\theta} = 0$.

2.2) You only need to have the right shape of your plot.

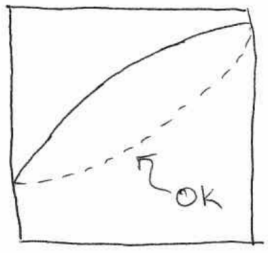

Figure 5: Problem 2.2

6 Spring 2016: Problem 4

4.1) (a) and (d) should be marked.

7 Spring 2016: Problem 5

5.1) $x \leq -1$.

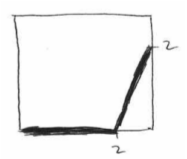

Figure 6: Problem 5.2

5.2)

5.3) Yes

5.4) $x \in (1, \infty)$

5.5) All weights stay the same except for w_0.

5.6) A,C,B

8 Spring 2017: Problem 1

1.1) θ_0 =-7

1.2) No

1.3) $\theta = [1 \quad 1], \theta_0 = -5$

1.4) Margin = $\frac{1}{\sqrt{2}}$

1.5) 0

1.6) 1.5

9 Spring 2017: Problem 2

1.1) $\theta \leftarrow (1 - \eta\lambda)\theta + \eta \begin{cases} y^{(i)}x^{(i)}, & \text{if } y^{(i)}\theta \cdot x^{(i)} \leq 1 \\ 0, & \text{otherwise} \end{cases}$

1.2) ((B)) small λ, small η;
((A)) small λ, large η;
((C)) large λ, large η.

10 Spring 2017: Problem 4

4.1) See the plot below.

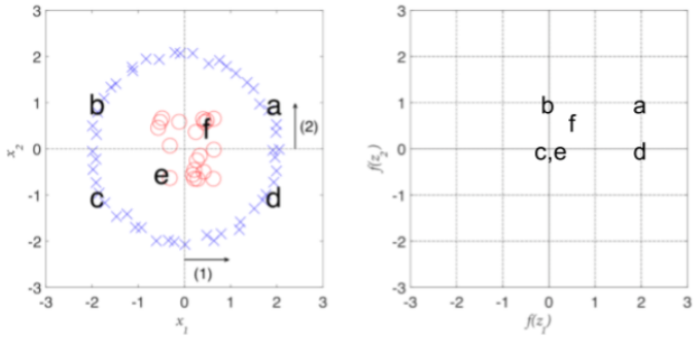

Figure 7: Problem 4.1

4.2) No

4.3) Yes

4.4) True, True, True

11 Fall 2017: Problem 1

1. (a) The second option should be checked (Linearly separable but not through the origin).

   (b) There are many examples of linear separators that properly classify these points. One possible answer:
   $(-1)x_1 + (-1)(x_2) + (0)x_1^2 + (0)x_2^2 + (4) = 0$

   (c) Check the third option (False for all data sets).

12 Fall 2017: Problem 4

4. (a) i. Yes
   ii. $(1, 1) : 0$,
   $(1, 3) : 1$,
   $(3, 2) : 2$,
   $(3, 4) : 0$
   iii. $J = 1.75$

   (b) (1): N/A or -1,
   (2): 1,
   (3): 1

(c) Select the third separator (3)

(d) Yes

13 Fall 2017: Problem 5

5. a) 3

b) $\frac{\partial L(f_2,y)}{\partial z_2} = w_2 \max(0, w_1 x + b_1) + b_2 - y$

c) $\frac{\partial L(f_2,y)}{\partial z_1} = \begin{cases} 0, & \text{if } w_1 x + b_1 < 0 \\ w_2(w_2 \max(0, w_1 x + b_1) + b_2 - y), & \text{o.w..} \end{cases}$

d) $w_1 = -5$

e) If $x = 0$, or if $w_2 = 0$, or if $w_1 x + b_1 < 0$, or if $f_2 = y$, which corresponds to the condition $\max(w_1 x + b_1, 0) w_2 + b_2 = y$

14 Fall 2017: Problem 6

6. a) $c_1 = 1, c_2 = 1$ OR $c_1 = \frac{1}{2}, c_2 = \frac{1}{2}$. (The rest of the answers below assume $c_1 = c_2 = 1$.)

c) $\theta = \theta - 2\eta x(g - y) \begin{cases} c_1, & \text{if } g > y \\ c_2 & \text{o.w.} \end{cases}$

$\theta_0 = \theta_0 - 2\eta(g - y) \begin{cases} c_1, & \text{if } g > y \\ c_2 & \text{o.w.} \end{cases}$

15 Fall 2017: Problem 7

7. a) i. One-hot encoding

ii. Divide by 50

iii. Divide by 1 billion

iv. Omit

b) i. α) 1
β) sigmoid
γ) NLL
Also okay: 1 unit, linear, hinge or 2 units, softmax, NLL

ii. α) 1
β) linear
γ) squared-error

iii. α) 100
β) sigmoid
γ) NLL (individually)

16 Fall 2017: Problem 9

9. (a) 0.01: Weight trajectory: (d) , Objective: (g)
   (b) 0.05: Weight trajectory: (a) , Objective: (h)
   (c) 0.50: Weight trajectory: (c) , Objective: (f)
   (d) 1.00: Weight trajectory: (b) , Objective: (e)